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MAGDM Model Using an Intuitionistic Fuzzy Matrix Energy Method and Its Application in the Selection Issue of Hospital Locations

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Abstract: Matrix energy is a valid mathematical tool for representing collective information. However, it is not used in the existing literature for fuzzy sets, intuitionistic fuzzy sets, and multi-attribute group decision making (MAGDM) problems, which highlights research gaps. Motivated by both matrix energy and the research gaps, this study aims to extend matrix energy to the energy of an intuitionistic fuzzy matrix (IFM) and to utilize the IFM energy method in the MAGDM problem, which fully contains all the IFM information on attribute weights, decision maker weights, and attribute values. To achieve these objectives, this paper first proposes IFM energy in terms of the true matrix energy and false matrix energy; then, it develops a MAGDM model using the IFM energy method and the score and accuracy equations of IFM energy. Then, the developed MAGDM model is applied to the selection problem of hospital locations in Shaoxing City, China to demonstrate the practicality and validity of the developed model. Compared with existing intuitionistic fuzzy MAGDM methods, the developed MAGDM model using the IFM energy method reveals its superiority and novelty in complete IFM information expressions and the MAGDM method because the existing MAGDM methods containing intuitionistic fuzzy set information have difficulty tackling MAGDM problems containing all the IFM information on attribute weights, decision maker weights, and attribute values.

Keywords: intuitionistic fuzzy matrix; intuitionistic fuzzy matrix energy; intuitionistic fuzzy weight; group decision making

MSC: 03E72; 91B06

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1. Introduction

Multi-attribute (group) decision making (MADM/MAGDM) has become a critical research topic in operations research. Indeterminacy and vagueness in real decision making scenarios can be represented by fuzzy information [1]. Due to the lack of non-membership degrees in the fuzzy set [1], Atanassov [2,3] presented an intuitionistic fuzzy set (IFS) in terms of membership and non-membership degrees as an extension of the fuzzy set. Since IFSs have much more information to represent uncertain and vague information, IFSs have been applied to various decision making problems [3–8]. Many researchers have developed various group decision making methods and applications [9–14]. In contrast, intuitionistic fuzzy matrices (IFMs) were introduced by Pal et al. [15], and some researchers have mainly investigated some properties and operations of IFMs [16–22]. Recently, IFMs have been applied to IFM games [23,24] and MADM [25].

Balakrishnan [26] introduced the energy of a graph, which is defined as the absolute sum of the eigenvalues of its adjacency matrix. Then, Anjali and Sunil [27] extended the graph energy to the energy of a fuzzy graph. Praba et al. [28] further extended the energy of the fuzzy graph to the energy of an intuitionistic fuzzy graph (IFG) and introduced its lower and upper bounds. Bravo et al. [29] presented matrix energy as an extension of graph energy and introduced upper and lower bounds for matrix energy. Oboudi [30] further investigated matrix energy bounds. Recently, Donbosco and Ganesan [31] proposed rough neutrosophic matrix energy and its application regarding the choice problem of building construction sites. However, the existing literature does not extend matrix energy to the IFM energy and its MAGDM model. Furthermore, existing decision making methods containing IFS information are not able to represent group/collective evaluation information for all the IFM information on attribute weights, decision maker weights, and attribute values, nor are they able to deal with a MAGDM problem containing all the IFM information on attribute weights, decision maker weights, and attribute values. Therefore, there are research gaps in the environment of IFMs. In this case, it is necessary to propose IFM energy and its MAGDM model for solving a MAGDM problem containing all the IFM information on attribute weights, decision maker weights, and attribute values. Based on the motivation of both matrix energy and the research gaps, this study aims to extend matrix energy to the energy of IFM and to use the IFM energy method for the MAGDM problem with all the IFM information on attribute weights, decision maker weights, and attribute values, which existing MAGDM methods containing IFS information cannot handle. To do so, we first define the energy of IFM. Then, we develop a MAGDM model using the IFM energy method and score and accuracy equations. In the MAGDM process, we first established the IFMs of attribute weights and the IFMs of alternatives satisfying the attributes provided by a group of decision makers. Then, considering the relationships between the decision maker weights and the IFMs of the alternative evaluations and between the attribute weights and the IFMs of the alternative evaluations, the weighted IFMs were established and divided into their true and false square matrices. Next, we presented the IFM energy, which includes the truth matrix energy and the false matrix energy, and defined the score and accuracy equations of the IFM energy to rank the alternatives and determine the best one. Finally, the developed MAGDM model was applied to the selection problem of hospital locations in Shaoxing City, China to demonstrate the practicality and validity of the developed model. Compared with existing MAGDM methods containing IFS information, the main highlights and novelties of the developed MAGDM model are revealed by the following two aspects: (1) the IFM information fully represents all decision maker weights, attribute weights, and evaluation values of alternatives satisfying attributes in the assessment process to satisfy the full expression of group evaluation information; (2) the IFM energy and its score and accuracy equations are used to solve intuitionistic fuzzy MAGDM problems in the scenario of IFMs.

The remainder of this paper is divided into the following sections. Section 2 presents some basic notions of IFSs. In Section 3, IFM energy is proposed as an extension of matrix energy. In Section 4, a new MAGDM model is developed based on the IFM energy method and score and accuracy equations. In Section 5, the developed MAGDM model is applied to an actual example to validate its practicality and validity. Section 6 presents the conclusions and future work.

2. Some Basic Notions of IFSs

Atanassov [2,3] introduced the notion of IFS. He denoted IFS as $O = \{<y, t_o(y), f_o(y)> \mid y \in Y\}$ in a fixed set Y where $t_o(y) \in [0, 1]$ and $f_o(y) \in [0, 1]$ are a membership degree and a non-membership degree, respectively, subject to $0 \leq t_o(y) + f_o(y) \leq 1$ for $y \in Y$. In the IFS O, its element $<y, t_o(y), f_o(y)>$ is simply represented as an intuitionistic fuzzy element (IFE) $o = <t_o, f_o>$.

For two IFEs, $o_1 = <t_{o1}, f_{o1}>$ and $o_2 = <t_{o2}, f_{o2}>$, these are their operation relationships [3–5]:

(i). $o_1 \oplus o_2 = \langle t_{o1} + t_{o2} - t_{o1}t_{o2}, f_{o1}f_{o2} \rangle$;
(ii). $o_1 \otimes o_2 = \langle t_{o1}t_{o2}, f_{o1} + f_{o2} - f_{o1}f_{o2} \rangle$;
(iii). $\omega o_1 = \left\langle 1 - (1 - t_{o1})^{\omega}, f_{o1}^{\omega} \right\rangle$ for $\omega > 0$;
(iv). $o_1^{\omega} = \left\langle t_{o1}^{\omega}, 1 - (1 - f_{o1})^{\omega} \right\rangle$ for $\omega > 0$.

3. Energy of IFM

This section proposes IFM energy as an extension of matrix energy [29].

We first introduce the notion of matrix energy [29] to propose the energy of IFM.

Set $M(s_{kl})$ $(k, l = 1, 2, \ldots, q)$ as a matrix with the order of $q \times q$. Then, the matrix energy of $M(s_{kl})$ is defined by the following equation [29]:

$$E(M(s_{kl})) = \sum_{k=1}^{q}\left|\tau_k - \frac{tr(M(s_{kl}))}{q}\right| = \sum_{k=1}^{q}\left|\tau_k - \frac{1}{q}\sum_{k=1}^{q}\tau_k\right|, \tag{1}$$

where τ_k $(k = 1, 2, \ldots, q)$ are the eigenvalues of $M(s_{kl})$ and $\mathrm{tr}(M(s_{kl}))$ is the trace of $M(s_{kl})$. In particular, when $M(s_{kl})$ is the adjacency matrix of a graph G or $\mathrm{tr}(M(s_{kl})) = 0$, $E(M(s_{kl}))$ is reduced to the energy of the graph G [26]:

$$E(G) = \sum_{k=1}^{q}|\tau_k|. \tag{2}$$

In terms of matrix energy, we can present IFM energy, which is composed of the energy of the true matrix and the energy of the false matrix.

Definition 1. *Set $M(o_{kl})$ as IFM with the order of $q \times q$. It consists of the matrix of the membership degrees t_{kl} $(k, l = 1, 2, \ldots, q)$ (simply called the true matrix $M(t_{kl})$) and the matrix of the non-membership degrees f_{kl} (simply called the false matrix $M(f_{kl})$) which is denoted by $M(o_{kl}) = <M(t_{kl}), M(f_{kl})>$. The energy of IFM $M(o_{kl})$ is defined as:*

$$E(M(o_{kl})) = \langle E(M(t_{kl})), E(M(f_{kl}))\rangle = \left\langle \sum_{k=1}^{q}|\tau_k - m_\tau|, \sum_{k=1}^{q}|\kappa_k - m_\kappa| \right\rangle, \tag{3}$$

where τ_k and κ_k are the eigenvalues of the true and false matrices $M(t_{kl})$ and $M(f_{kl})$ and m_τ and m_κ are the average values of the eigenvalues $m_\tau = \frac{1}{q}\sum_{k=1}^{q}\tau_k$ and $m_\kappa = \frac{1}{q}\sum_{k=1}^{q}\kappa_k$, respectively.

Example 1. *Set $M(o_{kl})$ as the following IFM with the order of 3×3:*

$$M(o_{kl}) = \begin{bmatrix} <0.6, 0.2> & <0.7, 0.3> & <0.8, 0.1> \\ <0.7, 0.2> & <0.9, 0.1> & <0.8, 0.2> \\ <0.5, 0.3> & <0.9, 0.1> & <0.6, 0.3> \end{bmatrix}.$$

Then, $M(o_{kl})$ can be represented as the following true and false matrices:

$$M(t_{kl}) = \begin{bmatrix} 0.6 & 0.7 & 0.8 \\ 0.7 & 0.9 & 0.8 \\ 0.5 & 0.9 & 0.6 \end{bmatrix} \text{ and } M(f_{kl}) = \begin{bmatrix} 0.2 & 0.3 & 0.1 \\ 0.2 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.3 \end{bmatrix}.$$

Using Equation (3), the energy of $M(o_{kl})$ is given as follows:

$$E(M(o_{kl})) = \langle E(M(t_{kl})), E(M(f_{kl}))\rangle = \left\langle \sum_{k=1}^{3}|\tau_k - m_\tau|, \sum_{k=1}^{3}|\kappa_k - m_\kappa| \right\rangle = <2.9763, 0.8137>.$$

4. MAGDM Model Using the IFM Energy Method

This section proposes a MAGDM model based on IFM energy and the score and accuracy equations of IFM energy to determine the best choice in the decision set of alternatives $Q = \{Q_1, Q_2, \ldots, Q_m\}$. In the assessment process, the alternatives must satisfy the set of attributes $H = \{h_1, h_2, \ldots, h_p\}$. Then, the satisfaction levels of the alternatives with respect to the attributes are assessed by a group of decision makers $E = \{E_1, E_2, \ldots, E_q\}$ and their weight vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_q)$ which is composed of the intuitionistic fuzzy weights ω_k = <$t_{\omega k}$, $f_{\omega k}$> (k = 1, 2, ..., q). Regarding the MAGDM problem, we develop a MAGDM model based on IFM energy and the score and accuracy equations of IFM energy and share the decision steps below.

Step 1: A group of decision makers/experts gives the intuitionistic fuzzy weight w_{kj} = <t_{wkj}, f_{wkj}> (j = 1, 2, ..., p; k = 1, 2, ..., q) of each attribute and establishes the IFM of the attribute weights:

$$W = \begin{array}{c} \\ E_1 \\ E_2 \\ \vdots \\ E_q \end{array} \begin{array}{c} \begin{array}{cccc} h_1 & h_2 & \ldots & h_p \end{array} \\ \begin{bmatrix} w_{11} & w_{12} & \ldots & w_{1p} \\ w_{21} & w_{22} & \cdots & w_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ w_{q1} & w_{q2} & w_{q3} & w_{qp} \end{bmatrix} \end{array}. \quad (4)$$

Step 2: A group of decision makers/experts gives the IFEs o_{ikj} = <t_{ikj}, f_{ikj}> (i = 1, 2, ..., m; j = 1, 2, ..., p; k = 1, 2, ..., q) based on the true and false evaluation values of each alternative over the attributes and then establishes the IFMs $M(o_{ikj})$ for Q_i:

$$M(o_{ikj}) = \begin{array}{c} \\ E_1 \\ E_2 \\ \vdots \\ E_q \end{array} \begin{array}{c} \begin{array}{cccc} h_1 & h_2 & \ldots & h_p \end{array} \\ \begin{bmatrix} < t_{i11}, f_{i11} > & < t_{i12}, f_{i12} > & \ldots & < t_{i1p}, f_{i1p} > \\ < t_{i21}, f_{i21} > & < t_{i22}, f_{i22} > & \cdots & < t_{i2p}, f_{i2p} > \\ \vdots & \vdots & \vdots & \vdots \\ < t_{iq1}, f_{iq1} > & < t_{iq2}, f_{iq2} > & \cdots & < t_{iqp}, f_{iqp} > \end{bmatrix} \end{array}. \quad (5)$$

Step 3: Considering the relationship between the decision maker weights and the IFMs $M(o_{ikj})$, we can establish the weighted IFMs:

$$M_H(\omega_k \otimes o_{ikj}) = \\ \begin{array}{c} \\ E_1 \\ E_2 \\ \vdots \\ E_q \end{array} \begin{array}{c} \begin{array}{cccc} h_1 & h_2 & \ldots & h_p \end{array} \\ \begin{bmatrix} < t_{\omega 1}t_{i11}, f_{\omega 1} + f_{i11} - f_{\omega 1}f_{i11}) > & < t_{\omega 1}t_{i12}, f_{\omega 1} + f_{i12} - f_{\omega 1}f_{i12} > & \ldots & < t_{\omega 1}t_{i1p}), f_{\omega 1} + f_{i1p} - f_{\omega 1}f_{i1p} > \\ < t_{\omega 2}t_{i21}, f_{\omega 2} + f_{i21} - f_{\omega 2}f_{i21} > & < t_{\omega 2}t_{i22}, f_{\omega 2} + f_{i22} - f_{\omega 2}f_{i22} > & \cdots & < t_{\omega 2}t_{i2p}), f_{\omega 2} + f_{i2p} - f_{\omega 2}f_{i2p} > \\ \vdots & \vdots & \vdots & \vdots \\ < t_{\omega q}t_{iq1}, f_{\omega q} + f_{iq1} - f_{\omega q}f_{iq1} > & < t_{\omega q}t_{iq2}, f_{\omega q} + f_{iq2} - f_{\omega q}f_{iq2} > & \cdots & < t_{\omega q}t_{iqp}, f_{\omega q} + f_{iqp} - f_{\omega q}f_{iqp} > \end{bmatrix} \end{array} \quad (6)$$

Considering the relationship between the attribute weights and the IFMs $M(o_{ikj})$ (i = 1, 2, ..., m), we can establish the weighted IFMs:

$$M_Q(w_{kj} \otimes o_{ikj}) = \\ \begin{array}{c} \\ E_1 \\ E_2 \\ \vdots \\ E_q \end{array} \begin{array}{c} \begin{array}{cccc} h_1 & h_2 & \ldots & h_p \end{array} \\ \begin{bmatrix} < t_{w11}t_{i11}, f_{w11} + f_{i11} - f_{w11}f_{i11} > & < t_{w12}t_{i12}, f_{w12} + f_{i12} - f_{w12}f_{i12} > & \ldots & < t_{w1p}t_{i1p}, f_{w1p} + f_{i1p} - f_{w1p}f_{i1p} > \\ < t_{w21}t_{i21}, f_{w21} + f_{i21} - f_{w21}f_{i21} > & < t_{w22}t_{i22}, f_{w22} + f_{i22} - f_{w22}f_{i22} > & \cdots & < t_{w2p}t_{i2p}, f_{w2p} + f_{i2p} - f_{w2p}f_{i2p} > \\ \vdots & \vdots & \vdots & \vdots \\ < t_{wq1}t_{iq1}, f_{wq1} + f_{iq1} - f_{wq1}f_{iq1} > & < t_{wq2}t_{iq2}, f_{wq2} + f_{iq2} - f_{wq2}f_{iq2} > & \cdots & < t_{wqp}t_{iqp}, f_{wqp} + f_{iqp} - f_{wqp}f_{iqp} > \end{bmatrix} \end{array} \quad (7)$$

Step 4: The weighted IFMs $M_H(\omega_{k\otimes}o_{ikj})$ and $M_Q(w_{kj\otimes}o_{ikj})$ are divided into the true matrices $M_H(t_{\omega k}t_{ikj})$ and $M_Q(t_{wkj}t_{ikj})$ and the false matrices $M_H(f_{\omega k} + f_{ikj} - f_{\omega k}f_{ikj})$ and $M_Q(f_{wkj} + f_{ikj} - f_{wkj}f_{ikj})$, respectively. Then, the true and false square matrices $M(t_{ikl})$ and $M(f_{ikl})$ (i = 1, 2, ..., m; k, l = 1, 2, ..., q) are obtained by the following calculations:

$$M(t_{ikl}) = M_Q(t_{wkj}t_{ikj}) \times [M_H(t_{\omega k}t_{ikj})]^T = \begin{bmatrix} t_{w11}t_{i11} & t_{w12}t_{i12} & \ldots & t_{w1p}t_{i1p} \\ t_{w21}t_{i21} & t_{w22}, t_{i22} & \cdots & t_{w2p}t_{i2p} \\ \vdots & \vdots & \vdots & \vdots \\ t_{wq1}t_{iq1} & t_{wq2}, t_{iq2} & \cdots & t_{wqp}t_{iqp} \end{bmatrix} \times \begin{bmatrix} t_{\omega 1}t_{i11} & t_{\omega 2}t_{i21} & \cdots & t_{\omega q}t_{iq1} \\ t_{\omega 1}t_{i12} & t_{\omega 2}t_{i22} & \cdots & t_{\omega q}t_{iq2} \\ \vdots & \vdots & \vdots & \vdots \\ t_{\omega 1}t_{i1p} & t_{\omega 2}t_{i2p} & \cdots & t_{\omega q}t_{iqp} \end{bmatrix}, \quad (8)$$

$$M(f_{ikl}) = M_Q(f_{wkj} + f_{ikj} - f_{wkj}f_{ikj}) \times [M_H(f_{\omega k} + f_{ikj} - f_{\omega k}f_{ikj})]^T$$
$$= \begin{bmatrix} f_{w11} + f_{i11} - f_{w11}f_{i11} & f_{w12} + f_{i12} - f_{w12}f_{i12} & \cdots & f_{w1p} + f_{i1p} - f_{w1p}f_{i1p} \\ f_{w21} + f_{i21} - f_{w21}f_{i21} & f_{w22} + f_{i22} - f_{w22}f_{i22} & \cdots & f_{w2p} + f_{i2p} - f_{w2p}f_{i2p} \\ \vdots & \vdots & \vdots & \vdots \\ f_{wq1} + f_{iq1} - f_{wq1}f_{iq1} & f_{wq2} + f_{iq2} - f_{wq2}f_{iq2} & \cdots & f_{wqp} + f_{iqp} - f_{wqp}f_{iqp} \end{bmatrix} \tag{9}$$
$$\times \begin{bmatrix} f_{\omega 1} + f_{i11} - f_{\omega 1}f_{i11} & f_{\omega 2} + f_{i21} - f_{\omega 2}, f_{i21} & \cdots & f_{\omega q} + f_{iq1} - f_{\omega q}f_{iq1} \\ f_{\omega 1} + f_{i12} - f_{\omega 1}f_{i12} & f_{\omega 2} + f_{i22} - f_{\omega 2}, f_{i22} & \cdots & f_{\omega q} + f_{iq2} - f_{\omega q}f_{iq2} \\ \vdots & \vdots & \vdots & \vdots \\ f_{\omega 1} + f_{i1p} - f_{\omega 1}f_{i1p} & f_{\omega 2} + f_{i2p} - f_{\omega 2}, f_{i2p} & \cdots & f_{\omega q} + f_{iqp} - f_{\omega q}f_{iqp} \end{bmatrix}.$$

Step 5: Using Equation (3), we can obtain the IFM energy for each alternative Q_i:

$$E(M(o_{ikl})) = \langle E(M(t_{ikl})), E(M(f_{ikl})) \rangle. \tag{10}$$

Step 6: The score and accuracy (if some of the score values are equal) values are obtained by the following defined score and accuracy equations of IFM energy:

$$S[E(M(o_{ikl}))] = E(M(t_{ikl})) - E(M(f_{ikl})), \tag{11}$$

$$H[E(M(o_{ikl}))] = E(M(t_{ikl})) + E(M(f_{ikl})). \tag{12}$$

Step 7: The ranking order of the alternatives Q_i (i = 1, 2, . . ., m) and the best choice are given in terms of the score values and the accuracy values (if necessary).

Step 8: End.

5. Actual Example

5.1. Selection of Hospital Locations

In this part, we apply the developed MAGDM model to a selection of hospital locations in Shaoxing City, China to demonstrate the practicality and validity of the developed model.

A local investor wants to build a hospital in the best location in Shaoxing City. In the decision making problem of hospital locations, the local investor first provides four potential locations, denoted as a set of alternatives $Q = \{Q_1, Q_2, Q_3, Q_4\}$. Then, they must satisfy the four attributes: construction cost (h_1), zonal population (h_2), transport facility (h_3), and zonal environment (h_4). Regarding this MAGDM problem of hospital locations, a group of decision makers $E = \{E_1, E_2, E_3\}$ is invited with their intuitionistic fuzzy weight vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$ = (<0.9, 0.1>, <0.8, 0.2>, <0.7, 0.3>).

Thus, the developed MAGDM model can be applied to solve the hospital location selection problem and can be addressed using the following decision process.

Step 1: The three decision makers give the intuitionistic fuzzy weights $w_{kj} = <t_{wkj}, f_{wkj}>$ of the four attributes subject to $t_{wkj}, f_{wkj} \in [0, 1]$ and $0 \leq t_{wkj} + f_{wkj} \leq 1$ (j = 1, 2, 3, 4; k = 1, 2, 3) and establish the IFM of the attribute weights:

$$W = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array} \\ \begin{bmatrix} <0.8, 0.1> & <0.9, 0.1> & <0.8, 0.2> & <0.7, 0.3> \\ <0.8, 0.2> & <0.7, 0.3> & <0.8, 0.1> & <0.8, 0.2> \\ <0.9, 0.1> & <0.6, 0.3> & <0.7, 0.3> & <0.8, 0.1> \end{bmatrix} \end{array}.$$

Step 2: The three decision makers give the IFEs $o_{ikj} = <t_{ikj}, f_{ikj}>$ (i, j = 1, 2, 3, 4; k = 1, 2, 3) based on the true and false evaluation values of each alternative Q_i over the four attributes h_j and establish the following four IFMs:

$$M(o_{1kj}) = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array} \\ \begin{bmatrix} <0.7, 0.2> & <0.8, 0.1> & <0.7, 0.2> & <0.6, 0.4> \\ <0.8, 0.2> & <0.7, 0.3> & <0.6, 0.4> & <0.6, 0.3> \\ <0.6, 0.3> & <0.7, 0.1> & <0.8, 0.2> & <0.9, 0.1> \end{bmatrix} \end{array},$$

$$M(o_{2kj}) = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array}\begin{array}{c} \begin{array}{cccc} \quad h_1 \quad & \quad h_2 \quad & \quad h_3 \quad & \quad h_4 \quad \end{array} \\ \begin{bmatrix} <0.6, 0.3> & <0.8, 0.2> & <0.7, 0.3> & <0.7, 0.3> \\ <0.7, 0.2> & <0.7, 0.3> & <0.6, 0.4> & <0.7, 0.2> \\ <0.8, 0.1> & <0.7, 0.1> & <0.8, 0.2> & <0.8, 0.1> \end{bmatrix} \end{array},$$

$$M(o_{3kj}) = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array}\begin{array}{c} \begin{array}{cccc} \quad h_1 \quad & \quad h_2 \quad & \quad h_3 \quad & \quad h_4 \quad \end{array} \\ \begin{bmatrix} <0.8, 0.2> & <0.7, 0.1> & <0.8, 0.2> & <0.7, 0.3> \\ <0.7, 0.3> & <0.6, 0.3> & <0.7, 0.2> & <0.7, 0.2> \\ <0.7, 0.3> & <0.6, 0.4> & <0.7, 0.3> & <0.8, 0.1> \end{bmatrix} \end{array},$$

$$M(o_{4kj}) = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array}\begin{array}{c} \begin{array}{cccc} \quad h_1 \quad & \quad h_2 \quad & \quad h_3 \quad & \quad h_4 \quad \end{array} \\ \begin{bmatrix} <0.7, 0.3> & <0.6, 0.2> & <0.7, 0.1> & <0.6, 0.3> \\ <0.6, 0.3> & <0.7, 0.1> & <0.7, 0.2> & <0.7, 0.3> \\ <0.8, 0.2> & <0.8, 0.1> & <0.9, 0.1> & <0.7, 0.2> \end{bmatrix} \end{array}.$$

Step 3: Considering the relationship between the decision maker weights and the four IFMs $M(o_{ikj})$ (i = 1, 2, 3, 4), we can establish the weighted IFMs:

$$M_H(\omega_k \otimes o_{1kj}) = \begin{bmatrix} <0.63, 0.28> & <0.72, 0.19> & <0.63, 0.28> & <0.54, 0.46> \\ <0.64, 0.36> & <0.56, 0.44> & <0.48, 0.52> & <0.48, 0.44> \\ <0.42, 0.51> & <0.49, 0.37> & <0.56, 0.44> & <0.63, 0.37> \end{bmatrix},$$

$$M_H(\omega_k \otimes o_{2kj}) = \begin{bmatrix} <0.54, 0.37> & <0.72, 0.28> & <0.63, 0.37> & <0.63, 0.37> \\ <0.56, 0.36> & <0.56, 0.44> & <0.48, 0.52> & <0.56, 0.36> \\ <0.56, 0.37> & <0.49, 0.37> & <0.56, 0.44> & <0.56, 0.37> \end{bmatrix},$$

$$M_H(\omega_k \otimes o_{3kj}) = \begin{bmatrix} <0.72, 0.28> & <0.63, 0.19> & <0.72, 0.28> & <0.63, 0.37> \\ <0.56, 0.44> & <0.48, 0.44> & <0.56, 0.36> & <0.56, 0.36> \\ <0.49, 0.51> & <0.42, 0.58> & <0.49, 0.51> & <0.56, 0.37> \end{bmatrix},$$

$$M_H(\omega_k \otimes o_{4kj}) = \begin{bmatrix} <0.63, 0.37> & <0.54, 0.28> & <0.63, 0.19> & <0.54, 0.37> \\ <0.48, 0.44> & <0.56, 0.28> & <0.56, 0.36> & <0.56, 0.44> \\ <0.56, 0.44> & <0.56, 0.37> & <0.63, 0.37> & <0.49, 0.44> \end{bmatrix}.$$

Considering the relationship between the attribute weights and the four IFMs $M(o_{ikj})$ (i = 1, 2, 3, 4), we can establish the weighted IFMs:

$$M_Q(w_{kj} \otimes o_{1kj}) = \begin{bmatrix} <0.56, 0.28> & <0.72, 0.19> & <0.56, 0.36> & <0.42, 0.58> \\ <0.64, 0.36> & <0.49, 0.51> & <0.48, 0.46> & <0.48, 0.44> \\ <0.54, 0.37> & <0.42, 0.37> & <0.56, 0.44> & <0.72, 0.19> \end{bmatrix},$$

$$M_Q(w_{kj} \otimes o_{2kj}) = \begin{bmatrix} <0.48, 0.37> & <0.72, 0.28> & <0.56, 0.44> & <0.49, 0.51> \\ <0.56, 0.36> & <0.49, 0.51> & <0.48, 0.46> & <0.56, 0.36> \\ <0.72, 0.19> & <0.42, 0.37> & <0.56, 0.44> & <0.64, 0.19> \end{bmatrix},$$

$$M_Q(w_{kj} \otimes o_{3kj}) = \begin{bmatrix} <0.64, 0.28> & <0.63, 0.19> & <0.64, 0.36> & <0.49, 0.51> \\ <0.56, 0.44> & <0.42, 0.51> & <0.56, 0.28> & <0.56, 0.36> \\ <0.63, 0.37> & <0.36, 0.58> & <0.49, 0.51> & <0.64, 0.19> \end{bmatrix},$$

$$M_Q(w_{kj} \otimes o_{4kj}) = \begin{bmatrix} <0.56, 0.37> & <0.54, 0.28> & <0.56, 0.28> & <0.42, 0.51> \\ <0.48, 0.44> & <0.49, 0.37> & <0.56, 0.28> & <0.56, 0.44> \\ <0.72, 0.28> & <0.48, 0.37> & <0.63, 0.37> & <0.56, 0.28> \end{bmatrix}.$$

Step 4: The weighted IFMs $M_H(\omega_{k\otimes}o_{ikj})$ and $M_Q(w_{kj\otimes}o_{ikj})$ (i, j = 1, 2, 3, 4; k = 1, 2, 3) are divided into the true matrices $M_H(t_{\omega k}t_{ikj})$ and $M_Q(t_{wkj}t_{ikj})$ and the false matrices $M_H(f_{\omega k} + f_{ikj} - f_{\omega k}f_{ikj})$ and $M_Q(f_{wkj} + f_{ikj} - f_{wkj}f_{ikj})$, respectively. Then, the true and false square matrices $M(t_{ikl})$ and $M(f_{ikl})$ (i = 1, 2, 3, 4; k, l = 1, 2, 3) are obtained by the following calculations:

$$M(t_{1kl}) = M_Q(t_{wkj}t_{1kj}) \times [M_H(t_{\omega k}t_{1kj})]^T = \begin{bmatrix} 0.56 & 0.72 & 0.56 & 0.42 \\ 0.64 & 0.49 & 0.48 & 0.48 \\ 0.54 & 0.42 & 0.56 & 0.72 \end{bmatrix} \times \begin{bmatrix} 0.63 & 0.64 & 0.42 \\ 0.72 & 0.56 & 0.49 \\ 0.63 & 0.48 & 0.56 \\ 0.54 & 0.48 & 0.63 \end{bmatrix} = \begin{bmatrix} 1.4508 & 1.2320 & 1.1662 \\ 1.3176 & 1.1448 & 1.0801 \\ 1.3842 & 1.1952 & 1.1998 \end{bmatrix},$$

$$M(t_{2kl}) = M_Q(t_{wkj}t_{2kj}) \times [M_H(t_{\omega k}t_{2kj})]^T = \begin{bmatrix} 0.48 & 0.72 & 0.56 & 0.49 \\ 0.56 & 0.49 & 0.48 & 0.56 \\ 0.72 & 0.42 & 0.56 & 0.64 \end{bmatrix} \times \begin{bmatrix} 0.54 & 0.56 & 0.56 \\ 0.72 & 0.56 & 0.49 \\ 0.63 & 0.48 & 0.56 \\ 0.63 & 0.56 & 0.56 \end{bmatrix} = \begin{bmatrix} 1.4391 & 1.2152 & 1.2096 \\ 1.3104 & 1.1320 & 1.1361 \\ 1.4472 & 1.2656 & 1.2810 \end{bmatrix},$$

$$M(t_{3kl}) = M_Q(t_{wkj}t_{3kj}) \times [M_H(t_{\omega k}t_{3kj})]^T = \begin{bmatrix} 0.64 & 0.63 & 0.64 & 0.49 \\ 0.56 & 0.42 & 0.56 & 0.56 \\ 0.63 & 0.36 & 0.49 & 0.64 \end{bmatrix} \times \begin{bmatrix} 0.72 & 0.56 & 0.49 \\ 0.63 & 0.48 & 0.42 \\ 0.72 & 0.56 & 0.49 \\ 0.63 & 0.56 & 0.56 \end{bmatrix} = \begin{bmatrix} 1.6272 & 1.2936 & 1.1662 \\ 1.4238 & 1.1424 & 1.0388 \\ 1.4364 & 1.1584 & 1.0584 \end{bmatrix},$$

$$M(t_{4kl}) = M_Q(t_{wkj}t_{4kj}) \times [M_H(t_{\omega k}t_{4kj})]^T = \begin{bmatrix} 0.56 & 0.54 & 0.56 & 0.42 \\ 0.48 & 0.49 & 0.56 & 0.56 \\ 0.72 & 0.48 & 0.63 & 0.56 \end{bmatrix} \times \begin{bmatrix} 0.63 & 0.48 & 0.56 \\ 0.54 & 0.56 & 0.56 \\ 0.63 & 0.56 & 0.63 \\ 0.54 & 0.56 & 0.49 \end{bmatrix} = \begin{bmatrix} 1.2240 & 1.1200 & 1.1746 \\ 1.2222 & 1.1320 & 1.1704 \\ 1.4121 & 1.2808 & 1.3433 \end{bmatrix}.$$

$$M(f_{1kl}) = M_Q(f_{wkj} + f_{1kj} - f_{wkj}f_{1kj}) \times [M_H(f_{\omega k} + f_{1kj} - f_{\omega k}f_{1kj})]^T$$
$$= \begin{bmatrix} 0.28 & 0.19 & 0.36 & 0.58 \\ 0.36 & 0.51 & 0.46 & 0.44 \\ 0.37 & 0.37 & 0.44 & 0.19 \end{bmatrix} \times \begin{bmatrix} 0.28 & 0.36 & 0.51 \\ 0.19 & 0.44 & 0.37 \\ 0.28 & 0.52 & 0.44 \\ 0.46 & 0.44 & 0.37 \end{bmatrix} = \begin{bmatrix} 0.4821 & 0.6268 & 0.5861 \\ 0.5289 & 0.7868 & 0.7375 \\ 0.3845 & 0.6084 & 0.5895 \end{bmatrix}$$

$$M(f_{2kl}) = M_Q(f_{wkj} + f_{2kj} - f_{wkj}f_{2kj}) \times [M_H(f_{\omega k} + f_{2kj} - f_{\omega k}f_{2kj})]^T$$
$$= \begin{bmatrix} 0.37 & 0.28 & 0.44 & 0.51 \\ 0.36 & 0.51 & 0.46 & 0.36 \\ 0.19 & 0.37 & 0.44 & 0.19 \end{bmatrix} \times \begin{bmatrix} 0.37 & 0.36 & 0.37 \\ 0.28 & 0.44 & 0.37 \\ 0.37 & 0.52 & 0.44 \\ 0.37 & 0.36 & 0.37 \end{bmatrix} = \begin{bmatrix} 0.5668 & 0.6688 & 0.6228 \\ 0.5794 & 0.7228 & 0.6575 \\ 0.4070 & 0.5284 & 0.4711 \end{bmatrix}$$

$$M(f_{3kl}) = M_Q(f_{wkj} + f_{3kj} - f_{wkj}f_{3kj}) \times [M_H(f_{\omega k} + f_{3kj} - f_{\omega k}f_{3kj})]^T$$
$$= \begin{bmatrix} 0.28 & 0.19 & 0.36 & 0.51 \\ 0.44 & 0.51 & 0.28 & 0.36 \\ 0.37 & 0.58 & 0.51 & 0.19 \end{bmatrix} \times \begin{bmatrix} 0.28 & 0.44 & 0.51 \\ 0.19 & 0.44 & 0.58 \\ 0.28 & 0.36 & 0.51 \\ 0.37 & 0.36 & 0.37 \end{bmatrix} = \begin{bmatrix} 0.4040 & 0.5200 & 0.6253 \\ 0.4317 & 0.6484 & 0.7962 \\ 0.4269 & 0.6700 & 0.8555 \end{bmatrix}$$

$$M(f_{4kl}) = M_Q(f_{wkj} + f_{4kj} - f_{wkj}f_{4kj}) \times [M_H(f_{\omega k} + f_{4kj} - f_{\omega k}f_{4kj})]^T$$
$$= \begin{bmatrix} 0.37 & 0.28 & 0.28 & 0.51 \\ 0.44 & 0.37 & 0.28 & 0.44 \\ 0.28 & 0.37 & 0.37 & 0.28 \end{bmatrix} \times \begin{bmatrix} 0.37 & 0.44 & 0.44 \\ 0.28 & 0.28 & 0.37 \\ 0.19 & 0.36 & 0.37 \\ 0.37 & 0.44 & 0.44 \end{bmatrix} = \begin{bmatrix} 0.4572 & 0.5664 & 0.5944 \\ 0.4824 & 0.5916 & 0.6277 \\ 0.3811 & 0.4832 & 0.5202 \end{bmatrix}$$

Step 5: Using Equations (3) and (10), we can obtain the IFM energy values for the four alternatives as follows:

$E(M(o_{1kl}))$ = $<E(M(t_{1kl})), E(M(f_{1kl}))>$ = <4.6124, 2.1876>, $E(M(o_{2kl}))$ = $<E(M(t_{2kl})), E(M(f_{2kl}))>$ = <4.7408, 2.1678>, $E(M(o_{3kl})) =< E(M(t_{3kl})), E(M(f_{3kl})) > =< 4.7483, 2.2356 >$, and $E(M(o_{4kl})) =< E(M(t_{4kl})), E(M(f_{4kl})) > =< 4.6112, 1.9445 >$.

Step 6: Using Equation (11), we give the score values of the four IFM energies $E(M(o_{ikl}))$ (i = 1, 2, 3, 4):

$$S[E(M(o_{1kl}))] = 2.4248,\ S[E(M(o_{2kl}))] = 2.5730,\ S[E(M(o_{3kl}))] = 2.5127, \text{ and } S[E(M(o_{4kl}))] = 2.6667.$$

Step 7: The ranking order of the four alternatives is $Q_4 > Q_2 > Q_3 > Q_1$. Therefore, the best hospital location is Q_4.

5.2. Discussion

From this actual MAGDM example, we can see that the developed MAGDM model using the IFM energy method can ensure decision making rationality and validity in the scenario of all IFMs. Therefore, the developed MAGDM model solves the MAGDM problem with all the IFM information on attribute weights, decision maker weights, and attribute values, which the existing MAGDM methods containing IFS information cannot tackle.

Since the existing intuitionistic fuzzy MAGDM methods cannot tackle a MAGDM problem containing all the IFM information on attribute weights, decision maker weights, and attribute values, it is difficult to compare the decision results based on the existing methods with this actual example. In this case, we only provide a qualitative comparison.

Through comparative analysis between the new model and existing intuitionistic fuzzy MAGDM methods, the main advantages of the new model are summarized below:

(a) The developed MAGDM model can fully express all the IFM information on attribute weights, decision maker weights, and attribute values in the group evaluation process which can compensate for the information representation problem of existing MAGDM methods containing the information of IFSs.

(b) The developed MAGDM model using the IFM energy method ensures the validity and rationality of the decision results based on the sufficient expression of all the IFM information in the MAGDM problem which can overcome the insufficiency of the existing MAGDM methods containing the information of IFSs.

Overall, the developed MAGDM model demonstrates its superiority over existing intuitionistic fuzzy MAGDM methods and provides a new and effective way to tackle MAGDM problems containing the complete IFM information on attribute weights, decision maker weights, and attribute values.

6. Conclusions

As an extension of matrix energy, this paper originally proposed IFM energy, which is composed of true matrix energy and false matrix energy; then, it developed a MAGDM model using the IFM energy method and score and accuracy equations to compensate for the existing research gaps. In the group decision making process, the proposed IFMs can fully represent the group evaluation information of decision maker weights, attribute weights, and attribute values to solve the full expression problems of intuitionistic fuzzy collective evaluation information. Then, the defined score and accuracy equations of the IFM energy can effectively rank all the alternatives and decide the best one. Furthermore, the application of the proposed MAGDM model in the selection problem of hospital locations demonstrated the practicality and validity of the proposed model in the scenario of complete IFMs.

Although the IFM energy method is used in this study to solve the MAGDM problem containing complete IFM information, it cannot be used for image processing, clustering analysis, slope stability analysis, and risk assessment in IFM scenarios, which means that there are some limitations in this paper. Therefore, we need to expand the scope of research and applications. In future research, it is necessary to apply the proposed IFM energy to image processing, clustering analysis, slope stability analysis, and risk assessment in IFM scenarios. Furthermore, the proposed IFM energy should be further extended to picture fuzzy matrix energy, Pythagorean fuzzy matrix energy, and simplified neutrosophic matrix energy and their applications in the fields of MAGDM, medicine, pattern recognition, clustering analysis, engineering management, and so on.

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