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Editorial

Recent Advances in Fractional Calculus

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1. Introduction

This Special Issue of the scientific journal *Axioms*, entitled "Recent Advances in Fractional Calculus", is dedicated to one of the most dynamic areas of mathematical sciences today. For 50 years, the number of researchers and scientific productions dealing with this topic has been increasing day by day, which clearly demonstrates the growing interest in fractional calculus, both from a practical and a theoretical point of view.

Fractional calculus is important because it expands the scope of classical calculus, enabling the modeling and analysis of a wide range of complex phenomena in fields such as physics, engineering, biology, economics, and others. Its flexibility and explanatory power make it an invaluable tool in scientific research and practical application.

The diversity of fractional calculus, and thus of this Special Issue, is well illustrated by the various types of fractional operators considered in the published contributions, such as Caputo-type, Hilfer-type, and Riemann–Liouville-type, and the various types of inequalities presented, such as Bullen-type, Jensen–Mercer-type, and Hermite–Hadamard-type, in addition to the examined fractional-order differential equations and boundary value problems.

2. Overview of the Published Papers

This Special Issue contains 10 articles that were accepted for publication after a rigorous review process.

Ogbu F. Imaga, Samuel A. Iyase, and Peter O. Ogunniyi (Contribution 1) consider the existence of solutions for a mixed fractional-order boundary value problem at resonance on the half line, in which Caputo and Riemann–Liouville fractional derivatives appear. Conditions for the existence of solutions to the problem are given using Mawhin's coincidence degree theory when the dimensions of the kernel of the linear fractional differential operator are two. At the end of the paper, the main result is applied to an example boundary value problem.

Sheza M. El-Deeb and Luminiţa-Ioana Cotîrlă (Contribution 2) introduce and investigate the properties of some new subclasses of the class of meromorphic p-valent functions in the punctured open unit disk. To define these subclasses, a new linear differential operator is presented by using the combination of q-derivative and convolution. Various properties are studied, and results are given for coefficient estimation, distortion bounds, convex family, and the concept of neighborhoods and partial sums of analytic functions for the class in question.

Ayub Samadi, Sotiris K. Ntouyas, Bashir Ahmad, and Jessada Tariboon (Contribution 3) investigate a non-linear, non-local, and fully coupled boundary value problem containing a generalized Hilfer fractional derivative and generalized Riemann–Liouville fractional integral operators. Existence and uniqueness results are established by transforming the given problem into a fixed-point problem, which facilitates the application of

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fixed-point theorems. The main results are accompanied by three examples. The paper concludes with some new results arising from the findings as special cases.

Ahmed Salem and Kholoud N. Alharbi (Contribution 4) analyze an infinitely delaying system of Caputo fractional evolution equations with an infinitesimal generator operator. The authors examine a moderate controllability solution based on two different arguments, one using compactness technology and the other using non-compactness. The first argument is based on Krasnoselskii's theorem, while the second one is rooted in the Kuratowski measure of non-compactness and the Sadovskii fixed-point theorem. They achieve the mild solution by assuming that the generator is an infinitesimal generator of a strongly continuous cosine family of uniformly bounded linear operators. Finally, the results are illustrated with a numerical example.

Isa A. Baba, Usa W. Humphries, Fathalla A. Rihan, and Juan E. Nápoles Valdés (Contribution 5) construct a fractional-order COVID-19 model consisting of six compartments in Caputo sense. The model integrates the indirect mode of transmission of the virus, which a result of the shedding effect. The main achievement of the article is the mathematical demonstration of the fact that an uninfected population can become infected via both direct and indirect methods by the exposed or infected class. In addition to the analysis of model's mathematical properties (positivity and boundedness, computation of equilibria, basic reproduction number, existence and uniqueness analysis of the solution of the model, local stability analysis), optimal control analysis and numerical simulations are provided.

Constantin Fetecău and Costică Moroşanu (Contribution 6) address two main topics in their paper. The first topic is a rigorous qualitative study of a second-order reaction–diffusion problem with non-linear diffusion and cubic-type reactions, as well as inhomogeneous dynamic boundary conditions. They extend previously known results by enabling new mathematical models to be more suitable to describe the complexity of a wide class of different physical phenomena in life sciences, including moving interface problems, material sciences, digital image processing, and others. The second topic is the development of an iterative fractional step-type scheme which approximates the non-linear second-order reaction–diffusion problem. Convergence and error estimates are established for the proposed numerical scheme, and a conceptual numerical algorithm is formulated.

Bahtiyar Bayraktar, Péter Kórus, and Juan Eduardo Nápoles Valdés (Contribution 7) consider convex functions, general convex functions, and differentiable functions whose derivatives, in absolute value, are generally convex. They obtain various relevant Hermite–Hadamard-type fractional inequalities via non-conformable fractional integrals, using the classical Jensen–Mercer inequality and its variants for general convex functions. In addition to showing that the main results extend previously known results from the literature, their three examples illustrate the scope and strength of their results.

Mohammad Faisal Khan, Suha B. Al-Shaikh, Ahmad A. Abubaker, and Khaled Matarneh (Contribution 8), starting from the known theory of q-calculus, define a differintegral operator for m-fold symmetric functions and obtain a new class of close-to-convex bi-univalent functions. The authors estimate the general Taylor–Maclaurin coefficient bounds, the initial coefficients, and the Fekete–Szegö functional for this class of functions using the Faber polynomial expansion method. The results obtained are novel and consistent with previous research, which is highlighted by some of the obtained corollaries.

Asfand Fahad, Saad Ihsaan Butt, Bahtiyar Bayraktar, Mehran Anwar, and Yuanheng Wang (Contribution 9) establish a new fractional Bullen-type identity for twice-differentiable functions in terms of fractional integral operators. Using convexity properties, the authors obtain some generalized Bullen-type inequalities, which are supplemented with concrete examples with graphical representations. They provide an analysis of the estimates of boundaries and show that the improved Hölder and power mean inequalities give better results in the upper limit than classical inequalities. Some applications with respect to quadrature rules, modified Bessel functions, and digamma functions are provided at the end of the article.

Muhammad Aamir Ali, Thanin Sitthiwirattham, Elisabeth Köbis, and Asma Hanif (Contribution 10) present an integral identity that incorporates a twice-differentiable function. After presenting this equality, some new Hermite–Hadamard–Mercer-type inequalities are given for twice-differentiable convex functions. Furthermore, it is demonstrated that the newly introduced inequalities serve as generalizations of certain inequalities previously established in the literature. Finally, the authors provide some applications which illustrate the scope and usefulness of their results.

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List of Contributions

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