



# Article The Universe in Leśniewski's Mereology: Some Comments on Sobociński's Reflections

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**Abstract:** Stanisław Leśniewski's mereology was originally conceived as a theory of foundations of mathematics and it is also for this reason that it has philosophical connotations. The 'philosophical significance' of mereology was upheld by Bolesław Sobociński who expressed the view in his correspondence with J.M. Bocheński. As he wrote to Bocheński in 1948: "[...] it is interesting that, being such a simple deductive theory, mereology may prove a number of very general theses reminiscent of metaphysical ontology". The theses which Sobociński had in mind were related to the mereological notion of "the Universe". Sobociński listed them in the letter adding his philosophical commentary but he did not give proofs for them and did not specify precisely the theory lying behind them. This is what we want to supply in the first part of our paper. We indicate some connections between the notion of the universe and other specific mereological notions. Motivated by Sobociński's informal suggestions showing his preference for mereology over the axiomatic set theory in application to philosophy we propose to consider Sobociński's formalism in a new frame which is the ZFM theory—an extension of Zermelo-Fraenkel set theory by mereological axioms, developed by A. Pietruszczak. In this systematic part we investigate reasons of 'philosophical hopes' mentioned by Sobociński, pinned on the mereological concept of "the Universe".

Keywords: Leśniewski; Sobociński; ontology; mereology; the universe

### 1. Introduction

There is definitely some scepticism as to whether it is possible to establish the so-called intended interpretation of a formalized theory. This does not mean, however, that any studies into such an interpretation must be considered aimless. We would rather say that, in this case, our expectations should not be too high: a reconstruction of intended meanings of the terms belonging to a given system might not expose the intended model, but it still increases the pragmatic value of the examined text.

Stanisław Leśniewski's mereology was originally conceived as a theory of foundations of mathematics and it is also for this reason that it has philosophical connotations. We may look for its new philosophical interpretation and ask whether and to what extent (from the perspective of a given purpose) mereology is really an interesting theory of part-whole. The 'philosophical significance' of mereology was upheld by Bolesław Sobociński who expressed the view in his correspondence with J.M. Bocheński [1] (Sobociński and Bocheński together with J. Drewnowski and J. Salamucha formed the so-called Cracow Circle, which was a branch of the Lvov-Warsaw School, interested in modern analytical tools used in Christian philosophy. For the richer historical context we refer the reader to [2]). As he wrote to Bocheński in 1948:

[...] it is interesting that, being such a simple deductive theory, mereology may prove a number of very general theses reminiscent of metaphysical ontology. [1]

The theses which Sobociński had in mind were related to the mereological notion of "the Universe". Sobociński listed them in the letter adding his philosophical commentary. However, he did not give proofs for them and did not define precisely the theory lying behind them. This is what we want to supply in the first part of our paper. We focus on the deductive minimum for the mereological theses listed by Sobociński and indicate some connections between the notion of the universe and other specific mereological notions. In the considered letter Sobociński expressed his preference for mereology over the axiomatic set theory in application to philosophy, This motivates us to look for a frame in which both mereological and set theoretical notions may be expressed. We choose for this aim the ZFM system, which is an extension of Zermelo-Fraenkel set theory by mereological axioms, developed by A. Pietruszczak [3]. In this systematic part we reconsider reasons of 'philosophical hopes' mentioned by Sobociński, pinned on the mereological concept of "the Universe".

#### 2. The Universe in Mereology with Ontology

The concept of the universe which Sobociński explained in a letter to Bocheński was already introduced by Leśniewski in his early mereology, where he put forward the following definition

**Definition VII.** I use the expression 'universe' to denote the class of objects.

and proved theorems on the existence and uniqueness of the universe:

**Theorem XLIII.** Some object is the class of non-contradictory objects. [...] **Theorem XLIV.** The class of non-contradictory objects is the universe. [...] **Theorem XLV.** If P is the universe, and  $P_1$  is the universe, then P is  $P_1$ .

[4] (159–160), [5] (L2: 31–32)

The question of the provability of theorems XLIII and XLIV requires a commentary on the ontological commitment of mereology: if it requires that the domain of "objects" should be nonempty. In the proofs of XLIII and XLIV the existential assumption is used that there is at least one "object" (or "non-contradictory object" (there is no explicit definition of a non-contradictory object but certainly it is dependent on the notion of an object)) (comments on provability of XLIII and XLIV are to be found in [6] (128–129). However, we would not agree with the opinion that including XLIII and XLIV as theses shows that "Leśniewski was not clear as to the logical foundations of his system". We would rather say that he simply changed the opinion about the ontological commitment of his theory). Indeed, Leśniewski in his early studies believed that the sentence "no object contains contradictions" is true and may be proved ([4] (46) ([5] (L1: 226))). If he could use a strong interpretation of universal negative sentences (as he declared in [4] (231), [5] (L3: 264)), it follows that there exists at least one object (non-contradictory object) (theorem I "No object is a part of itself." [4] (131) ([5] (L2: 9)) of early mereology already implies the existence of an object). In his later works, however, Leśniewski changed his opinion on the matter and decided not to assert conclusively whether any objects exist at all [4] (232) ([5] (L3: 265)). Ontology with mereology in their later version have models with an empty domain of individuals where formulas XLIII and XLIV are not true (in this sense Leśniewski's system is not ontologically committed to any object (we follow Urbaniak referring to [7])).

Sobociński essentially took over the notion of the universe from Leśniewski, but did not formulate any existential theses about it. Perhaps he assumed the later version of ontology with mereology. In his correspondence with Bocheński, he included many more theses about the universe than Leśniewski did, considering them interesting for philosophical reasons.

We will reconstruct Sobociński's exposition in Leśniewski's assumed system.

We expound Sobociński's approach in mereology based on ontology using the same method as the one employed by Sobociński himself in [8].

The assumed theory is expressed in the first-order language, whose vocabulary comprises: individual variables: x, y, z, ...; name-forming functor pt (*part of*); inherence two-place predicate  $\varepsilon$  (*is*); logical connectives:  $\neg$ ,  $\land$ ,  $\lor$ ,  $\rightarrow$ ,  $\leftrightarrow$ ; quantifiers  $\forall$ ,  $\exists$  and parentheses (, ). The terms of our language

are individual variables and expressions  $pt(\tau)$ , where  $\tau$  is a term. Atomic formulas are of the shape  $\tau \varepsilon \tau'$ , where  $\tau$  and  $\tau'$  are terms. Other formulas are built in a standard way. (In the original manuscript Sobociński used the style of notation from *Principia Mathematica*. Like Leśniewski, he applied two types of variables *A*, *B*, *a* (which are of the same sort). The same notation is used in [8], with one exception: in the manuscript there is *cz* instead of *pt* (from the Polish "część" meaning "part"). Cf. also [9]. )

Mereology with ontology OML is characterized by:

- theorems of first-order logic (QL)
- axiom of ontology
- $AO. \quad \forall x, y \ (x \varepsilon y \leftrightarrow \exists z (z \varepsilon x) \land \forall z, u \ (z \varepsilon x \land u \varepsilon x \to z \varepsilon u) \land \forall z \ (z \varepsilon x \to z \varepsilon y))$

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- axioms of mereology
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AM1. \forall x, y, z \ (x \varepsilon pt(y) \land y \varepsilon pt(z) \rightarrow x \varepsilon pt(z))
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- *AM2.*  $\forall x, y \ (x \varepsilon p t(y) \rightarrow y \varepsilon \sim p t(x))$
- AM3.  $\forall x, y, z \ (x \in Kl(z) \land y \in Kl(z) \rightarrow x = y)$
- $AM4. \quad \forall x, y \ (x \varepsilon y \to \exists z (z \varepsilon K l(y)))$

where:

| (non-y)   | $\forall x \ (x\varepsilon \sim y \leftrightarrow x\varepsilon x \land \neg (x\varepsilon y))$   | $(\sim)$ |
|---|--|----------|
| (identity)  | $\forall x, y \ (x = y \leftrightarrow x \varepsilon y \land y \varepsilon x)$   | (=)      |
|   | $\forall x, y \ (x \in Kl(y) \leftrightarrow x \in x \land \forall z (z \in y \rightarrow z \in ing(x)) \land \forall z (z \in ing(x) \rightarrow z \in ing(x)) \land \forall z (z \in ing(x) \rightarrow z \in ing(x)) \land \forall z \in X \land x \in X \land \forall z \in X \land \forall z \in X \land x \in X$ | (Kl)     |
| (mereological class)                                | $\exists u, v(u \in y \land v \in ing(u) \land v \in ing(z))))$  |          |
| ( <i>ingredient</i> , in the manuscript <i>el</i> ) | $\forall x, y \ (x \varepsilon ing(y) \leftrightarrow x = y \lor x \varepsilon pt(y))$   | (ing)    |

(We delete the redundant part of the conjunction occurring on the right side of the definition of *Kl* from [8] (219):  $\exists z(z\varepsilon y)$ .)

We use also the following definitions:

| (inclusion)                                       | $\forall x, y \ (x \subset y \leftrightarrow \forall z (z \varepsilon x \to z \varepsilon y))$   | $(\subset)$ |
|---|--|-------------|
| (intersection)                                    | $\forall x, y \ (x \Delta y \leftrightarrow \exists z (z \varepsilon x \land z \varepsilon y))$  | $(\Delta)$  |
| $\cdot z \varepsilon \sim (ing(x)))))$            | $\forall x, y \ (x \varepsilon extr(y) \leftrightarrow x \varepsilon x \land \exists z (z \varepsilon ing(y)) \land \forall z (z \varepsilon ing(y) \rightarrow z) $ | (extr)      |
| ( <i>exterior</i> , in the manuscript <i>zw</i> ) |  |             |
| (sum)   | $\forall x, y, z \ (x\varepsilon(y \cup z) \leftrightarrow x\varepsilon x \land (x\varepsilon y \lor x\varepsilon z))$   | (∪)         |
|   | $\forall x, y, z \ (x\varepsilon(y+z) \leftrightarrow x\varepsilon x \land x\varepsilon Kl(y \cup z) \land y\varepsilon extr(z))$                                    | (+)         |
| (sum of exterior objects)                         |  |             |
| (subtration)                                      | $\forall x, y, z \ (x\varepsilon(y-z) \leftrightarrow x\varepsilon x \wedge y\varepsilon(x+z))$  | (-)         |
| (object)  | $\forall x \ (x \varepsilon \lor \leftrightarrow x \varepsilon x)$   | (∨)         |
| <i>(contradictory object)</i>                     | $\forall x \ (x \varepsilon \wedge \leftrightarrow x \varepsilon x \wedge \neg (x \varepsilon x))$   | $(\wedge)$  |

The definitions listed above are special cases of three definitional schemata, which enable us to introduce predicates (symbolized by F), function constants (f) and individual constants (n):

def-p  $F(x_1...x_n) \leftrightarrow \phi_F$ def-f  $\forall u(u\varepsilon f(x_1...x_n) \leftrightarrow u\varepsilon x_i \wedge \phi_f)$   $x_i \in \{u, x_1, ..., x_n\}$ def-n  $\forall u(u\varepsilon n \leftrightarrow u\varepsilon u \wedge \phi_n)$ 

(Formulas  $\phi_F, \phi_f, \phi_n$  contain the same free variables as the left sides of the equivalences.) (cf. [10]) Primitive rules of OML are **MP**:  $\vdash \phi \rightarrow \psi, \phi \implies \vdash \psi$  and **Gen**:  $\vdash \phi \implies \vdash \forall x\phi$ .

Let us now in our notation retype formulas listed by Sobociński with his original comments (we also change notation of symbols in the quoted commentaries).

$$\begin{aligned} W & \forall x (x \varepsilon W \leftrightarrow x \varepsilon K l(\vee)) \\ & (x \text{ is the Universe} \leftrightarrow x \varepsilon K l(objects).) \end{aligned}$$

 $\forall x(x \in W \leftrightarrow \neg \exists y(x \in pt(y)))$ 

S1

|             | (The Universe is not a part of anything.)   |
|-------------|---|
| <i>S</i> 2  | $\forall x, y(x \in W \land x \in ing(y) \rightarrow y \in W)$  |
|             | (If x is the Universe and x is an element [ingredient of] (B), then B is the Universe.)                       |
| <i>S</i> 3  | $\forall x(x\Delta W \to Kl(x) \subset W)$  |
|             | (If some x is W, then any $Kl(x)$ is W.)  |
| S4          | $\forall x (x \Delta W \rightarrow K l(x) = W)$   |
|             | (If $x\Delta W$ , then $Kl(x)$ is identical with W.)  |
| <i>S</i> 5  | $\forall x (x \varepsilon W \leftrightarrow \forall y (x \varepsilon \sim pt(y))$                             |
|             | (Definition of W using the term "pt".)  |
| <i>S</i> 6  | $\forall x (x \varepsilon \lor \land \lor \subset ing(x) \to x \varepsilon W)$                                |
|             | (If every object is $ing(x)$ , then $x \in W$ .)  |
| <i>S</i> 7  | $\forall x, y, z(x \in Kl(z) \land y \in Kl(\sim (z)) \to x \in W \lor y \in W)$                              |
| S8          | $\forall x (x \varepsilon W \leftrightarrow x \varepsilon \lor \land \lor \subset ing(x))$                    |
|             | (Definition of W using the term "ing".)   |
| <i>S</i> 9  | $\forall x (x \varepsilon W \rightarrow \neg y \varepsilon extr(x))$  |
|             | (Nothing is exterior to W.)   |
| <i>S</i> 10 | $\forall x (x \varepsilon W \leftrightarrow x \varepsilon \lor \land \forall y (\neg y \varepsilon extr(x)))$ |
|             | (Definition of W using "extr".)   |
| 044         |   |

*S11*  $\forall x (x \in W \leftrightarrow x \in \bigvee \land \forall z, y (\neg y \in (z + x)))$ (*Definition of W using "+"; addition of anything to W is not possible.*)

In our derivations of formulas written down by Sobociński we will use the following OML theses:

| T1  | $\forall x(x = x \leftrightarrow x\varepsilon x)$  | (=)                                    |
|-----|--|--|
| T2  | $\forall x \neg x \varepsilon p t(x)$  | ( <i>AM</i> 2,∼)                       |
| T3  | $\forall x(x \varepsilon ing(x) \leftrightarrow x \varepsilon x)$  | ( <i>ing</i> , <i>T</i> 2, <i>T</i> 1) |
| T4  | $\forall x, y(x \varepsilon p t(y) \to x \varepsilon x \land y \varepsilon y \land \neg y \varepsilon x)$                                      | ( <i>AM</i> 2, ∼)                      |
| T5  | $\forall x, y(x \varepsilon x \land y \varepsilon x \to x = y)$  | ( <i>AO</i> ,=)                        |
| T6  | $\forall x, y(x \varepsilon ing(y) \rightarrow y \varepsilon \lor)$  | $(ing, T4, =, \lor)$                   |
| T7  | $\forall x (x \varepsilon x \leftrightarrow \exists z (z \varepsilon x) \land \forall u, v (u \varepsilon x \land v \varepsilon x \to u = v))$ | (AO)                                   |
| T8  | $\forall x, y(x \in Kl(y) \to x = Kl(y))$  | ( <i>AM</i> 3, <i>T</i> 7, <i>T</i> 5) |
| T9  | $\forall x, y, z (x \varepsilon y \land y \varepsilon z \to x \varepsilon z)$  | ( <i>AO</i> )                          |
| T10 | $\forall x, y(x\varepsilon(y\cup \sim y) \leftrightarrow x\varepsilon \vee)$   | $(\cup, \sim, \lor)$                   |
| T11 | $\exists x(x \varepsilon \lor) \to Kl(\lor) \varepsilon \lor$  | $(AM4, T8, =, \lor)$                   |
| T12 | $\forall x, y(x \in pt(y) \to \exists z(z \in pt(y) \land z \in extr(x)))$   | ( <i>AM</i> 2, <i>AM</i> 3)            |
| T13 | $\forall x, y(x \varepsilon p t(y) \to \exists z(z \varepsilon e x t r(x)))$   | (T12)                                  |

(T12 expresses the *Weak Supplementation Principle* accepted by Simons in [11] (p. 28) as a mereological axiom. For possible connections between this principle and other mereological assumptions cf. [3] (pp. 71–72).)

Now we are immediately able to notice that the definition of the universe (*W*) is equivalent to def-n for *W* because of (*Kl*).

Actually, formula S1 brings problems because:

**Fact 1.** *S1 added to* OML +*W causes a contradiction.* 

From *S*1 we have  $\wedge \varepsilon W \leftrightarrow \neg \wedge \varepsilon pt(\wedge)$ . Because the OML thesis is  $\neg \wedge \varepsilon pt(\wedge)$ , we get  $\wedge \varepsilon W$  and with *AO*:  $\exists z(z\varepsilon \wedge)$ . But the OML thesis is  $\neg \exists z(z\varepsilon \wedge)$ .

Perhaps Sobociński's original comment to *S*1: "The Universe is not a part of anything" should be understood as weaker than *S*1 but only as the implication

 $S1^{\rightarrow}$ .  $\forall x(x \in W \rightarrow \neg \exists y(x \in pt(y)))$ .

Formula  $S1^{\rightarrow}$  is derivable in OML+W and the same is to be said about other formulas listed by Sobociński:

**Fact 2.**  $S1^{\rightarrow}$  and S2–S11 are theses of OML extended by W.

We formulate the above fact giving the following proofs:

 $S1^{\rightarrow}$ .  $\forall x(x \in W \rightarrow \neg \exists y(x \in pt(y)))$ 

| <ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> <li>5.</li> <li>6.</li> <li>7.</li> <li>8.</li> <li>9.</li> <li>10.</li> </ol> | $\begin{aligned} x \varepsilon W \wedge x \varepsilon pt(y) &\to \forall z(z \varepsilon \lor \to z \varepsilon ing(x)) \wedge y \varepsilon y \wedge \neg y \varepsilon pt(x) \\ x \varepsilon Kl(\lor) \wedge x \varepsilon pt(y) \to (y \varepsilon \lor \to y \varepsilon ing(x)) \wedge y \varepsilon \lor \wedge \neg y \varepsilon pt(x) \\ x \varepsilon Kl(\lor) \wedge x \varepsilon pt(y) \to y = x \\ x \varepsilon Kl(\lor) \wedge x \varepsilon pt(y) \to y = x \wedge x \varepsilon pt(y) \\ x \varepsilon Kl(\lor) \wedge x \varepsilon pt(y) \to y \varepsilon pt(y) \\ x \varepsilon Kl(\lor) \to \neg x \varepsilon pt(y) \\ x \varepsilon Kl(\lor) \to \neg \exists y(x \varepsilon pt(y)) \\ x \varepsilon W \to \neg \exists y(x \varepsilon pt(y))) \\ \forall x(x \varepsilon W \to \neg \exists y(x \varepsilon pt(y))) \end{aligned}$ | $ \begin{array}{l} (W, Kl, T4) \\ (x) & (1, \lor) \\ (2) \\ (3, ing) \\ (4) \\ (5) \\ (6, T2) \\ (7) \\ (8, W) \\ (9) \end{array} $ |
|---|---|---|
| <i>S</i> 2. `   | $\forall x, y(x \varepsilon W \land x \varepsilon ing(y) \to y \varepsilon W)$  |   |
| 1.  | $x \in W \land x \in ing(y) \rightarrow (x = y \lor x \in pt(y)) \land \neg x \in pt(y)$  | $(ing, S1^{\rightarrow})$   |
| 2.  | $x \in W \land x \in ing(y) \to x = y$  | (1)   |
| 3.  | $x \varepsilon W \wedge x \varepsilon ing(y) \rightarrow y \varepsilon x \wedge x \varepsilon W$  | (2,=)   |
| 4.  | $x \varepsilon W \wedge x \varepsilon ing(y) \rightarrow y \varepsilon W$   | (3 <i>, T</i> 9)  |
| 5.  | $\forall x, y(x \in W \land x \in ing(y) \to y \in W)$  | (4)   |
| <i>S</i> 3.   | $\forall x (x \Delta W \to K l(x) \subset W)$   |   |
| 1.  | $z \varepsilon K l(\lor) \to K l(\lor) \varepsilon z$   | ( <i>T</i> 8, =)  |
| 2.  | $z \varepsilon x \wedge z \varepsilon K l(\lor) \to K l(\lor) \varepsilon z \wedge z \varepsilon x$   | (1)   |
| 3.  | $z \varepsilon x \wedge z \varepsilon K l(\lor) \to K l(\lor) \varepsilon x$  | (2, T9)   |
| 4.  | $y \in Kl(x) \rightarrow \forall z(z \in x \rightarrow z \in ing(y))$   | (Kl)  |
| 5.  | $y \in Kl(x) \to (Kl(\vee) \in x \to Kl(\vee) \in ing(y))$  | (4)   |
| 6.  | $z \varepsilon x \wedge z \varepsilon Kl(\lor) \wedge y \varepsilon Kl(x) \rightarrow Kl(\lor) \varepsilon ing(y)$  | (3,5)   |
| 7.  | $z\varepsilon x \wedge z\varepsilon Kl(\vee) \wedge y\varepsilon Kl(x) \to Kl(\vee) = y \vee Kl(\vee)\varepsilon pt(y)$   | (6, ing)  |
| 8.  | $Kl(\lor)\varepsilon W \to \neg Kl(\lor)\varepsilon pt(y)$  | $(S1^{\rightarrow}, x/Kl(\vee), W)$   |
| 9.  | $z\varepsilon x \wedge z\varepsilon Kl(\vee) \wedge y\varepsilon Kl(x) \to Kl(\vee) = y$  | (8 <i>, T</i> 11 <i>,</i> 7)  |
| 10.   | $z \varepsilon x \wedge z \varepsilon Kl(\lor) \to (y \varepsilon Kl(x) \to y \varepsilon Kl(\lor))$  | (9,=)   |
| 11.   | $z\varepsilon x \wedge z\varepsilon Kl(\vee) \to \forall y(y\varepsilon Kl(x) \to y\varepsilon Kl(\vee))$   | (10)  |
| 12.   | $z \varepsilon x \wedge z \varepsilon Kl(\lor) \to Kl(x) \subset Kl(\lor)$  | (11,⊂)  |
| 13.   | $\exists z(z\varepsilon x \wedge z\varepsilon W) \to Kl(x) \subset W$   | (12, W)   |
| 14.   | $x\Delta W \to Kl(x) \subset W$   | $(13, \Delta)$  |
| 15.   | $\forall x (x \Delta W \to K l(x) \subset W)$   | (14)  |

S4.  $\forall x(x\Delta W \rightarrow Kl(x) = W)$ 1.  $z \varepsilon x \wedge z \varepsilon Kl(\lor) \wedge y \varepsilon Kl(x) \rightarrow Kl(\lor) = y$  $(S3, \Delta, W, T8)$ (1, T8)2.  $z \varepsilon x \wedge z \varepsilon Kl(\lor) \wedge y \varepsilon Kl(x) \rightarrow Kl(\lor) = y \wedge y = Kl(x)$ 3.  $z \varepsilon x \wedge z \varepsilon K l(\lor) \wedge y \varepsilon K l(x) \rightarrow K l(x) = K l(\lor)$ (2)4.  $z \varepsilon x \wedge z \varepsilon K l(\vee) \wedge y \varepsilon K l(x) \rightarrow K l(x) = W$ (3, W)5.  $z \varepsilon x \wedge z \varepsilon K l(\vee) \rightarrow (\exists y (y \varepsilon K l(x)) \rightarrow K l(x) = W)$ (4)6.  $z \varepsilon x \to \exists y (y \varepsilon K l(x))$ (AM4)7.  $z \varepsilon x \wedge z \varepsilon K l(\lor) \to K l(x) = W$ (5, 6)8.  $\exists z(z\varepsilon x \wedge z\varepsilon W) \to Kl(x) = W$ (7, W)9.  $x \Delta W \rightarrow K l(x) = W$  $(8, \Delta)$ 10.  $\forall x (x \Delta W \rightarrow K l(x) = W)$ (9) S5.  $\forall x(x \in W \leftrightarrow \forall y(x \in p t(y)))$ 1.  $\neg \exists y(x \in pt(y)) \rightarrow \forall y(x \in ing(y) \rightarrow x = y)$ (ing) 2.  $\neg \exists y(x \in pt(y)) \rightarrow (x \in ing(W) \rightarrow x = W)$ (1, y/W)3.  $x \in \bigvee \to \exists z (z \in Kl(\lor))$ (AM4) $x \in \bigvee \to \exists z (z = K l(\bigvee))$ 4. (T8, 3)5.  $x \in \bigvee \to Kl(\bigvee) \in Kl(\bigvee)$ (=, 4)6.  $x \in \bigvee \to \forall z (z \in \bigvee \to z \in ing(Kl(\bigvee)))$ (5, Kl)7.  $x \in \bigvee \to (x \in \bigvee \to x \in M(W))$ (6, W)8.  $x \in \bigvee \to x \in M(W)$ (7)9.  $x \in x \land \neg \exists y (x \in pt(y)) \to x = W$  $(\vee, 8, 2)$ 10.  $\forall y(x \in x \land \neg x \in pt(y)) \to x = W$ (9) 11.  $\forall y(x \varepsilon \sim pt(y)) \rightarrow x \varepsilon W$  $(10, \sim)$ 12.  $x \in W \to x \in x \land \neg \exists y (x \in pt(y))$  $(S1^{\rightarrow})$ 13.  $x \in x \to \forall y (x \in x \land \neg x \in pt(y))$ (12)14.  $x \in W \to \forall y (x \in v \sim y)$ (13)15.  $\forall x(x \in W \leftrightarrow \forall y(x \in \sim pt(y)))$ (11, 14)S6.  $\forall x(x \in V \land V \subset ing(x) \rightarrow x \in W)$ 1.  $x \in V \land V \subset ing(x) \rightarrow (x \in V \land \forall z (z \in V \rightarrow z \in ing(x)))$  $(\subset)$ 2.  $x \in \bigvee \land \bigvee \subset ing(x) \to (x \in \bigvee \land Kl(\bigvee) \in \bigvee \to Kl(\bigvee) \in ing(x))$ (1)3.  $x \in V \land V \subset ing(x) \rightarrow Kl(V) \in ing(x)$ (2, T11)4.  $x \in \forall \land \forall \subset ing(x) \rightarrow x = Kl(\forall) \lor Kl(\forall) \in pt(x)$ (ing,3)

5.  $x \in V \land V \subset ing(x) \rightarrow x = Kl(V)$ 

6.  $x \in V \land V \subset ing(x) \to x \in W$ 

7.  $\forall x(x \in V \land V \subset ing(x) \rightarrow x \in W)$ 

 $(4, S1^{\rightarrow}, x/Kl(\lor))$ (5, =, W)(6)

S7.  $\forall x, y, z(x \in Kl(z) \land y \in Kl(\sim (z)) \rightarrow x \in W \lor y \in W)$  $x \in Kl(z) \land y \in Kl(\sim z) \rightarrow$ 1.  $\forall u(u\varepsilon z \to u\varepsilon ing(x)) \land \forall u(u\varepsilon \sim z \to u\varepsilon ing(y))$ 

| 2.          | $x \varepsilon K l(z) \land y \varepsilon K l(\sim z) \rightarrow$  |                                     |
|-------------|---|-------------------------------------|
|             | $\forall u(u \varepsilon u \land (u \varepsilon z \lor u \varepsilon \sim z) \rightarrow u \varepsilon ing(x) \lor u \varepsilon ing(y))$ | (1)                                 |
| 3.          | $x \in Kl(z) \land y \in Kl(\sim z) \rightarrow$  |                                     |
|             | $\forall u(u\varepsilon(z\cup \sim z) \rightarrow (u\varepsilon ing(x) \lor u\varepsilon ing(y)))$  | (2,∪)                               |
| 4.          | $x \in Kl(z) \land y \in Kl(\sim z) \rightarrow$  |                                     |
|             | $\forall u(u \in \bigvee \rightarrow u \in ing(x) \lor u \in ing(y))$   | (3, T10)                            |
| 5.          | $x \in Kl(z) \land y \in Kl(\sim z) \rightarrow$  | . ,                                 |
|             | $Kl(\forall) \varepsilon \forall \rightarrow (Kl(\forall) \varepsilon ing(x) \lor Kl(\forall) \varepsilon ing(y))$                        | $(4, u/Kl(\vee))$                   |
| 6.          | $x \in V \to Kl(V) \in Kl(V)$   | (T11)                               |
| 7.          | $x \varepsilon \lor \to K l(\lor) \varepsilon \lor$   | (T11)                               |
| 8.          | $x \in Kl(z) \land y \in Kl(\sim z) \rightarrow Kl(\lor) \in ing(x)) \lor Kl(\lor) \in ing(y))$   | (5,7)                               |
| 9.          | $x \in Kl(z) \land y \in Kl(\sim z) \to (Kl(\lor) = x \lor Kl(\lor) \in pt(x))$   |                                     |
|             | $\vee (Kl(\vee) = y \vee (Kl(\vee)\varepsilon pt(y))$   | (8, ing)                            |
| 10.         | $Kl(\forall) \varepsilon Kl(\forall) \rightarrow \neg \exists y (Kl(\forall) \varepsilon pt(y))$  | $(S1^{\rightarrow}, x/Kl(\lor), W)$ |
| 11.         | $x \in \bigvee \to \neg Kl(\bigvee) \in pt(x) \land \neg Kl(\bigvee) \in pt(y)$   | (10,6)                              |
| 12.         | $x \in Kl(z) \land y \in Kl(\sim z) \to Kl(\lor) = x \lor Kl(\lor) = y$   | (9,11)                              |
| 13.         | $x \in Kl(z) \land y \in Kl(\sim z) \to x \in W \lor y \in W$   | (12)                                |
| 14.         | $\forall x, y(x \in Kl(z) \land y \in Kl(\sim z) \to x \in W \lor y \in W)$   | (13)                                |
| <i>S</i> 8. | $\forall x (x \varepsilon W \leftrightarrow x \varepsilon \lor \land \lor \subset ing(x))$  |                                     |
| 1.          | $x \varepsilon W \to x \varepsilon x \land \forall z (z \varepsilon \lor \to z \varepsilon ing(x))$                                       | (Kl, W)                             |
| 2.          | $x \varepsilon W \to x \varepsilon \lor \land \lor \subset ing(x)$  | $(1, \lor, \subset)$                |
| 3.          | $\forall x (x \varepsilon W \leftrightarrow x \varepsilon \lor \land \lor \subset ing(x))$  | (2, S6)                             |
| <u>co</u>   | $\forall u(u_0   M ) = u_0 out_u(u_0)$  |                                     |
| 37.         | $\forall x(xevv \rightarrow \neg yeextr(x))$  |                                     |
|             |   |                                     |

- 1.  $x \in Kl(\vee) \land y \in extr(x) \rightarrow [\forall z(z \in \vee \rightarrow z \in ing(x)) \land y \in y \land$  $\forall z(z\varepsilon ing(x) \rightarrow z\varepsilon \sim ing(y))]$ (Kl, extr)2.  $x \in W \land y \in extr(x) \rightarrow$
- $(y \in \bigvee \to y \in ing(x)) \land y \in \bigvee \land (y \in ing(x) \to y \in \sim ing(y)) \quad (1, z/y, \bigvee)$ 3.  $x \in W \land y \in extr(x) \rightarrow$  $y \varepsilon ing(x) \land (y \varepsilon ing(x) \rightarrow y \varepsilon \sim ing(y))$ (2)(3)
- 4.  $x \in W \land y \in extr(x) \rightarrow y \in cong(y)$
- 5.  $x \in W \land y \in extr(x) \rightarrow y \in y \land \neg(y \in ing(y))$ (4,~) (5, T3)
- 6.  $x \in W \land y \in extr(x) \rightarrow y \in y \land \neg y \in y$ 7.  $\forall x(x \in W \rightarrow \neg y \in extr(x))$ (6)

*S*10.  $\forall x(x \in W \leftrightarrow x \in \bigvee \land \forall y(\neg y \in extr(x)))$ 

1.  $x \in \bigvee \land \forall y (\neg y \in extr(x))) \rightarrow \neg x \in pt(y)$ (T13)2.  $x \in \bigvee \land \forall y (\neg y \in extr(x))) \rightarrow \forall y (x \in x \land \neg x \in pt(y))$ (1)3.  $x \in \bigvee \land \forall y (\neg y \in extr(x))) \rightarrow \forall y (x \in \sim pt(y))$  $(2, \sim)$ 4.  $x \in \bigvee \land \forall y (\neg y \in extr(x))) \to x \in W$ (3*, S*5) 5.  $x \in W \to x \in x \land \forall y (\neg y \in extr(x))$ (S10)6.  $\forall x(x \in W \leftrightarrow x \in x \land \forall y(\neg y \in extr(x)))$ (4, 5)

(Kl)

*S*11.  $\forall x(x \in W \leftrightarrow x \in \bigvee \land \forall z, y(\neg y \in (z + x)))$ 

| 1.  | $x \in W \land y \in (z + x) \rightarrow z \in extr(x) \land z \in ing(x)$   | (+, S8)     |
|-----|--|-------------|
| 2.  | $x \varepsilon W 	o \neg y \varepsilon (z + x)$  | (1)         |
| 3.  | $\forall x (x \varepsilon W \to x \varepsilon \lor \land \forall y \forall z (\neg y \varepsilon (z + x)))$  | (2)         |
| 4.  | $x \varepsilon x \wedge u \varepsilon W \to x \varepsilon ing(u)$  | (W)         |
| 5.  | $(x \varepsilon x \wedge u \varepsilon W \to \neg (x = u)) \to x \varepsilon p t(u)$   | $(4, \sim)$ |
| 6.  | $(x \varepsilon x \wedge u \varepsilon W \to \neg (x = u)) \to \exists z (z \varepsilon extr(x))$  | (5, T13)    |
| 7.  | $(x\varepsilon x \land \land u\varepsilon W \to \neg (x = u)) \to \exists z \exists y (z\varepsilon extr(x) \land y\varepsilon kl(z \cup x) \land y\varepsilon y)$ | (6)         |
| 8.  | $(x\varepsilon x \wedge u\varepsilon W \to \neg (x=u)) \to \exists z \exists y (y\varepsilon(z+x))$  | (7)         |
| 9.  | $x \varepsilon x \land \forall z, y \neg (y \varepsilon z + x) \land u \varepsilon W \to x = u$  | (8)         |
| 10. | $x \varepsilon x \wedge \forall z, y \neg (y \varepsilon z + x) \wedge u \varepsilon W \rightarrow x \varepsilon W$  | (9)         |
| 11. | $x \varepsilon x \land \forall z, y \neg (y \varepsilon z + x) \land \exists u (u \varepsilon W) \to x \varepsilon W$  | (10)        |
| 12. | $\forall x (x \varepsilon x \land \forall z, y \neg (y \varepsilon z + x) \to x \varepsilon W)$  | (11)        |
| 13. | $\forall x (x \varepsilon W \leftrightarrow x \varepsilon \lor \land \forall y \forall z (\neg y \varepsilon (z + x)))$  | (3,12)      |
|     |  |             |

As we have said, Leśniewski's ontology has interpretations in an empty set of individuals and this is not changed in the case of OML. This is why the counterparts of Leśniewski's theorems XLIII and XLIV are not theses of OML+W but only their weaker versions:

Fact 3. It is derivable in OML+W that

| xliii. | $\exists x(x \varepsilon \lor) \to \lor \Delta K l(\lor)$ | $(AM4, \lor)$ |
|--------|---|---------------|
| xliv.  | $\exists x(x \varepsilon \lor) \to Kl(\lor)\varepsilon W$ | (AM4, T8, W)  |

Let us sketch the following model. We take a set of individuals *D*. The power set of *D* is a domain of a valuation of individual variables,  $\varepsilon^*$  is a semantical counterpart of the inherence predicate - it is a certain set of order pairs, where the first element of every pair is a singleton made of an inividual and the second element is any of its supersets;  $pt^*$  is an operation which for every singleton assigns a set of all parts of the element of this singleton. We can sketch the following model for OML + W which falsifies formula  $\exists x(x\varepsilon W): \langle D, \varepsilon^*, pt^*, v \rangle$ , such that  $D = \varepsilon^* = pt^* = \emptyset$ . In such a model all axioms of OML are true and  $v(\bigvee) = v(\bigwedge) = v(\bigotimes) = v(\bigotimes) = 0$ .

However, we may easily obtain the counterpart of Theorem XLV on the uniqueness of the universe:

**Fact 4.** In OML+W the following formula is derivable

$$xlv. \quad \forall x, y(x \in W \land y \in W \to x = y) \tag{AM3, W}$$

Sobociński himself formulated theses about the universe also in [8,12]. He noted that everything which is exterior to W is a contradictory object (in the sense of  $\Lambda$ ) and that universe W by itself corresponds to a Boolean-algebraic unit element.

Let us add further theses expressed by Sobociński:

**Fact 5.** *In* OML+W *the following formulas are derivable* 

$$\begin{array}{ll} \text{S12.} & \forall x (x \varepsilon W \leftrightarrow x \varepsilon x \land \forall y (y \sub{extr}(x) \rightarrow y \sub{\wedge})) & (\text{cf. [8] (221)}) \\ \text{S13.} & \forall x (x \varepsilon x \rightarrow x \varepsilon ing(W)) & \\ \text{S14.} & \forall x (x \varepsilon x \rightarrow W = ((W - x) + x)) & ([12] (93, A18, A19)) \end{array}$$

The universe may possess more interesting properties in atomistic mereology which we obtain from OML by adding the following axiom:

AM5. 
$$\forall x(x \in x \to \exists z(x \in Kl(z) \land \forall y \forall u(y \in z \land u \in ing(y) \to u = y)))$$

and the definition of an atom:

(At)  $\forall x(x \in At \leftrightarrow x \in x \land \forall z(z \in ing(x) \to z = x))$ 

(x is an atom if every of its ingredient is identical with x)

(We can use also more intuitive definition:  $x \in At \leftrightarrow x \in x \land \neg \exists z (z \in pt(x))$  (*x* is an atom if *x* is an object which does not have parts).)

We note

**Fact 6.** OML+ $\{AM5, At\}$  theses are

S15at.  $\exists x(x \in \forall) \rightarrow \exists z(Kl(z) = Kl(\forall) \land \forall y(y \in z \rightarrow y \in At))$  (T11, T8, AM5, At) S16at.  $\exists x(x \in \forall) \rightarrow W = Kl(At)$  (cf. [12] (V2, 96))

In other words, if there is at least one object in the atomic universe, everything which is the universe (or, should we say, 'universal') is a compound of atoms. Moreover, the universe is identical with the mereological class of all atoms.

Regardless of how the universal set of individuals is structured — moving still within OML — we may identify yet another feature of the mereological class of all objects. As it can be demonstrated, in OML, the following formula is the thesis:

(\*\*\*) 
$$Kl(z)\varepsilon z \to \forall x(x\varepsilon z \to (x\varepsilon Kl(z) \leftrightarrow \forall y(y\varepsilon z \to (x\varepsilon \sim pt(y)))))$$

To prove (\*\*\*) we use *AO*, *AM*2, *Kl* and *ing*; the part of predecessor  $Kl(z)\varepsilon z$  is essential in a proof of  $\leftarrow$  (from  $Kl(z)\varepsilon z$  and  $t\varepsilon Kl(z)$  we get:  $t\varepsilon z$  and from  $\forall y(y\varepsilon z \rightarrow x\varepsilon \sim pt(y))$  we have  $x\varepsilon \sim pt(t)$ ).

Finally we note:

**Fact 7.** Formula S5 follows from (\*\*\*) in OML+(W).

The formula (\*\*\*) says that for every z which fulfills  $Kl(z)\varepsilon z$  we can consider a 'local' (restricted to z) universe which is a mereological class of all z with the same property as is expressed for W in S5.

#### 3. Universes in ZFM

Sobociński was convinced about the advantage of mereology over Zermelo's set theory in application to philosophical issues. He expressed this conviction in his letter to Bocheński giving a theological example:

If somebody takes a position of Zermelo's set theory, he can draw conclusions that are grossly in relation to theological opinions, eg. we assume that *God* exists, and so the object  $\{God\}$  exists [...] and this object  $\{God\} \neq God$ , ect.  $\{\{God\}\}, \{\{\{God\}\}\}, \ldots$  and they are all different, concretely existent objects! [...] In mereology it is not the case, because  $Kl\{God\} = God$ , and so it is only a different way of speaking and this is always permitted [...] Pay attention to the consequences of Zermelo's system: before the creation of anything, there have been existent already an infinite amount of other objects  $\neq God$  !!! [1].

The question which we want to put now is: how does this preference occur when the philosophical notion of the universe is considered? In other words: in which sense does the notion called by Sobociński "the Universe" (we would say: the world, totum) identified with *W* in OML have more significant philosophical content than its set theoretical counterpart? We will analyze this issue in the frame of a richer system than OML, which gives the possibility of speaking about both types of multitudes: mereological collections and distributive sets—in the ZFM theory proposed by A. Pietruszczak ([3] (pp. 172–181)).

ZFM is expressed in a first order-language with the following primitive symbols: *Z* (*set*),  $\in$  (for *being an element*), = (first-order identity) and  $\Box$ — symbol for a part relation. ZFM is built on first-order predicate logic with identity with proper axioms of Zermelo-Fraenkel and the following axioms for  $\Box$ :

 $\begin{array}{lll} AM1_{\square} & \forall x \forall y \forall z (x \sqsubseteq y \land y \sqsubset z \to x \sqsubset z) \\ AM2_{\square} & \forall x \forall y (x \sqsubset y \to \neg (y \sqsubset x)) \\ AM3_{\square} & \forall x \forall y \forall z (xSumz \land ySumz \to x = y) \\ AM4_{\square} & \forall z \forall y (y \in z \to \exists x (xSumz)) \\ AM5_{\square} & \forall_z (Zz \to \neg \exists x (x \sqsubset z)) \end{array}$ 

where:

```
\begin{array}{ll} (Sum) & xSumz \leftrightarrow Zz \land \forall y (y \in z \rightarrow y \sqsubseteq x) \land \forall y (y \sqsubseteq x \rightarrow \exists u \exists v (u \in z \land v \sqsubseteq u \land v \sqsubseteq y)) \\ (\sqsubseteq) & x \sqsubseteq y \leftrightarrow x \sqsubset y \lor x = y \end{array}
```

The idea of the interpretation of elementary mereology in the set theoretical frame is obviously realizable because of Tarski's well-known observation concerning close connection between the so-called mereological structures (which are models of elementary mereology) and complete Boolean algebras (an extensive description of this topic is given e.g., in [3] (especially Chapter 3 (pp. 91–107))).

Our aim will be now to interpret the formalism of Sobociński in ZFM and to reconsider his definition of the universe.

We take mereology expressed in a slightly different language than OML. We use a first-order language with two primitive predicates  $\varepsilon$  and  $\Box^*$ . The second one may be understood in OML as a part relation by:

 $(\sqsubset^{\star}) \quad x \sqsubset^{\star} y \leftrightarrow x \varepsilon p t(y).$ 

We call this version of mereology  $OML_{\perp^*}$  and characterize it by all theorems of first-order logic (QL), specific axiom of ontology *AO* and the following counterparts of *AM*1–*AM*4:

 $\begin{array}{lll} AM1_{\sqsubset^{\star}} & \forall x \forall y \forall z (x \sqsubset^{\star} y \land y \sqsubset^{\star} z \rightarrow x \sqsubset^{\star} z) \\ AM2_{\sqsubset^{\star}} & \forall x \forall y (x \sqsubset^{\star} y \rightarrow \neg (y \sqsubset^{\star} x)) \\ AM3_{\sqsubset^{\star}} & \forall x \forall y \forall z (xSum^{\star}z \land ySum^{\star}z \rightarrow x =^{\star} y) \\ AM4_{\sqsubset^{\star}} & \forall z \forall y (y\varepsilon z \rightarrow \exists x (xSum^{\star}z)) \end{array}$ 

where:

```
 \begin{array}{ll} (=^{\star}) & \forall x \forall y (x =^{\star} y \leftrightarrow x \varepsilon y \wedge y \varepsilon x) \\ (Sum^{\star}) & xSum^{\star}z \leftrightarrow x \varepsilon x \wedge \forall y (y \varepsilon z \rightarrow y \sqsubseteq^{\star} x) \wedge \forall y (y \sqsubseteq^{\star} x \rightarrow \exists u \exists v (u \varepsilon z \wedge v \sqsubseteq^{\star} u \wedge v \sqsubseteq^{\star} y)) \\ (\sqsubseteq^{\star}) & x \sqsubseteq^{\star} y \leftrightarrow x \sqsubset^{\star} y \vee x =^{\star} y \end{array}
```

We accept all  $\sqsubset^*$  counterparts of the OML definitions mentioned above. Primitive rules are as of OML.

Actually, we want to define in ZFM predicates  $\varepsilon$ ,  $\Box^*$  and the notion of the universe that depends on them.

We start with the extension of ZFM by the following equivalence introducing predicate M:

$$(M) \quad \forall z (Mz \leftrightarrow Zz \land \forall x \forall y (ySumx \land x \subseteq z \rightarrow y \in z) \land \forall x \forall y (y \in z \land x \sqsubset y \rightarrow x \in z))$$

Predicate *M* is applied to every object *z* which is also a set, every mereological sum of each its subset is an element of *z*, every part of every element of *z* is an element of *z*.

We note:

**Fact 8.** *In* ZFM+*M it is derivable that*  $\exists x M x$ .

From axioms of ZF we get:  $\exists ! u(Zu \land \forall y(\neg y \in u))$ . We name this set  $\emptyset$ .

 $\emptyset$  fulfills *M* because  $\neg \exists z(zSum\emptyset)$  and  $\neg \exists y(y \in \emptyset)$ .

Let us fix any element  $\mathbf{z}$  fulfilling M.

Depending on this choice we define predicate  $U^{\mathbf{z}}$ :

 $(U^{\mathbf{z}}) \quad \forall x (U^{\mathbf{z}} x \leftrightarrow M \mathbf{z} \land x \subseteq \mathbf{z})$ 

We also know that  $\exists y \ U^z y$ , because  $\forall x (Zx \rightarrow \exists y \ y \subseteq x)$ . We take two axioms more:

 $\begin{array}{ll} (\varepsilon) & \forall x \forall y (x \varepsilon y \leftrightarrow U^{\mathbf{z}} x \wedge U^{\mathbf{z}} y \wedge x \subseteq y \wedge \exists ! v (v \in x)) \\ (\sqsubset^*) & \forall x \forall y (x \sqsubset^* y \leftrightarrow U^{\mathbf{z}} x \wedge U^{\mathbf{z}} y \wedge \exists ! u \exists ! v (v \in x \wedge u \in y \wedge v \sqsubset u)). \end{array}$ 

We consider an interpretation function of the OML<sup> $\perp$ </sup> language in a fragment of the ZFM+{ $M, \mathbf{z}, \varepsilon, \sqsubset^*$ } language which we name  $I^{\mathbf{z}}$  (we follow [13] (pp. 61–65)). For every formula A of the OML<sup> $\perp$ </sup> language we define formula  $I^{\mathbf{z}}(A)$  belonging to the ZFM+{ $M, \mathbf{z}, \varepsilon, \sqsubset^*$ } language in the following way: (*i*) every subformula of A of the shape  $\forall xB$  or  $\exists xB$  we retype with a modification, respectively:  $\forall x(U^{\mathbf{z}}x \rightarrow B), \exists x(U^{\mathbf{z}}x \wedge B)$  and (*ii*) every subformula B of A with { $x_1, ..., x_n$ } = FV(B) we retype with prefix:  $U^{\mathbf{z}}x_1 \rightarrow (U^{\mathbf{z}}x_2 \rightarrow (\cdots \rightarrow (U^{\mathbf{z}}x_n \rightarrow B)...))$ .

Let us take the name ZFMMz for the considered extension of ZFM. Now we can observe that

**Fact 9.** For every axiom A of  $OML_{r}$ :  $ZFMMz \vdash I^{z}(A)$ .

To prove  $I^{z}(AO)$  we need only ( $\varepsilon$ ).  $I^{z}(AM1_{{}^{+}})$  and  $I^{z}(AM2_{{}^{-}})$  are derivable using ( ${}^{-}$ ),  $AM1_{{}^{-}}$  and  $AM2_{{}^{-}}$ . To prove  $I^{z}(AM3_{{}^{-}})$  and  $I^{z}(AM4_{{}^{-}})$  we use the following:  $\forall x \forall y \forall v (U^{z}x \land U^{z}y \land v \in x \land \forall u (u \in x \rightarrow u = v) \rightarrow (xSum^{*}y \leftrightarrow vSumy))$  (here we use essentially the second and third part of the conjunction occurring on the right side of equivalence M).

Because of the interpretation theorem [13] (pp. 62–63), for any chosen **z** fulfilling *M* we can speak about theory  $I^{z}(OML_{\perp^{*}})$  which consists of all  $I^{z}$  interpretations of theorems of  $OML_{\perp^{*}}$ . Of course:  $I^{z}(OML_{\perp^{*}}) \subsetneq ZFMMz$ .

Let us now come back to our notion of the universe considered by Sobociński. We introduce constant  $\bigvee^{z}$  dependent on z:

 $(\bigvee^{\mathbf{z}}) \quad \forall x (x \varepsilon \bigvee^{\mathbf{z}} \leftrightarrow U^{\mathbf{z}} x \wedge x \varepsilon x)$ 

and constant W<sup>z</sup> representing the universe dependent on z:

 $(W^{\mathbf{z}}) \quad \forall x (x \in W^{\mathbf{z}} \leftrightarrow U^{\mathbf{z}} x \wedge x Sum^{\star} \vee \mathbf{z})$ 

We take the symbol  $\sqcup$ :  $\sqcup(x) = y \leftrightarrow ySumx$ . The abstract operator is a metatheoretical symbol used just as in [3] (p. 175).

Now we can speak about different 'Universes', depending on the chosen **z**. Remember that in OML we have already considered 'local' universes which fulfilled condition  $Kl(z)\varepsilon z$  (cf. (\*\*\*), Fact 7)). Now every universe **z** is 'global' and we could speak about 'local' universes which are certain subsets of **z**.

Let us give selected examples of chosen z.

**Example 1.** At first we consider an extension of ZFM : the Unitary Theory of Individuals and Sets (UTIS) described in [3] (pp. 172–181).

In UTIS ur-elements called *individuals* are considered in the following sense:

(Ind) Ind $x \leftrightarrow \forall y (y \sqsubseteq x \rightarrow \neg Zy)$ 

To get UTIS from ZFM we add two axioms concerning the existence of individuals which form a set:

(*i*1)  $\exists x Indx$ 

(*i*2)  $\exists y \forall x (x \in y \leftrightarrow Indx)$ 

From the extensionality axiom we know that  $\exists ! y \forall x (x \in y \leftrightarrow Indx)$  and we name the set of all individuals **i**. The set **i** fulfills *M*. We can prove in UTIS both  $\forall x \forall y (ySumx \land x \subseteq \mathbf{i} \rightarrow y \in \mathbf{i})$  ([3] (FT', 181)) and  $\forall x \forall y (y \in \mathbf{i} \land x \sqsubset y \rightarrow x \in \mathbf{i})$  (directly from *Ind*).

Now we choose  $\mathbf{z} = \mathbf{i}$  and from  $(\bigvee^i)$  and  $(W^i)$  we obtain  $\bigvee^i = \mathbf{i}$  and  $x \in W^i \leftrightarrow x Sum^* \bigvee^i$ . Just because  $\mathbf{i} \neq \emptyset$  and  $AM4_{\Box^*}$ , we also know that  $\exists x(x \in W^i)$ . By the way, although  $W^i$  is composed of individuals, they do not need to be atoms.

**Example 2.** Let us stay in ZFMMz and take  $z = \emptyset$ . We know that  $M\emptyset$ . Now  $\bigvee^{\emptyset} = \emptyset$  and  $\neg \exists x(x \in W^{\emptyset})$ 

**Example 3.** In ZFM we can prove the existence of the set  $\{\emptyset, \{\emptyset\}, \bigsqcup(\emptyset, \{\emptyset\})\}$  which fulfills M. Now we choose  $z = \{\emptyset, \{\emptyset\}, \bigsqcup(\emptyset, \{\emptyset\})\}$ .

In this case:  $x \in W^{\mathbf{z}} \leftrightarrow \exists ! y (y \in x \land y = \bigsqcup (\emptyset, \{\emptyset\})).$ 

As we wrote at the beginning of our article, Sobociński claimed that the mereological tools are more suitable for philosophical investigations than set theoretical ones. Actually, the connotations linked with the term "the Universe" and expressed in quoted theses are not dependent on some specific properties of individuals or their mereological whole. As it can be seen their proofs may be presented using only two steps. In the first step we use the fact that the mereological class of all objects is an object (if there is at least one object). Then we take (\* \* \*) and get implications with consequents of the same structure as the appropriate theorems of Sobociński, with a restricted quantification to *z* and with antecedent  $Kl(z)\varepsilon z$ . Now we obtain  $S1^{\rightarrow} - S11$  via *dictum de omni* and taking (*W*).

ZFM gave us the possibility of looking at Sobociński's approach from a wider perspective, but also showed that the questioned philosophical expectations linked with (W) would be too high. Although we could find the intended interpretation of the notion of W described in Example 1, we found also some undesirable cases: in Example 2 the universe is empty and in Example 3 the universe consists of distributive sets. After all, Example 1 also is far insufficient to realize the idea of 'the Universe created' by *God* expressed in Sobociński's quoted reflection. In this case, *God* would need to be singled out from the set of all individuals and to stay in some causal relation to other individuals. The given characteristics of W of course does not depend on any such a construction and can be treated at most as the starting point of next philosophical investigation.

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