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Structural Design with Self-Weight and Inertial Loading Using Simulated Annealing for Non-Gradient Topology Optimization

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Abstract: This paper explores implementation of self-weight and inertial loading in topology optimization (TO) employing the Simulated Annealing (SA) algorithm as a non-gradient-based technique. This method can be applied to find optimum design of structures with no need for gradient information. To enhance the convergence of the SA algorithm, a novel approach incorporating the crystallization factor is introduced. The method is applied in a benchmark problem of a cantilever beam. The study systematically examines multiple scenarios, including cases with and without self-weight effects, as well as varying point loads. Compliance values are calculated and compared to those reported in existing literature to validate the accuracy of the optimization results. The findings highlight the versatility and effectiveness of the SA-based TO methodology in addressing complex design challenges with considerable self-weight or inertial effect. This work can contribute to structural design of systems where only the objective value is available with no gradient information to use sensitivity-based algorithms.

Keywords: topology optimization (TO); simulated annealing (SA); self-weight; inertial load; structural design

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1. Introduction

Topology optimization (TO) stands as a pivotal mathematical method within structural design, strategically utilized to attain optimal performance relative to predefined objectives and constraints. This method intricately endeavors to ascertain the most efficient allocation of material resources within a specified design domain. The evolution of manufacturing technologies, including additive manufacturing and CNC machining, has notably heightened the acclaim of this design methodology, attributed to its inherent capacity to seamlessly align with the manufacturability requirements of the final design [1,2]. Diverse optimization methodologies find application in TO, their selection contingent upon the unique characteristics of the problem and its accompanying constraints. Broadly speaking, TO can be classified into two overarching approaches: gradient-based and non-gradient-based [3].

Gradient-based TO uses derivatives of the objective function and problem constraints, utilizing an iterative process to generate successive designs until a notable enhancement in the optimal solution is no longer attainable. Notable examples of gradient-based TO methods encompass level set, moving morphable components, and evolutionary structural optimization (ESO) [4].

Conversely, non-gradient-based TO performs optimization techniques independent of gradient information, making them applicable to scenarios where the objective function lacks differentiability. Predominantly employing stochastic optimization algorithms, such as genetic algorithms, particle swarm, and simulated annealing, these methods converge

towards an optimal design [5,6]. Although they may not guarantee an exact optimum solution, non-gradient-based TO methods exhibit a high level of accuracy. For instance, in optimization problems characterized by uncertainties in material properties, these methods efficiently converge to an optimal solution [7,8]. Furthermore, in discrete structural optimization problems, such as truss assembly design, probabilistic methods have demonstrated the capability to attain the optimum solution [9]. Non-gradient based methods have demonstrated success in a wide range of optimization problems [10]. Nevertheless, they are still evolving and require further development to address diverse design challenges. In the most of design using TO for additive manufacturing, this limit can change the final design considerably and imposes hard limits to the optimization process [4,11]. Deviation of the manufactured parts from the initial design is another challenge in TO and metrology problems that can be addressed with smart search and advanced evaluation techniques [12]. Researchers are actively working on enhancing these algorithms for various applications, with improvements achieved through parameter tuning and the use of intelligent search algorithms [13,14].

However, prior investigations in TO mostly focused on loads originating from external sources. In such instances, the load remains consistent and is either applied at a fixed point or varies along a boundary. Contrastingly, when contending with self-weight loads or forces stemming from inertial sources, the design problem becomes inherently more intricate. In these scenarios, the load and stiffness of the structure undergo dynamic changes as material is either added or removed from the design. Essentially, augmenting mass in a specific region not only increases stiffness but also introduces inertial load. This dynamic interplay holds noteworthy significance, especially in the structural design of elements such as bridges and buildings, where the weight of the structure itself constitutes a substantial portion of the applied load [15,16]. Another pertinent application of this loading scenario arises in the design of inertial sensors, where the challenge involves simultaneously increasing inertial force while decreasing stiffness. This conflicting set of objectives gives rise to a complex TO problem, for which the utilization of non-gradient-based algorithms becomes imperative to converge towards viable solutions [17].

In gradient-based TO, the literature presents various approaches to tackle the impact of self-weight and inertial loading. Initial efforts involved applying TO methods to design tensegrity structures, taking into account the weight of struts. These early endeavors employed mixed-integer linear programming to ascertain the configuration that minimized strain energy in the equilibrium state. However, it is important to highlight that this gradient-based approach had limitations in its applicability, primarily stemming from constraints associated with the structure's member count. Additionally, it did not address the stability of the final design [18]. In another research, the TO of solid structures, encompassing arches and beams, has been specifically addressed with a focus on accommodating inertial loads. This was accomplished through the application of a non-linear semi-definite programming method, extending the solution beyond the conventional eigenvalue problem. To address challenges arising from multiple eigenvalues due to variable loading from inertial forces, gradient vectors undergo a smoothing process via a linear local averaging design variable filter. This filtering approach is instrumental in bolstering convergence towards a local optimum while concurrently curtailing computational costs. However, it is crucial to acknowledge that this method still grapples with limitations in identifying the global optimum in non-convex problems and remains primarily applicable to scenarios providing gradient information [19].

An alternative approach in the literature involves the utilization of the guide-weight algorithm to consider the impact of self-weight when minimizing compliance in structural design. Given the non-monotonous nature of the gradient in compliance optimization, guided search proves efficient convergence to the optimum solution. This is accomplished through the application of the Optimality Criteria (OC) method, coupled with the maintenance of a comprehensive record of the convergence history. It is noteworthy that this algorithm is particularly well-suited for solving convex problems, such as compliance min-

imization, owing to the specific characteristics of the optimization process it employs [20]. Similar methodologies have found application in the TO of diverse structures, incorporating considerations for the impact of self-weight. These approaches often involve the imposition of specific constraints on geometry or stress [21,22].

Another gradient-based TO method, specifically tailored for problems incorporating self-weight and inertial forces, involves the adaptation of the Bi-directional Evolutionary Structural Optimization (BESO) method for continuum structures [23]. In this particular approach, the compliance selected as the objective function to be minimized. Notably, the sensitivity of compliance can display non-monotonous behavior, as it may be either positive or negative. To tackle this issue, especially within the context of convex problems such as compliance minimization, the literature employs a power law material interpolation scheme along with an adaptive penalty factor [24]. The extension of the objective function to similar criteria, such as von Mises stress, allows for having viable solution while accounting for the self-weight effect [25]. While these techniques expedite the convergence to optimal solutions, it is important to note that they are still constrained to convex problems and rely on gradient information derived from the objective function.

Non-gradient based TO methods play a crucial role in addressing a broad range of problems, particularly in situations where gradient information is either unavailable or the problem lacks convexity. Consider, for instance, an experimental case where the objective is directly measured, rendering gradient information inaccessible for parameter design or TO. In such scenarios, it becomes feasible to model the system using neural networks and subsequently optimize it using non-gradient techniques such as simulated annealing. This approach facilitates effective optimization in situations where traditional gradient-based methods are not applicable [26].

Certainly, there are cases where the objective function is both available and differentiable, but the utility of gradient information is limited. Consider the design of an inertial sensor with the goal of enhancing sensitivity to external accelerations. In such scenarios, where the addition of mass to a specific region within the design domain increases the load while simultaneously decreasing flexibility, the gradient of the objective function becomes less informative for guiding the optimization process effectively. In these complex and non-linear cases, non-gradient-based algorithms have demonstrated success in navigating the design space to reach an optimal solution. They excel at accommodating intricate trade-offs and non-linearities more effectively than gradient-based methods [27].

While non-gradient-based TO algorithms, such as genetic algorithms and simulated annealing, are acknowledged for their efficiency and, in certain cases, their necessity in solving TO design problems characterized by complex objective functions and constraints, it is important to note that they are also linked with high computational costs [28–30]. The elevated computational cost stems from the necessity to explore a larger number of potential solutions within the design domain, presenting a notable drawback of these methods.

Recent advancements in computational cost reduction, including the utilization of smart search algorithms and emerging technologies such as quantum computing, have begun to address this drawback [31]. Non-gradient-based TO algorithms are gaining popularity for addressing large-scale and commercial design problems involving TO. With a growing demand for the application of non-gradient-based TO methods in real-world design challenges, especially in structural design, it becomes imperative to incorporate considerations related to self-weight problems within these algorithms [32].

Moreover, the groundbreaking implementation of simulated annealing on quantum computers, known as quantum annealers, holds the potential to significantly accelerate the optimization process. This advancement makes it feasible for a broader spectrum of design problems [33]. This includes scenarios where self-weight or inertial forces play a significant role in the applied loading, such as design of structural elements of buildings and bridges.

The simulated annealing algorithm is already used in the TO of structures with different objective functions and constraints [34,35]. To the best knowledge of the authors, the impact of self-weight and inertial forces as variable loads has not been considered in

TO with simulated annealing. The primary contribution of this paper is to address TO with simulated annealing while accounting for the effects of inertial or self-weight loads in the design. This algorithm holds the potential to contribute to a wide range of problems where the weight of the structure constitutes a significant source of loading.

The subsequent chapter of this paper will define the problem and present the formulation used in the TO problem. Following that, simulations will be conducted using a benchmark problem involving a cantilever beam, and the results will be compared with a similar case from gradient-based TO in the existing literature. In the final chapter, the research will conclude with an analysis of the obtained results and provide recommendations for future research directions.

2. Materials and Methods

2.1. Topology Optimization Algorithm

A topology optimization (TO) problem typically entails optimizing the distribution of material within a designated design domain to either maximize or minimize an objective function while adhering to specific constraints. In the context of this paper, the defined TO problem focuses on maximizing the stiffness of a cantilever beam while being subject to a volume fraction constraint. The design domain for this benchmark problem is shown in Figure 1, with a point load applied at the top end of the beam.

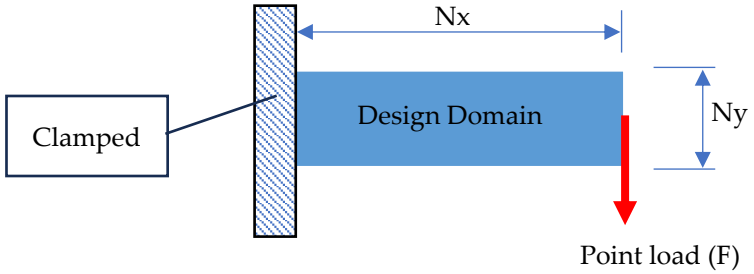

Figure 1. Design domain for the cantilever beam with the point load.

In TO, there are different approaches to representing the design domain. One approach involves treating the domain as a continuous area with changing borders. Another approach discretizes the domain into smaller elements that can be manipulated by a discrete optimization algorithm. In the case of TO utilizing simulated annealing as a non-gradient based method, the discretization method is often more compatible with the optimization algorithm. For this problem, the design domain is discretized into smaller elements, with Nx elements in the horizontal direction and Ny elements in the vertical direction, as illustrated in Figure 1. To simplify the problem and make it comparable with existing literature, the thickness of the beam is assumed to be uniform across the entire domain, effectively reducing it to a 2D problem. Each element in the discretized domain is a square with unit length.

The primary objective of the optimization problem is to maximize the stiffness of this structure. The stiffness maximization is directly proportional to minimize strain energy of the system, as shown in Equation (1):

$$\begin{gathered} Min \quad S = \frac{1}{2} U(x)^T K U(x) \\ \text{Subjected to } F = KU(x) \end{gathered} \tag{1}$$

The formulation of the TO problem involves the strain energy (S), displacement matrix ($U(x)$), stiffness matrix (K), and applied forces (F) within the design domain. In the case of a discretized system comprising Nx by Ny elements, the total strain energy can be expressed as the sum of strain energies for each individual element. To ensure a smooth and continuous objective function, the density of each element is considered as a continuous value ranging from zero to one. Consequently, the material properties of each element are

specified in proportion to its density, with the introduction of a penalization factor. This approach, commonly referred to as Solid Isotropic Material with Penalization (SIMP), has been successfully utilized in TO with simulated annealing [13]. Thus, the problem can be simplified as shown in Equation (2), where p represents the penalization factor, and it is selected as 3 in accordance with recommendations from the literature:

$$Min \quad S = \sum_{i=1}^{N} \frac{1}{2} x_i^p u(x_i)^T k_i u(x_i) \tag{2}$$

where the index "i" represents the i-th element within the design domain, with i ranging from one to the total number of elements ($N = Nx \times Ny$). Each element corresponds to a square region, and nodes are located at the corners of these square elements.

To account for self-weight or inertial loads, these loads can be applied at each node based on the average density of the four surrounding elements. These nodal forces are directly incorporated into the finite element code as the force matrix to compute the displacement of each element. Subsequently, these displacements are used in Equation (2) to determine the total strain energy of the entire structure.

2.2. Simulated Annealing in TO

Simulated Annealing (SA) is a stochastic optimization algorithm that converges to an optimum solution by accepting or rejecting new solutions using the Metropolis–Hastings algorithm. In the SA optimization algorithm applied to TO, a critical aspect is generating new solutions within the design domain. A common way to generate these new solutions is by selecting a random element and altering its density, a process that introduces stochasticity into the search [36]. However, employing a purely random search can lead to relatively high computational costs and may not efficiently converge to the optimal solution. To enhance both computational efficiency and the convergence to the optimum solution, an auxiliary variable known as the crystallization factor can be used for each element. This crystallization factor can be interpreted as the significance or importance of each element within the objective function and is utilized to guide the generation of new solutions. The TO process begins with an initial solution, which consists of randomly distributed densities for each element (denoted as x_j) and crystallization factors (denoted as C_k) for each element. A new solution is then generated using the previous solution and the crystallization factor, as outlined in Equation (3):

$$x_{j,new} = x_j + \frac{1}{C_k} \sum_{1}^{C_k} rand\left(-\frac{1}{2}, \frac{1}{2}\right) \Delta r \tag{3}$$

where generation of a new solution involves selecting a specific element denoted as 'j' within the design domain, k represents an index signifying the elements surrounding the target element with a distance less than the effective radius, and the variable Δr denotes the difference between the effective radius and the distance to the target element. It is important to note that the minimum range of the crystallization factor is set to 1, and the maximum range should be high enough to prevent changes in elements that are not particularly sensitive to optimization. Based on empirical experience and parameter tuning in SA, a value of 20 is selected as the maximum value for the crystallization factor. After generating a new solution using Equation (3), the SA algorithm evaluates the objective function to determine whether to accept or reject it.

If the newly generated solution, obtained by altering the density of an element using Equation (3), results in an improved objective function, the new solution is accepted, and the density is replaced for that element. In such cases where changing the target element leads to an enhancement in the objective function, the crystallization factor for the target element and its neighbors is decreased by one unit. If the crystallization factor is already one for an element, it cannot be decreased with improvement of the objective function.

The SA algorithm incorporates the possibility of accepting a new solution even when it is worse than the current solution. This feature is essential for escaping from local optima.

However, to ensure convergence in the optimization process, this possibility gradually decreases over the optimization process. Equation (4) illustrates how this possibility is calculated using the Boltzmann probability function. If it is less than a random number ranging from zero to one, the new solution is accepted. Conversely, if the calculated probability is higher, the new solution is rejected, and the crystallization factor for the target element and its neighbors is increased by one unit until reaching the maximum value.

This mechanism of accepting less favorable solutions early in the optimization process, with a gradual reduction in acceptance probability, helps the SA algorithm explore the solution space comprehensively and ultimately converge to an optimal solution.

$$P(T) = e^{-\frac{\Delta E}{T}} \tag{4}$$

where ΔE represents the change in the objective function from the previous to the new solution and the parameter 'T' denotes the temperature, which is a decreasing parameter in the SA algorithm. The TO process begins with a relatively high temperature, and at each temperature level, several new solutions are evaluated. Subsequently, the temperature decreases by a cooling factor, typically a value between 0.8 and 0.99 in most cases. As the temperature decreases in each step, the probability of accepting less favorable solutions decreases as well. This gradual reduction in temperature helps the SA algorithm converge towards the optimal point.

The selection of the initial and final temperature and the cooling factor is typically determined through an analysis of convergence behavior and prior experience. The incorporation of the crystallization factor further aids in enhancing convergence and can be utilized as a tool for analyzing the optimization process.

Figure 2 provides a visual representation of TO process using the SA algorithm as applied in this research, highlighting the progressive steps involving temperature reduction and crystallization factor adjustments, all contributing to the convergence towards an optimal solution.

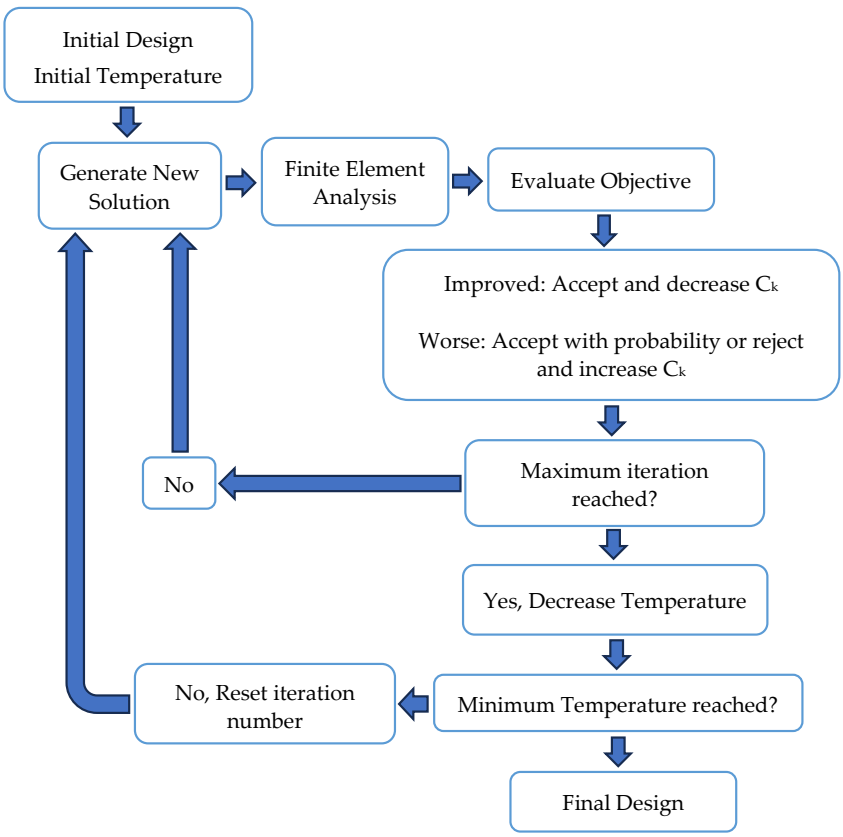

Figure 2. The process of topology optimization with simulated annealing.

3. Results

Implementation of the proposed methos is shown in this section for the cantilever beam to maximize its stiffness with a volume fraction constraint while a point load is applied, as shown in Figure 1, and self-weighted load. Design parameter for the cantilever beam are selected equal or scaled to values of the benchmark problem in the literature for a comparison [23]. Table 1 shows design parameters for the cantilever beam and the parameters of the SA algorithm. The volume fraction for this case study is 0.5 of the design domains and the point load applied with a magnitude as a fraction of the weight.

Table 1. Design parameters for the cantilever beam problem and simulated annealing.

Parameter	Symbol	Value
Number of elements in horizontal direction	Nx	160
Number of elements in vertical direction	Ny	100
Thickness	t	1
Density range	ρ	[0, 1]
Poisson's ratio	υ	0.3
Young's modulus	E	1
Penalization factor	p	3
Initial temperature	T_{max}	10000
Minimum temperature	T_{min}	0.00001
Cooling factor	α	0.9
Maximum iteration	n	1000
Crystallization factor range	Ci	(1, 20)

The selection of parameters, as presented in Table 1, has been guided by values found in the existing literature to ensure consistency and verification in the research. Parameters related to the SA algorithm for TO, such as temperature and the number of iterations, have been chosen based on a thorough analysis of the objective function range and convergence behavior.

The research explores several scenarios for the optimization of the cantilever beam, each characterized by different loading conditions. These cases are as follows:

(a) Case with no self-weight effect and only a point load equal to the weight of the structure;
(b) Case with self-weight effect and a point load equal to 25% percent of the weight;
(c) Another scenario with self-weight effect and a point load equal to a 50% of the weight;
(d) Yet another scenario with self-weight effect and a point load equal to the weight;
(e) Lastly, a case with self-weight effect and a point load equal to 200% of the weight.

The results of the TO for the specified scenarios are shown in Figure 3. Due to the stochastic nature of the SA method, there may be regions with gray areas in the results. These areas may require further post-processing and refinement to be suitable for manufacturing purposes [37]. Post-processing techniques can be employed to refine the obtained designs, ensuring that they meet specific manufacturing constraints and requirements. This step may involve smoothing the obtained topologies or adjusting densities. Another approach to get more clear results with higher quality for manufacturing is using binary elements. This method was verified for TO problems with binary elements and its efficiency for structural design problems is shown. By examining these various cases, the research aims to gain insights into the impact of self-weight on the optimized topology of the cantilever beam. The results from the TO with SA are presented in Figure 3 for different scenarios of loading condition and self-weight. The results have gray areas with intermediate density, which is common in TO with SA. Those elements can be post-processed for the manufacturing process, which is not the main focus of this research.

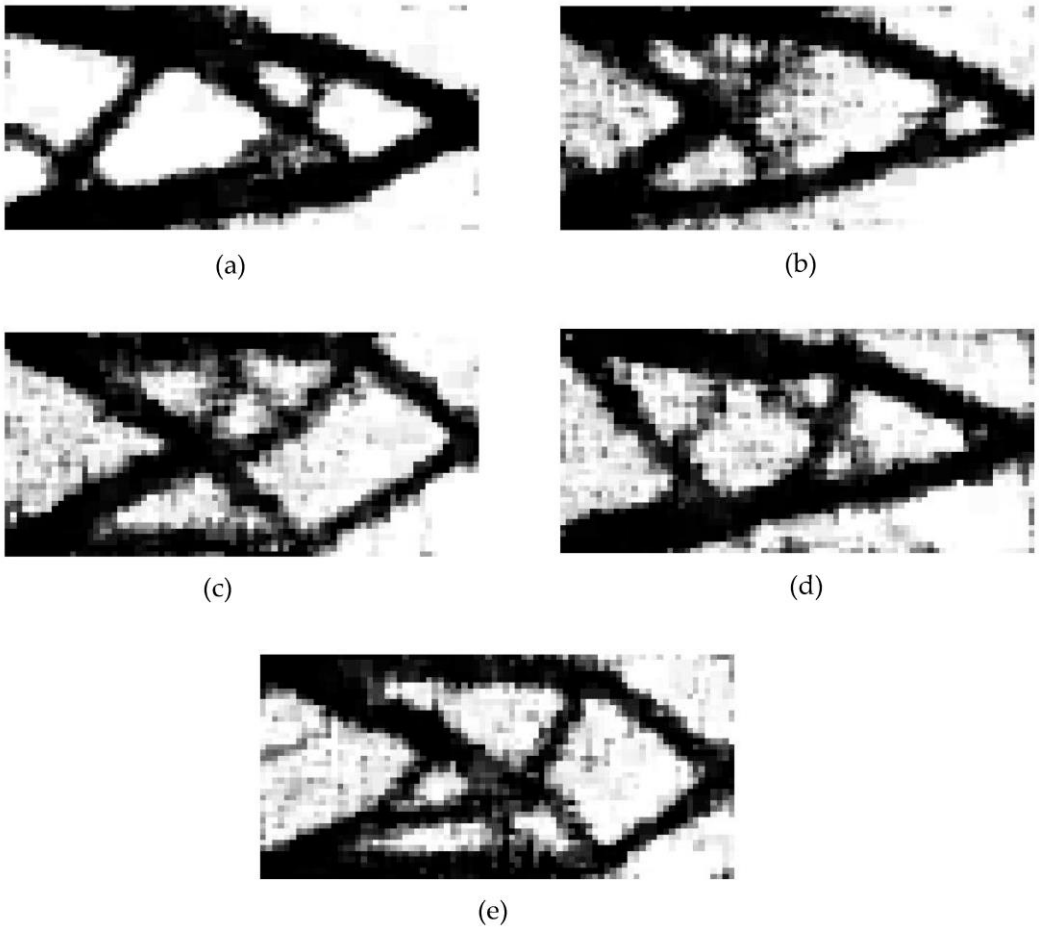

Figure 3. Results from topology optimization with simulated annealing for a 2D cantilever beam before post-processing. (**a**) Only point load applied at the top with no self-weight effect. (**b**) The self-weight load applied with a point load 25% of weight. (**c**) Self-weight applied with a point load 50% of weight. (**d**) Self-weight applied with a point load equal to weight. (**e**) Self-weight applied with a point load 200% of the weight.

Compliance values have been calculated based on the input parameters specified in Table 1 and Equation (2) for the different loading conditions. These compliance values are then presented in Table 2 alongside results reported in the existing literature.

Table 2. Comparison of compliance from the proposed method and literature [23].

Loading Condition	Compliance from Reference [23]	Calculated Compliance from the Proposed Method
Without self-weight and only point load equal to weight	4.9062×10^5	5.5563×10^5
Self-weight with point load 25% of weight	5.0291×10^4	7.6035×10^4
Self-weight with point load 50% of weight	1.8725×10^5	2.2347×10^5
Self-weight with point load 100% of weight	6.7959×10^5	7.1928×10^5
Self-weight with point load 200% of weight	2.4241×10^6	2.5515×10^6

As shown in Table 2, the results from the literature and the proposed method present similar compliance for different loading scenarios. Since the results from TO with simulated annealing are based on the stochastic search, they present results with intermediate density elements in the void area that increases the compliance. These points can be removed using

a proper post-processing method. To verify the convergence of the objective function to the optimum solution, Figure 4 shows the change of the scaled best objective function versus the logarithm of temperature.

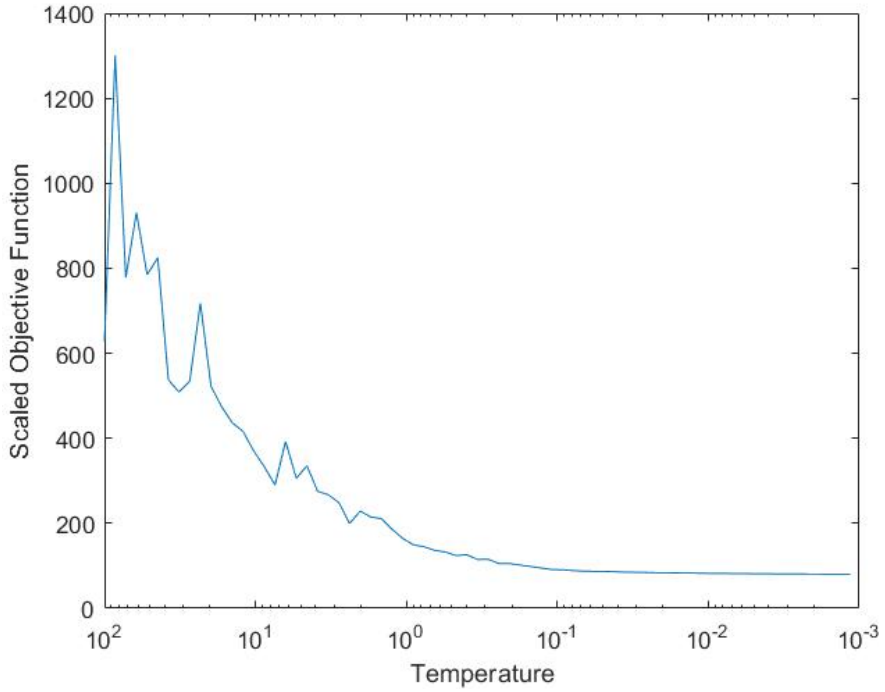

Figure 4. Objective function scaled to the percentage of final compliance versus the logarithm of temperature.

As shown in Figure 4, the objective function is converging to the optimum point near the minimum temperature. Similarly, the number of accepted and rejected solutions are shown for each temperature in Figure 5.

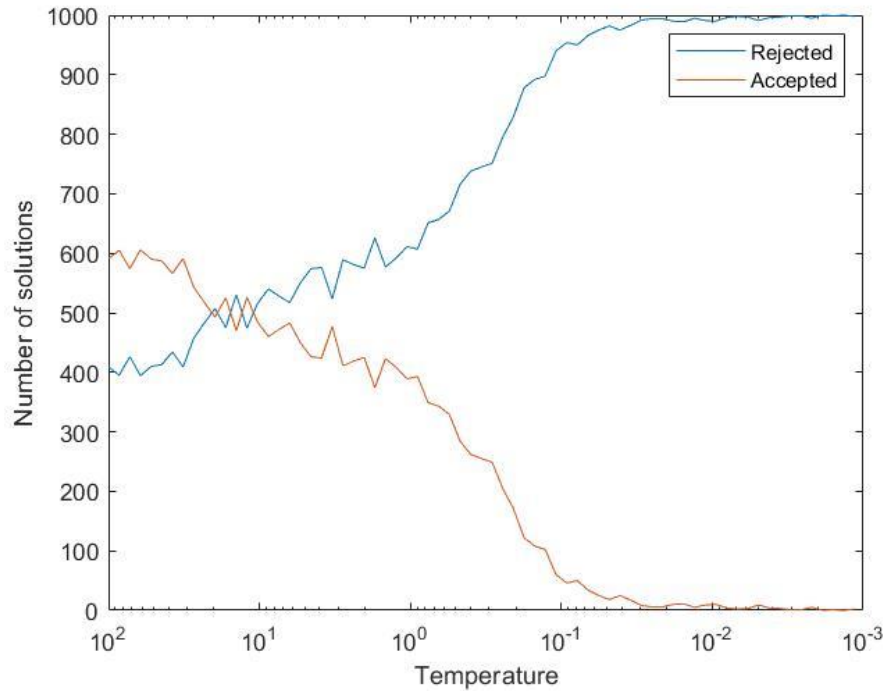

Figure 5. Number of accepted and rejected solutions versus the logarithm of temperature.

As shown in Figure 5, all the new solutions are accepted at the beginning of the optimization and reduced during the optimization. Near the minimum temperature, major portions of the solutions rejected. Number of rejected and accepted solutions analyzed to find the convergence and tune parameters of topology optimization with simulated annealing.

4. Discussion

According to the graphical results shown in Figure 3 and numerical values of the compliances in Table 2, the self-weight effect can considerably change the topology of a structure. As shown in Figure 3, increasing the effectiveness of the self-weight in the design moves the material more to the center where the weight load can be cancelled out easier. In case (a), there is no self-weight effect, and the only point load is applied to the middle of right surface of the beam. This results in a structure with mass distributed on the top and bottom to reduce the deflection of beam for this beam. In case (b), the highest effect of the weight is applied, with a point load with only 25% of the self-weight. In this case, sloid elements are moved to the center and left side of the beam to reduce the weight effect on deflection. From case (c) to (e), increasing the point load shows the distribution of solid elements to be changed accordingly toward more symmetry and having more solid elements at the top and bottom. It should be noted that there are some gray areas remaining in the design domain that are inevitable while using stochastic methods such as SA. To avoid grays areas, one strategy is to increase the number of iterations and decrease the minimum temperature, but that will add substantially to the computational costs with no considerable change in the optimum design. Another approach is to use post-processing techniques to obtain a clearer design for manufacturing. According to the simulation results of compliance in Table 2, the proposed method can give similar and improved compliance in the minimization problems. The results clearly demonstrate the versatility and effectiveness of the SA algorithm in tackling complex design problems with the self-weight effect. In addition, where self-weight is a considerable part of loading, the proposed method shows improved compliance in comparison to the results with the gradient-based method in the literature. This is due to the capability of finding a global optimum in SA regardless of the sensitivity and initial design.

5. Conclusions

In conclusion, this study has delved into the application of topology optimization (TO) using the Simulated Annealing algorithm for designing structures affected by self-weight loading. The research shows the effectiveness and versatility of the Simulated Annealing algorithm, particularly when enhanced with the crystallization factor, in addressing intricate TO problems. Systematically exploring different scenarios, including cases with and without self-weight effects and varying point loads, has provided results for verification of the proposed method.

The primary contribution of this work lies in applying the effect of self-weight and inertial forces in TO with simulated annealing. As evidenced by the results, the compliance values closely align with those found in the literature, validating the applicability of this method. Consequently, the algorithm proves adaptable for diverse problems, regardless of the objective function and constraints.

The results of this study, corroborated by comparisons to compliance values from existing literature, affirm the accuracy and reliability of the proposed TO approach. Moreover, the ability to post-process the obtained design for manufacturability underscores the practicality of the optimization results. While the stochastic nature of Simulated Annealing introduces variability, our findings demonstrate that it can be used efficiently to generate innovative and efficient engineering solutions. This research makes a notable contribution to the field of structural design optimization by highlighting the potential of Simulated-Annealing-based topology optimization for addressing the self-weight effect in non-gradient-based problems. It establishes a foundation for the application of this methodology in real-world engineering scenarios, paving the way for more efficient and

practical designs across various domains. Future research could further refine the optimization process, address the influence of additional parameters, and explore applications in diverse structural systems.

Author Contributions: Methodology, validation, Software development and writing: H.R.N.; Conceptualization, Supervision and review: T.C.M. Conceptualization and Review M.S.G.T.; Conceptualization, Methodology, Supervision and review: A.B. All authors have read and agreed to the published version of the manuscript.

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