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Review

Fifty Years of Eclipsing Binary Analysis with the Wilson–Devinney Model

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Abstract: The Wilson–Devinney model has—over the last 50 years—become the standard in analyzing eclipsing binary observations. To provide orientation for both active binary and non-binary researchers, it is presented here in historical and on-going as well as astrophysical perspectives. Among the important advances that originated with the model are: the representation of star surfaces as equipotentials for circular and eccentric orbits, leading to four morphological types; simultaneous least-squares light and velocity curve analyses; efficient reflection computation, including multiple reflection; disk theory and disk modeling. Solutions in physical units allowed for the accurate estimation of parameters such as stellar masses and photometric distances; inclusion of types of observables, properly weighted.

Keywords: binary stars; eclipsing binaries; Wilson–Devinney model; light-curve analysis; parameter estimation

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1. Introduction

Binary stars, especially eclipsing binaries (*EBs*),[1] provide rich data of various kinds. The term *observables* can refer to all *EB* observations, such as light and radial-velocity (RV) curves, polarization curves, and line profiles—measured at given times—as well as pulse-arrival times, conjunction times and timing residuals.

As readers may be active binary and non-binary researchers, the second group may want to start with the orientation in the Appendix A or enjoy the publication by Wilson [1], which covers the basics of light curve analysis fairly extensively, and then return to this point.

As the title may suggest, this review article is a combination of my own personal experiences (becoming familiar with the model and program, using it for decades) and the binary community's collective experiences that I have gleaned from interactions with others and/or from my reading of the literature. Mostly, the paper on the Wilson–Devinney (WD) model is based on what it can do and how it can do it. Section 5 also contains a summary of how the various capabilities have advanced the field—from my experience, perspective, and interactions with others.

The Wilson–Devinney Model and Key Binary Analysis Ideas

The Wilson–Devinney model (WDM) covers the geometric and physical principles required to analyze binaries. While originally developed for eclipsing systems, it also applies to non-eclipsing binaries for which we have at least RV curves, and to triple systems (a binary plus a third component). The basic ideas are in the original papers (WD71) by Wilson and Devinney [2], with eccentric orbit generalization and simultaneous least-squares analysis of light and RV curves in W79 (Wilson [3]). Accuracy and numerical efficiency in the reflection effect followed in W90 (Wilson [4]), and can also be found in the book *Eclipsing Binary Stars: Modeling and Analysis* by Kallrath and Milone [5]. The three W–D model papers mentioned above have been cited about 4000 times, the 1971 paper alone approximately 2300 times.

The *current citations per year* in Table 1 might be even more relevant than total citations, since a paper from long ago has had many years to gather citations, thus placing more recent papers at a disadvantage. WD71's average per year is 47 over the span of 1971 to 2021. Now it is about twice that. It is very remarkable to find an astronomy/astrophysics paper from approximately half a century ago with so many *current* citations per year.

Table 1. Current citation counts per year for three major publications related to the Wilson–Devinney model (last two years, only).

	2020	**2021**
WD71	97	101
W79	47	41
W90	42	43

A key idea of the Wilson–Devinney model is the explicit representation of star surfaces by equipotentials. This strategy assures consistent treatment of all morphological types (detached, semi-detached, over-contact, and double contact)—a major advantage over pre-selected shapes such as spheres or ellipsoids. Other central ideas are:

- Automatic simultaneous parameter estimation by adjustment of a physical model to observations with a least-squares fitting criterion: This procedure incorporates measurement error statistics into solutions with proper weighting and produces parameter uncertainties. This has advanced the field significantly as it became the de facto standard of the analysis of EB observations;
- The generic character of the physical model allows subsequent extension. This explains why the Wilson–Devinney model has been around for 50 years and still is marching forward into new territories, for example, adding a realistic model of a disk in a binary system;
- Eccentric orbit generalization for an equipotential model (Wilson [3]) (i.e., unification of potential theory covering both asynchronous and synchronous rotation as well as eccentric and circular orbits in any combination). Now, with the generalization as an accepted standard it seems natural to do so, but in 1979 this was a major step and breakthrough that made the model suitable for analyzing many kinds of binary systems;
- Simultaneous analysis of RV and multiband photometric observations (Wilson [3]). Here the same comment applies: This approach appears so natural, but it was a conceptual breakthrough;
- Constrained solutions, including all morphological types based on Roche geometry, that is, the surfaces of both stars are modeled as equipotential surfaces: Detached, semi-detached, over-contact types (see Wilson [3] for the basic idea) and the fourth morphological type (double contact, DBC) (Wilson [6]—paragraph 5, Wilson [7]). Embedding Roche geometry appropriately into the model and program, leads to an important example of improved astrophysical understanding through *EB* light curve analysis: The successful modeling of W UMa stars as over-contact systems. These very abundant binaries are excellent laboratories for convection in stars. Their fast orbital motion makes them attractive candidates for gravitational wave astronomy;
- Solutions in standard physical units so as to produce photometric distances (Wilson [8], Wilson and Hamme [9]). This can be a great advantage when analyzing huge numbers of EB stars observed in surveys;
- Efficient reflection effect, including multiple reflection (Wilson [4]). In terms of accuracy, many situations require multiple reflection to be done correctly, especially where the effective temperatures of the two stars are nearly equal;
- Disk theory and disk modeling (Wilson [10,11]) leading to a better understanding of cataclysmic variables and W Serpentis binaries;

- Kinematic third body parameters from photometric light and RV curves (Van Hamme and Wilson [12], Vaccaro et al. [13]);
- Unification of ephemeris analysis (Wilson and van Hamme [14], Wilson and Honeycutt [15]).

There are other valuable insights into astrophysical processes obtained by work on specific topics, such as analyzing and modeling Algol's polarization curves (Wilson and Liou [16]) or exploiting X-ray pulse-arrival times by Wilson et al. [17].

We focus mainly on modeling ideas and principles rather than the WD computer program (briefly covered in Section 4) that is based on the WDM. Because the WDM and WD programs are the most widely used light curve modeling tools, it seems appropriate to describe its foundations, features, capabilities, and continuing development in some detail; also see Wilson [7] for a recent publication on desirable developments for the field.

Since the Kepler satellite was launched, thousands of gigabytes of data are becoming available for astronomers all over the world to analyze in order to help find and verify newly discovered exoplanets. With so much data it is imperative to streamline analysis. The WDM was originally developed to analyze binary star systems quickly and accurately. However, there is no reason to exclude other systems that share similar characteristics, such as exoplanet systems. The WD program (WDP, 2010 version and newer) can accurately identify exoplanet system parameters.

To readers outside the EB community, the nomenclature may require some caution when it comes to primary and secondary stars. Another surprise could be that most *(Eclipsing) Binary* models and programs nowadays do not use radii as adjustable parameters but rather the Roche potentials Ω_1 and Ω_2, which cover the full range of morphological types from detached and semi-detached to over-contact binaries. The star-planet systems in extra-solar planet research are detached. For those, the relative radii r_j and Roche potentials are related by $r_j \sim 1/\Omega_j$.

2. History, Present and Future

2.1. The Transition from Rectification Techniques to Physical Models

In the first half of the twentieth century, eclipsing binary light curves were analyzed with rectification techniques based on ellipsoidal star figures and questionable reflection treatment that changed not only the model parameters but also the observations.

The WDM is one of the earliest paradigm shifts exploiting the benefits of direct (rectification-free) models, and fundamental four-type binary system morphology: detached, semi-detached, over-contact, and double contact. The first analysis comparing analyses of light curves with photometric anomalies by modeling starspots to those of light curves cleared by rectification techniques is by Milone et al. [18]. A delightful review of early history and models is also done by Wilson [19].

The early 1970s was a transition time into the *age of computational astrophysics* when models and programs were developed to compute (synthetic) light and RV curves directly. Some computer models and programs were still based on spherical stars, as in EBOP, the *Eclipsing Binary Orbit Program* (Nelson and Davis [20], Etzel [21,22]) or ellipsoidal geometry, as in WINK developed by Wood [23] and later versions of EBOP. Lucy [24], Hill and Hutchings [25], Wilson and Devinney [2], Mochnacki and Doughty [26], and Mochnacki and Doughty [27] produced models and programs based on the Roche geometry. Lucy's was the first attempt at direct calculation of light curves; it was limited to over-contact systems describable by a single value of the potential. Only bolometric light curves were computed and effects of mutual irradiation were neglected. Hill and Hutchings [25] provided an early calculation of irradiation effects, assuming a spherical primary for *Algol*. These new approaches permitted the computation of light curves on the basis of complex physical models describing the dynamic forces controlling the stellar mass distributions and radiation transport in the atmospheres. Physical models based on equipotentials and Roche geometry are implemented in the Wilson–Devinney program (see Wilson and Devinney [2], Wilson [3]) and in LIGHT2 (Hill and Rucinski [28] and citations contained therein); see Section 6.3 in

Kallrath and Milone [5] for more references to Roche model-based models and programs. The development of physical models and programs led to least-squares determinations of light curve parameters. The first use of least-squares for a physical light curve model was by Wilson and Devinney [2,29,30], the next was Lucy [31]. EB computer programs based on Roche models and least-squares analyses really started the age of computational astrophysics in *EB* research.

2.2. The WDM Becoming the Most Often Adopted EB Tool: 1980–Present

When I started my first eclipsing binary research studies at the Bonn University Astronomy department in the early 1980s, the standard was to run the WD program on mainframes. Departments were happy to receive a copy from R.E. Wilson shipped as magnetic tapes. Astronomy departments typically would have a copy of the WD program, although not necessarily the most recent version. Email attachment became available a few years later.

Since 1971 the WDM has been extended, mainly by R.E. Wilson, cf. Wilson (1979 [3], 1990 Wilson [4], 1998 [32], 2003, 2007 [33]), but also in close collaboration with Walter van Hamme, cf. Van Hamme and Wilson [12,34–37]. Note that Van Hamme has done the programming related to stellar atmosphere emission for the last 30 years. All these updates led to the current version covering estimation of:

- System parameters (systemic velocity, third light, distance, extinction);
- Orbital parameters, period change, and apsidal motion;
- Stellar parameters (volume-equivalent radii, masses, surface-averaged effective temperatures);
- Stellar atmospheres parameters and physics (passbands and atmospheres, limb darkening, gravity brightening—sometimes, especially in the older literature, called *gravity darkening*).

This list includes only the major parameters and physical effects.

2.3. Recent Progress: 2008–2020

Two connected features coming up in that period are *Direct distance estimation (DDE)* and the use of absolute units tied into the calibrated fluxes for a large number of photometric passbands (see Wilson [8], Wilson and Hamme [9], and the adaptation to the observables of X-ray binaries by Wilson et al. [38]). DDE was quickly adopted, cf. Milone and Schiller [39] or Milone et al. [40] to illustrate the use of DDE and provide detailed descriptions of experiments with many model variations including composition, interstellar extinction, and spot arrangements.

Time-related parameters (ephemeris, apsidal motion, and light travel time) are used Wilson and van Hamme [14] with a unified algorithm that processes photometric light and RV curves, as well as pre-existing eclipse timings simultaneously, without the need to compute any new timings. Wilson and Honeycutt [15] use this procedure to analyze outburst-related period changes of the recurrent Nova CI Aquilae. The most recent progress and extension to the WDM is the circumstellar accretion disk model (Wilson [11,41]).

2.4. Future Model Features

A major extension of the model developed by Wilson [11,41] is inclusion of the reflection effect on the disk and by the disk, that is, multiple reflection of and by the disk. Despite the intricacies in treating attenuation of light that penetrates a semi-transparent disk (going in and coming out) and connecting this with differential-corrections, a publication on this is expected soon (Wilson (2022), *private communication*).

A conceptual outline of the analysis of binary components that are intrinsic variables, for example, pulsating stars, is Wilson et al. [42].

In the view of precise space photometry, including some (by now) neglected phenomena might also find a way into the WDM. The future will show whether one day we find

Doppler boosting (relativistic beaming), gravitational lensing and other phenomena related to general relativity being included in the WDM.

3. A Closer Look at the Physical Contributions of the WDM to Astrophysics

Many important contributions of the WDM to astrophysics are covered in more detail in this section.

3.1. Simultaneous RV and Multiband Light Curve Solutions

Considering all available observations systematically leads to improved and reliable binary star parameters if one carefully treats weighting—and reports how the weighting is done.

The foundations of weighting photometric light-curve points are in Wilson [3], including curve-dependent, level-dependent, and individual data point weights. Weighting is important in simultaneous least-squares analyses, that is, in the presence of observables of different types. Unfortunately, many publications lack information about how the weighting is done. We therefore encourage authors to think carefully about proper weighting and to be explicit. Without proper weighting the weakest observations are likely to have the strongest influence. In contrast to photometric light curves, RV weighting and eclipse timing weighting are not level-dependent but otherwise are done in the same way as light-curve weighting. Only weight ratios matter among the points of a given data subset such as a light or RV curve, since curve-dependent weights are computed subsequently by the WDP, taking account of individual weights. Accordingly, in that case the scaling factor for individual weights is arbitrary and usually set to unity. Note that proper relative curve weighting of the RV and photometric curves is a very critical issue. Level-dependent weights are generated within the WD approach (WDA) from applicable statistics that are assumed to be dominated by atmospheric extinction fluctuations and scintillation for bright sources or by photon counting statistics for fainter sources. If the individual errors of observed data points are known, the weights can follow from the variances of the individual data points, that is, individual data points are weighted by a scaled inverse square of their individual errors as in Milone et al. [40], who provide a full discussion of all the weighting factors in the context of the 2013 WDP.

It must be strongly stressed that all physical parameters in the WDA are derived from consistently weighted observables. So all that has been said in the previous paragraph for photometric light and RV curves also holds for other observables such as spectral line profiles or polarization curve analysis, leading to physical parameters with proper variances or uncertainties. Besides the rich physics in the WDM, it is this rigorous analysis that produces great value to the stellar astrophysics community.

3.2. Eccentric Orbit Generalization for an Equipotential Model

The earliest generalization of the Roche potential for eccentric binaries is, to our best knowledge, by Avni [43]. This generalization is based on an approximated quasi-static equilibrium—and found its way into the Wilson [3] model. To include the effects of both eccentric orbits and nonsynchronous rotation, it introduces optional enforcement of physical constraints (semi-detached condition, etc.). The extended WDM is based on a unification of potential theory covering both asynchronous and synchronous rotation as well as eccentric and circular orbits in any combination. It allows the modeling of semi-detached, detached, double-contact, and X-ray binaries for arbitrary rotation rates and orbital eccentricity, and over-contact binaries for the synchronous, circular orbit case. In all cases, the shapes of the components and the surface gravity fields are described by surfaces of constant potential energy.

3.3. Constrained Morphological Solutions

The term *constrained solutions* refers to least-squares analyses subject to explicit constraints. Examples are lobe filling constraints, X-ray eclipse duration constraints, and common envelope constraints (for over-contact binaries). The application of a constraint may be indicated by observational evidence. Constraints simplify analyses by reducing the number of free parameters according to the underlying physics or geometry.

Following upon Kopal's detached, semi-detached, and contact types, Wilson [3] introduced automated modes of operation, with each mode allowing only solutions that fulfill a particular constraint. Wilson and Devinney [30] and Lucy [31] produced the first impersonal over-contact solutions.

The fourth morphological type, double-contact, was introduced by Wilson [3], (Wilson [6], paragraph 5) and Wilson [7]. This previously unrecognized morphological type, for which the name double-contact binary was suggested by Wilson, is a binary in which both stars fill their limiting lobes and at least one spins faster than synchronously, so that the components do not touch, even at one point (rotation reduces lobe size). The proposed members of this group include β Lyr, V356 Sgr, and perhaps U Cep. Terrell and Nelson [44] give an impressive illustration of how a double-contact solution finally gave a consistent picture of the short-period Algol TT Herculis that had a century of widely varying analysis results.

3.4. Solutions in Standard Physical Units

EB distance estimates differ from those of standard candles in being individually measurable—without reliance on (usually nearby) objects that are assumed to be similar. That makes EBs very important.

To determine photometric distances, that is, to derive distances and effective temperatures from EB light curves, Wilson [8] and Wilson and Hamme [9] suggest the use of light curves that are in standard flux units, as opposed to traditional dimensionless, normalized and somewhat arbitrary units. Using standard flux units, distance becomes a systemic parameter with a standard error, effective temperatures of both stars may be derivable in favorable circumstances and semi-detached and over-contact binaries suffer no loss of distance accuracy (when compared to well-detached binaries). Flux calibrations are only needed for the observations, while theoretical fluxes are naturally in standard units. Thus, the comparison of theory and observation is direct, and semi-empirical quantities based on color-temperature relations are not required. This integrated process—called *direct distance estimation*—also reduces time and effort by avoiding separate distance estimation steps and enables the application of routine distance measurements for large numbers of EBs found, for instance, in surveys.

3.5. Efficient Reflection Effect, Including Multiple Reflection

Wilson [4] presented a rigorous treatment of the geometric and irradiation heating problems for the binary star reflection effect. The geometric and surface gravity features of the theory are developed in terms of equipotential level surfaces and are sufficiently general to include eccentric orbits and nonsynchronous (even centrifugally limited) rotation and to treat multiple reflection. Explicit introduction of a reflection ratio (R-function) allows for a simple and natural separation of the bolometric heating problem (invoking conservation of energy) from the wavelength-specific reradiation problem. The R-function also covers multiple reflection in a simple, flexible and numerically efficient iterative way.

3.6. Disk Theory and Disk Modeling

An accretion–decretion (A-D) circumstellar disk model, suitable for the analysis of light and RV curves, has been developed by Wilson [11] for application to double contact binaries. An earlier paper is by Wilson [10]. The ideas are based on the assumption that some highly evolved systems, such as cataclysmic variables and W Serpentis binaries, may have the morphological type of double-contact and the related evolutionary property of having A-D disks. The core of the model is a globally self-gravitating equipotential disk. The disk can be semi-transparent with attenuation of internal disk light and the light of both binary components. Tidal stretching of the disk with implied brightness variation, as in the *ellipticity* effect for ordinary binaries, is a natural consequence of the disk's tidally extended structure.

It is important to understand the distinct roles of fluid dynamic, structural, and analytic models. The disk is a volume emitter with the attenuation of internally generated light. Computations intrinsically produce phenomena that are characteristic of circumstellar disks in binaries, in particular tidal and rotational gravity brightening and an outer effective gravity null point that do not occur in the common axisymmetric disk model.

3.7. Kinematic Third Body Parameters from Light and RV Curves

Simultaneous analysis of RV curves, multiband light curves, and eclipse timings permits one to treat stellar, orbital, and ephemeris parameters coherently in one conceptual step; see Van Hamme and Wilson [12] and Vaccaro et al. [13].

3.8. Unification of Ephemeris Analysis

The orbital period, its rate of change, apsidal motion, and variable light-time delay due to a third body are time-related binary system parameters that have usually been determined solely from eclipse timings. An alternative was proposed in 2014 by Wilson and van Hamme [14] as an extension to the simultaneous light and RV solutions of Wilson [3], now including eclipse timings. Soon after, the procedure was applied by Wilson and Honeycutt [15] to outburst-related period changes of recurrent nova CI Aquilae. A central motivation is to avoid the quandary of how to relate time of minima information to light curve information. It is far more elegant and consistent to combine the three statistically justifiable ways, with proper data point weighting.

Each data source has its benefits, for example, eclipse timings typically cover relatively long time spans while complete light curves often have densely grouped data within specific intervals and allow access to systemic properties that carry additional timing information. The benefit of the unification is, similarly as in Section 3.1, that these different kinds of observables are input to a simultaneous least-squares analysis, with automated weighting based on their standard deviations. Simultaneous light-velocity-timing analyses treat parameters of apsidal motion and the light-time effect consistently with those of period and period change, and yield parameter standard errors based on the quantities and variances (transformed into weights) of the curve data and timings. With this unified approach, the WDM and WDP can support the analysis of survey missions containing millions of EB systems.

3.9. Other Contributions

In the overview presented in the previous subsections we have only covered the most important contributions of the WDA to astrophysics. Other WDM contributions and features are just mentioned here: spot modeling, blended spectral line profiles, and polarization curve analysis. It might be helpful, in addition to the original publications by Wilson and co-authors, to consult the book *Eclipsing Binary Stars: Modeling and Analysis* by Kallrath and Milone [5] for material published before 2009.

4. Software and Programs

The WDP consists of two programs—LC and DC—for modeling and analyzing binary star observables, cf. Wilson [7]. Program LC generates light and RV curves, spectral line profiles, images, conjunction times, and timing residuals. Program DC handles differential-corrections, including a Levenberg–Marquardt (L-M) scheme, performing parameter adjustment of light curves, RV curves, and eclipse timings by the least-squares criterion. WDP handles eccentric orbits and asynchronous rotation, and can compute velocity curves (with proximity and eclipse effects). It offers options for detailed reflection and nonlinear (square-root and logarithmic law) limb darkening, adjustment of spot parameters, an optional provision for spots to drift over the surface, and can follow light curve development over large numbers of orbits. Absolute flux solutions allow direct distance estimation, cf. Wilson [8], and, vice versa, as demonstrated by Wilson et al. [45], there are improved solutions for ellipsoidal variables and for *EBs* with very shallow eclipses if parallax distances are available because several photometric bands and RV curves can be analyzed simultaneously in terms of absolute flux. Absolute flux solutions can also estimate effective temperatures of both *EB* components under suitable circumstances.

The WDP has seen continual improvements, comes with a well-structured and carefully maintained documentation and the current version with its powerful features provides the opportunity to extract maximum information from a variety of observational types.

Without going into specific details, we stress that the WDP has been extremely carefully programmed when it comes to numerical stability and efficiency. First, this applies to representing star surfaces with a near-uniform distribution of grid points. In a comparison and test, the accuracy is the same as in the well-known Gauß and Chebyshev schemes for integration of a function over the unit spheres, but they required approximately the same number of grid points and were not better. Second, we mention storing two-dimensional arrays as one-dimensional vectors. Two more examples justifying this statement about numerical stability and efficiency are enhancements of the differential-correction method: The *Method of Multiple Subsets* (MMS) introduced by Wilson and Biermann [46] and the *Vector Length Reduction* (VLR) found in the documentation of the WDP and Section 6.1 in Wilson [47].

MMS is for reducing the correlations among parameters. MMS works in pre-selected lower-dimensional subspaces of parameters, the other fitting parameters are fixed. The parameters in these subspaces are comparatively correlation free, and convergence is fast. The parameters selected for each subset can be changed as considered most advantageous. Although uniqueness of the finally converged result is not guaranteed, in most practical EB examples this method works fine and produces the unique local minimum of the overall least-squares problem. Note that probable errors of the fitted parameters have to be computed using the full parameter set.

Although VLR serves the same purpose—dealing with convergence problems when solving the least-squares problem using differential-corrections—the angle of attack is very different from MMS. VLR belongs to the class of *damped least-squares methods* in which one moves only a fractional part, α, in the correction vector obtained in each differential-correction iteration. The damping or reduction constant α can be set to numbers between very small, for example, 0.1 and 1. Using VLR, there is no need to think about subsets, and the method works more automatically. Selecting the value of α requires some care and possibly numerical trials: Small values lead to more iterations caused by smaller step lengths, larger values may not significantly reduce the problem of non-linearity in parameter space. One might ask: Which of the three techniques, L-M, MMS, or VLR is preferable? The answer is: L-M, MMS, and VLR can be used separately or together, according to situation. It is helpful, to have all of them at hand.

The WDM and WDP have stimulated development of public programs that adopt the WDP in their kernels, or at least LC, and sometimes, DC, e.g.,

- WDx95 by Kallrath et al. [48], WDx98k93h, by Milone et al. [49] adding Kurucz atmospheres,WDx03 by Milone and Kallrath [50], WDx2007 by Kallrath and Milone [5], Chap. 7.
- WDwint by Nelson [51] (see also Nelson [52] for graphics output produced by WDwint or http://binaries.boulder.swri.edu/binaries/ (accessed on 18 Janaury 2022).
- phoebe by Prša [53]; see also http://phoebe-project.org/releases (accessed on 18 Janaury 2022).
- Binary Maker 3.0 by Bradstreet and Steelman [54]; see http://daniel.eastern.edu/faculty_personal/dbradstr/ (accessed on 18 Janaury 2022).

These programs offer data input and output advantages. They provide help pages and have error traps to prevent illegal model parameters and so forth. The plotting facilities coming with these tools permit one to adjust WD parameters and quickly see the effect on light curves in LC, thereby enabling one to zoom in to plausible parameters before resorting to DC.

Some programs provide additional ways to solve the inverse problem of parameter estimation. WDx95 by Kallrath et al. [48] uses a Simplex method based on Nelder and Mead [55]) and a Levenberg–Marquardt scheme inspired by Levenberg [56]. The Levenberg–Marquardt can be understood as damped least squares including a damping constant to facilitate the adjustment of partially correlated parameters. WD03 by Milone and Kallrath [50] offers a simulated annealing algorithm (cf. Pardalos and Mavridou [57]) to provide good initial values for DC. Several such programs with new innovative features or added functionality now coexist with WDP. Some of these features were developed independently in several models or programs, including WD.

For a more complete list of other eclipsing binary models and software—not related to the WDM—see Chapter 6 in Kallrath and Milone [5].

5. Conclusions

The WDM is a good example of how a well founded model can become a quasi-standard in analyzing astrophysical observations. A large community of analyzers using the WDP produce valuable binary parameters. The model and program are still maintained by one of the original authors and is contained in variants and modifications of other packages.

The fundamental ideas of the WDA, that is, the computational implementation of Roche models coupled with least-squares analyses, really started the age of computational astrophysics in *(Eclipsing) Binary* research. Due to its generic framework and broad range of geometric and physical situations, it has also been successfully used to analyze transit data in exoplanet systems, cf. Milone et al. [58] or Vaccaro and Van Hamme [59]. Another example is the successful modeling of Algols as semi-detached. This gave quantitative reinforcement to the already accepted solution of the *Algol paradox*: The hotter, more massive primaries were clearly main sequence stars, but the less massive secondaries had radii much too large to be on the main sequence (i.e., they were evolved subgiants or giants). This finding initially appeared to contradict the well accepted picture that more massive stars evolve faster than less massive stars.

The accretion–decretion (A–D) circumstellar disk model for application to double contact binaries may lead to an improved understanding of cataclysmic variables and W Serpentis binaries.

The open character of WDP (it comes as a collection of Fortran routines) allowed others to add enhancements such as refined limb-darking laws or a detailed model of the Kurucz atmospheres) as well as to develop their own user-interfaces. Due to the generosity of Robert E. Wilson, everyone who wanted the source programming would receive it across national borders or across communities. Graphical user interfaces have been built around the WDP, which explains why the model and program are used not only in the main binary community, but also in exoplanet research and among amateur astronomers analyzing their own EB observations. Amateur astronomers may consult the valuable publication by Wilson [1].

Although thirty years ago EB research seemed a classical and mature field, somewhat detached from the mainstream (galaxies, cosmology, etc.), it has always proven a lively field in close contact with cutting-edge research. Gravitational wave experiments on W UMa systems or automatic EB-analysis on the fly may soon be standard in large surveys, even in the design stages. In the EB community, the WDM and WDP have been drivers of scientific progress because of their profound astrophysical and technical basis and strong linkage to the EB community, leading to lively discussions and inspiring ideas.

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Appendix A. An Elementary Introduction and Orientation for Non-Binary Researchers

The analysis of a photometric light curve alone cannot provide absolute dimensions of the stars (masses, radii, effective temperatures, etc.) or the orbit (semi-major axis).[2] This is because of a scaling property: if all geometric properties of a binary system are doubled, the original light curves can be reproduced by shifting to a larger distance. The distance and surface brightnesses, which would determine the scale, are known only under exceptional circumstances, e.g., if the binary is a member of a well-studied star cluster.

Before going into details we collect the symbols used in this paper and, especially, in this appendix:

a	semi-major axis of the relative orbit, in units of solar radii
a_j	semi-major axis of the absolute orbit of component j, in units of solar radii
d	distance of the binary
e	the orbital eccentricity = separation of foci / 2a
F	rotation parameter
F_j	rotation parameter for binary star component j
g_j	gravity brightening coefficient of component j
i	orbital inclination; angle between orbital plane and plane-of-sky (angular degree)
ℓ_3	third light, sum of all contributions from any systems parts beyond the binary pair (usually assumed to be constant)
$\mathcal{L}_j$	bolometric luminosity (radiant power in Watts, over 4π steradians, units could be W/micron or in units of solar luminosity)
L_j	monochromatic luminosity (in a specified passband) of component j over 4π steradians
$\mathcal{M}_j$	mass of component j (in units of solar masses)
P	binary orbital period
q	binary system mass ratio: $q = \mathcal{M}_2/\mathcal{M}_1$
$q_{\rm ph}$	photometric mass ratio
$q_{\rm sp}$	spectroscopic mass ratio
r_j	relative radius of component j
$\mathcal{R}_j$	mean radius of component j; usually the "equal volume radius" in units of solar radii
T_j	mean effective temperature of component j
x_j	limb darkening coefficient of component j
α	damping factor in VLR algorithm
γ	radial velocity of the center-of-mass of a binary system
ω	argument of periastron
Ω_j	Roche potential of component j

Light curves can provide relative quantities (radii in terms of the semi-major axis a, information on effective temperatures, relative luminosities, perhaps a photometric mass ratio, star shapes) and the orientation (inclination, i, argument of periastron, ω) and eccentricity. Radial velocity curves can provide $a \sin i$, i.e., a in physical units if i is known. When a and the mass ratio q are known, the masses can be found. Note that in order to derive definite masses and orbital dimensions from a RV curve, the inclination needs to be known. Thus, the absolute determination of *EB* parameters in many cases requires at least one light curve and RV curves for both components unless the photometric mass ratio is accurate. In that case, the absolute parameters for a single-lined spectroscopic binary can be determined.

Table A1 lists combinations of observables needed to derive certain parameters in favorable cases:

1 = at least one light curve;
2 = only one RV curve;
3 = both RV curves, but no light curve;
4 = at least one photometric light and one RV curve; and
5 = at least one photometric light curve and both RV curves:

Table A1. Combinations of observables needed to derive certain parameters in favorable cases. The symbol $\surd$ indicates a sufficient combination of observables.

	1	**2**	**3**	**4**	**5**
$a_1 \sin i$ or $a_2 \sin i$		$\surd$	$\surd$	$\surd$	$\surd$
$a \sin i$, $a_{1,2} \sin i$, $\mathcal{M}_{1,2} \sin^3 i$			$\surd$		$\surd$
a, $a_{1,2}$, $\mathcal{M}_{1,2}$, $\mathcal{R}_{1,2}$, $\mathcal{L}_{1,2}$, d				$(\surd)$	$\surd$
e, ω, P	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
γ		$\surd$	$\surd$		$\surd$
$q_{\rm sp}$			$\surd$		$\surd$
$q_{\rm ph}$	$(\surd)$			$(\surd)$	$(\surd)$
i, $\mathcal{R}_{1,2}/a$, L_2/L_1, $g_{1,2}$, $A_{1,2}$, $F_{1,2}$, $x_{1,2}$, ℓ_3	$\surd$			$\surd$	$\surd$
T_2	$\surd$	(?)	(?)	$\surd$	$\surd$

The photometric mass ratio $q_{\rm ph}$ indicated by $(\surd)$ can be determined with high confidence only for lobe-filling or over-contact systems. Unfortunately, the incorrect notion that $q_{\rm ph}$ derives mainly from ellipsoidal variation has gained moderately widespread acceptance. The actual situation is that $q_{\rm ph}$ comes from coupling the Roche configuration to the radii of the stars. Thus, the same light curve characteristics that define the radii[3] also define $q_{\rm ph}$, but only when the radii can be related to the equipotential configuration. There are two distinct cases. In a semi-detached binary the radius of the lobe-filling star fixes the lobe radius, and thus the mass ratio. Best results are obtained for complete eclipses. In an over-contact binary with complete eclipses the ratio of the radii is fixed by elementary considerations. The ratio of radii in turn is essentially a unique function of the mass ratio for over-contact equipotentials, with minor dependence on the degree of over-contact (Wilson [60]).

Nearly all published values of photometric F's are for rapidly rotating Algols. Two conditions help in determining the values of F. First, eclipse circumstances (shape and depth) are altered by rotationally induced oblateness. Second, and more subtle, the proximity effects due to reflection and ellipsoidal variation of the secondary star are effectively enhanced by the reduced brightness of the fast rotating primary for observers near the orbit plane.

The question mark in parentheses (?) indicates that the effective temperatures can be derived from spectral features. In order to compute the temperature T_2 from a light curve solution, the temperature T_1 has to be known in advance (e.g., derived from color or spectral type); of course, we could also fix T_2 and adjust T_1. The determination of both temperatures T_1 and T_2 is possible when RV curves and two light curves are analyzed and the photometry has been fully calibrated. This is discussed in great depth by Wilson [8] in the context of the $T-d$ theorem. The quantity $a_1 \sin i$ can be obtained from a single-lined system, and $a \sin i$ from a double-lined system to give lower limits for the orbital size $a = a_1 + a_2$. In ideal cases, an eclipsing, double-lined system therefore provides everything needed. Whether or not this is true for a given system is a matter to be determined. The light curve of an *EB* depends nonlinearly on the parameters, so solving the inverse problem, and thus minimizing the sum of squared-residuals, requires to navigate all the pitfalls of nonlinear multi-parameter fitting.

Notes

[1] However depending on context, the statements are also valid for non-eclipsing binaries.

[2] For reader convenience, this appendix is, by courtesy and under the license 5181321004554 of Springer Nature, a close excerpt of pages 173 and 174 of the book *Eclipsing Binary Stars: Modeling and Analysis* by Kallrath and Milone [5].

[3] In detached systems, correlations of q_{ph} with Ω_1, Ω_2, and other parameters make it almost impossible to derive meaningful photometric mass ratios. In lobe-filling or over-contact binaries either of Ω_1 or Ω_2 is eliminated from the adjustable parameter list.

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