galaxies

MDPI

Article

Spin and Maximal Acceleration

Giorgio Papini

Department of Physics and Prairie Particle Physics Institute, University of Regina, Regina, SK S4S 0A2, Canada; papini@uregina.ca

Received: 17 November 2017; Accepted: 13 December 2017; Published: 20 December 2017

Abstract: We study the spin current tensor of a Dirac particle at accelerations close to the upper limit introduced by Caianiello. Continual interchange between particle spin and angular momentum is possible only when the acceleration is time-dependent. This represents a stringent limit on the effect that maximal acceleration may have on spin physics in astrophysical applications. We also investigate some dynamical consequences of maximal acceleration.

Keywords: alternative gravity; maximal acceleration; spin; astrophysics

PACS: 04.62.+v; 95.30.Sf

1. Introduction

Recently, several aspects of spin physics have been actively investigated. The interaction of photon [1] and neutron spins [2] with non-inertial fields like rotation [3–6] has been experimentally verified at the quantum level and spin-induced effects for macroscopic objects have been observed [7–9]. It is also known that spin is not a constant of motion when a particle interacts with external fields, either electromagnetic [10], or gravitational [11] and that continual interchange between spin and orbital angular momentum is possible.

The purpose of this work is to study the behaviour of spin at large accelerations, those that may be met close to a black hole and to the maximal acceleration (MA), an upper limit introduced by Caianiello in his geometrical formulation of quantum mechanics [12–14]. In Caianiello's model, in fact, the absolute value of a particle proper acceleration satisfies the inequality $a \leq \mathcal{A}_m$, where $\mathcal{A}_m = 2mc^3/\hbar$ is the upper limit mentioned and m the particle mass. No counterexamples are known to the validity of this inequality. The limit $\hbar \to 0$ restores $\mathcal{A}_m$ to its infinite, classical limit. The value of $\mathcal{A}_m$ is mass dependent and very large even for the lightest particles. It leads to violations of the equivalence principle, also a subject of great interest.

Classical and quantum arguments supporting the existence of a MA have been given in the literature [4,15–46]. MA is also found in the context of Weyl space [47–50] and of a geometrical analogue of Vigier's stochastic theory [51]. It rids black hole entropy of ultraviolet divergences [52] and is a regularization procedure [53] that avoids the introduction of a fundamental length [54], thus preserving the continuity of space-time.

A MA also exists in string theory [55–61] when the acceleration induced by a background gravitational field reaches the critical value $a_c = \lambda^{-1} = (\tilde{m}\alpha)^{-1}$ where λ, $\tilde{m}$ and α^{-1} are string size, mass and tension. At accelerations larger than a_c the string extremities become casually disconnected.

Applications of Caianiello's model include cosmology [62–64], the dynamics of accelerated strings [65,66], neutrino oscillations [67–69] and the determination of a lower neutrino mass bound [70]. The model also makes the metric observer–dependent, as conjectured by Gibbons and Hawking [71].

The model has been applied to classical [72] and quantum particles [73] falling in the gravitational field of a collapsing, spherically symmetric object described by the Schwarzschild metric and also to the Reissner-Nordström [74] and Kerr [75] metrics. In the model, the end product of stellar collapse is represented by compact, impenetrable astrophysical objects whose radiation characteristics are similar to those of known bursters [76].

The consequences of MA for the classical electrodynamics of a particle [77], the mass of the Higgs boson [78,79], the Lamb shift in hydrogenic atoms [80], muonic atoms [81], the helicity and chirality of particles [82] and the temperature [83] have also been investigated.

Most recently Rovelli and Vidotto have found evidence for MA and singularity resolution in covariant loop quantum gravity [84,85].

Caianiello's model is based on an embedding procedure [72] that stipulates that the line element experienced by an accelerating particle is represented by

$$d\tau^2 = \left(1 + \frac{g_{\mu\nu}\ddot{x}^\mu \ddot{x}^\nu}{\mathcal{A}_m^2}\right) g_{\alpha\beta} dx^\alpha dx^\beta = \left(1 - \frac{|a(x)|^2}{\mathcal{A}_m^2}\right) ds^2 \equiv f(x) ds^2 \,, \tag{1}$$

where $g_{\alpha\beta}$ is a background gravitational field and $|a|$ the absolute value of the acceleration. The value $f = 1$ corresponds to the classical limit $\mathcal{A} \to \infty$ and $f = 0$ to the MA limit. A particle therefore experiences acceleration as if subjected to an external gravitational field represented by the metric $g_{\mu\nu} = f(x)\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric (of signature-2), if the background is flat. Particles of different mass experience different metrics, hence different effective gravitational fields, but their kinematics is characterized by the same velocity field. The metric (1) lends support to geometrical models of confinement in the strong interactions and hadronization processes. If an effective space-time curvature can be generated by acceleration, then confinement inside hadrons can affect only quarks that are strongly accelerated by the strong interactions, while other particles, leptons for instance, that are not affected by the strong interactions, experience a geometry identical to that of an inertial observer. Since Caianiello's model of quantum geometry offers a metric to work with, it is convenient to use it in conjunction with covariant wave equations that are the byproduct of minimal coupling and Lorentz invariance.

Covariant wave equations that apply to particles with, or without spin, have solutions [86–90] that are exact to first order in the metric deviation $\gamma_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ and have been applied to problems like geometrical optics [89], interferometry and gyroscopy [87], the spin-flip of particles in gravitational and inertial fields [91], radiative processes [92,93] and spin currents [11]. We are interested in spin-1/2 particles, in particular. The covariant Dirac equation [94]

$$\left[i\gamma^\mu(x)\mathcal{D}_\mu - m\right]\Psi(x) = 0 \,, \tag{2}$$

is remarkably successful in dealing with all inertial and gravitational effects discussed in the literature [95–99]. The notations and units ($\hbar = c = 1$) are as in [91], in particular $\mathcal{D}_\mu = \nabla_\mu + i\Gamma_\mu(x)$, ∇_μ is the covariant derivative, $\Gamma_\mu(x)$ the spin connection, commas indicate partial derivatives and the matrices $\gamma^\mu(x)$ satisfy the relations $\{\gamma^\mu(x), \gamma^\nu(x)\} = 2g^{\mu\nu}$. In the absence of external fields, (2) reduces to the free Dirac equation

$$\left(i\gamma^{\hat{\mu}}\partial_\mu - m\right)\psi_0(x) = 0 \,, \tag{3}$$

where $\gamma^{\hat{\mu}}$ are the usual constant Dirac matrices.

The first order solution of (2) is of the form

$$\Psi(x) = \hat{T}(x)\psi_0(x) \,, \tag{4}$$

where $\psi_0(x)$ is a solution of (3), the operator $\hat{T}$ is given by

$$\hat{T} = -\frac{1}{2m}\left(-i\gamma^\mu(x)\mathcal{D}_\mu - m\right) e^{-i\Phi_T} \,, \tag{5}$$

$$\Phi_T = \Phi_S + \Phi_G, \Phi_S(x) = \int_P^x dz^\lambda \Gamma_\lambda(z), \tag{6}$$

and

$$\Phi_G(x) = -\frac{1}{4}\int_P^x dz^\lambda \left[\gamma_{\alpha\lambda,\beta}(z) - \gamma_{\beta\lambda,\alpha}(z)\right] \left[(x^\alpha - z^\alpha) k^\beta - \left(x^\beta - z^\beta\right) k^\alpha\right] + \frac{1}{2}\int_P^x dz^\lambda \gamma_{\alpha\lambda}(z) k^\alpha. \tag{7}$$

It is convenient to choose $\psi_0(x)$ in the form of plane waves, but wave packets can also be used.

In (6) and (7), the path integrals are taken along the classical world line of the fermion, starting from an arbitrary reference point P that will be dropped in the following. Only the path to $\mathcal{O}(\gamma_{\mu\nu})$ needs to be known in the integrations indicated because (4) already is a first order solution. The positive energy solutions of (3) are given by

$$\psi(x) = u(\mathbf{k}) e^{-ik_\alpha x^\alpha} = N \begin{pmatrix} \phi \\ \frac{\sigma\cdot\mathbf{k}}{E+m}\phi \end{pmatrix} e^{-ik_\alpha x^\alpha}, \tag{8}$$

where $N = \sqrt{\frac{E+m}{2E}}$, $u^+ u = 1$, $\bar{u} = u^+\gamma^0$, $u_1^+ u_2 = u_2^+ u_1 = 0$ and σ are the Pauli matrices. In addition ϕ can take the forms ϕ_1 and ϕ_2 where $\phi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and $\phi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Solution (2) contains that of the covariant Klein-Gordon equation that, neglecting curvature dependent terms becomes to $\mathcal{O}(\gamma_{\mu\nu})$

$$\left(\nabla_\mu \nabla^\mu + m^2\right)\phi(x) \simeq \left[\eta_{\mu\nu}\partial^\mu\partial^\nu + m^2 + \gamma_{\mu\nu}\partial^\mu\partial^\nu\right]\phi(x) - \frac{1}{2}\eta^{\sigma\rho}\left(2\gamma^\mu_{\rho,\mu} - \gamma_{,\rho}\right)\phi = 0, \tag{9}$$

where $\gamma \equiv \gamma^\rho_\rho$. The solution of (9) is obtained by solving the Volterra equation

$$\phi(x) = \phi_0(x) - \int_P^x d^4x' G(x,x')\gamma_{\mu\nu}(x')\partial'^\mu\partial'^\nu\phi(x'), \tag{10}$$

along the particle world-line, where P is again a fixed reference point, x a generic point in the physical future along the world-line, $G(x,x')$ is the causal Green function with $(\partial^2 + m^2)G(x,x') = \delta^4(x-x')$. The free Klein-Gordon equation is

$$(\partial^2 + m^2)\phi_0 = 0. \tag{11}$$

In first approximation ϕ_0 can be substituted for ϕ in (10) and the integrations can then be carried following [87,88]. The solution of (9) is

$$\phi(x) = (1 - i\Phi_G(x))\,\phi_0(x), \tag{12}$$

which is contained in $\exp(-i\Phi_T)$. Higher order approximations to the solution of (12), therefore of (2), can be obtained by writing

$$\phi(x) = \Sigma_n \phi_{(n)}(x) = \sum_n e^{-i\Phi_G}\phi_{(n-1)}(x). \tag{13}$$

Because of the structure of (13), the higher order corrections are expected to be well behaved and to not affect the conclusions.

2. Spin Currents

The transfer of angular momentum between the external field and the fermion spins can be calculated using the spin current tensor [10]

$$S^{\rho\mu\nu} = \frac{1}{4im}\left[\left(\nabla^\rho\bar{\Psi}\right)\sigma^{\mu\nu}(x)\Psi - \bar{\Psi}\sigma^{\mu\nu}(x)\left(\nabla^\rho\Psi\right)\right], \tag{14}$$

that satisfies the conservation law $S^{\rho\mu\nu}{}_{,\rho} = 0$ when all $\gamma_{\alpha\beta}(x)$ vanish and yields in addition the expected result $S^{\rho\mu\nu} = \frac{1}{2}\bar{u}_0\sigma^{\hat{\mu}\hat{\nu}}u_0$ in the rest frame of the particle. Writing $\sigma^{\mu\nu}(x) \approx \sigma^{\hat{\mu}\hat{\nu}} + h^{\mu}_{\hat{\tau}}\sigma^{\hat{\tau}\hat{\nu}} + h^{\nu}_{\hat{\tau}}\sigma^{\hat{\mu}\hat{\tau}}$, where $\sigma^{\hat{\alpha}\hat{\beta}} = \frac{i}{2}[\gamma^{\hat{\alpha}}, \gamma^{\hat{\beta}}]$, substituting (3) and (5) in (14) one obtains, to $\mathcal{O}(\gamma_{\alpha\beta})$,

$$\begin{aligned} S^{\rho\mu\nu} = \tfrac{1}{16im^3}\bar{u}_0 \Big\{ & 8im^2k^\rho\sigma^{\hat{\mu}\hat{\nu}} + 8imk^\rho h^{[\mu}_{\hat{\tau}}\sigma^{\hat{\tau}\hat{\nu}]} + 4imk^\rho\left(\Phi_{G,\alpha} + k_\sigma h^\sigma_{\hat{\alpha}}\right)\{\sigma^{\hat{\mu}\hat{\nu}}, \gamma^{\hat{\alpha}}\} \\ & -8imk^\rho\Phi_G k^{[\mu}\gamma^{\hat{\nu}]} + 4mk^\rho k_\alpha\left[\sigma^{\hat{\mu}\hat{\nu}}, \left(\gamma^{\hat{\alpha}}\Phi_S - \gamma^{\hat{0}}\Phi_S^+\gamma^{\hat{0}}\gamma^{\hat{\alpha}}\right)\right] \\ & +4m^2k^\rho\left[\sigma^{\hat{\mu}\hat{\nu}}, \left(\Phi_S - \gamma^{\hat{0}}\Phi_S^+\gamma^{\hat{0}}\right)\right] - 8m^2k^\rho h^0_{\hat{\alpha}}\left[\gamma^{\hat{0}}, \left[\sigma^{\hat{0}\hat{\alpha}}, \sigma^{\hat{\mu}\hat{\nu}}\right]\right] \\ & -8im^2k_\sigma\left(\Gamma^\sigma_{\alpha\beta}\eta^{\beta\rho} + \partial^\rho h^\sigma_{\hat{\alpha}}\right)\eta^{\alpha[\mu}\gamma^{\hat{\nu}]} + 8im^2\partial^\rho\Phi_G\left(4m\sigma^{\hat{\mu}\hat{\nu}} - 2ik^{[\mu}\gamma^{\hat{\nu}]}\right) \\ & +4im^2\gamma^{\hat{0}}\Gamma^{\rho+}\gamma^{\hat{0}}\left\{\left(\gamma^{\hat{\alpha}}k_\alpha + m\right), \sigma^{\hat{\mu}\hat{\nu}}\right\}\Gamma^\rho\Big\} u_0 \end{aligned} \tag{15}$$

where use has been made of the relation

$$\Phi_{G,\mu\nu} = k_\alpha\Gamma^\alpha_{\mu\nu}\,. \tag{16}$$

It is therefore possible to separate $S^{\rho\mu\nu}$ in inertial and non-inertial parts. The first term on the r.h.s. of (15) gives the usual result in the particle rest frame, when the external field vanishes. From (1) we get

$$\gamma_{\mu\nu} = (f(x) - 1)\eta_{\mu\nu}\,. \tag{17}$$

To first order the tetrad is given by

$$e^\mu_{\hat{\alpha}} \approx \delta^\mu_\alpha + h^\mu_{\hat{\alpha}}\,, h^\nu_{\hat{\alpha}} = \delta^\nu_\alpha\left(\frac{1}{\sqrt{f}} - 1\right), \tag{18}$$

from which the spinorial connection can be calculated using the relations

$$\gamma^\mu(x) = e^\mu_{\hat{\alpha}}(x)\gamma^{\hat{\alpha}}\,, \qquad \Gamma_\mu(x) = -\frac{1}{4}\sigma^{\hat{\alpha}\hat{\beta}}e^\nu_{\hat{\alpha}}\nabla_\mu e_{\nu\hat{\beta}}\,. \tag{19}$$

The result is

$$\Gamma_\mu = \sigma^{\hat{\alpha}\hat{\beta}}\eta_{\alpha\mu}\left(\frac{1}{2}\ln f\right)_{,\beta}\,. \tag{20}$$

The choice $\phi = \phi_1$ corresponds to u_1 and $\phi = \phi_2$ to u_2. Substituting in (8), one finds

$$u_1 = N\begin{pmatrix} 1 \\ 0 \\ \frac{k^3}{E+m} \\ \frac{k^1+ik^2}{E+m} \end{pmatrix}, u_2 = N\begin{pmatrix} 0 \\ 1 \\ \frac{k^1-ik^2}{E+m} \\ \frac{-k^3}{E+m} \end{pmatrix},$$

that are not eigenspinors of the matrix $\Sigma^3 = \sigma^3 I$ whose eigenvalues represent the spin components in the z-direction, but become eigenspinors of Σ^3 when $k^1 = k^2 = 0$, or in the rest frame of the fermion $\mathbf{k} = 0$. By performing a transformation of coordinates $x_\mu \to x_\mu + \xi_\mu$, where ξ_μ is small of first order, we obtain $\gamma_{\mu\nu} \to \gamma_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu}$ and write the Lanczos-DeDonder condition in the form

$$\gamma^\nu_{\alpha,\nu} - \frac{1}{2}\gamma_{,\alpha} \to \gamma^\nu_{\alpha,\nu} - \frac{1}{2}\gamma_{,\alpha} - \partial_\nu\partial^\nu\xi_\alpha - f_{,\alpha} = 0\,. \tag{21}$$

By choosing ξ_α to satify $\partial_\beta\partial^\beta\xi_\alpha + f_{,\alpha} = 0$, we get $\partial^\mu\Phi_{G,\mu} = k_\alpha\Gamma^\alpha_{\mu\nu}\eta^{\mu\nu} = 0$. We also find $\partial_\alpha\partial^\alpha\Phi_{G,\beta} = k_\beta\partial_\alpha\partial^\alpha f/2\,, \Gamma^\beta_{\alpha\varrho} = \eta^{\beta\sigma}(\eta_{\sigma\alpha}f_{,\varrho} + \eta_{\sigma\varrho}f_{,\alpha} - \eta_{\alpha\varrho}f_{,\sigma})/2$, and the spinorial connection gives

$\partial^\rho \Gamma_\rho = 0$ except at $f = 0$ for which the external field approximation breaks down. With these simplifications, setting $k_1 = k_2 = k_3 = 0$ and by differentiating (14) with respect to x_ρ, we finally find

$$\begin{aligned}\partial_\rho S^{\rho\mu\nu} = \tfrac{\bar{u}_0}{16im^3} &\Big[8im^3\partial_0 B^{\mu\nu} + 4im^2\left(\Phi_{G,\alpha 0} + m\partial_0 h^0_{\hat{\alpha}}\right)\left\{\sigma^{\hat{\mu}\hat{\nu}}, \gamma^{\hat{\alpha}}\right\} \\ &-8im^2\left(k^\mu\gamma^{\hat{\nu}} - k^\nu\gamma^{\hat{\mu}}\right) + 4m^3\left(\sigma^{\hat{\mu}\hat{\nu}}\gamma^{\hat{0}}\Gamma_0 - \gamma^{\hat{0}}\Gamma_0^\dagger\sigma^{\hat{\mu}\hat{\nu}}\right) \\ &+4m^3\left(\sigma^{\hat{\mu}\hat{\nu}}\Gamma_0 - \gamma^{\hat{0}}\Gamma_0^\dagger\gamma^{\hat{0}}\sigma^{\hat{\mu}\hat{\nu}}\right) + 4im^3\partial_0 h^0_{\hat{\alpha}}\left(\gamma^{\hat{0}}\gamma^{\hat{\alpha}}\sigma^{\hat{\mu}\hat{\nu}} - \sigma^{\hat{\mu}\hat{\nu}}\gamma^{\hat{\alpha}}\gamma^{\hat{0}}\right) \\ &-4im\partial_\rho\partial^\rho\Phi_{G,\alpha}\left(\eta^{\alpha\mu}\gamma^{\hat{\nu}} - \eta^{\alpha\nu}\gamma^{\hat{\mu}}\right) - 8im^3\left(\sigma^{\hat{\mu}\hat{\nu}}\Gamma^\rho_{0\rho} + \sigma^{\hat{\alpha}\hat{\nu}}\Gamma^\mu_{\alpha 0} + \sigma^{\hat{\mu}\hat{\alpha}}\Gamma^\nu_{\alpha 0}\right)\Big] u_0\,,\end{aligned} \tag{22}$$

where $B^{\mu\nu} \equiv h^\mu_{\hat{\tau}}\sigma^{\hat{\tau}\hat{\nu}} + h^\nu_{\hat{\tau}}\sigma^{\hat{\mu}\hat{\tau}}$. The only non-vanishing components of (22) are

$$\partial_\rho S^{\rho 12} = -\frac{3f_{,0}}{4f^{\frac{3}{2}}} - f_{,0} \simeq -\frac{9}{8}f_{,0}\,, \partial_\rho S^{\rho 01} \simeq \frac{1}{4}f_{,2}\,, \partial_\rho S^{\rho 02} \simeq -\frac{1}{4}f_{,1}\,. \tag{23}$$

The derivatives of $f = 1 - (|a|/\mathcal{A})^2$, rather than f itself, are responsible for the interchange of spin and angular momentum. In fact, no interchange is possible for a strictly uniform acceleration [100]. The interchange can take place for any value of the acceleration for $0 < f < 1$. We notice that in order to alter the component S^ρ_{12}, that in the rest system of the particle corresponds to the spin density in the direction of motion, simple flow of momentum between field and particle is not sufficient. A time-dependent acceleration is necessary, and that requires that energy be transferred from the external accelerating agent. It is therefore not so much the acceleration that affects the spin-angular momentum interchange, as the way acceleration is applied. The remaining two components of $\partial_\rho S^{\rho\mu\nu}$ refer to the motion of the particle as a whole and do not require any time dependence of f. These results do not depend on any specific model for f.

3. Dispersion Relations and Particle Motion

Some considerations about particle motion in the MA model are now in order. We are not concerned with spin in this section and consider a spinless, uncharged particle for simplicity. By using Schroedinger's logarithmic transformation $\phi = e^{-iS}$ [101], we can pass from the KG Equation (9) to the quantum Hamilton-Jacobi equation. We find to $O(\gamma_{\mu\nu})$

$$i(\eta^{\mu\nu} - \gamma^{\mu\nu})\partial_\mu\partial_\nu S - (\eta^{\mu\nu} - \gamma^{\mu\nu})\partial_\mu S\partial_\nu S + m^2 = 0\,, \tag{24}$$

where

$$S = k^\beta\left\{x_\beta + \frac{1}{2}\int^x dz^\lambda\gamma_{\beta\lambda}(z) - \frac{1}{2}\int^x dz^\lambda\left(\gamma_{\alpha\lambda,\beta}(z) - \gamma_{\beta\lambda,\alpha}(z)\right)(x^\alpha - z^\alpha)\right\}. \tag{25}$$

It is known that the Hamilton-Jacobi equation is equivalent to Fresnel's wave equation in the limit of large frequencies [101]. However, at smaller, or moderate frequencies the complete Equation (24) should be used. We follow this path. By substituting (25) into the first term of (24), we obtain

$$i(\eta^{\mu\nu} - \gamma^{\mu\nu})\partial_\mu\partial_\nu S = i\eta^{\mu\nu}\partial_\mu(k_\nu + \Phi_{G,\nu}) - i\gamma^{\mu\nu}\partial_\mu k_\nu = i\eta^{\mu\nu}\Phi_{G,\mu\nu} = ik_\alpha\eta^{\mu\nu}\Gamma^\alpha_{\mu\nu} = 0\,, \tag{26}$$

on account of (21). This part of (24) is usually neglected in the limit $\hbar \to 0$. Here it vanishes as a consequence of (25). The remaining terms of (24) yield the classical Hamilton-Jacobi equation

$$(\eta^{\mu\nu} - \gamma^{\mu\nu})\partial_\mu S\partial_\nu S - m^2 = \gamma^{\mu\nu}k_\mu k_\nu - 2k^\mu\Phi_{G,\mu} = 0\,, \tag{27}$$

because $k^\mu\Phi_{G,\mu} = 1/2\gamma^{\mu\nu}k_\mu k_\nu$. Equation (12) is therefore a solution of the more general quantum Equation (24). It also follows that the particle acquires a generalized "momentum"

$$P_\mu = k_\mu + \Phi_{G,\mu} = k_\mu + \frac{1}{2}\gamma_{\alpha\mu}k^\alpha - \frac{1}{2}\int^x dz^\lambda\left(\gamma_{\mu\lambda,\beta}(z) - \gamma_{\beta\lambda,\mu}(z)\right)k^\beta\,, \tag{28}$$

that describes the geometrical optics of particles correctly and gives the correct deflection predicted by general relativity. It is Feynman's "p-momentum" in the case of gravity and gravity-like fields. On using the relation $\Phi_{G,\mu\nu} = k_\alpha \Gamma^\alpha_{\mu\nu}$ and differentiating (28) we obtain the covariant derivative of P_μ

$$\begin{aligned} \frac{DP_\mu}{Ds} &= m\left[\frac{du_\mu}{ds} + \frac{1}{2}\left(\gamma_{\alpha\mu,\nu} - \gamma_{\mu\nu,\alpha} + \gamma_{\alpha\nu,\mu}\right) u^\alpha u^\nu\right] \\ &= m\left(\frac{du_\mu}{ds} + \Gamma_{\alpha,\mu\nu} u^\alpha u^\nu\right) = \frac{Dk^\mu}{Ds}. \end{aligned} \tag{29}$$

This result is independent of any choice of field equations for $\gamma_{\mu\nu}$. We see from (29) that if k_μ follows a geodesic, then $DP_\mu/Ds = 0$ and $D(P_\alpha P^\alpha)/Ds = 0$. The classical equations of motion are therefore contained in (29), but it would require the particle described by (9) to just choose a geodesic, among all paths allowed to a quantum particle. It also follows from (28) that

$$\frac{D(P_\alpha P^\alpha)}{Ds} = 2m^2 \frac{df}{ds}, \tag{30}$$

and that, therefore, $P_\alpha D(P^\alpha/Ds) \neq 0$. For massless bosons, however, P_α and DP_α/Ds are still orthogonal. Remarkably, (28) is an exact integral of (29) which can itself be integrated to give the particle's motion

$$X_\mu = x_\mu + \frac{1}{2}\int^x dz^\lambda \left\{\gamma_{\mu\lambda} - \left(\gamma_{\alpha\lambda,\mu} - \gamma_{\mu\lambda,\alpha}\right)\left(x^\alpha - z^\alpha\right)\right\}. \tag{31}$$

By substituting the explicit espressions for $\gamma_{\mu\nu}$ in (28) and (31), we obtain

$$P_\mu = k_\mu + \frac{m}{2}\int^s ds f_{,\mu}\ , X_\mu = x_\mu + \frac{1}{2m}\int^s ds \left\{k_\mu (f-1) - k_\alpha \left(x^\alpha - z^\alpha\right) f_{,\mu} + k_\mu f_{,\alpha}\left(x^\alpha - z^\alpha\right)\right\}. \tag{32}$$

There is no back reaction to the motion described by (31) and (28). The field experienced by the particle is its own acceleration and that takes into account any back reaction automatically. This can also be seen as follows. The back reaction is normally introduced by means of a four-vector [102] g_μ such that $g_\mu P^\mu = 0$. A natural choice for g_μ is

$$g_\mu = m^2 \frac{D^2 P_\mu}{Ds^2} - P_\mu P_\alpha \frac{D^2 P\alpha}{Ds^2} \approx m^2 \frac{D^2 P_\mu}{Ds^2} - k_\mu k_\alpha \frac{D^2 P^\alpha}{Ds^2} \tag{33}$$

to $O(\gamma_{\alpha\beta})$. We obtain, in fact,

$$g_\mu = m^2\left[\Phi_{G,\mu\nu\alpha} u^\alpha u^\nu + \Phi_{G,\mu\nu}\frac{du^\nu}{ds}\right] - k_\mu k^\alpha \left[\Phi_{G,\alpha\nu\sigma} u^\sigma u^\nu + \Phi_{G,\alpha\nu}\frac{du^\nu}{ds}\right], \tag{34}$$

from which $g_\mu P^\mu = 0$ follows. By substituting (17) in (34), we obtain

$$g_\mu = \frac{m^2}{2}\left(\frac{df_{,\mu}}{ds} - k_\mu \frac{d^2 f}{ds^2}\right), \tag{35}$$

which along a particle world-line gives $g_\mu k^\mu/m = 0$ and also $\int g_\mu dx^\mu = 0$, as expected, because no energy-momentum dissipation mechanism is provided. The situation would of course change if the particle were charged.

4. Conclusions

We have investigated a particle spin at accelerations close to the MA limit. The model incorporates MA by assuming that particles are subjected to a metrical field that has a gravity-like behaviour that changes from particle to particle according to a particle's mass. The model also provides a comprehensive framework to treat accelerations with values between the classical and the MA limits.

The solutions of the covariant Dirac equation, consisting of plane wave solutions of the free Dirac equation and appropriate spin terms, have been applied to the third rank spin current tensor. The calculations are performed in the rest frame of the fermion and confirm that continual interchange between spin and angular momentum occurs in the case of MA fields, but only if the acceleration is time-dependent ($f_{,0} \neq 0$). This requires that a transfer of energy from the agent responsible for the acceleration, say a very compact star, or a black hole and the particle take place because the field is not stationary. The non vanishing components $\partial_\rho S^{\rho 01}$ and $\partial_\rho S^{\rho 02}$ of the spin current tensor refer to the motion of the particle as a whole and are present whatever the nature of the acceleration. Even in the case of MA, uniform acceleration produces no observable effects on the particle spin, in agreement with [100]. These results are independent of any model for f and provide a stringent limit on the the astrophysical applications of spin physics, even in the case of MA.

The back reaction of the MA field on the particle vanishes along the particle world-line and momentum cannot therefore be dissipated unless the particle is charged, or another specific radiation mechanism is provided.

Conflicts of Interest: The author declare no conflict of interest.

References

1. Ashby, N. Relativity in the global positioning system. *Liv. Rev. Relativ.* **2003**, *6*, 1.
2. Demirel, B.; Sponar, S.; Hasegawa, Y. Measurement of the spin-rotation coupling in neutron polarimetry. *New J. Phys.* **2015**, *17*, 023065.
3. Mashhoon, B. Neutron interferometry in a rotating frame of reference. *Phys. Rev. Lett.* **1988**, *61*, 2639.
4. Mashhoon, B. Limitations of spacetime measurements. *Phys. Lett. A* **1990**, *143*, 176.
5. Mashhoon, B. The hypothesis of locality in relativistic physics. *Phys. Lett. A* **1990**, *145*, 147–153.
6. Papini, G. Spin–gravity coupling and gravity-induced quantum phases. *Gen. Relativ. Grav.* **2008**, *40*, 1117–1144.
7. Everitt, C.W.F.; DeBra, D.B.; Parkinson, B.W.; Turneaure, J.P.; Conklin, J.W.; Heifetz, M.I.; Keiser, G.M.; Silbergleit, A.S.; Holmes, T.; Kolodziejczak, J.; et al. Gravity Probe B: Final results of a space experiment to test general relativity. *Phys. Rev. Lett.* **2011**, *106*, 221101.
8. Iorio, L.; Ruggiero, M.L.; Corda, C. Novel considerations about the error budget of the LAGEOS-based tests of frame-dragging with GRACE geopotential models. *Acta Astronaut.* **2013**, *91*, 141–148.
9. Iorio, L.; Lichtenegger, H.I.M.; Ruggiero, M.L.; Corda, C. Phenomenology of the Lense-Thirring effect in the Solar System. *Astrophys. Space Sci.* **2011**, *331*, 351–395.
10. Grandy, W.T., Jr. *Relativistic Quantum Mechanics of Leptons and Fields*; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1990.
11. Papini, G. Spin-rotation coupling in compound spin objects. *Phys. Lett. A* **2013**, *377*, 960–963.
12. Caianiello, E.R. Geometry from quantum mechanics. *Nuovo Cimento B* **1980**, *59*, 350–366.
13. Caianiello, E.R. Is there a maximal acceleration? *Lett. Nuovo Cimento* **1981**, *32*, 65–70.
14. Caianiello, E.R. Quantum and other physica as systems theory. *Riv. Nuovo Cimento* **1992**, *15*, 1–65.
15. Caianiello, E.R. Maximal acceleration as a consequence of Heisenberg's uncertainty relations. *Lett. Nuovo Cimento* **1984**, *41*, 370–372.
16. Wood, W.R.; Papini, G.; Cai, Y.Q. Maximal Acceleration and the Time-Energy Uncertainty Relation. *Nuovo Cimento B* **1989**, *104*, 361–369.
17. Papini, G. Revisiting Caianiello's Maximal Acceleration. *Nuovo Cimento B* **2002**, *117*, 1325–1331.
18. Das, A. Extended phase space. I. Classical fields. *J. Math. Phys.* **1980**, *21*, 1506.
19. Caianiello, E.R.; Vilasi, G. Extended Dirac particles and their spectra in curved phase-space. *Lett. Nuovo Cimento* **1981**, *30*, 469–473.
20. Caldirola, P. On the existence of a maximal acceleration in the relativistic theory ofelectron. *Lett. Nuovo Cimento* **1981**, *32*, 264–266.
21. Caianiello, E.R.; de Filippo, S.; Vilasi, G. Extended Dirac particles and their spectra in curved phase-space. *Lett. Nuovo Cimento* **1982**, *33*, 555–558.

22. Caianiello, E.R.; de Filippo, S.; Marmo, G.; Vilasi, G. Remarks on the maximal-acceleration hypothesis. *Lett. Nuovo Cimento* **1982**, *34*, 112–114.
23. Caianiello, E.R.; Landi, G. Maximal acceleration and Sakharov's limiting temperature. *Lett. Nuovo Cimento* **1985**, *42*, 70–72.
24. Caianiello, E.R.; Marmo, G.; Scarpetta, G. (Pre)quantum geometry. *Nuovo Cimento A* **1985**, *86*, 337–355.
25. Sharma, C.S.; Srirankanatham, S. On Caianiello's maximal acceleration. *Lett. Nuovo Cimento* **1985**, *44*, 275–276.
26. Gasperini, M. Very early cosmology in the maximal acceleration hypothesis. *Astrophys. Space Sci.* **1987**, *138*, 387–391.
27. Toller, M. Test particles with acceleration-dependent Lagrangian. *J. Math. Phys.* **2006**, *47*, 022904.
28. Toller, M. A Theory of Gravitation Covariant under Sp(4,R). *arXiv* **2017**, arXiv:hep-th/0312016.
29. Toller, M. Lagrangian and Presymplectic Particle Dynamics with Maximal Acceleration. *arXiv* **2004**, arXiv:hep-th/0409317.
30. Toller, M. Theories with limited acceleration: Free fields. *Nuovo Cimento B* **1988**, *102*, 261–308.
31. Toller, M. Maximal acceleration, maximal angular velocity and causal influence. *Int. J. Theor. Phys.* **1990**, *29*, 963–984.
32. Toller, M. Supersymmetry and maximal acceleration. *Phys. Lett. B* **1991**, *256*, 215–217.
33. Falla, D.F.; Landsberg, P.T. A black-hole minimum mass. *Nuovo Cimento B* **1991**, *106*, 669–671.
34. Pati, A.K. On the maximal acceleration and the maximal energy loss. *Nuovo Cimento B* **1992**, *107*, 895–901.
35. Pati, A.K. A note on maximal acceleration. *Europhys. Lett.* **1992**, *18*, 285.
36. Parentani, R.; Potting, R. Accelerating observer and the Hagedorn temperature. *Phys. Rev. Lett.* **1989**, *63*, 945.
37. Rama, S.K. Classical velocidy in κ-deformed Poincaré algebra and maximum aceleration. *Mod. Phys. Lett. A* **2003**, *18*, 527.
38. Schuller, F. Born-Infeld kinematics. *Ann. Phys.* **2002**, *299*, 174–207.
39. Schuller, F. Born-Infeld kinematics and correction to the Thomas precession. *Phys. Lett. B* **2002**, *540*, 119–124.
40. Schuller, F.; Wohlfarth, N.R.; Grimm, T.W. Pauli-Villars regularization and Born-Infeld kinematics. *Class. Quant. Grav.* **2003**, *20*, 4269.
41. Torrome, R.G. On a covariant version of Caianiello's Model. *arXiv* **2007**, arXiv:gr-qc/0701091.
42. Torrome, R.G. Emergent phenomenology from deterministic Cartan-Randers systems. *arXiv* **2005**, arXiv:gr-qc/0501094.
43. Friedman, Y.; Gofman, Y. A new relativistic kinematics of accelerated systems. *arXiv* **2005**, arXiv:gr-qc/0509004.
44. Misner, C.W.; Thorne, K.S.; Wheeler, J.A. *Gravitation*; W.H. Freeman and Company: San Francisco, CA, USA, 1973.
45. Brandt, H.E. Maximal proper acceleration relative to the vacuum. *Lett. Nuovo Cimento* **1983**, *38*, 522–524.
46. Brandt, H.E. Maximal proper acceleration and the structure of spacetime. *Found. Phys. Lett.* **1989**, *2*, 39–58.
47. Papini, G.; Wood, W.R. Maximal Acceleration of Thin Shells in Weyl Space. *Phys. Lett. A* **1992**, *170*, 409–412.
48. Wood, W.R.; Papini, G. Breaking Weyl Invariance in the Interior of a Bubble. *Phys. Rev. D* **1992**, *45*, 3617.
49. Wood, W.R.; Papini, G. A Geometric Formulation of the Causal Interpretation of Quantum Mechanics. *Found. Phys. Lett.* **1993**, *6*, 207–223.
50. Papini, G. Maximal Proper Acceleration. *Math. Jpn.* **1995**, *41*, 81.
51. Vigier, J.P. Explicit mathematical construction of relativistic nonlinear De Broglie waves described by three-dimensional (wave and electromagnetic) solitons "piloted" (controlled) by corresponding solutions of associated linear Klein-Gordon and Schrödinger equations. *Found. Phys.* **1991**, *21*, 125–148.
52. McGuigan, M. Finite black hole entropy and string theory. *Phys. Rev. D* **1994**, *50*, 5225.
53. Nesterenko, V.V.; Feoli, A.; Lambiase, G.; Scarpetta, G. Regularizing properties of the maximal acceleration principle in quantum field theory. *Phys. Rev. D* **1998**, *60*, 065001.
54. Breckenridge, J.C.; Elias, V.; Steele, T.G. Massless scalar field theory in a quantized spacetime. *Class. Quantum Grav.* **1995**, *12*, 637.
55. Sanchez, N.; Veneziano, G. Jeans-like instabilities for strings in cosmological backgrounds. *Nucl. Phys. B* **1990**, *333*, 253–266.
56. Gasperini, M.; Sanchez, N.; Veneziano, G. Self-sustained inflation and dimensional reduction from fundamental strings. *Nucl. Phys. B* **1991**, *364*, 365–380.

57. Gasperini, M.; Sanchez, N.; Veneziano, G. Highly unstable fundmental strings in inflationary cosmologies. *Int. J. Mod. Phys. A* **1991**, *6*, 3853.
58. Gasperini, M. Kinematic interpretation of string instabilities in a background gravitational field. *Phys. Lett. B* **1991**, *258*, 70–74.
59. Gasperini, M. Causal horizons and strings. *Gen. Rel. Grav.* **1992**, *24*, 219.
60. Frolov, V.P.; Sanchez, N. Instability of accelerated strings and the problem of limiting acceleration. *Nucl. Phys. B* **1991**, *349*, 815–838.
61. Sanchez, N. *Structure: From Physics to General Systems*; Marinaro, M., Scarpetta, G., Eds.; World Scientific: Singapore, 1993; Volume 1, p. 118.
62. Caianiello, E.R.; Gasperini, M.; Scarpetta, G. Inflation and singularity prevention in a model for extended-object-dominated cosmology. *Class. Quantum Grav.* **1991**, *8*, 659.
63. Gasperini, M. *Advances in Theoretical Physics*; Caianiello, E.R., Ed.; World Scientific: Singapore, 1991; p. 77.
64. Capozziello, S.; Lambiase, G.; Scarpetta, G. Cosmological perturbations in singularity-free deflationary models. *Nuovo Cimento B* **1999**, *114*, 93–104.
65. Feoli, A. String dynamics in Rindler spacein a model with maximal acceleration. *Nucl. Phys. B* **1993**, *396*, 261–269.
66. Capozziello, S.; Lambiase, G.; Scarpetta, G. Generalized uncertainty principle from quantum geometry. *Int. J. Theor. Phys.* **2000**, *39*, 15–22.
67. Caianiello, E.R.; Gasperini, M.; Scarpetta, G. Phenomenological consequences of a geometric model with limited proper acceleration. *Nuovo Cimento B* **1990**, *105*, 259–278.
68. Bozza, V.; Lambiase, G.; Papini, G.; Scarpetta, G. Quantum Violations of the Equivalence Principle in a modified Schwarzschild Geometry. Neutrino Oscillations. *Phys. Lett. A* **2001**, *279*, 163–168.
69. Bozza, V.; Capozziello, S.; Lambiase, G.; Scarpetta, G. Neutrino oscillations in Caianiello quantum geometry model. *Int. J. Theor. Phys.* **2001**, *40*, 849–859.
70. Lambiase, G.; Papini, G.; Punzi, R.; Scarpetta, G. Lower neutrino mass bound from SN1987A data and quantum geometry. *Class. Quant. Grav.* **2006**, *23*, 1347.
71. Gibbons, G.; Hawking, S.W. Cosmological event horizons, thermodynamics and particle creation. *Phys. Rev. D* **1977**, *15*, 2738.
72. Feoli, A.; Lambiase, G.; Papini, G.; Scarpetta, G. Schwarzschild Field with Maximal Acceleration Corrections. *Phys. Lett. A* **1999**, *263*, 147–153.
73. Capozziello, S.; Feoli, A.; Lambiase, G.; Papini, G.; Scarpetta, G. Massive Scalar Particles in a Modified Schwarzschild Geometry. *Phys. Lett. A* **2000**, *268*, 247–254.
74. Bozza, V.; Feoli, A.; Papini, G.; Scarpetta, G. Maximal Acceleration Effects in Reissner-Nordstrom Space. *Phys. Lett. A* **2000**, *271*, 35–43.
75. Bozza, V.; Feoli, A.; Lambiase, G.; Papini, G.; Scarpetta, G. Maximal Acceleration Effects in Kerr Space. *Phys. Lett. A* **2001**, *283*, 53–61.
76. Papini, G.; Scarpetta, G.; Bozza, V.; Feoli, A.; Lambiase, G. Radiation Bursts from Particles in the Field of Compact, Impenetrable Astrophysical Objects. *Phys. Lett. A* **2002**, *300*, 603–610.
77. Feoli, A.; Lambiase, G.; Papini, G.; Scarpetta, G. Classical Electrodynamics of a Particle with Maximal Acceleration Corrections. *Nuovo Cimento B* **1997**, *112*, 913–922.
78. Lambiase, G.; Papini, G.; Scarpetta, G. Maximal Acceleration Limits on the Mass of the Higgs Boson. *Nuovo Cimento B* **1999**, *114*, 189–197.
79. Kuwata, S. Higgs-boson mass and the modification of the Higgs-fermion interaction owing to the existence of a maximal acceleration. *Nuovo Cimento B* **1996**, *111*, 893–899.
80. Lambiase, G.; Papini, G.; Scarpetta, G. Maximal Acceleration Corrections to the Lamb Shift of Hydrogen, Deuterium and He+. *Phys. Lett. A* **1998**, *244*, 349–354.
81. Chen, C.X.; Lambiase, G.; Mobed, G.; Papini, G.; Scarpetta, G. Helicity and Chirality Precessions of Maximally Accelerated Fermions. *Nuovo Cimento B* **1999**, *114*, 1335–1343.
82. Chen, C.X.; Lambiase, G.; Mobed, G.; Papini, G.; Scarpetta, G. Maximal Acceleration Corrections to the Lamb Shift of Muonic Hydrogen. *Nuovo Cimento B* **1999**, *114*, 199–205.
83. Benedetto, E.; Feoli, A. Unruh temperature with maximal acceleration. *Mod. Phys. Lett. A* **2015**, *30*, 1550075.
84. Rovelli, C.; Vidotto, F. Evidence for maximal acceleration and singularity resolution in covariant loop quantum gravity. *Phys. Rev. Lett.* **2013**, *111*, 091303.

85. Rovelli, C.; Vidotto, F. *Covariant Loop and Quantum Gravity*; Cambridge University Press: Cambridge, UK, 2015.
86. Papini, G. Spin-Gravity Coupling and Gravity-Induced Quantum Phases. *Gen. Relativ. Gravit.* **2008**, *40*, 1117–1144.
87. Cai, Y.Q.; Papini, G. Particle Interferometry in Weak Gravitational Fields. *Class. Quantum Grav.* **1989**, *6*, 407.
88. Cai, Y.Q.; Papini, G. Neutrino Helicity Flip from Gravity-Spin Coupling. *Phys. Rev. Lett.* **1991**, *66*, 1259.
89. Papini, G.; Scarpetta, G.; Feoli, A.; Lambiase, G. Optics of Spin-1 Particles from Gravity-Induced Phases. *Int. J. Mod. Phys. D* **2009**, *18*, 485.
90. Papini, G. Spin-2 Particles in Gravitational Fields. *Phys. Rev. D* **2007**, *75*, 044022.
91. Lambiase, G.; Papini, G.; Punzi, R.; Scarpetta, G. Neutrino Optics and Oscillations in Gravitational Fields. *Phys. Rev. D* **2005**, *71*, 073011.
92. Papini, G. Radiative Processes in External Gravitational Fields. *Phys. Rev. D* **2010**, *82*, 024041.
93. Papini, G. Fermion antifermion mixing in gravitational fields. *Mod. Phys. Lett. A* **2013**, *28*, 1350071.
94. De Oliveira, C.G.; Tiomno, J. Representations of Dirac equation in general relativity. *Nuovo Cimento* **1962**, *24*, 672–687.
95. Colella, R.; Overhauser, A.W.; Werner, S.A. Observation of gravitationally induced quantum interferometry. *Phys. Rev. Lett.* **1975**, *34*, 1472.
96. Page, L.A. Effect of Earth's rotation in neutron interferometry. *Phys. Rev. Lett.* **1975**, *35*, 543. Werner, S.A.; Staudenmann, J.L.; Colella, R. Effect of Earth's rotation on the quantum mechanical phase of the neutron. *Phys. Rev. Lett.* **1979**, *42*, 1103.
97. Bonse, V.; Wroblewski, T. Measurement of the neutron quantum interference in noninertial frames. *Phys. Rev. Lett.* **1983**, *51*, 1401.
98. Hehl, F.W.; Ni, W.-T. Inertial effects of a Dirac particle. *Phys. Rev. D* **1990**, *42*, 2045.
99. Singh, D.; Papini, G. Spin-1/2 Particles in Noninertial Reference Frames: Low- and High-Energy Approximations. *Nuovo Cimento B* **2000**, *115*, 223.
100. Bini, D.; Cherubini, Ch.; Mashhoon, B. Spin, acceleration and gravity. *Class. Quantum Grav.* **2004**, *21*, 3893.
101. Lanczos, C. *The Variational Principles Of Mechanics*; Dover Publications: New York, NY, USA, 1970.
102. Landau, L.D.; Lifshitz, E.M. *The Classical Theory of Fields*; Pergamon Press: Oxford, UK, 1975.