

Article

A Simple Model for Explaining Galaxy Rotation Curves

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Abstract: A new simple expression for the circular velocity of spiral galaxies is proposed and tested against HI Nearby Galaxy Survey (THINGS) data set. Its accuracy is compared with the one coming from MOND.

Keywords: rotation curves; spiral galaxies; circular velocity; THINGS; MOND

1. Introduction

The current Standard Cosmological Model, the so-called Λ CDM model [1,2], together with astronomical observations, indicates that there is about 30% of dark matter which we know exists. From it we are able to detect only 20% which is baryonic described by the Standard Model of particle physics. The rest of it is so-called Dark Matter [3–6] which is supposed to explain the flatness of rotational galaxies' curves [7–11].

Nowadays, there are two main competing ideas for explaining the Dark Matter problem. The first one consists in modifying the geometric part of the gravitational field equations (see e.g., [12–15]) while the other one introduces weakly interacting particles which have failed to be detected [16]. Despite this, it is also believed that these two ideas do not contradict each other and could be combined together in some future successful theory.

If Dark Matter exists, it interacts only gravitationally with visible parts of our universe, and it seems to also have an effect on the large scale structure of our Universe [17,18]. There are some models which have faced the problem of this unknown ingredient. The famous one is called Modified Newtonian Dynamics (MOND) [19–25]—it has already predicted many galactic phenomena and this is why it is very popular among astrophysicists. It has already a relativistic version: the so-called Tensor/Vector/Scalar (TeVeS) theory of gravity [26,27]. The MOND result is also obtained when one considers an effect caused by a reaction of dark energy to the presence of baryonic particles [28,29]. Another approach is to consider Extended Theories of Gravity (ETGs) in which one modifies the geometric part of the field equations [30–32]. There were also attempts to obtain MOND result from ETGs, see for example [33–38]. The Weyl conformal gravity [39–41] is another interesting proposal for explaining rotation curves. Moreover, we would also like to mention the existence of a model based on large scale renormalization group effects and a quantum effective action [42–44]. In this work we will not consider any concrete theory of gravitation from which we provide the equation ruling the motion of galactic stars. Starting from the standard form of the geodesic equation, a formula for the rotational velocity will be derived. We will also present how our simple model matches the astrophysical data and how it possesses some similarities to the ones appearing in the literature. At the end we will draw our conclusions. The metric signature convention is $(-, +, +, +)$.

2. Proposed Model

The standard expression of the quadratic velocity for a star moving on a circular trajectory around the galactic center is simply obtained from the GR in the weak field and small velocity approximations. One assumes that the orbit of a star in a galaxy is circular which is in a good agreement with astronomical observations [45]. Thus the relation between the centripetal acceleration and the velocity is simply:

$$a = -\frac{v^2}{r}. \quad (1)$$

A test particle as we treat a single star in our considerations satisfies the geodesic equation

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\nu\sigma}^\mu \frac{dx^\nu}{ds} \frac{dx^\sigma}{ds} = 0. \quad (2)$$

Although the velocity of stars moving around the galactic center is very high, when compared with the speed of light, it turns out that they are still much smaller so we deal with the condition $v \ll c$. It means that in the spherical-symmetric parametrization the velocities satisfy

$$v^i = \left(\frac{dr}{dt}, r \frac{d\theta}{dt}, r \sin \theta \frac{d\varphi}{dt} \right) \ll \frac{dx^0}{dt}, \quad (3)$$

where $x^0 = ct$. Taking into account Equation (3) and considering the weak field limit of Equation (2) together with $\Gamma_{00}^0 = 0$ (static spacetime), we obtain

$$\frac{d^2 x^r}{dt^2} = -c^2 \Gamma_{00}^r. \quad (4)$$

Inserting Equation (4) into (1) one gets

$$v^2(r) = rc^2 \Gamma_{00}^r = r \frac{d\Phi(r)}{dr} \quad (5)$$

with $\Phi(r)$ being a Newtonian potential (see for example [46]) such that finally we have

$$v^2(r) = \frac{GM}{r} \quad (6)$$

where G is gravitational constant while the mass M is usually assumed to be r -dependent, that is, one deals with some matter distribution depending on a concrete model. Let us assume the following simple distribution of mass in a galaxy [47]

$$M(r) = M_0 \left(\sqrt{\frac{R_0}{r_c}} \frac{r}{r + r_c} \right)^{3\beta} \quad (7)$$

with M_0 the total galaxy mass, r_c the core radius and R_0 the observed scale length of the galaxy. The matter distribution in Equation (7) without the term containing the square root was also used in Reference [48]. Since the GR prediction on the shapes of galaxies' curves coming from (6) failed against the observation data, one looks for some modification. The first one which appears in one's mind is to consider a bit more complicated mass distribution which can also include Dark Matter halo in his form as well as different galaxy structure, for example disk, or other shapes.

We would like to perform a bit of a different approach, that is, let us modify the geometry part by, for example, considering effective quantities that could be obtained from Extended Theories of Gravity. There are many works following this approach which inspired us to examine a below toy

model. The most interesting ones which do not assume the existence of any Dark Matter according to the authors are the following:

- The Modified Newtonian Dynamics (MOND) [22] (see also similar result in [49] and reviews in [19,24,25,38]). It is the most spread modification among astronomers since it is very simple, does not include any exotic ingredients (Dark Matter) and the most important, it is in good agreement with observations. The MOND velocity is given by

$$v^2(r) = \frac{GM}{r} \frac{1}{\sqrt{2}} \left[1 + \sqrt{1 + r^4 \left(\frac{2a_0}{GM} \right)^2} \right]^{1/2} \quad (8)$$

where $a_0 \approx 1.2 \times 10^{-10} \text{ ms}^{-2}$ is the critical acceleration. Equation (8) is obtained from the Milgrom's acceleration formula

$$a = \frac{MG}{r^2} \mu \left(\frac{MG}{r^2 a_0} \right) \quad (9)$$

using the standard interpolation function

$$\mu(x) = \frac{x}{\sqrt{1+x^2}} \quad (10)$$

In the limit $a_{\text{Newt}} \gg a_0$, the MOND formalism gives asymptotic constant velocities

$$v_c^2 = \sqrt{a_0 GM}. \quad (11)$$

- Coming from $f(R)$ gravity (metric formalism) examined by [32,50]. Here, they used the ansatz $f(R) \sim R^n$, to obtain:

$$v^2(r) = \frac{GM}{2r} \left[1 + (1-\beta) \left(\frac{r}{r_c} \right)^\beta \right] \quad (12)$$

where β is a function of the slope n of the Lagrangian while r_c is a scale length depending on gravitational system properties.

- Given by Scalar-Vector-Tensor Gravity [27,48] which is in very good agreement with the RC Milky Way data

$$v^2(r) = \frac{GM}{r} [1 + \alpha - \alpha(1 + \mu r)e^{-\mu r}] \quad (13)$$

where the two free parameters allow the fitting of galaxy rotation curves.

- Our previous result [47], coming from Starobinsky model [51] $f(\hat{R}) = \hat{R} + \gamma \hat{R}^2$ considered in Palatini formalism which is the simplest example of EPS interpretation [52]

$$v^2 \approx \frac{GM(r)}{r} \left(1 + \frac{2GM(r)}{c^2 r} - \frac{2\pi\kappa\gamma c^2 r^3 \rho^2}{M(r)(1 + 2\kappa\gamma c^2 \rho)^2} \right), \quad (14)$$

where we assumed the order of γ as 10^{-10} taken from cosmological considerations [53], ρ is energy density obtained from mass distribution provided by the model and (7), see the details in [47].

We immediately observe that all these modifications coming from different models of gravity possess a feature which can be simply written as

$$v^2(r) = \frac{GM}{r} (1 + A(r)) \quad (15)$$

where the unknown function $A(r)$ depends on the radial coordinate and some parameters. In this manner, the function $A(r)$ is treated as a deviation from the Newtonian limit of General Relativity.

Our task now is to find a suitable function $A(r)$ which takes into account and reproduces the observed flatness of galaxy rotation curves. Moreover, at short distances (at least the size of the Solar System) the velocity from Equation (15) should have as a limit the Newtonian result $v^2(r) = GM/r$. This imposes some constraints on the function $A(r)$.

3. A Particular Example

We have seen in the previous section that there are many alternatives to General Relativity which possess extra terms that improve the behavior of the galaxy curves. Moreover, many of them can have the same weak field limit producing the same result (15). Thus, one can explain the observed galaxy rotation curves using the Equation (15) without the assumption on the existence of Dark Matter.

In this section we would like to propose a model for fitting the galaxy rotation curves data observed astronomically. As we will see, the model fits quite well the data set of galaxies obtained from THINGS: The HI Nearby Galaxy Survey catalogue [54,55], on which our analysis is performed.

A very simple model that fits well the data (as can be seen from Figures 1 and 2) is obtained by choosing

$$A(r) = b \left(\frac{r + r_0}{r_0} \right) \quad (16)$$

where b and r_0 are two parameters. The idea of including additionally $1/r$ term in the gravitational force without or with the Yukawa potential, which improves the behaviour of the theoretically obtained curves, was already discussed in [56–59].

Inserting the Equation (16) into the velocity Formula (15) we obtain

$$v^2(r) = \frac{GM}{r} \left[1 + b \left(1 + \frac{r}{r_0} \right) \right]. \quad (17)$$

In the non-relativistic limit, the circular velocity and the gravitational potential are related through the usual formula $v^2(r) = r \frac{d\Phi}{dr}$, from which it follows immediately that

$$\Phi(r) = -\frac{GM}{r} \left\{ 1 + b \left[1 - \frac{r}{r_0} \ln \left(\frac{r}{r_0} \right) \right] \right\}. \quad (18)$$

The dependence on $\ln(r/r_0)$ in the potential was also reported in References [42–44,49,60]. Moreover, we observe that in the limit $b \rightarrow 0$ both Equations (17) and (18) reduce to their usual Newtonian expressions. It should also be mentioned that the considered form (17) can cause problems with lensing effect if one finds out the relativistic version of the model. There is a proposed mechanism in [61] which allows to interpolate between the short and long distance ranges in order to avoid the divergence of the effective mass when $r \rightarrow \infty$. It also takes into account the necessity of rescaling the gravitational mass (which appears because of the constant b) in order to save the desired properties of the model when we consider it in the case of the Solar System.

Using the matter distribution (7) and identifying the parameter r_0 contained in the Equation (17) with the galaxy scale length R_0 , the final rotational velocity of stars moving in circular orbits is

$$v^2(r) = \frac{GM_0}{r} \left(\sqrt{\frac{R_0}{r_c}} \frac{r}{r + r_c} \right)^{3\beta} \left[1 + b \left(1 + \frac{r}{R_0} \right) \right] \quad (19)$$

One can immediately deduce an important feature of the above formula, namely that in the limit of large radii we obtain flat rotation curves, similar to what happens in MOND theories [22–24] (see also Equation (11) above)

$$v_0 = \sqrt{\frac{GMb}{R_0} \left(\frac{R_0}{r_c} \right)^{3\beta/2}} \quad (20)$$

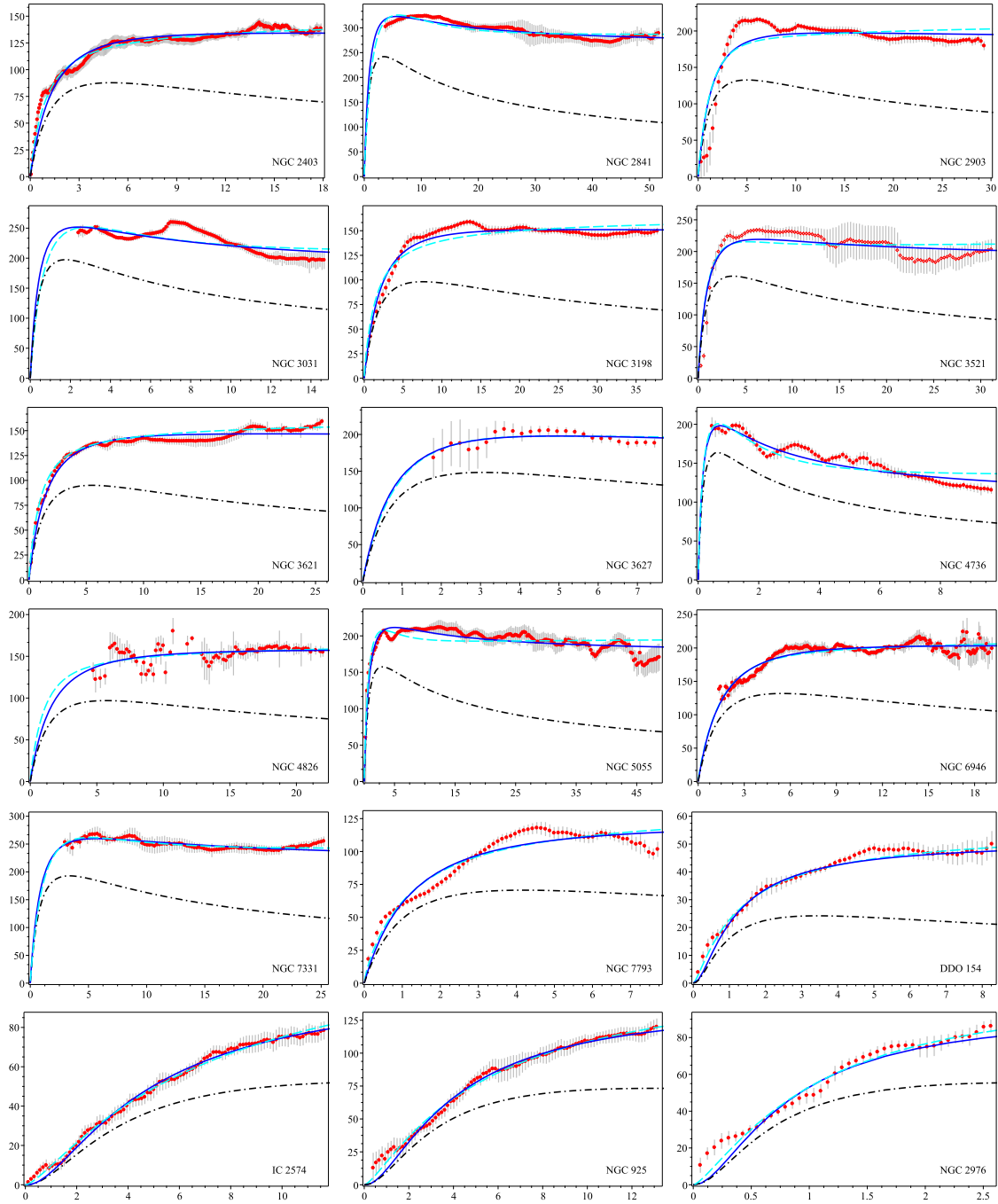


Figure 1. (color online) Rotational velocities in km/s (y axis) at a certain distance in kpc (x axis) from the center of the galaxy. The blue curves RC are obtained from the parametric fit of Equation (19) in the case of 18 THINGS galaxies. The proprieties of the galaxies in the sample can be found in Table I from Reference [55]. The full (blue) curve are the rotation curves obtained using Equation (19); the (red) full circles are the observed data points where the vertical (grey) lines represent the error bars; the contribution due to the Newtonian term is given by the dash-dotted (black) lines, while the dashed (cyan) lines give the MOND rotation curves. The numerical values resulted from the fits are given in Table 1.

From the analysis of the 18 THINGS galaxies sample we have found $b = 0.352 \pm 0.08$ to give good fit results for the rotation curves. The plots in Figure 1 and the best fit results from Table 1 are obtained using the value $b = 0.352$. As in [48] the value $\beta = 1$ (for HSB galaxies) and $\beta = 2$ (LSB galaxies) give good fit results. By allowing β to be a free parameter, slightly better fits results can be obtained. In this

case a preliminary analysis indicates that $0.75 < \beta < 1.25$ for HSB galaxies and $1.9 < \beta < 2.1$ for LSB galaxies. However, in order to keep the free parameters to a minimum we have chosen here to fix the value of β .

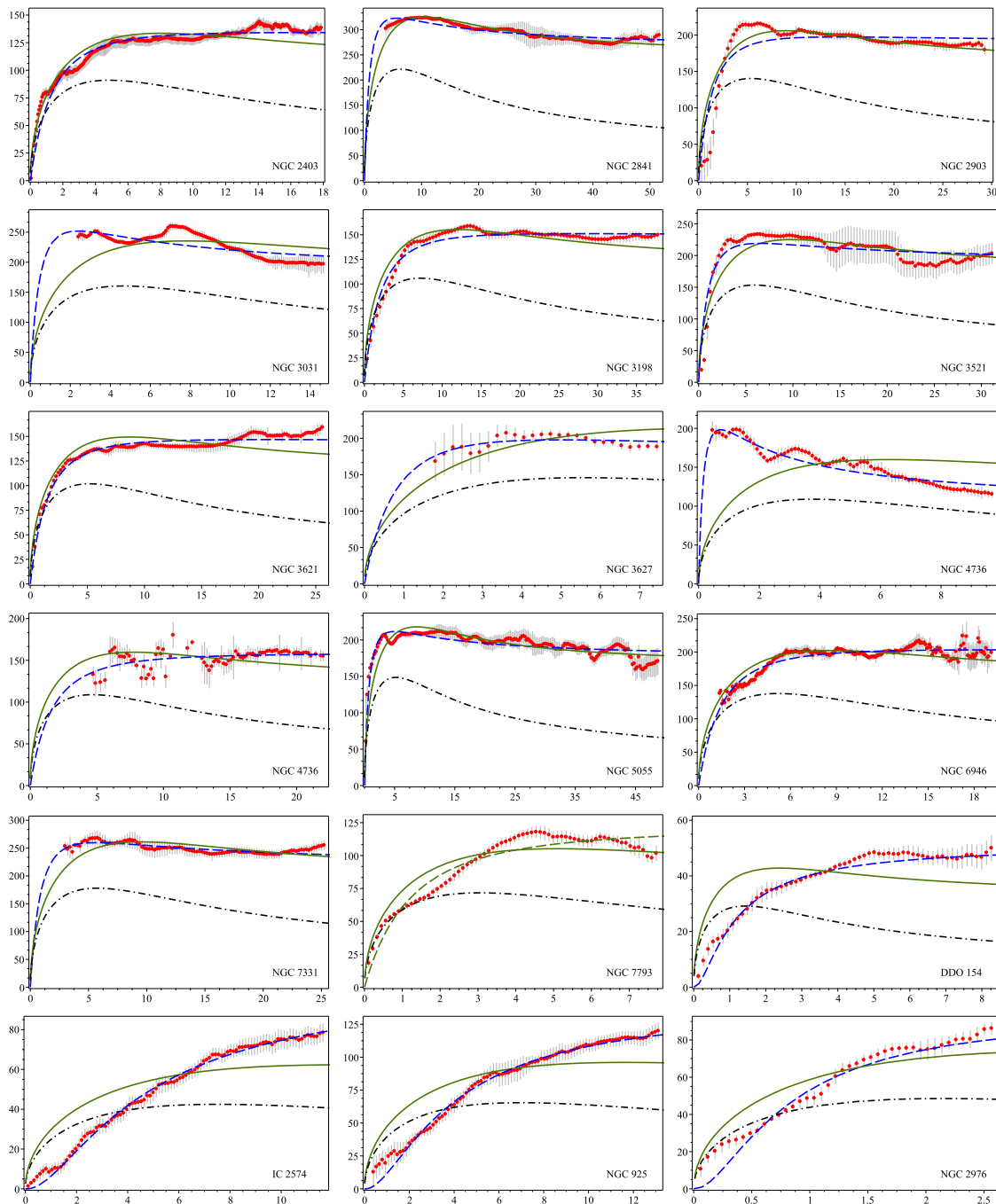


Figure 2. (color online) Rotational velocities in km/s (y axis) at a certain distance in kpc (x axis) from the center of the galaxy. The blue curves RC are obtained from the parametric fit of Equation (19) in the case of HSB THINGS galaxies. The full (green) curve is the rotation curves obtained using the spherical mass distribution (21); the (red) full circles are the observed data points where the vertical (grey) lines represent the error bars; the contribution due to the Newtonian term is given by the dash-dotted (black) lines, while the dashed (blue) lines give rotation curves obtained using the mass distribution in Equation (7). The numerical values resulted from the fits are given in Table 2.

Table 1. Best fit results according to Equation (19) using the parametric mass distribution Equation (7). These numerical values correspond to rotation curves presented in Figure 1. Column (1) name of galaxy; Column (2) distance; Column (3) measured scale length of the galaxy; Column (4) base ten logarithm of total gas mass given by $M_{gas} = 4/3 M_{HI}$, with the M_{HI} data taken from [55]; Column (5) galaxy luminosity in the B-band calculated from [55]; Column (6) base ten logarithm of the predicted stellar mass M_* of the galaxy (obtained by subtracting M_{gas} from the best-fit results for the total mass M_0); Column (7) the predicted core radius r_c ; Column (8) reduced χ_r^2 ; Column (9) the stellar mass-to-light ratio calculated by subtracting the mass of the gas from the total mass and then dividing it by the B-band luminosity; Column (10) base ten logarithm of MOND predicted mass of the galaxy; Column (11) the MOND predicted core radius r_c ; Column (12) MOND reduced χ_r^2 ; and Column (13) the MOND stellar mass-to-light ratio.

Galaxy	D	R_0	$\log M_{gas}$	L_B	$\log M_*$	r_c	χ_r^2	M_*/L	MOND			
									$\log M_*$	r_c	χ_r^2	M_*/L
	Mpc	kpc	$M_\odot$	$10^{10} L_\odot$	$M_\odot$	kpc		$M_\odot/L_\odot$	$M_\odot$	kpc		$M_\odot/L_\odot$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
HSB type												
NGC 2403	3.2	2.7	9.53	0.921	10.36	2.48	1.32	2.49	10.21	2.06	0.69	1.78
NGC 2841	14.1	3.5	10.06	4.742	10.64	1.73	1.11	0.92	11.50	2.81	1.71	6.71
NGC 2903	8.9	3.0	9.76	3.664	10.67	2.51	5.30	1.29	11.06	2.85	7.94	3.14
NGC 3031	3.6	2.6	9.68	3.049	9.76	0.88	5.63	0.19	10.66	1.37	6.07	1.52
NGC 3198	13.8	4.0	10.13	3.106	10.58	3.76	1.61	1.23	10.19	2.96	3.99	0.50
NGC 3521	10.7	3.3	10.03	3.698	10.31	1.84	5.19	0.55	10.78	2.09	6.31	1.65
NGC 3621	6.6	2.9	9.97	1.629	10.42	2.75	1.49	1.63	10.28	2.29	0.85	1.17
NGC 3627	9.3	3.1	9.04	3.076	10.23	1.53	0.83	0.56	10.68	1.87	0.91	1.59
NGC 4736	4.7	2.1	8.72	1.294	8.42	0.32	2.50	0.02	8.93	0.34	5.18	0.07
NGC 4826	7.5	2.6	8.86	2.779	10.67	2.85	1.57	1.71	10.61	2.27	1.61	1.46
NGC 5055	10.1	2.9	10.08	4.365	9.98	1.50	1.24	0.22	10.35	1.47	2.54	0.51
NGC 6946	5.9	2.9	9.74	2.729	10.80	2.74	1.52	2.31	11.26	3.29	1.61	6.70
NGC 7331	14.7	3.2	10.08	7.244	10.40	1.68	0.37	0.35	11.13	2.31	0.24	1.86
NGC 7793	3.9	1.7	9.07	0.511	10.32	2.09	4.65	4.13	10.53	2.29	4.23	6.73
LSB type												
DDO 154	4.3	0.8	8.68	0.007	8.62	0.69	1.01	6.00	8.34	0.83	0.59	3.14
IC 2574	4.0	4.2	9.29	0.273	9.98	3.07	0.52	3.49	10.49	4.86	0.30	11.40
NGC 925	9.2	3.9	9.78	1.614	9.34	2.62	0.31	0.54	10.55	3.61	0.25	2.22
NGC 2976	3.6	1.2	8.27	0.201	8.71	0.58	2.19	0.26	9.41	0.75	1.37	1.30

Table 2. Best fitting results using Equations (17) and (21). The corresponding rotation curves are given in Figure 2. Column (1) is the name of the galaxy; Column (2) the galaxy type; Column (3) gives best-fit results for the predicted galaxy stellar mass; Column (4) gives the values of reduced χ_r^2 ; and Column (5) gives the stellar mass-to-light ratio.

Galaxy	Type	$\log M (M_\odot)$	χ_r^2	$M/L (M_\odot/L_\odot)$
(1)	(2)	(3)	(4)	(5)
NGC 2403	HSB	10.15	3.88	1.51
NGC 2841	HSB	11.08	0.55	2.58
NGC 2903	HSB	10.60	2.10	1.09
NGC 3031	HSB	10.67	15.01	1.55
NGC 3198	HSB	10.33	3.55	0.69
NGC 3521	HSB	10.69	6.18	1.34
NGC 3621	HSB	10.14	10.41	0.86
NGC 3627	HSB	10.70	4.94	1.63
NGC 4736	HSB	10.27	-	1.46
NGC 4826	HSB	10.36	3.14	0.83
NGC 5055	HSB	10.57	2.99	0.86
NGC 6946	HSB	10.57	2.55	1.37
NGC 7331	HSB	10.82	1.84	0.92
NGC 7793	HSB	9.75	12.47	1.09
DDO 154	LSB	7.69	21.17	0.71
IC 2574	LSB	9.59	18.47	1.43
NGC 925	LSB	9.83	10.98	0.42
NGC 2976	LSB	9.34	12.98	1.10

If we replace the matter distribution (7) in the Equation (17) with the one coming from the spherical version of the exponential disc profile [45]

$$M(r) = M_0 \left[1 - \left(1 + \frac{r}{R_0} \right) \exp \left(-\frac{r}{R_0} \right) \right], \quad (21)$$

we can then fit the rotation curves using only M/L as a free parameter. The resulted predicted values for the stellar mass of the galaxies are given in Table 2 together with the corresponding rotation curves in Figure 2. By combining the mass distributions (7) and (21) a “core-disk” model can be obtained. However, the resulted masses will be much lower than in the cases when the core mass distribution and the disk profile are considered to be independent. In Figure 3 we compare the resulted rotation curves using the three mass distributions for two galaxies.

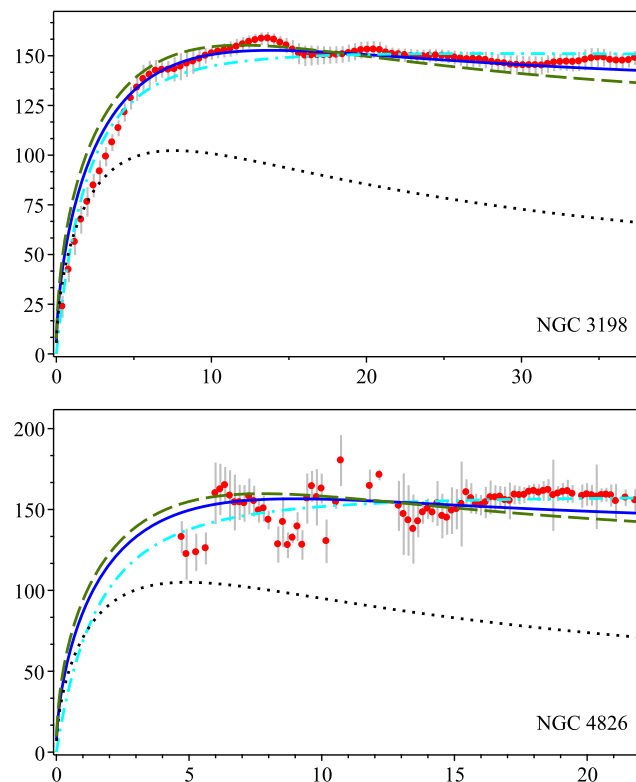


Figure 3. (color online) Comparison of rotation curves obtained using the “core” mass distribution (7) (cyan dash-dot curve), the observed data (red full circles with vertical grey lines (error bars)), the disk profile (21) (green dash curve) and the “core-disk” mass distribution (blue solid curve) together with its Newtonian contribution (black dotted curve).

The Tully-Fisher Relation

The empirical observational relation between the observed luminosity of a galaxy and the fourth power of the last observed velocity point is known as the Tully-Fisher relation [62]

$$L \propto v_{last}^4, \quad (22)$$

which can be rewritten as

$$\log(M) = a \log(v) + b. \quad (23)$$

In the Figure 4 we have presented the observational Tully-Fisher relation (top-left panel) together with the fits of the parametric model given by the Equation (17) using the mass distribution (7) in

the right-top panel and the spherical version of the exponential disk mass distribution (21) in the right-bottom panel, respectively. The left-bottom panel presents the Tully-Fisher relation coming from MOND mass predictions.

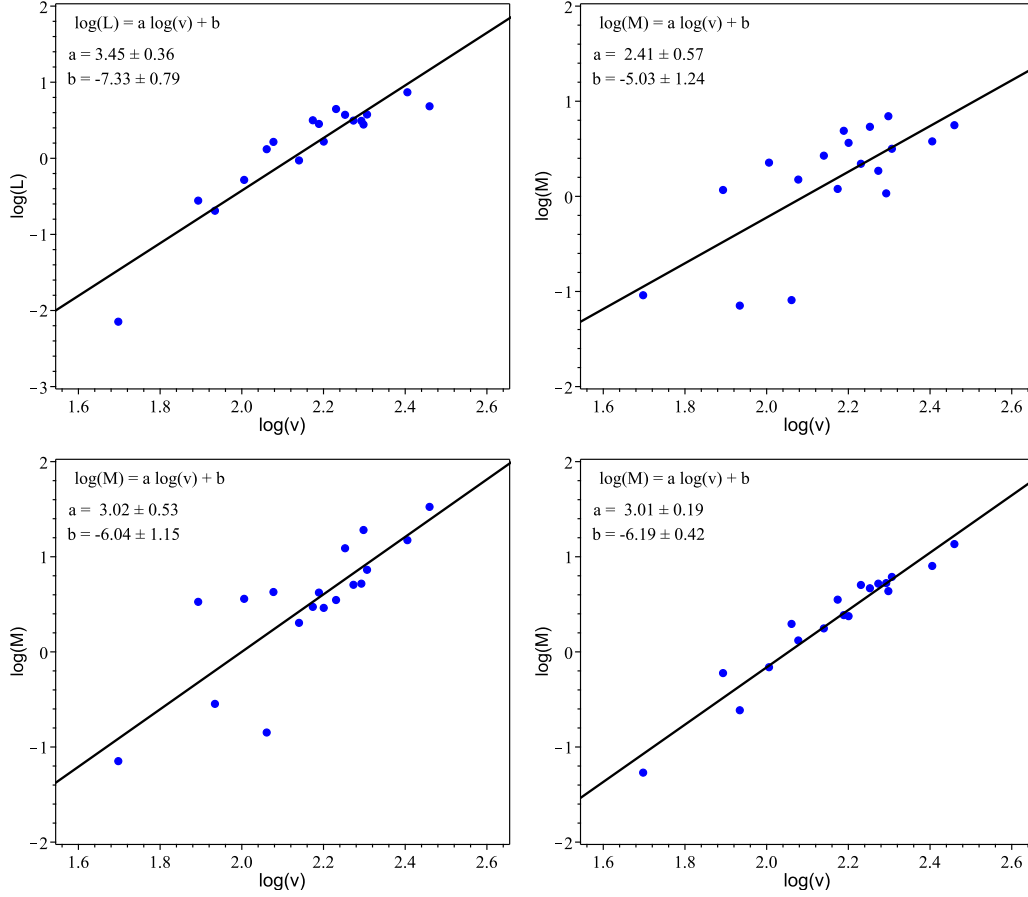


Figure 4. (color online) The Tully-Fisher relation. **Left-top panel:** the observed B-band Tully-Fisher relation. Vertical axis gives the base 10 logarithm of the observed luminosity (in units of $10^{10} L_{\odot}$, respectively $10^{10} M_{\odot}$) and the horizontal axis is the base 10 logarithm of the last observed velocity (in km/s). **Left-bottom panel:** best fit Tully-Fisher relation parameterized by $\log(M) = a \log(v) + b$ in the case of MOND. **Right-top panel:** Tully-Fisher best fit for the masses resulted from the parametric model given by Equation (17) using the mass distribution (7), respectively. **Right-bottom panel:** Tully-Fisher relation obtained using the spherical version of the exponential disk mass distribution (21). The value of M used in the plots is the total mass of a given galaxy: $M = M_{*} + M_{gas}$.

4. Discussion and Conclusions

In the presented paper we have considered the possible explanation of observed galactic rotation curves by the assumption that the observed effect of the flatness can be explained by some alternative theory of gravity which introduces an extra term which we called $A(r)$. This term can be treated as a deviation from the Newtonian limit of GR.

Our results are presented in Tables 1 and 2 together with the plots in Figures 1 and 2. Although we would like to consider this contribution like something coming from a slightly different geometry appearing in the modified Einstein field equations, it can be also thought of as some extra field, for example the scalar one which has recently been considered as an agent of the cosmological inflation [63–66]. This choice for $A(r)$ in (16) could be explained by considering two conformally related metrics (the GR metric $g_{\mu\nu}$ and a “dark metric” $h_{\mu\nu}$ [67,68]) as proposed in [47]. However, so far

we have not been able to find a suitable metric $h_{\mu\nu}$. This means that one needs to know a form of a lagrangian in the case of Palatini gravity in order to know the form of the dark metric.

From now on we shall compare the new phenomenological model proposed in Section 2 for explaining flat galaxy rotation curves with the widely accepted MOND model.

Let us start analyzing the predictions from Table 1. Comparing Column (7) and Column (3) from the Table 1 we observe that in all galaxies of the sample (excepting NGC4826 and NGC7793) the predicted core radius r_c is smaller than the galaxy length scale R_0 . The same is true for MOND (excepting galaxies NGC7793, DDO154 and IC2574). The ratio between the predicted MOND mass in Column (10) and the predicted mass in Column (6) is in the interval (0.4, 8.1) such that for 13 out of 18 galaxies the MOND mass is higher.

The stellar mass-to-light ratio M/L (denoted Y_*) is usually estimated in the literature [69–71] by using color-to-mass-to-light ratio relations (CMLR) of the type

$$\log Y_*^i = a_i + b_i \cdot color \quad (24)$$

a , b are two parameters and i is the band of the measured data. Then using the observed luminosity in the corresponding band, an estimate of the stellar mass is obtained. In [70] the authors use CMLR and four stellar population synthesis models [71–74] to compute the stellar mass for a sample of 40 galaxies, including 13 of the THINGS galaxies used in this paper. Comparing our predicted stellar mass from the Table 1, Column (6) with the values from the Table 3 in [70] and/or the values from the Tables 3 and 4 in [54] we have found that for five galaxies the predicted mass in Column (6) is in very good agreement, for seven galaxies the mass is higher, while for four of the galaxies the mass is slightly lower. Looking now at the values of Column (9) in the Table 1 and Column (5) in the Table 2 we can say that the values of Y_* are in agreement with what is expected based on stellar population models [70]. However, using the spherical mass distribution (21) for LSB galaxies does not result in good fits for the rotational curves.

In Column (8) and Column (12) of Table 1 the values of reduced χ^2 are presented. These values were computed using the standard definition: $\chi_r^2 = \chi^2 / (N - n)$, where N is the number of observational velocity data points; n is the number of parameters to be fitted; and

$$\chi^2 = \sum_i^N \left(\frac{V_i^{obs} - V_{model}(R_i)}{error_i} \right)^2. \quad (25)$$

Taking all of the above into account, one arrives to the conclusion that the new model (which does not assume the existence on any type of Dark Matter) proposed in this paper gives very good flat rotation curves fits of the 18 THINGS galaxies in the data sample. Moreover, when compared with MOND, the difference between the two set of fits is small and thus one is not able to say which model is better than the other one for the explanation of the rotation curves.

We had not had any concrete theory in mind when we wanted to check our assumptions on the modification term $A(r)$. Since we have been influenced by the results obtained by the others (briefly described in Section 2), we wanted to find much simpler modification apart from MOND which also provides a required shape of the galaxies' curves. Therefore now, when we have shown that observational data does not exclude the obtained result (19), it is stimulating to think about existing theories of gravity.

The proposed model presented in this paper (enclosed in Equation (19)) can be viewed for now as a phenomenological model, until a concise theory of gravity from which it can be derived, will be found or constructed. We started to tackle this task, thus working on a given theory of gravity which produces a simple modification of the quadratic velocity is a topic of our current research.

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References

- Ostriker, J.P.; Steinhardt, P.J. The observational case for a low-density Universe with a non-zero cosmological constant. *Nature* **1995**, *377*, 600–602. [CrossRef]
- Peebles, P.J.E. Tests of cosmological models constrained by inflation. *Astrophys. J.* **1984**, *284*, 439–444. [CrossRef]
- Joyce, A.; Lombriser, L.; Schmidt, F. Dark Energy Versus Modified Gravity. *Ann. Rev. Nucl. Part. Sci.* **2016**, *66*, 95–122. [CrossRef]
- Planck 2015 Results. Available online: <https://www.cosmos.esa.int/web/planck/publications> (accessed on 13 July 2018).
- Sanders, R.H. *The Dark Matter Problem: A Historical Perspective*; Cambridge University Press: Cambridge, UK; New York, NY, USA, 2010.
- Trimble, V. Existence and nature of dark matter in the universe. *Annu. Rev. Astron. Astrophys.* **1987**, *25*, 425–472. [CrossRef]
- Rubin, V.C.; Kent Ford, W., Jr. Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions. *Astrophys. J.* **1970**, *159*, 379. [CrossRef]
- Rubin, V.C.; Kent Ford, W., Jr.; Thonnard, N. Rotational properties of 21 SC galaxies with a large range of luminosities and radii, from NGC 4605 ($R = 4$ kpc) to UGC 2885 ($R = 122$ kpc). *Astrophys. J.* **1980**, *238*, 471–487. [CrossRef]
- Karukes, E.V.; Salucci, P. The universal rotation curve of dwarf disc galaxies. *Mon. Not. R. Astron. Soc.* **2017**, *465*, 4703–4722. [CrossRef]
- Salucci, P.; Lapi, A.; Tonini, C.; Gentile, G.; Yegorova, I.; Kleinet, U. The universal rotation curve of spiral galaxies—II. The dark matter distribution out to the virial radius. *Mon. Not. R. Astron. Soc.* **2007**, *378*, 41–47. [CrossRef]
- Sofue, Y.; Rubin, V. Rotation Curves of Spiral Galaxies. *Ann. Rev. Astron. Astrophys.* **2001**, *39*, 137–174. [CrossRef]
- Capozziello, S.; De Laurentis, M. Extended Theories of Gravity. *Phys. Rep.* **2011**, *159*, 167–321. [CrossRef]
- Capozziello, S.; Faraoni, V. *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics*; Springer Science and Business Media: Berlin, Germany, 2010; Volume 170.
- Nojiri, S.; Odintsov, S.D. Dark energy, inflation and dark matter from modified $F(R)$ gravity. In *Problems of Modern Theoretical Physics : A Volume in Honour of Prof. I.L.Buchbinder in the Occasion of His 60th Birthday*; Tomsk State Pedagogical University Press: Tomsk, Russia, 2008.
- Nojiri, S.; Odintsov, S.D. Unified cosmic history in modified gravity: From $F(R)$ theory to Lorentz non-invariant models. *Phys. Rep.* **2011**, *505*, 59–144. [CrossRef]
- Bertone, G.; Hooper, D.; Silk, J. Particle dark matter: Evidence, candidates and constraints. *Phys. Rep.* **2005**, *405*, 279–390. [CrossRef]
- Davis, M.; Efstathiou, G.; Frenk, C.S.; White, S.D. The evolution of large-scale structure in a universe dominated by cold dark matter. *Astrophys. J.* **1985**, *292*, 371–394. [CrossRef]
- Refregier, A. Weak Gravitational Lensing by Large-Scale Structure. *Annu. Rev. Astron. Astrophys.* **2003**, *41*, 645–668. [CrossRef]
- Bekenstein, J.D. The modified Newtonian dynamics—MOND and its implications for new physics. *Contemp. Phys.* **2006**, *47*, 387–403. [CrossRef]
- McGaugh, S.S.; De Blok, W.J.G. Testing the Hypothesis of Modified Dynamics with Low Surface Brightness Galaxies and Other Evidence. *Astrophys. J.* **1998**, *499*, 66. [CrossRef]
- McGaugh, S.S. Novel Test of Modified Newtonian Dynamics with Gas Rich Galaxies. *Phys. Rev. Lett.* **2011**, *106*, 121303. [CrossRef] [PubMed]

22. Milgrom, M. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophys. J.* **1983**, *270*, 365–370. [[CrossRef](#)]
23. Milgrom, M. A modification of the Newtonian dynamics—Implications for galaxies. *Astrophys. J.* **1983**, *270*, 371–389. [[CrossRef](#)]
24. Milgrom, M. The MOND paradigm. *arXiv* **2008**, arXiv:0801.3133.
25. Sanders, R.H.; McGaugh, S.S. Modified Newtonian Dynamics as an Alternative to Dark Matter. *Annu. Rev. Astron. Astrophys.* **2002**, *40*, 263–317. [[CrossRef](#)]
26. Bekenstein, J.D. Relativistic gravitation theory for the modified Newtonian dynamics paradigm. *Phys. Rev. D* **2004**, *70*, 083509. [[CrossRef](#)]
27. Moffat, J.W.; Toth, V.T. Rotational velocity curves in the Milky Way as a test of modified gravity. *Phys. Rev. D* **2015**, *91*, 043004. [[CrossRef](#)]
28. Cadoni, M.; Casadio, R.; Giusti, A.; Muck, W.; Tuveri, M. Effective fluid description of the dark universe. *Phys. Lett. B* **2018**, *776*, 242–248. [[CrossRef](#)]
29. Cadoni, M.; Casadio, R.; Giusti, A.; Tuveri, M. Emergence of a dark force in corpuscular gravity. *Phys. Rev. D* **2018**, *97*, 044047. [[CrossRef](#)]
30. Iorio, L.; Ruggiero, M.L. Solar System tests of some models of modified gravity proposed to explain galactic rotation curves without dark matter. *Sch. Res. Exchange* **2008**, *2008*, 968393. [[CrossRef](#)]
31. Sebastiani, L.; Vagnozzi, S.; Myrzakulov, R. Static spherically symmetric solutions in mimetic gravity: Rotation curves and wormholes. *Class. Quantum Gravity* **2016**, *33*, 125005.
32. Capozziello, S.; Cardone, V.F.; Trois, A. Dark energy and dark matter as curvature effects? *J. Cosmol. Astropart. Phys.* **2006**, *2006*, 001. [[CrossRef](#)]
33. Barrientos, E.; Mendoza, S. A relativistic description of MOND using the Palatini formalism in an extended metric theory of gravity. *Eur. Phys. J. Plus* **2016**, *131*, 367. [[CrossRef](#)]
34. Barvinsky, A.O. Dark matter as a ghost free conformal extension of Einstein theory. *J. Cosmol. Astropart. Phys.* **2014**, *2014*, 014. [[CrossRef](#)]
35. Bernal, T.; Capozziello, S.; Hidalgo, J.C.; Mendoza, S. Recovering MOND from extended metric theories of gravity. *Eur. Phys. J. C* **2011**, *71*, 1794. [[CrossRef](#)]
36. Bruneton, J.P.; Esposito-Farese, G. Field-theoretical formulations of MOND-like gravity. *Phys. Rev. D* **2007**, *76*, 124012. [[CrossRef](#)]
37. Esposito-Farese, G. Motion in Alternative Theories of Gravity. *Fundam. Theor. Phys.* **2011**, *162*, 461–489.
38. Famaey, B.; McGaugh, S.S. Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions. *Living Rev. Relativ.* **2012**, *15*, 10. [[CrossRef](#)] [[PubMed](#)]
39. Mannheim, P.D.; O'Brien, J.G. Impact of a Global Quadratic Potential on Galactic Rotation Curves. *Phys. Rev. Lett.* **2011**, *106*, 121101. [[CrossRef](#)] [[PubMed](#)]
40. Mannheim, P.D.; O'Brien, J.G. Fitting galactic rotation curves with conformal gravity and a global quadratic potential. *Phys. Rev. D* **2012**, *85*, 124020. [[CrossRef](#)]
41. Mannheim, P.D. Alternatives to dark matter and dark energy. *Prog. Part. Nucl. Phys.* **2006**, *56*, 340–445. [[CrossRef](#)]
42. Rodrigues, D.C.; Letelier, P.S.; Shapiro, I.L. Galaxy rotation curves from general relativity with renormalization group corrections. *J. Cosmol. Astropart. Phys.* **2010**, *2010*, 020. [[CrossRef](#)]
43. Rodrigues, D.C.; de Oliveira, P.L.; Fabris, J.C.; Gentile, G. Modified gravity models and the central cusp of dark matter haloes in galaxies. *Mon. Not. R. Astron. Soc.* **2014**, *445*, 3823–3838. [[CrossRef](#)]
44. Rodrigues, D.C.; Mauro, S.; de Almeida, A.O.F. Solar System constraints on renormalization group extended general relativity: The PPN and Laplace-Runge-Lenz analyses with the external potential effect. *Phys. Rev. D* **2016**, *94*, 084036. [[CrossRef](#)]
45. Binney, J.; Tremaine, S. *Galactic Dynamics*, 2nd ed.; Princeton University Press: Princeton, NY, USA, 2008.
46. Weinberg, S. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, 1st ed.; John Wiley & Sons: Hoboken, NJ, USA, 1972.
47. Sporea, C.A.; Borowiec, A.; Wojnar, A. Galaxy rotation curves via conformal factors. *Eur. Phys. J. C* **2018**, *78*, 308. [[CrossRef](#)] [[PubMed](#)]
48. Brownstein, J.R.; Moffat, J.W. Galaxy Rotation Curves without Nonbaryonic Dark Matter. *Astrophys. J.* **2006**, *636*, 721–741. [[CrossRef](#)]

49. Mendoza, S.; Olmo, G.J. Astrophysical constraints and insights on extended relativistic gravity. *Astrophys. Space Sci.* **2015**, *357*, 133. [\[CrossRef\]](#)
50. Capozziello, S.; Cardone, V.F.; Trois, A. Low surface brightness galaxy rotation curves in the low energy limit of R^n gravity: No need for dark matter? *Mon. Not. R. Astron. Soc.* **2007**, *375*, 1423–1440. [\[CrossRef\]](#)
51. Starobinsky, A.A. A new type of isotropic cosmological models without singularity. *Phys. Lett. B* **1980**, *91*, 99–102. [\[CrossRef\]](#)
52. Ehlers, J.; Pirani, F.A.E.; Schild, A. *The Geometry of Free Fall and Light Propagation, in General Relativity*; ORaifeartaigh, L., Ed.; Clarendon: Oxford, UK, 1972.
53. Stachowski, A.; Szydłowski, M.; Borowiec, A. Starobinsky cosmological model in Palatini formalism. *Eur. Phys. J. C* **2017**, *77*, 406. [\[CrossRef\]](#)
54. De Blok, W.J.G.; Walter, F.; Brinks, E.; Trachternach, C.; Oh, S.H.; Kennicutt, R.C., Jr. High-resolution rotation curves and galaxy mass models from THINGS. *Astron. J.* **2008**, *136*, 2648. [\[CrossRef\]](#)
55. Walter, F.; Brinks, E.; De Blok, W.J.G.; Bigiel, F.; Kennicutt, R.C., Jr.; Thornley, M.D.; Leroy, A. THINGS: The HI Nearby Galaxy Survey. *Astrophys. J.* **2008**, *136*, 2563. [\[CrossRef\]](#)
56. Finzi, A.; Pirani, F.A.E. On the Validity of Newton's Law at a Long Distance. *Mon. Not. R. Astron. Soc.* **1963**, *127*, 21–30 [\[CrossRef\]](#)
57. Tohline, J.E. Stabilizing a Cold Disk with a $1/r$ Force Law. In *Symposium-International Astronomical Union*; Cambridge University Press: Cambridge, UK, 1983; Volume 100, pp. 205–206.
58. Sanders, R.H. Anti-gravity and galaxy rotation curves. *Astron. Astrophys.* **1984**, *136*, L21–L23.
59. Kuhn, J.R.; Kruglyak, L. Non-Newtonian Forces and the Missing Mass Problem. *Astrophys. J.* **1987**, *313*, 1–12. [\[CrossRef\]](#)
60. Minguzzi, E. Possible relation between galactic flat rotational curves and the Pioneers' anomalous acceleration. *New Astron.* **2006**, *12*, 142–145. [\[CrossRef\]](#)
61. Acedo, L. Modified Newtonian Gravity as an Alternative to the Dark Matter Hypothesis. *Galaxies* **2017**, *5*, 74. [\[CrossRef\]](#)
62. Tully, R.B.; Verheijen, M.A.; Pierce, M.J.; Huang, J.S.; Wainscoat, R.J. The Ursa Major Cluster of Galaxies. I. Cluster Definition and Photometric Data. *Astrophys. J.* **1996**, *112*, 2471.
63. Linde, A.D. A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems. *Phys. Lett. B* **1982**, *108*, 389–393. [\[CrossRef\]](#)
64. Linde, A.D. Chaotic inflation. *Phys. Lett. B* **1983**, *129*, 177–181. [\[CrossRef\]](#)
65. Linde, A.D. Hybrid inflation. *Phys. Rev. D* **1994**, *49*, 748. [\[CrossRef\]](#)
66. Guth, A.H. Inflationary universe: A possible solution to the horizon and flatness problems. *Phys. Rev. D* **1981**, *23*, 347–356. [\[CrossRef\]](#)
67. Capozziello, S.; De Laurentis, M.; Francaviglia, M.; Mercadante, S. From Dark Energy & Dark Matter to Dark Metric. *Found. Phys.* **2009**, *39*, 1161–1176.
68. Fatibene, L.; Francaviglia, M. Mathematical Equivalence versus Physical Equivalence between Extended Theories of Gravitations. *Int. J. Geom. Methods Mod. Phys.* **2014**, *11*, 1450008. [\[CrossRef\]](#)
69. Bell, E.F.; de Jong, R.S. Stellar Mass-to-Light Ratios and the Tully-Fisher Relation. *Astrophys. J.* **2001**, *550*, 212. [\[CrossRef\]](#)
70. McGaugh, S.S.; Schombert, J.M. Color-Mass-to-Light-Ratio Relations for Disk Galaxies. *Astrophys. J.* **2014**, *148*, 77. [\[CrossRef\]](#)
71. Zibetti, S.; Charlot, S.; Rix, H.-W. Resolved stellar mass maps of galaxies—I. Method and implications for global mass estimates. *Mon. Not. R. Astron. Soc.* **2009**, *400*, 1181–1198. [\[CrossRef\]](#)
72. Bell, E.F.; McIntosh, D.H.; Katz, N.; Weinberg, M.D. The Optical and Near-Infrared Properties of Galaxies. I. Luminosity and Stellar Mass Functions. *Astrophys. J. Suppl. Ser.* **2003**, *149*, 289. [\[CrossRef\]](#)
73. Into, T.; Portinari, L. New colour-mass-to-light relations: The role of the asymptotic giant branch phase and of interstellar dust. *Mon. Not. R. Astron. Soc.* **2013**, *430*, 2715–2731. [\[CrossRef\]](#)
74. Portinari, L.; Sommer-Larsen, J.; Tantalo, R. On the mass-to-light ratio and the initial mass function in disc galaxies. *Mon. Not. R. Astron. Soc.* **2004**, *347*, 691–719. [\[CrossRef\]](#)

