galaxies

MDPI

Article

Thermodynamic Constraints on the Non-Baryonic Dark Matter Gas Composing Galactic Halos

Anne M. Hofmeister

Department of Earth and Planetary Science, Washington University, St Louis, MO 63130, USA; hofmeist@wustl.edu; Tel.: +1-314-935-7440

Received: 17 August 2020; Accepted: 5 November 2020; Published: 8 November 2020

Abstract: To explain rotation curves of spiral galaxies through Newtonian orbital models, massive halos of non-baryonic dark matter (NBDM) are commonly invoked. The postulated properties are that NBDM interacts gravitationally with baryonic matter, yet negligibly interacts with photons. Since halos are large, low-density gaseous bodies, their postulated attributes can be tested against classical thermodynamics and the kinetic theory of gas. Macroscopic models are appropriate because these make few assumptions. NBDM–NBDM collisions must be elastic to avoid the generation of light, but this does not permit halo gas temperature to evolve. If no such collisions exist, then the impossible limit of absolute zero would be attainable since the other available energy source, radiation, does not provide energy to NBDM. The alternative possibility, an undefined temperature, is also inconsistent with basic thermodynamic principles. However, a definable temperature could be attained via collisions with baryons in the intergalactic medium since these deliver kinetic energy to NBDM. In this case, light would be produced since some proportion of baryon collisions are inelastic, thereby rendering the halo detectable. Collisions with baryons are unavoidable, even if NBDM particles are essentially point masses. Note that <0.0001 × the size of a proton is needed to avoid scattering with γ-rays, the shortest wavelength used to study halos. If only elastic collisions exist, NBDM gas would collapse to a tiny, dense volume (zero volume for point masses) during a disturbance—e.g., cosmic rays. NBDM gas should occupy central galactic regions, not halos, since self-gravitating objects are density stratified. In summary, properties of NBDM halos as postulated would result in violations of thermodynamic laws and in a universe unlike that observed.

Keywords: rotation curves; kinetic theory of gases; thermodynamic laws; inelastic collisions; blackbody radiation; non-baryonic dark matter; non-luminous matter

1. Introduction

Following Rubin and Ford's [1] discovery that the rotation curves (RCs, measurements of tangential velocity v as a function of equatorial radius r) of the many billions of stars distributed in spiral galaxies do not exhibit a Keplerian orbital pattern as do the few tiny planets of our Solar System, spherical halos of dark matter (Figure 1a) have been proposed to account for the mass discrepancy (e.g., [2,3]). The concept of dark matter (DM) traces to the 1930s and efforts of Zwicky and others, and originally denoted material that was neither visibly detected nor luminous. Consequently, considerable efforts in observational astronomy were expended to confirm the existence of the halos using something other than visible wavelengths of electromagnetic (EM) radiation.

Because no halo matter was detected and laboratory experiments show that ordinary matter (e.g., baryons) interacts with EM radiation, the description of halo material evolved to become non-baryonic dark matter (NBDM) sometime in the 1990s. This mysterious matter is purported to interact gravitationally with ordinary matter, but not with EM radiation in any appreciable amount (e.g., [2,4]). Recent efforts were directed toward unexplored, very high energies. Probes of dwarf spheroidal galaxies in the γ-ray spectral region (~10 TeV) failed to provide evidence of NBDM [5]. This was perplexing because analyses of RC data, using the standard approach after Ref. [1,2] of multicomponent fitting in forward Newtonian orbital models (NOMs), suggest that these objects are ~99% dark matter [6]. Gamma-ray studies contributed to neutrinos falling out of favor in the last decade as halo constituents [7,8].

The failure of observational astronomy to provide direct evidence for NBDM on even larger scales [9] motivated a re-direction of efforts towards experimental particle physics, whereby properties of hypothetical non-baryons might be ascertained (e.g., [7,10]). Three different experiments utilizing large hadron colliders failed to provide evidence that the other popular postulated entity termed weakly interacting massive particles (WIMPs [3]) exist [11]. Proposals now invoke a different hypothetical particle (axions) and gravitational lensing models as a test [12]. This approach, similar to fitting RCs, is a gravitation model, and thus cannot address the fundamental question: "What type of matter does not interact with light?".

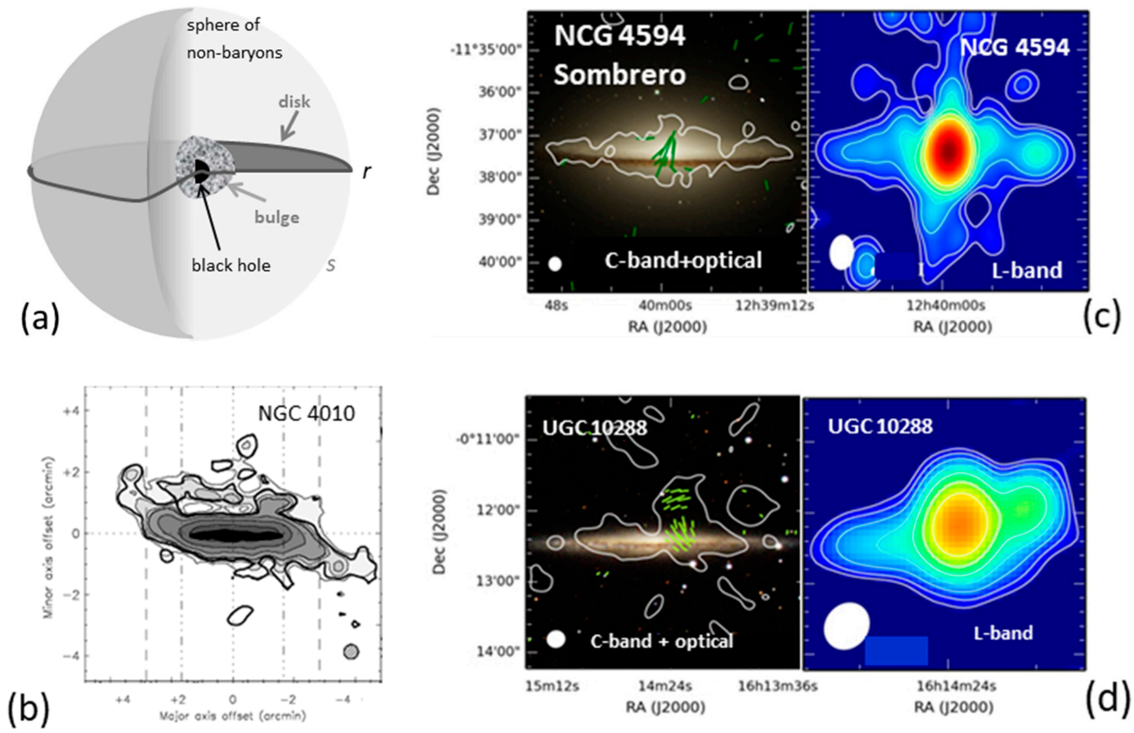

Figure 1. Postulated and measured distributions of baryonic and non-baryonic types of matter in spiral galaxies: (**a**) schematic of a spiral galaxy with a purported halo of non-baryonic dark matter (NBDM), as used in Newtonian Orbit Models (NOMs). Components in this perspective view are labeled. For simplicity, the disk represents stars along with detectable H atoms. Other figure parts show cross sections; (**b**) contours of H atoms surrounding an edge-on spiral galaxy (NGC 4010, also known as UCC 6964) detected using a 21-cm line. The black lens shape shows the extent of the visibly detected galaxy. Panel (b) is modified after Figure 13 (panel 11) of Reference [13], with permission. This distribution of H atoms is analogous to an atmosphere around a star or planet; (**c**,**d**) radio contours of two additional edge-on galaxies in the C- and L-bands, with superimposed optical images, modified after Wigert et al. [14]. Both panels are publically available from CHANG-ES [15].

Ambiguities in the indirect evidence for NBDM halos around galaxies from NOMs are well known [16]. Multicomponent fits are inherently ambiguous, so it is common practice to minimize the amount of dark matter [17]. This procedure is needed because 100% dark matter reproduces RCs in all cases [18]. The lack of independent and direct confirmation of either halo material or NBDM particles has fueled many challenges to the assumptions underlying NOMs (e.g., [17–27]; also see the recent

Special Issue of Galaxies, "Debate on the Physics of Galactic Rotation and the Existence of Dark Matter"). Previous investigations of galactic rotation have focused on how gravitational forces could produce the observed RC without NBDM halos. Because this "missing mass" problem is relevant to cosmological models, which suggest that the mass of the universe is larger than current estimates of total baryonic mass [28], exploring the properties of halos via diverse avenues is warranted.

Theoretical efforts on the nature of dark matter have not provided a satisfactory description [29]. Most models are microscopic, focusing on hypothetical behavior and properties of individual particles [4,8,10,12]. The alternative approach, namely macroscopic modeling, offers many advantages: (1) macroscopic models provide a simple description of phenomena; (2) they require no special assumptions concerning the nature of matter [30]; (3) they provide straightforward, testable predictions; (4) they disclose connections between measurable quantities; (5) because few assumptions and parameters are needed, the macroscopic approach is central to research problems where constraints are few [30].

Tolman's [31] long-standing advice is also relevant, namely that rough methods of scientific deduction are the only means to address situations lacking adequate models and information. For the above reasons, the present paper centers on the macroscopic thermodynamic implications of NBDM halos that stem from their properties as claimed by its proponents. The claimed NBDM properties can be recast as the three postulates listed in Table 1:

Table 1. Properties of dark matter halos according to NOMs.

Postulated Properties of Non-Baryons	Key Consequences [1]
Gravitationally interacts with ordinary matter	NBDM has mass; collisions must occur
Negligibly interacts with photons	Nearly point masses; elastic collisions
Distributed in a large halo around galaxies	A dilute assembly of particles (a gas)

[1] Details are in Section 3.

Because halos are considered to be large, macroscopic models clearly apply. An inverse approach is used. In this thought experiment, we adopt the postulated NBDM properties (Table 1) as truth, and derive behavior expected for NBDM halos in view of (1) classical thermodynamics (the theory of heat), in combination with (2) the experimentally confirmed equivalence of heat and light in dilute media (gas or the "vacuum" of space), and (3) kinetic descriptions of ideal point-mass gases [32] and real gases [33] (ch. 5). Section 2 provides background on the semi-classical physical description of a gas, thermodynamic laws, and the simplifying conditions of space. Section 3 first explains why NBDM halos, as postulated, must have the properties listed in the left-hand side of Table 1, and then applies the principles of Section 2 to infer the behavior of this gassy medium. Section 4 summarizes. We find that NBDM halos cannot exist without violating some laws of thermodynamics. Simply put: motions of matter cannot exist without producing heat, which is intimately linked to both temperature and light production.

2. Theory

This paper addresses ordinary, classical situations where mass is not converted to energy; this case is exemplified by an unconsolidated gas, of mostly H atoms, medium located towards the outskirts of spiral galaxies [34]. The 21-cm line associated with H atoms (Figure 1b) is important for RC determinations [35]. This section focuses on the behavior of both baryonic gas (e.g., monatomic H) and on an idealized gas of particles, as was considered in the 1800s prior to knowledge of atomic structure, in order to portray NBDM constituents.

2.1. Classical Thermodynamics with and without Radiative Transfer

Classical thermodynamics resulted from efforts to understand thermal energy in the era of steam engines. Hence, gas behavior was the focus. Modest temperatures accessible in laboratories of that era were considered, which involve blackbody emissions mainly in the infrared region. Unfortunately, Fourier's 1822 equations for heat transfer were not incorporated [36], which greatly contributed to depicting real processes in terms of idealized, time-reversible processes [33]. This approach is a pillar of classical thermodynamics [37], and the meaning of thermodynamic reversibility is debated today [38]. Radiative heat transfer was also not considered, due to Melloni's ~1843 experimental evidence for the equivalence of heat and light, which was debated for ~50 more years [39]. These omissions stem from the rudiments of atomic structure being likewise debated up until the famous works of Thomson and Rutherford ca 1900 [40]. In addition, blackbody radiation was viewed as a property of condensed matter, but not of gas, well into the 1900s [41]. Hence, the classical laws of thermodynamics (Table 2) center on heat content, rather than on photons and heat transfer.

Table 2. Laws of thermodynamics.

Law No.	Classical Statement [1]
0th	Equilibrium between systems is communicable
1st	Energy is conserved if heat is accounted for
2nd	Flow of heat from a colder to a hotter body cannot occur as a sole result
3rd	Absolute zero is unattainable by processes involving finite steps

[1] See, e.g., Pippard [42], Purrington [40], or Fegley [37].

Importantly, thermodynamics is a macroscopic theory directed towards explaining observations with minimal assumptions. Temperature (T) is thus a macroscopic property arising from the thermal energy of an object, although temperature is not identical to heat.

Temperature is ascertained remotely by fitting the frequency dependence of the photons emitted by a real body to Planck's blackbody curve ($I_{BB}(\nu,T)$, where ν is frequency), allowing for the material property of emissivity (Section 2.3). In 1893, Wein showed that the wavelength ($\lambda = c/\nu$) of the blackbody peak is inversely proportional to T:

$$\lambda_{\max} = \frac{b}{T} = \frac{c}{\nu_{\max}} \text{ or } \nu_{\max} = \frac{w_3 k_B}{h} T \tag{1}$$

where b = 2897.8 μm K^{-1} was experimentally determined, c = lightspeed, k_B is Boltzman's constant, h = Planck's constant., and w_3 is near 2.821439 (see below). Wein's derivation and spectral measurements ended the debate regarding the equivalence of heat and light [39].

Importantly, Equation (1) requires a continuous distribution of photon energies ($E = h\nu$), not an isolated emission line as that produced by a laser, since T is a macroscopic, continuous quantity [33] (ch. 8). The right-hand side (RHS) of Equation (1) is theoretical and was derived from Planck's blackbody curve by numerically solving a transcendental equation [43,44]. This is the origin of the irrational number w_3. From Marr and Wilkin [45], the average frequency, $<\nu>$ is constrained by:

$$\langle E \rangle = \frac{\text{Total energy}}{\text{Number of photons}} = \frac{\int_0^\infty I_{BB}(\nu,T)d\nu}{\int_0^\infty \frac{I_{BB}(\nu,T)}{h\nu} d\nu} = \frac{\pi^4}{30\zeta(3)} k_B T \equiv h\langle \nu \rangle \tag{2}$$

where the Rieman zeta function ζ(3) is an irrational number (~1.20206), as are π and w_3.

Regardless of how many decimal places are used, the number of photons composing blackbody emissions cannot be an integer, unless the temperature itself is quantized in a manner that somehow offsets the irrational numbers. Contrary to the standard view, Planck's formula is not consistent with the discretization of light via thermal processes, because an integer number of photons (Nj) should then exist for each ν_j, and the sum of these integers ($N = \Sigma N$j) should also be an integer. In fact, Planck's derivation of I_{BB} in 1900 did not invoke energy quantization [46]. Planck portrayed his 1901 formulation, which added constants, as a mathematical construction [47]. Quantization of photons is of course associated with processes such as transitions between electronic or vibrational states, but from the above does not describe the thermal state of a body. Ref. [33] provides details and also discusses the connection of blackbody emissions with thermodynamic laws.

Section 2.2. below considers the T of the distributed NBDM in the halo, in view of Equation (2) and collisions, which are the means of energy exchange in a gas. It is also relevant that radiative transfer moves heat between well-separated objects in space, including between bodies of baryonic gas. Temperatures in space are cold—even in molecular clouds, which contain young, hot stars and where radiative transfer is limited by optical depths (i.e., heat is locally retained)—and the T inferred from emission is low—e.g., ~8 K [48] and centers in the microwave region.

For all baryonic gases, collisions occur that are associated with the thermal energy of such gases (e.g., [32]). Collisions also exchange kinetic energy. Investigating this exchange requires a microscopic model and involves additional assumptions. Below, we considered one of the first microscopic models, the kinetic theory of gas, as this makes few assumptions.

2.2. Kinetic Theories of Gas, Elastic and Inelastic

The kinetic theory of gas (KTG) was developed when the nature of baryonic matter was unclear. Hence, this is a reasonable first approximation for the unknown particles in galactic halos.

In the 1800s, point mass particles were proposed as gas constituents, but because these cannot collide, this assumption morphed early-on to finite-sized, hard spherical particles (e.g., [32]). This classical model of Clausius, Maxwell, and Boltzmann, is denoted EKTG, where E indicates that *elastic* collisions of small particles were assumed. Considering collisions as elastic largely stems from laboratory experiments being performed near equilibrium, since virtually any thermal gradient causes convection in a gas. Because temperature cannot evolve during solely elastic collisions, which are conservative, this assumption has since been relaxed; such models are denoted IKTG, where "I" indicates *inelastic* Many consider inelastic collisions to be a minor occurrence—e.g., ~3% in polyatomics [49]. This fraction is underestimated, as indicated by experimental and theoretical studies of deformation of atoms during collisions in the gaseous state (e.g., [50]), and for additional reasons discussed below.

Assumptions in EKTG [32] are: (1) an average speed describes motions of the particles; (2) the particles are small compared to the volume occupied by the gas; (3) an average distance (the mean free path) describes travel of particles between collisions; (4) time during collisions (τ_{int}) is negligible compared to the lifetime (τ_c) between collisions; (5) collisions are perfectly elastic.

Equilibrium situations are not greatly affected by assuming elasticity, and so are discussed next.

2.2.1. Gas Temperature Depends on Kinetic Energy

How much thermal energy is present under isothermal conditions can be gleaned independent of the nature of the collisions. The behavior of an ideal gas (hard, small spheres) under isothermal conditions can be ascertained from the Virial theorem (VT) of Clausius [51] by assuming only that the particles have a mass m and move with an average velocity u [52]. Importantly, the VT is a mathematical identity.

An isothermal gas contained within a sphere of radius s has a surface area of $4\pi s^2$. Hence, the VT for this bound state can be expressed as $2 \langle K.E.\rangle = sF$, where F is the force governing the translational motions and $\langle K.E.\rangle$ is the average kinetic energy specifically of these translations. This is true for indivisible particles or for monatomic gas atoms, which lack internal molecular rotations or vibrations. The force need not be conservative, since isothermal conditions require constant energy inside the sphere. Equating pressure (P) with F divided by area introduces volume (V) into the VT:

$$2\langle \text{K.E.}\rangle_{\text{trans}} = sF = s\left(4\pi s^2 \frac{F}{area}\right) = 4\pi s^3 P = 3VP \quad (3)$$

Next, we invoke the equation of state (EOS) for an ideal gas, which is close to experimental determinations for many real gases:

$$PV = NR_{gc}T \tag{4}$$

where N is the number of moles and R_{gc} is the gas constant. Combining the above, while recognizing that all three translations involve the same average velocity u, yields:

$$\langle \text{K.E.} \rangle_{\text{trans}} = \frac{1}{2}m\left\langle u^2 \right\rangle = \frac{3}{2}NR_{gc}T = \frac{3}{2}NN_A k_B T \tag{5}$$

The result from [52] recapitulates the historical result of ETKG (e.g., [32]).

The meaning of Equations (4) and (5) is that the kinetic energy of the gas particles and rebounding collisions maintains a finite volume against some given pressure. Because indivisible ideal gas particles (which reasonably represent the behavior of He) only have translational motions, translational kinetic energy defines their thermal energy. The constants in Equation (4) are derived from experiments on real gases (baryons). Size of the interacting molecules has an effect on Equation (4), and thus on Equation (5), as exemplified by the modification known as the Van der Waals EOS [49]. Since the equation of state for NBDM is unknown, the proportionality constant may similarly differ from the product NR_{gc}. However, average velocities and temperature should be linked by some proportionality constant.

The finite size of the particles preventing collapse at a very low T (or at high P) is part of the Van der Waals EOS [53] and thus particle size is germane to compression. In classical thermodynamics, pressure was considered to be generated externally, as in a piston-cylinder apparatus, whereas gravitation was neglected during development of this theory and EKTG. For this reason, Section 3 also covers a possible compression of a gas of NBDM under the force of gravity.

The original model (EKTG) explains heat capacity data on real gases [32]. Monatomic H and He gases match nearly exactly the ideal gas EOS because their energies are predominantly translational [33]. Diatomic and more complex gases require additional terms to address substantial contributions of their molecular vibrations and rotations to heat capacity [33]. Other relevant EOS properties of ideal gases are thermal expansivity = $1/T$ and compressibility = $1/P$, which reasonably approximate simple gases. Again, temperature gradients are negligible and the EOS of ideal gas describes near-equilibrium conditions.

2.2.2. Inelastic Collisions and Blackbody Emissions

Blackbody emissions carry energy away from a body (thermal transport). This situation does not describe equilibrium unless the energy received balances the energy lost. Thermal emissions involve a continuous distribution of frequencies (Equations (1)–(3)) and require a universal process that is not quantized—the inelasticity of collisions is implicated since this also involves a continuous distribution of energies, and can be confirmed with data on transport properties [30]. This subsection provides a short summary and some evidence.

Because lifetimes inversely correlate with probabilities, shorter lifetimes dominate [32]. Hence, the brief interaction during the collision is crucial to heat transport and thermal evolution, rather than the time between collisions (computed from the mean free path and the speed u) dominating, as classically assumed. We recently argued [33] (ch. 5) that inelasticity accompanies all collisions of real gas atoms, because these are not rigid spheres, and have modified KTG formulae to account for a very small amount of heat being generated during each collision via deformation (Figure 2).

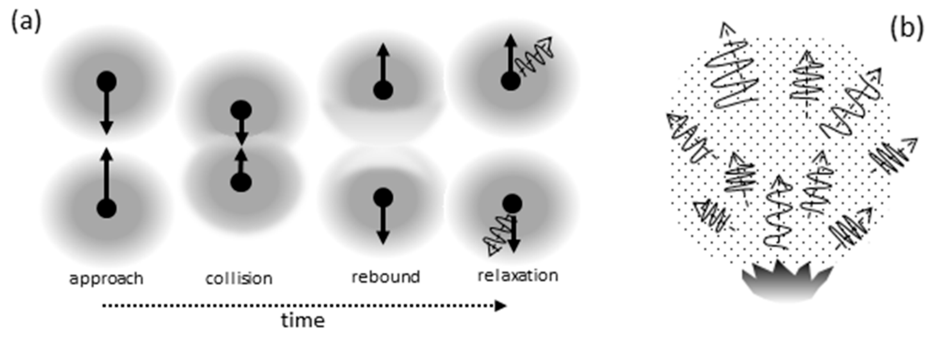

Figure 2. Schematics of inelastic collisions in a gas of finite-size atoms: (**a**) progression of an inelastic collision (viewed in the center of mass reference frame) with time. Black = nuclei. Grey = shells of electrons, forming a shielding cloud, which deforms during the collision, thereby consuming some of the energy of the collision. Squiggle arrow = thermal photon, emitted as the electron cloud adjusts to the reduction in force. Hence, each collision results in a tiny reduction in the thermal energy of the gas; (**b**) steady-state conditions in a body of gas. Applied heat (the fire) establishes T and a distribution of molecular velocities. The velocity distribution produces a thermal photon gas with its energy distribution linked to the velocities. Thermal photons escape, providing blackbody emissions, with this same distribution. To maintain constant T of the substance, heat must be continually supplied. Thus, isothermal conditions require that the heat input = heat output with a negligible thermal gradient. Reprinted from Figure 5.2 a,d of Reference [33], with permission from Elsevier.

Researchers in IKTG consider inelastic collisions to be infrequent (e.g., [49]). However, since atoms are not hard spheres (e.g., [50]), as is well-known from inelastic neutron scattering measurements, deformation exists during every collision and so heat should evolve in each interaction [33]. Following Fourier's theory of conduction, the heat evolved must be proportional to the energy of the collision. This deduction is supported by the energy in blackbody emissions being proportional to T (Equations (1) and (2)), and by the energy in the translations being proportional to T (Equation (5)). Furthermore, comparing the RHS terms of Equations (2) to (5) shows that the average energy loss per collision of ~2.7 $k_B T$ is about 90% of the K.E. of two atoms colliding ($3k_B T$). This proportion seems large, but gas atoms are far apart relative to their size and collisions involving any given atom are infrequent. Importantly, for a gas to be isothermal, it must have as much thermal energy coming in (the fire in Figure 2b) as leaving (the photons in Figure 2b). In summary, with particles being small compared to the volume of the gas body, collisions are infrequent, so the loss during all collisions at any given time is large per collision, but small compared to the total energy available. Flux differs fundamentally from energy.

Inelasticity being ubiquitous explains the now recognized fact that all experimentally explored media, including the gaseous media of monatomic atoms, emit blackbody radiation.

2.2.3. Cross Sections, Interactions during Collisions, and Transport Properties

Calculating the three transport properties (D_{heat} = thermal diffusivity, D_{mass} = mass diffusivity, and υ = kinematic viscosity) requires estimating both interaction times and cross sections during collisions. Classically, only head-on collisions as in Figure 2 were considered, where cross sections were equated to 2× particle area [32]. Classic EKTG predicts equal values for all three transport properties, which is not correct. Glancing collisions can also occur and will affect viscous drag, but this cross section only involves the area of one particle. Assuming that interaction times are the same in both types of collisions [33] leads to relative sizes for the transport properties in IKTG as being:

$$D_{heat} = D_{mass} = \frac{3}{2}\upsilon \quad (6)$$

Data on monatomic and diatomic gases (Figure 3a) confirm the ratio implicit in Equation (6). This ratio holds for diverse gases [33], thereby supporting the representation of collision cross sections by particle size and confirming the existence and importance of glancing collisions.

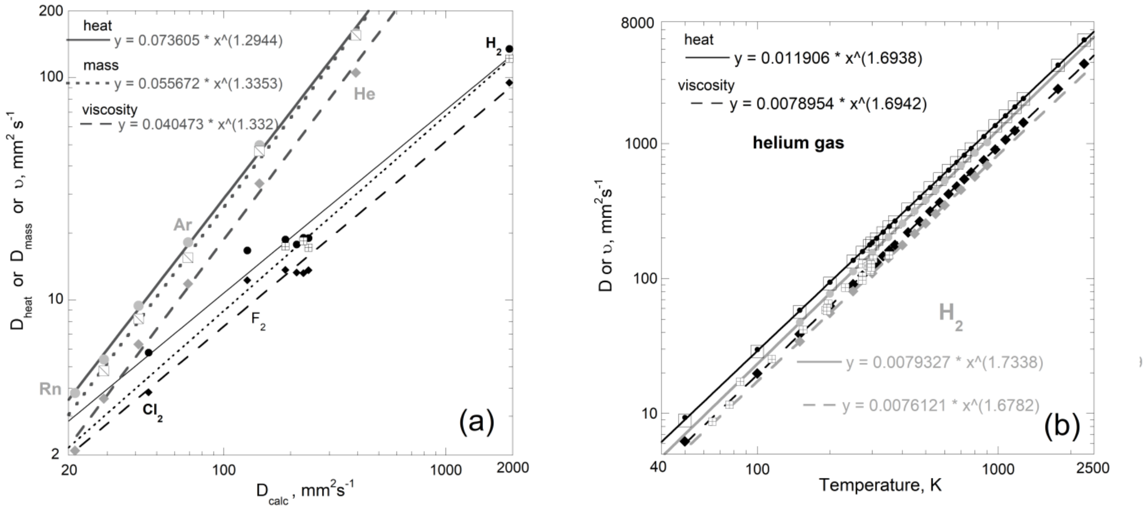

Figure 3. Data on transport properties. Circles and solid line = thermal diffusivity. Squares and dotted line = mass self-diffusivity. Diamond and dashed line = kinematic viscosity. Least squares fits are shown: (**a**) comparison of physical properties at ambient condition to EKTG of hard spheres (Equation (7)). From data compiled in [33] (Table 5.2). For clarity, fits are shown for noble gas only; (**b**) dependence of measured transport properties on temperature. Data on He from Kestin et al. [54]. Data on H_2 from the compilation of Suárez-Iglesias et al. [55] and various sources, listed in [33] (Table 5.2). For clarity, fits for D_{mass} are omitted.

For comparison, values for the two diffusivities in Figure 3a were calculated from EKTG for hard-sphere particles in ideal gas, wherein lifetimes were determined from the average speed and mean free path between collisions:

$$D_{\mathrm{EKTG}} \cong \frac{k_B T}{2\pi r_{\mathrm{atom}}^2 P}\sqrt{\frac{3N_M R_{gc} T}{m}}; D_{\mathrm{IKTG}} = \frac{3N_M R_{gc} T}{m}\tau_{\mathrm{interaction}} \qquad (7)$$

where m is particle mass and r is particle radius [32,33]. Agreement is better for monatomics than diatomics, as expected. However, transport properties do not go as $T^{1.5}$ (Figure 3b). This discrepancy and both diffusivities having a $1/m$ dependence, as in IKTG [33] (p. 176; Figure 5.12), point to problems accompanying the use of mean free paths and u between collisions to determine τ.

The relevant, shorter time of the interactions during gas collisions (including its T dependence) has not been studied to our knowledge. Yet, it should be evident that inelastic collisions are important for heat transport. Collisions between macroscopic particles demonstrably generate heat. Since kinetic theory describes particles of any size, and not necessarily atoms, heat generation is permissible. Allowing for inelasticity explains measured similarities and differences among all three transport properties for each of the various states of matter (gas, liquids, and solids) [33].

2.3. Interactions of Matter with Light (Heat)

Light interacts with ordinary matter in several ways. Energy is exchanged during absorption in particular, which is intimately tied to other processes (reflection, transmission, and emission). Diffusive radiative transfer consists of sequential emission and absorption in a medium, whereas ballistic radiative transfer occurs between two separated entities with negligible involvement of the intervening medium. Ballistic transfer occurs in absorption spectroscopy experiments, where the material is not in thermal equilibrium with the applied light (or heat). Such stimulated transitions produce line spectra for gas or narrow peaks in spectra for condensed matter.

For hot gases produced in Bunsen's burners, Kirchhoff discovered that their absorptivity (α) equals their emissivity (ε):

$$\alpha = \varepsilon \quad (8)$$

This statement stems from energy conservation when radiative transfer occurs under optically thin conditions. Equation (8) is appropriate for dilute gases in the laboratory because this state of matter lacks reflective surfaces and absorbs weakly to negligibly over wide frequency ranges. For additional discussion of the complications associated with condensed matter and with conditions that are not optically thin, see Bates [56] or the summary of [33] (ch. 2).

Although reflection can be neglected for gases of atoms, a medium of particles can scatter light when this has a wavelength similar to particle size. Diffraction and interference effects are also connected with size. A relevant example to NBDM is the scattering of X-rays or γ-rays by electrons, which was discovered by Compton in 1923.

2.4. Simplifications Associated with Astronomical Scales and Monatomic Baryonic Gases

Whether inelasticity describes a few percent of the collisions in a gas, 100% [33], or some other value [50], is immaterial over astronomical scales, due to the immense number of atoms present in the great expanses of space. The number of inelastic collisions between H atoms in the rarified intergalactic medium (IGM), which coexists with proposed halo matter, will be large irrespective of the percentage of collisions being inelastic because these regions are gargantuan. Densities are listed below.

Space contains the monatomic gases hydrogen and He. Diatomic H_2 exists, which has more complicated energetics associated with its molecular vibrations and rotations. Importantly, the proportionality between velocity and temperature was originally derived by considering gas to be made of particles without any prescribed properties. Real gas behaves in this manner because translational K.E. is the bulk of the thermal energy. On this basis, we approximate baryon gas in the IGM to consist of H or He atoms while considering NBDM in halos as being indivisible particles. Motions of the electrons around H or He nuclei are not considered for simplicity and because this behavior is not part of classical macroscopic models. The kinetic energy of these three gases being predominantly translational permits the use of Equation (5).

3. Thermodynamic Behavior of Non-Baryonic Dark Matter Halos

The postulates of NOMs are used here to infer the nature of NBDM halos. The key consequences listed in Table 1 and additional implications are discussed in detail below. Along with classical thermal physics, gravitation must be considered, since its effects constitute the sole source of information on the proposed halos.

3.1. NBDM Halos Are a Type of Gas

The mathematical constructs used to fit rotation curves in NOMs assume a spherical distribution of NBDM about the galactic center (Figure 1a), which is required for the halo to serve as the central point mass under Newtonian physics. Halos have mass, since Newton's law

$$F = \frac{GmM_H}{r^2} \quad (9)$$

is invoked, where M_H is the mass of the halo inside $r = s$ and m is the mass of baryons rotating at that equatorial radius. Halos occupy a volume and thus also have a density.

In the NOM construct, thin spherical shells of NBDM within halos each have a constant density; the thermodynamic variable, ρ = mass per volume, is germane. From Newton's work, the assumed geometry requires a medium that has homogeneous density as regards the two angular variables, although variation with spherical radius (s) can occur (Figure 1a). Other geometries with lower-than-spherical symmetry will result in non-central forces, as has been demonstrated for bars [57], disks [22], and oblate spheroids [58]. Distortion of the field provided by the baryons in a spiral galaxy is significant based on aspect ratios of their images (Figure 4). However, for the halo to act equivalently to a central point, it must have spherical symmetry.

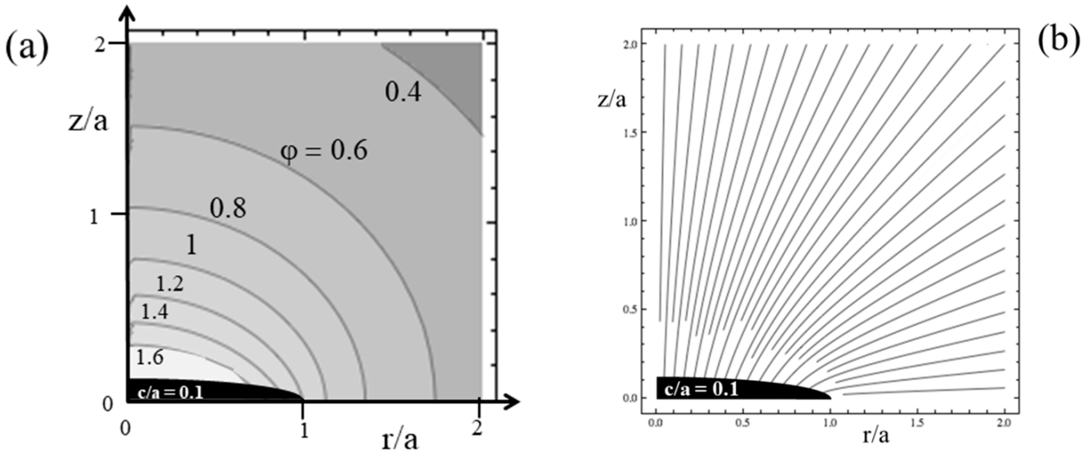

Figure 4. Newtonian attraction to a highly flattened constant density oblate spheroid (black shape) with a 1:10 axial ratio similar to those of spiral galaxies. One quadrant is shown: (**a**) contour plot of the Newtonian gravitational potential exterior to the oblate, shown in units of GMm/a; (**b**) lines of exterior force. Reprinted from Figure 7e,f in Reference [58], with permission from Elsevier.

NBDM halos are alleged to be transparent to photons, which constitutes end-member behavior under optically thin conditions [59,60]. Halos are rarefied and thus must be a gas. Halos cannot be some strange type of rarefied condensed matter, for two reasons: First, motions at contacts such as the interface between a solid halo and stars in the disk would involve friction and generation of heat (IR photons), which is not observed. Second, condensed matter would not transmit photons without some interaction, because solids have reflections at the interfaces or surfaces (e.g., [59]).

In detail, the dynamic (total) mass of large spirals is inferred to be roughly double that of the luminous mass [61], and so suggested halo mass is roughly similar to luminous mass. Since the maximum s of the halo is roughly similar to the maximum r of the detectable matter in the flattened shape of stars and H atoms (Figure 1a), density in the halo would be significantly lower than that inferred from the distributed mass of the visible galaxy. Ferrière [62] estimated density in the interstellar medium (ISM), which roughly represents galactic interiors, as 10^{-4} to 10^{6} atoms per cubic centimeter for its various components, which is much more rarified than the density associated with dispersing our Sun to Neptune's orbit, as the latter would yield a gas of 10^{12} H atoms per cm^3 or 10^{-8} kg per m^3. A cloud equivalent to the dispersed Sun would also be denser than astronomical environments such as molecular cloud cores, yet would be 10^{8} times more rarified than Earth's atmosphere. Thus, NBDM halos are a very dilute gas.

3.1.1. NBDM Gas Contains Particles

Halo gas must be composed of some type of particle, as the state of matter known as gas is defined as a largely non-interacting collection of particles. This assumption follows investigations of kinetic behavior in gases before atomic structure was known [40]. Section 2 describes the expected behavior of a gas composed of indivisible particles. In the remainder of this report, "particle" denotes NBDM constituents only. Constraints on the nature of these particles are discussed below.

3.1.2. Motions and Forces Inside an NBDM Halo

Obviously, rotation curves have not been measured for the undetected material composing a halo. However, NBDM matter must be moving in some way to avoid gravitational collapse to the center. Rotation (spin) is expected under Newtonian physics. If the luminous galaxy was not present and the rotation of the shells was slow and differential, the spherical shape might be maintained under Newtonian physics, or nearly so, as in elliptical galaxies.

However, spiral galaxies inside halos generate a non-central external field. Even for fairly round Saturn, non-central forces detectably affect motions of its moons out to ~4 body radii [58]. Halos have an s similar to the r of the disk, and so will be affected by non-central forces surrounding the disk. Extremely non-central forces exist around a flattened, constant density oblate with axial ratios similar to that of spirals (Figure 4). Orbital orientations of the dwarf satellites of the Milky Way and Andromeda are consistent with non-central force fields surrounding each of these large spirals [58]. Because an NBDM halo experiences non-central forces from its interior galaxy, it cannot be spherical as postulated, and thus cannot act as a central point mass.

Perhaps some unknown phenomenon maintains the spherical shape of an NBDM halo. If this were indeed the case, interactions involving NBDM particles are not entirely governed by the Newtonian physics exemplified by Equation (5). We cannot stipulate this mysterious hypothetical force without observations or experiments on NBDM. We return to this conundrum after discussing other inferred behaviors of halo gas.

3.2. Gravitation and Collisions

3.2.1. Gravitational Attraction of NBDM to Baryons Requires Collisions

Because gravity is an attractive force, collisions will occur between particles which gravitationally interact. Large-scale collisions inside spiral galaxies are deemed unimportant on the basis of their coherent internal rotational motions. Importantly, images utilizing light with ~21 cm wavelengths (Figure 1b) show that H atoms also surround spiral galaxies, essentially providing a galactic atmosphere. Radio images at similar wavelengths (L band) and shorter wavelengths (C band) provide confirmation and further show that cosmic rays behave likewise (Figure 1c,d). Cosmic rays are typically protons or He nuclei.

Thus, collisions of NBDM with cosmic ray baryons are unavoidable, even if NBDM consists of point masses, because baryons have a finite size, and these two entities coexist (cf. Figure 1a–d). Moreover, cosmic rays travel in paths other than rotating with the galaxy, as demonstrated by their detection on Earth, and so would cross paths of the coherently rotating halo particles.

Regarding H atoms in the IGM, random thermal motions exist locally. Local is defined by whatever volume is isothermal (Section 2.2.1). In low-density regions, the mean free path between collisions can be quite long. Thus, not only cosmic rays, but also H atoms in the IGM, can collide with NBDM particles (Figure 5). Despite the low estimated density of IGM gas (~3 H atoms per m^3 [34]), the scale length of this region (~10^{17} m for a small galaxy [63]) requires that the number of such collisions around a spiral galaxy be large. Since the mean free path between collisions is roughly particle separation distances (see [32] or Section 2.2), then roughly 10^{13} collisions should occur along a line-of-sight. Implications of baryon–NBDM particle collisions on halo energetics and stability are discussed below.

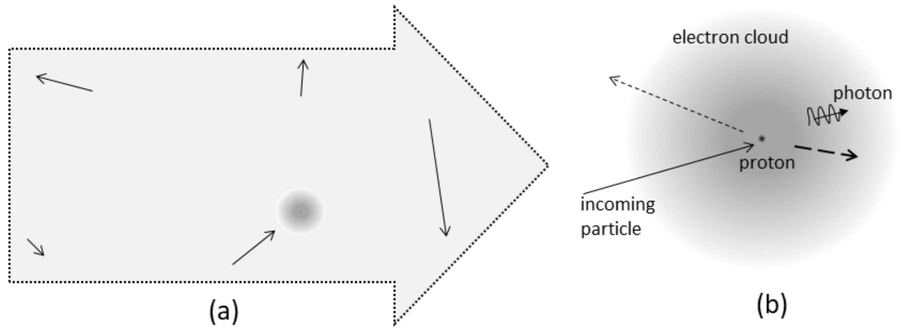

Figure 5. Schematics of collisions in a gas of NBDM: (**a**) section of the halo that is spinning (grey arrow) around the galaxy. The atom (spot) is immense compared to the particles (thin black arrows), offering a large cross section; (**b**) collision of particle with an H atom. The nucleus is the draw, which has $>10^4\times$ more mass. Distortions of the nucleus or of the electron cloud during the collision result in emission of a photon as the particle and atom each rebound (dotted arrows).

3.2.2. Limitations on the Size of NBDM Particles in View of Their Collisions

Only head-on collisions were considered in EKTG [32]. Glancing collisions of finite-size particles are confirmed by the ratios of transport properties (Figure 3a). Deformation of the electron cloud is unavoidable in either. A macroscopic analogy for this microscopic process is the surfaces of soft spheres rubbing during collisions and generating heat via friction (Section 2.2.3). Halos are not luminous, so NBDM particles should be point masses or very, very small—i.e., sizes must be less than those of γ-rays probed recently [5]. This deduction is consistent with proposals of neutrinos as halo constituents [3] and in the distant universe [9], since neutrinos are thought to occupy no space. However, probing dwarf spheroidal galaxies in the γ-ray EM region (~10 TeV and above) failed to provide evidence of NBDM [4], despite NOM fits to RC data suggesting these objects are ~99% dark matter [6]. The wavelengths explored (~10^{-10} nm) being four orders of magnitude smaller than the size of the H nucleus (~10^{-6} nm) supports approximating the particles as point masses. Glancing collisions being possible suggests that point masses are required to minimize and possibly to avoid any interaction of NBDM with baryons.

Notably, neutrinos were ruled out as the main type of NBDM particle due to various efforts in the past 15 years [7–10]. Another hypothesis, weakly interacting massive particles (WIMPs), has been a more recent focus [10,11]. The existence of WIMPs was subsequently ruled out by large hadron collider experiments [11]. In response, gravitational lensing models as a means to indirectly detect another hypothetical particle (axions) was proposed in 2019 [12]. Because considerations after 50 years of studies are relegated to hypothetical particles [29] and modeling efforts, it is valid to consider point masses, which are likewise hypothetical.

In considering NBDM particles as point masses, interactions with one another can be neglected since point masses cannot collide [32], despite having thermal motions and/or gravitational attraction. This lack of interactions (Figure 5) reduces, if not eliminates, the likelihood that some force in addition to Newtonian gravitation exists, which could maintain the shape of the halo (Section 3.1.2.). The reason underlying this statement is that the mystery force cannot affect the baryons in a spiral galaxy, or this too would be a sphere. Additional problems exist.

3.3. Thermal Consequences of Pure NBDM Halos Not Interacting with Photons

The lack of evidence for emission, absorption, or scattering of EM radiation from halos underlies the proposal of dark matter—i.e., that which does not interact with photons. Wavelengths ranging from those of radiowaves to γ-rays have been used to explore the surroundings of our Milky Way galaxy and other environments, so no spectral region remains explored under current technology. If both photons and NBDM particles are point “corpuscles”, no such interaction will occur. It is beyond the scope of this report to discuss the size of photons as particles—e.g., from slit experiments, or by considering the unresolved particle–wave duality. Perhaps the lack of photon interactions with NBDM particles may be interpreted as the wave-like character of photons persisting as EM radiation traverses the mysterious halo gas. To avoid making additional assumptions, we simply take non-interaction as a given and note that point mass particles would minimize such interactions, in view of laboratory experiments on baryonic matter.

Halo matter is presumed to be dark (Section 1). Because NBDM gas cannot absorb or emit light, it clearly has no blackbody spectrum. The peak in a blackbody curve trends to null frequency (and energy) as T approaches 0K, and likewise its intensity goes to zero, as established by both experiments and theory (see Equations (1) and (2)). Hence, pure NBDM halo gas devoid of collisions has a T exactly equal to 0K, a condition that violates the third law (Figure 6, right-hand side). The alternative explanation, namely that T of NBDM halo gas is undefined, is also inconsistent with thermodynamics and the kinetic theory of gas. In either case, the non-interaction of photons with NBDM particles, if present as a large halo, would violate the third law, which is listed in Table 2.

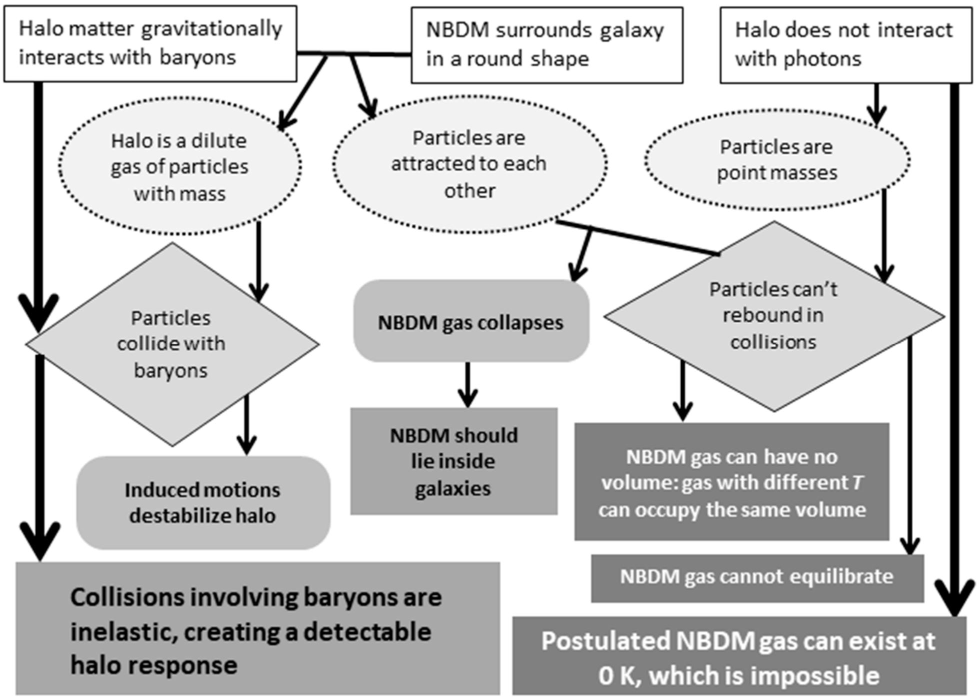

Figure 6. Schematic showing thermodynamic consequences of the postulated behavior of NBDM in galactic halos. Black arrows trace implications, where subhorizontal lines indicate considerations being applied simultaneously and heavy arrows emphasize key deductions. White boxes list postulated behavior of NBDM in halos; pale ovals indicate consequences for the nature of NBDM; various grey shapes indicate additional consequences; very dark grey boxes indicate violations of thermodynamic laws.

The second law is also violated for a medium that does not interact with photons. Since heat is not produced by the postulated NBDM gas, the direction of heat flow between two regions of NBDM gas with different temperatures is ambiguous or undefined.

3.3.1. Implications on Halo Gas Volume

NBDM consisting of point mass particles addresses its non-interaction with EM radiation, but this solution leads to a gas with particles that cannot collide. Various implications follow, as summarized in the schematic of Figure 6.

First, a gas solely composed of NBDM cannot equilibrate, because non-colliding particles cannot exchange energy. Fast, hot particles remain hot; slow, cold particles stay cold. Without the possibility of equilibration, what is the temperature? Such behavior violates the zeroth law. Second, this gas of point masses can occupy zero volume, as has been attributed to neutrinos. Consequently the pressure (force/area) is infinite. Defining any of the thermodynamic state variables (T, P, V, and entropy S) or state functions (internal energy, enthalpy or either free energy) for halo gas is problematic. This is obviated from considering T and V as the two independent state variables, as is appropriate for gravitational problems [64], which makes P and S the dependent variables [37]. The violation of virtually all statements of classical thermodynamics can be seen in Table 2.

3.3.2. Implications on Galaxy Structure

Because the NBDM point masses interact gravitationally, and such a gas is infinitely compressible, it should occupy zero volume in a self-gravitating body. This finding is underscored by the lack of a finite T, since thermal energy (K.E. of translations for indivisible particles) maintains a volume of any other gas against compression. Moreover, NDBM gas would have infinite density and so should lie at the center of a spiral galaxy, not on the outside. Given the rarified nature of the ISM and other regions of space, impediments against collapse are few. This distribution of masses for a galaxy composed of both NBDM and baryon types suggested here is consistent with what we know about stars and planets, but such a configuration cannot explain galactic rotation curves.

3.3.3. Can the Particles Be Very, Very Tiny Rather Than Being Point Masses?

The particles must be smaller than any EM waves that we can detect (~10^{-10} m wavelengths were recently explored [5]). Irrespective of the exact volume of a photon, it is thus possible that the volume occupied by NBDM particles would not be zero, but would be tiny. A high density relative to baryonic gas is still expected, and consequently NBDM should be preferentially located towards the center of a galaxy.

Another problem arises—interaction with light is not precluded for small particles. If NBDM exists, these particles have a size smaller than the wavelengths of γ-rays. Yet, EM radiation this short or this energetic is unknown [60]. The conundrum here is that the various regions of the EM spectrum [65] are associated with certain processes and length-scales. For example, molecular vibrations produce infrared light whereas atomic processes produce γ-rays. Should NBDM exist in the universe, its particles would have provided radiation at some frequency that is compatible with whatever processes exist in a halo. Given the large amounts of mass associated with halos and beyond [28], such radiation would have been detected. Yet, the EM photons that we observe in the universe are all consistent with processes occurring in baryonic matter.

3.3.4. Summary on the Thermodynamics of a Pure NBDM Gas

Thermodynamic principles prohibit the existence of a pure NBDM halo gas. Next, we consider if such a gas can exist if it is intermingled with baryons.

3.4. Thermal Consequences of Baryons Colliding with NBDM Particles

The inevitable collisions of NBDM halo particles with baryons in the IGM and in cosmic rays will transfer kinetic energy of the random baryon translations to NBDM particles, in accordance with the conservation of energy and momentum. However, baryons will deform during impacts and upon recovery will release a photon (Section 2; Figures 2 and 5). Photon characteristics are determined through conservation laws. Overall, H atoms colliding with halo particles will cool the IGM. This EM radiation will be abundant and detectable where halo gas and IGM overlap (Figure 1), but no evidence exists for these interactions, which would always cool the IGM. Hence, NBDM halo gas does not coexist with the baryonic gas composing the IGM (Figure 6, left side).

Intermixed gases can eliminate non-defined values of some thermodynamic state variables, since the T of halo gas will take on the T of the IGM under equilibrium. However, halo particles cannot equilibrate with each other and can still collapse to zero volume via gravitational attraction. Orbits are insufficient to stabilize a volume of NBDM gas because random motions are caused via interactions with the IGM, thereby perturbing NBDM particles from their stable orbits. This circumstance impedes and possibly prevents equilibration and can lead to other oddities (Figure 6) and violations of thermodynamic laws.

Furthermore, cosmic rays cross the volume purportedly occupied by the halo. Any collisions with NBDM would destabilize the halo, leading to its collapse.

4. Discussion and Conclusions

Attempts to explain galactic rotation curves when assuming Newtonian orbits about a dominating central mass (or spherical distribution) led to the hypothesis that spiral galaxies are surrounded by a halo of non-baryonic dark matter gas. This hypothesis, which requires gravitational interactions of halo particles with baryons but negligible interaction with photons, stipulates the behavior of halo gas. Above, we showed that these postulated properties for halo gas are inconsistent with thermodynamic laws and with the kinetic theory of gas. The classical kinetic theory of gas, which is based on elastic collisions and does not include the involvement of electromagnetic radiation, should describe a halo gas composed of point mass particles. Point masses may be problematic as regards the absence of collisions between NBDM particles, but are essential if interactions with photons are negligible or non-existent, as the presently popular descriptions assume. An intermingled gas of baryons and NBDM avoids some violations of thermodynamic laws but leads to an insurmountable problem: EM radiation would be produced in the numerous collisions of baryons and non-baryons, thereby making the halo detectable. This "light" is not seen, and its absence refutes the postulated behavior. Furthermore, should copious amounts of non-baryonic dark matter exist, such an entity would be most concentrated at the center of the galaxies due to its extreme compressibility. Rotation curves cannot be fitted under this circumstance. Notably, independent searches with large hadron colliders have yielded no evidence for the recently popular contender, hypothetical weakly interacting massive particles [12]. As noted above, the existence of NBDM particles smaller than protons requires radiation to exist at wavelengths smaller than are known, which should have arrived from space.

In short, evidence is lacking for halos on all fronts. Neither observational astronomy, nor experimental particle physics, nor classical thermodynamics provide support. In addition, the shape of an NBDM halo cannot be spherical around flat spirals with high aspect ratios, based on Gauss's formulae for attraction to an oblate spheroid (Figure 4). The non-central force field of oblate objects is confirmed by radio images of the hydrogen atmosphere around the visible disks of edge-on galaxies (Figure 1b–d), by the geometry of the orbits of dwarf satellite galaxies, and by analogy to the behavior of the moons closest to Saturn [58], the most oblate planet in the Solar System. Unambiguous evidence for halos did not exist in 1975 [16] and does not exist today, since the multicomponent forward fitting approach to RC (NOMs) utilizes too many free parameters compared to measurements (e.g., [18]). Halos cannot exist as currently postulated.

Funding: This research received no external funding.

Acknowledgments: I thank R.E. Criss (W.U.) and E.M. Criss (Panasonic Avionics) for stimulating discussions.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Rubin, V.C.; Ford, W.K. Rotation of the Andromeda nebula from a spectroscopic survey of emission regions. *Astrophys. J.* **1970**, *159*, 379. [CrossRef]
2. Faber, S.M.; Gallagher, J.S. Masses and mass-to-light ratios of galaxies. *Ann. Rev. Astron. Astrophys.* **1979**, *17*, 135–187. [CrossRef]

3. Trimble, V. Existence and nature of dark matter in the universe. *Ann. Rev. Astron. Astrophys.* **1987**, *25*, 425–472. [CrossRef]
4. Geist, K. Wimps and machos. In *Encyclopedia of Astronomy and Astrophysics*; Murdin, P., Ed.; Institute of Physics Publishing: Bristol, UK, 2006. [CrossRef]
5. Ackermann, M.; Albert, A.; Anderson, B.; Baldini, L.; Ballet, J.; Barbiellini, G.; Bastieri, D.; Bechtol, K.; Bellazzini, R.; Bissaldi, E.; et al. Dark matter constraints from observations of 25 Milky Way satellite galaxies with the Fermi Large Area Telescope. *Phys. Rev. D* **2014**, *89*, 042001. [CrossRef]
6. De Vega, H.J.; Salucci, P.; Sanchez, N.G. Observational rotation curves and density profiles versus the Thomas-Fermi galaxy structure theory. *Mon. Not. R. Astron. Soc.* **2014**, *442*, 2717–2727. [CrossRef]
7. Gaitskill, R.J. Direct detection of dark matter. *Annu. Rev. Nucl. Part. Sci.* **2004**, *54*, 315–359. [CrossRef]
8. Dark Matter. Available online: https://en.wikipedia.org/wiki/Dark_matter (accessed on 26 April 2020).
9. Ade, P.A.R.; Aghanim, N.; Armitage-Caplan, C.; Arnaud, M.; Ashdown, M.; Atrio-Barandela, F.; Aumont, J.; Baccigalupi, C.; Banday, A.J.; Barreiro, R.B.; et al. Planck 2013 results. XVI. Cosmological parameters. *Astron. Astrophys.* **2014**, *571*, A16.
10. Feng, J.L. Dark matter candidates from particle physics and methods of detection. *Annu. Rev. Astron. Astrophys.* **2010**, *48*, 495–545. [CrossRef]
11. Giagu, S. WIMP dark matter searches with the ATLAS detector at the LHC. *Front. Phys.* **2019**, *7*. [CrossRef]
12. Nagano, K.; Fujita, T.; Michimura, Y.; Obata, I. Axion dark matter search with interferometric gravitational wave detectors. *Phys. Rev. Lett.* **2019**, *123*. [CrossRef]
13. Garcia-Ruiz, I.; Sancisi, R.; Kuijken, K. Neutral hydrogen and optical observations of edge-on galaxies: Hunting for warps. *Astron. Astrophys.* **2002**, *394*, 769–789. [CrossRef]
14. Wiegert, T.; Irwin, J.; Miskolczi, A.; Schmidt, P.; Carolina Mora, S.; Damas-Segovia, A.; Stein, Y.; English, J.; Rand, R.J.; Santistevan, I. CHANG-ES IV. Radio continuum emission of 35 edge-on galaxies observed with the Karl G. Jansky very large array in D configuration—Data release 1. *Astronom. J.* **2015**, *150*, 81. [CrossRef]
15. CHANG-ES Continuum Halos in Nearby Galaxies- and EVLA Survey. Available online: http://www.queensu.ca/changes (accessed on 26 January 2020).
16. Burbidge, G. On the masses and relative velocities of galaxies. *Astrophys. J.* **1975**, *196*, L7–L10. [CrossRef]
17. Bottema, R.; Pestaña, J.L.G. The distribution of dark and luminous matter inferred from extended rotation curves. *Mon. Not. R. Astron. Soc.* **2015**, *448*, 2566–2593. [CrossRef]
18. Hofmeister, A.M.; Criss, R.E. Debated Models for Galactic Rotation Curves: A Review and Mathematical Assessment. *Galaxies* **2020**, *8*, 47. [CrossRef]
19. Milgrom, M. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophys. J.* **1983**, *270*, 365–370. [CrossRef]
20. Brownstein, J.R.; Moffat, J.W. Galaxy rotation curves without nonbaryonic dark matter. *Astrophys. J.* **2006**, *636*, 721–741. [CrossRef]
21. Lin, H.-N.; Li, M.-H.; Li, X.; Chang, Z. Galaxy rotation curves in the Grumiller's modified gravity. *Mon. Not. R. Astron. Soc.* **2013**, *430*, 450–458. [CrossRef]
22. Feng, J.Q.; Gallo, C.F. Mass distribution in rotating thin-disk galaxies according to Newtonian dynamics. *Galaxies* **2014**, *2*, 199–222. [CrossRef]
23. Pavlovich, K.; Pavlovich, A.; Sipols, A. Newtonian explanation of galaxy rotation curves based on distribution of baryonic matter. *arXiv* **2014**, arXiv:1406.2401P.
24. Marr, J.H. Galaxy rotation curves with lognormal density distribution. *Mon. Not. R. Astron. Soc.* **2015**, *448*, 3229. [CrossRef]
25. McGaugh, S.S. A tale of two paradigms, the mutual incommensurability of LCDM and MOND. *Can. J. Phys.* **2015**, *93*, 250–259. [CrossRef]
26. Hofmeister, A.M.; Criss, R.E. The physics of galactic spin. *Can. J. Phys.* **2017**, *95*, 156–166. [CrossRef]
27. Hofmeister, A.M.; Criss, R.E. Implications of Geometry and the Theorem of Gauss on Newtonian Gravitational Systems and a Caveat Regarding Poisson's Equation. *Galaxies* **2017**, *5*, 89. [CrossRef]
28. De Swart, J.G.; Bertone, G.; van Dongen, J. How dark matter came to matter. *Nat. Astron.* **2017**, *1*, 0059. [CrossRef]
29. Suleiman, R. A Model of Dark Matter and Dark Energy Based on Relativizing Newton's Physics. *World J. Condens. Matter Phys.* **2018**, *8*, 130–155. [CrossRef]
30. Zemansky, M.W.; Dittman, R.H. *Heat and Thermodynamics*, 6th ed.; McGraw-Hill: New York, NY, USA, 1981.

31. Tolman, R.C. *Relativity, Thermodynmaics and Cosmology*; Oxford University Press: Oxford, UK, 1934.
32. Reif, F. *Fundamentals of Statistical and Thermal Physics*; McGraw-Hill Book Company: St. Louis, MO, USA, 1965.
33. Hofmeister, A.M. *Measurements, Mechanisms, and Models of Heat Transport*; Elsevier: Amsterdam, The Netherlands, 2019; Chapters 1, 2, 5 and 8.
34. Fang, T.; Buote, D.A.; Humphrey, P.J.; Canizares, C.R.; Zappacosta, L.; Maiolino, R.; Tagliaferri, G.; Gastaldello, F. Confirmation of X-ray absorption by warm-hot intergalactic medium in the sculptor wall. *Astrophys. J.* **2010**, *714*, 1715–1724. [CrossRef]
35. Sofue, Y.; Rubin, V.C. Rotation curves of spiral galaxies. *Ann. Rev. Astron. Astrophys.* **2001**, *39*, 137–174. [CrossRef]
36. Truesdell, C. *The Tragicomical History of Thermodynamics*; Springer: New York, NY, USA, 1980.
37. Fegley, B., Jr. *Practical Chemical Thermodynamics for Geoscientists*; Academic Press/Elsevier: Waltham, MA, USA, 2015.
38. Norton, J.D. The impossible process: Thermodynamic reversibility. *Stud. Hist. Philos. Mod. Phys.* **2016**, *55*, 43–61. [CrossRef]
39. Barr, E.S. Historical survey of the early development of the infrared spectral region. *Am. J. Phys.* **1960**, *28*, 42–54. [CrossRef]
40. Purrington, R.D. *Physics in the Nineteenth Century*; Rutgers University Press: New Brunswick, NJ, USA, 1997.
41. McGucken, W. *Nineteenth-Century Spectroscopy*; The Johns Hopkins Press: Baltimore, MD, USA; London, UK, 1969.
42. Pippard, A.B. *The Elements of Classical Thermodynamics*; Cambridge University Press: London, UK, 1974.
43. Williams, B.W. A specific mathematical form for Wien's displacement law as νmax/T = constant. *J. Chem. Educ.* **2014**, *91*, 623. [CrossRef]
44. Valluri, S.R.; Corless, R.M.; Jeffrey, D.J. Some applications of the Lambert W function to physics. *Can. J. Phys.* **2000**, *78*, 823–831.
45. Marr, J.M.; Wilkin, F.P. A better presentation of Planck's radiation law. *Am. J. Phys.* **2012**, *80*, 339–405. [CrossRef]
46. Kangro, H. *Early History of Planck's Radiation Law*; Taylor and Francis: London, UK, 1976.
47. Kragh, H. Max Planck: The reluctant revolutionary. *Phys. World* **2000**, *13*, 31–35. [CrossRef]
48. Bergin, E.A.; Tafalla, M. Cold dark clouds: The initial conditions for star formation. *Ann. Rev. Astron. Astrophys.* **2007**, *45*, 339–396. [CrossRef]
49. Trusler, J.P.M. Kinetic Theory of Gases. Available online: http://www.thermopedia.com/content/907/ (accessed on 9 September 2018).
50. Brouard, M.; Chadwick, H.; Gordon, S.D.S.; Hornung, B.; Nichols, B.; Aoiz, F.J. Stereodynamics in NO(X)1+Ar inelastic collisions. *J. Chem. Phys.* **2016**, *144*, 224301. [CrossRef] [PubMed]
51. Clausius, R. On a mechanical theorem applicable to heat. *Phil. Mag.* **1870**, *40*, 122–127. [CrossRef]
52. Hofmeister, A.M.; Criss, R.E. Spatial and symmetry constraints as the basis of the virial theorem and astrophysical implications. *Can. J. Phys.* **2016**, *94*, 380–388. [CrossRef]
53. Berberan-Santos, M.N.; Bodunov, E.N.; Polliani, L. The van der Waals equation: Analytical and approximate solutions. *J. Math. Chem.* **2008**, *43*, 1437–1457. [CrossRef]
54. Kestin, J.; Knierrim, K.; Mason, E.A.; Najafi, B.; Ro, S.T.; Waldman, M. Equilibrium and transport properties of the noble gases and their mixtures at low density. *J. Phys. Chem. Ref. Data* **1984**, *13*, 229–303. [CrossRef]
55. Suárez-Iglesias, O.; Medina, I.; Sanz, M.; Pizarro, C.; Bueno, J.L. Self-diffusion in molecular fluids and noble gases: Available data. *J. Chem. Eng. Data* **2015**, *60*, 2757–2817. [CrossRef]
56. Bates, J.B. Infrared emission spectroscopy. *Fourier Transform. IR Spect.* **1978**, *1*, 99–142.
57. Kellogg, O.D. *Foundations of Potential Theory*; Dover Publications: New York, NY, USA, 1953.
58. Hofmeister, A.M.; Criss, R.E.; Criss, E.M. Verified solutions for the gravitational attraction to an oblate spheroid: Implications for planet mass and satellite orbits. *Planet. Space Sci.* **2018**, *152*, 68–81. [CrossRef]
59. Brewster, M.Q. *Thermal Radiative Transfer and Properties*; John Wiley & Sons: New York, NY, USA, 1992.
60. Siegel, R.; Howell, J.R. *Thermal Radiation Heat Transfer*; McGraw-Hill: New York, NY, USA, 1972.
61. Wiegert, T.; English, J. Kinematic classification of non-interacting spiral galaxies. *New Astron.* **2014**, *26*, 40–61. [CrossRef]
62. Ferrière, K. The interstellar environment of our galaxy. *Rev. Mod. Phys.* **2011**, *73*, 1031. [CrossRef]
63. NASA/IPAC Extragalactic Database. Available online: https://ned.ipac.caltech.edu/ (accessed on 10 January 2020).

64. Müller, I. Entropy: A subtle concept in thermodynamics. In *Entropy*; Greven, A., Keller, G., Warnecke, G., Eds.; Princeton University Press: Princeton, NJ, USA, 2003; pp. 17–36.
65. Electromagnetic Spectrum. Available online: https://en.wikipedia.org/wiki/Electromagnetic_spectrum (accessed on 30 April 2020).