Insects **Supplemental Material, accepted: 19 July 2020**

Wing design in flies: properties and aerodynamic function

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1. Ideal wing shape and Betz-Prandtl approximation

Prandtl [1] suggested that an elliptical wing produces minimum induced drag during translational motion. He argued that an elliptical distribution of circulation produces uniform downwash and thus minimizes the wake's kinetic energy. While elliptical wings face uniform inflow during forward motion, the inflow velocity in revolving and flapping wings increases with increasing distance from wing hinge to tip. This phenomenon requires adjustments in the ideal wing shape. The ideal circulation distribution for a revolving wing was first approximated by Betz and Prandtl [2] and later derived as a numerical exact solution by Goldstein [3] and Modarres and Peters [4]. For our review, we recalculated the ideal wing shape from Betz-Prandtl's approximation for propellers and compared the results with the solution by Goldstein. In our numerical framework we used Prandtl's lifting line theory of an inviscid fluid, and assumed a propeller with 2 blades and without geometrical twist in the spanwise direction. As hovering conditions cannot be considered by this approach, we calculated the ideal wing shape for small forward, v, and wake velocities, w.

According to Betz and Prandtl, circulation can be written as:

$$\frac{\Gamma\omega}{\pi w v} = \frac{2\mu^2}{\pi(1+\mu^2)}\cos^{-1}e^{-f}\ , \tag{1}$$

with $\Gamma\omega / \pi\omega v$ the velocity-normalized, nondimensional circulation, ω the angular velocity of the propeller, μ the translational speed of a wing blade normalized to forward speed, and f a correction factor (Fig. S1). The latter factor is defined as:

$$f = \left(1-\frac{\mu}{\mu_0}\right)\sqrt{1+\mu_0{}^2}\ , \tag{2}$$

with μ_0 the translational speed of the wing tip normalized to forward speed. The terms μ and μ_0 were derived from angular speed of the propeller, wing length, R, and the distance, r, between wing hinge and wing blade, and are written as:

$$\mu_0 = \frac{\omega R}{v} \text{ and } \mu(r) = \frac{\omega r}{v}\,. \tag{3-4}$$

As the terms μ and μ_0 are equal to the reciprocals of advance ratio, they are infinite at hovering conditions when forward speed is zero. In our calculations we thus allowed some forward speed, setting v = 0.75 ms^{-1} at a wake velocity behind the blade w =2.0 ms^{-1} (μ_0 =10). Sectional inflow velocity in a revolving wing is $u(r)=\omega r$ and calculated using values from the flight apparatus of *Calliphora* with *R*=0.01 m and 120 Hz flapping frequency (ω =754 rad s^{-1}, Fig. S2).

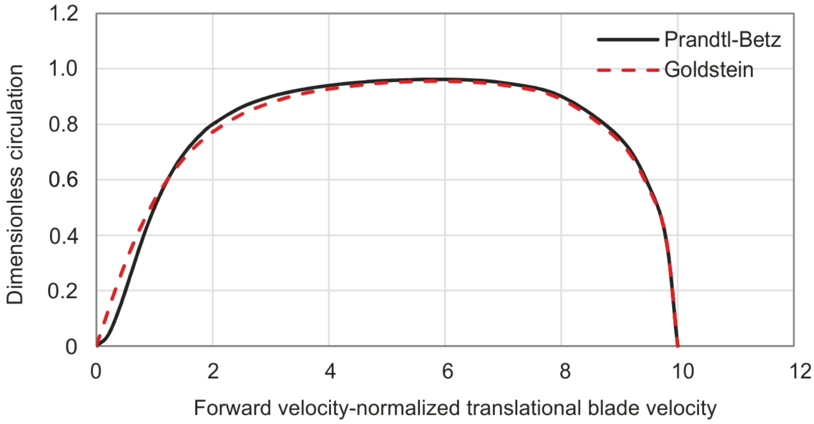

Figure S1. Spanwise distribution of normalized circulation (eq. 1) in a propeller with two blades, plotted against normalized blade velocity μ with $\mu_0=10$. Results are approximated using Prandl-Betz (black) or exactly derived using Goldstein's framework (red).

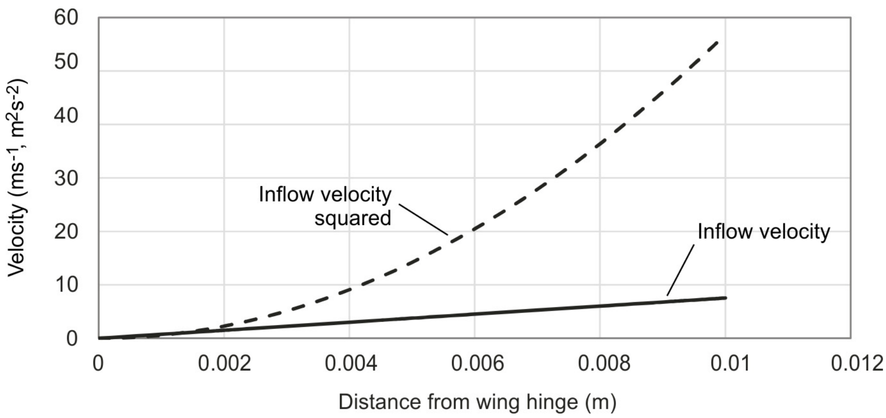

Figure S2. Oncoming air velocity (inflow) and squared inflow velocity in the revolving propeller is plotted as a function of the distance between wing blade and rotor axis (wing hinge).

2. Sectional lift production and wing chord

According to Kutta-Joukovski, lift per unit span (L' , sectional lift) is proportional to the product between sectional circulation, velocity, and air density ρ , and is written as:

$$L' = \rho u(r)\Gamma(r)\,. \tag{5}$$

Replacing circulation in equation 5 by the estimate of Prandtl and Betz, we may write equation 5 as:

$$L'(r)=\frac{2wv^2\mu(r)^3\rho}{\omega\left[1+\mu^2(r)\right]}\cos^{-1}e^{-f}\,. \tag{6}$$

To estimate sectional wing chord, c, for an ideal revolving wing, we compared the above relationship with a conventional quasi-steady model for lift production, i.e.:

$$L'(r) = \frac{1}{2} \rho \, c_L u^2(r) \, c(r) \, , \tag{7}$$

and replaced the sectional lift coefficient, c_L, by the theoretical value for a thin air foil that is $c_L = 2\pi\alpha$ with α_{eff} the wing's effective angle of attack (Fig. S3). Combining equations 6 and 7, and solving it towards c(r), sectional wing chord equals to (Fig. S4):

$$c(r) = \frac{2w\mu(r)}{\pi\omega\alpha_{eff}\left[1+\mu^2(r)\right]} \cos^{-1} e^{-f} \, . \tag{8}$$

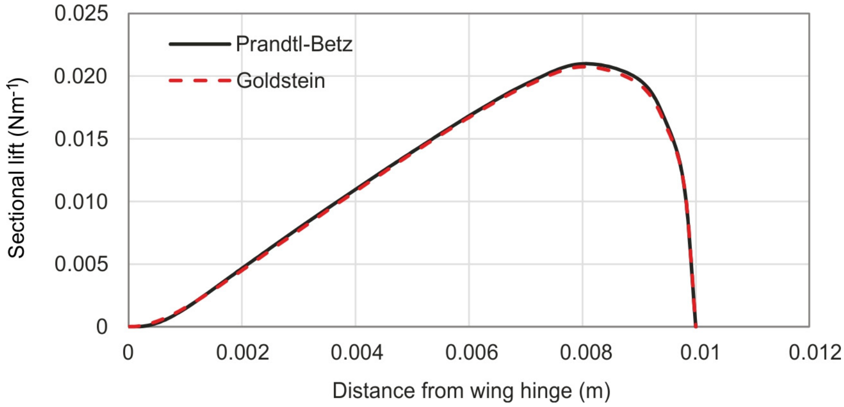

Figure S3. Spanwise distribution of sectional lift with $\mu_0 = 10$, $\upsilon = 0.7535ms^{-1}$, and $w = 2ms^{-1}$.

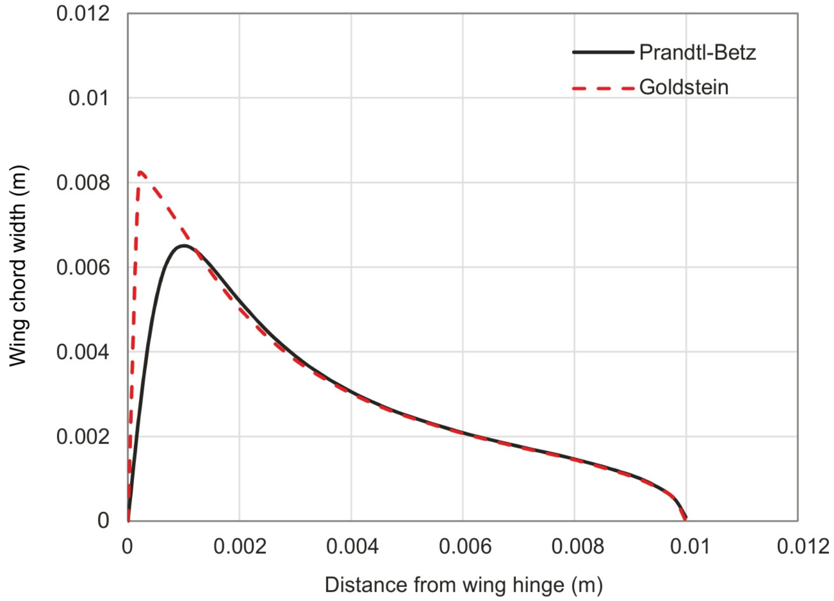

Figure S4. Sectional wing chord according to Prandtl-Betz and Goldstein. This wing shape is thought to produce an uniform vertical downwash in the wake. Effective angle of attack is 11.67° and the wing's aspect ratio is 3.44. See legends to figure S3 for other details.

3. Sectional lift production and wing chord

Prandtl and Betz did not show how the distribution of sectional wing chord leads to the planform of a wing [5]. We thus reconstructed the wing shape by distributing the ideal sectional chords above and below the wing's longitudinal axis (zero in Fig. S4). Previous data has shown that in *Calliphora* this axis lies approximately 10% behind the leading edge [6] and thus 10% of the sectional wing chord was distributed above the longitudinal axis rotational and 90% below. This procedure generated the final wing planform while maintaining the wing's aspect ratio of 3.44 (Fig. S4).

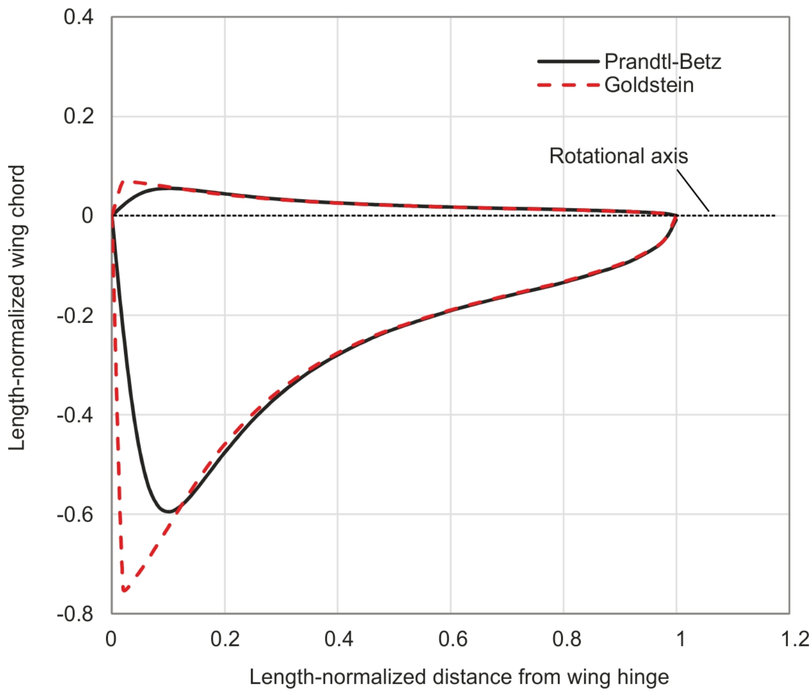

Figure S5. Wing planform using sectional chord values from figure S3 and assuming the rotational longitudinal wing axis 10% behind the leading edge. See figures S2 and S3 for more details on the numerical assumptions.

References

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