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Optimum Strength Distribution for Structures with Metallic Dampers Subjected to Seismic Loading

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Abstract: A key aspect of the seismic design of structures is the distribution of the lateral strength, because it governs the distribution of the cumulative plastic strain energy (i.e., the damage) among the stories. The lateral shear strength of a story i is commonly normalized by the upward weight of the building and expressed by a shear force coefficient α_i . The cumulative plastic strain energy in a given story i can be normalized by the product of its lateral strength and yield displacement, and expressed by a plastic deformation ratio η_i . The distribution α_i/α_1 that makes η_i equal in all stories is called the optimum yield-shear force distribution. It constitutes a major aim of design; a second aim is to achieve similar ductility demand in all stories. This paper proposes a new approach for deriving the optimum yield-shear force coefficient distribution of structures without underground stories and equipped with metallic dampers. It is shown, both numerically and experimentally, that structures designed with the proposed distribution fulfil the expected response in terms of both damage distribution and inter-story drift demand. Moreover, a comparison with other distributions described in the literature serves to underscore the advantages of the proposed approach.

Keywords: energy-based design; metallic dampers; optimum yield-shear force coefficient distribution; shaking table tests; damage concentration

1. Introduction

Conventional seismic design allows for damage to a structure under rare or very rare seismic events for economic reasons. The philosophy is to design the structural elements (e.g., beams and columns in frames) so that they can dissipate most of the energy introduced by the earthquake through plastic deformations, without leading to the collapse of the structure. To the present day, the seismic design of structures is made with the so-called force-based approach, which characterizes the loading effect of earthquakes on structures in terms of forces. The force-based approach implements the above philosophy through the use of ductility factors and the assumption of the “equal-displacement” rule. The origin of this approach can be found in the studies carried out for single degree of freedom (SDOF) systems by Veletsos and Newmark [1]. The equal displacement rule assumes that the maximum displacement of a SDOF elastoplastic system and that of the counterpart elastic system under a given ground motion are the same. Based on this assumption, the design lateral strength of the structure is reduced by a ductility-based factor μ (called q in Eurocode 8 [2]) with respect to the value that would be required for the structure to remain elastic. However, many difficulties arise when this approach is extended to multi-story structures. One resides in how to predict the distribution of the cumulative plastic energy dissipation demand E_{hi} among stories. Controlling this distribution is a key aspect of earthquake resistant design [3]. Force-based methods resolve the inability to deal explicitly with E_{hi} by prescribing rules aimed at avoiding/alleviating the concentration of plastic strain energy

demands in specific stories. These rules govern both the global (the whole structure) and the local level (the individual structural elements). Global level requirements include: (i) a minimum ratio between the sum of the flexural strengths of the columns, ΣM_{Rc} , and the flexural strengths of the beams, ΣM_{Rb} , that frame at the same joint; (ii) avoidance of abrupt variations in lateral stiffness and mass distributions, and (iii) ensuring that, at all end sections in a building where plastic hinges may form, the ratio between the maximum and minimum values of the parameter $\rho_i = D_i/C_i$ is between 2 and 3 [2]. Here, D_i is the demand obtained from a linear elastic analysis (e.g., the bending moment in moment frames), and C_i the capacity (e.g., the moment resistance in moment frames). Local level requirements include the capacity design of the sections. Yet, for economic reasons, the required minimum value of the ratio $\Sigma M_{Rc}/\Sigma M_{Rb}$ in codes (about 1.3) is far below the one that would guarantee the formation of a full strong-column/weak-beam mechanism (≥ 4) [4]. Hence, damage concentrations in given stories can occur to some extent.

Contrary to the presumption of the force-based approach that an earthquake exerts forces on the structure, in reality it imparts seismic energy [5]. Characterizing the loading effect of earthquakes in terms of energy instead of forces provides a more rational ground for seismic design and constitutes the basis of the so-called energy-based approach. This approach, first proposed by Housner [6] and later developed by Akiyama [7], Bertero [3] and others, has been gaining attention in the last two decades. It was implemented in 2009 in the Japanese Building Code [8]. Energy-based concepts are crucial also to analyse and overview strategies for designing buildings with energy dissipation devices, as shown for example in a recent review on different strategies for the optimal placement of viscous dampers in buildings structures [9]. The fact that the energy-based approach addresses the prediction of E_{hi} directly constitutes the core of the methodology. For convenience, E_{hi} can be normalized by the product of the lateral strength Q_{yi} and yield displacement δ_{yi} of the story and expressed by the parameter $\eta_i = E_{hi}/(Q_{yi}\delta_{yi})$. The ratio η_i has been widely used in the past as an index of damage. Akiyama [7] showed that the distribution of η_i among stories depends on the extent to which the so-called yield-shear force coefficient of the story, α_i , deviates from an “optimum” value $\alpha_{opt,i}$. Here, α_i is defined as Q_{yi} normalized by the weight above the story, i.e., $\alpha_i = Q_{yi} / \sum_{j=i}^N m_j g$, where m_j is the story mass, g the acceleration of gravity and N the total number of stories. $\alpha_{opt,i}$ is defined as the value of α_i that makes η_i equal in all stories (i.e., uniform distribution of damage). The distribution of $\alpha_{opt,i}$, i.e., $\bar{\alpha}_i = \alpha_{opt,i}/\alpha_1$, is referred to as the “optimum yield-shear force coefficient distribution”, and is a key aspect of seismic design methodology based on the energy approach. While several general expressions of $\bar{\alpha}_i$ have been proposed in the past for conventional structures [7,8,10,11], optimum distributions specifically for structures with energy dissipation systems are very scarce [12], in particular for those using metallic energy dissipation devices (dampers). The structures with metallic dampers constitute an innovative seismic design strategy whose use has increased exponentially in recent decades. Structures with metallic dampers solve one of the main drawbacks of conventional structures—the occurrence of plastic deformations in beams and columns of the frame—and concentrate the plastic-strain energy dissipation demands in specific elements (dampers) specially designed for that purpose and easily replaceable.

This paper proposes a new expression for the optimum yield-shear force coefficient distribution of structures without underground stories and equipped with metallic dampers, for application in the framework of the energy-based seismic design approach. In contrast to previous approaches that resort to regression analyses of the responses obtained through iterative trial-and-error calculations of structures subjected to a set of ground motions, the optimum yield-shear force distribution proposed in this study involves applying modal analysis formulation and two basic assumptions: (i) damage basically spreads out evenly among the stories regardless of the level of plastic deformation; and (ii) the ductility demand is approximately the same in all stories. The validity of these assumptions for structures designed with the optimum distribution has been demonstrated in past studies [7]. The optimum yield-shear force distribution proposed in this study applies a novel approach in comparison with the optimum distribution developed by the authors in previous studies [12]. In Benavent-Climent [12] the building was represented by an equivalent shear strut (continuous model

with distributed mass and stiffness) fixed at the base and the optimum strength distribution was obtained assuming that it coincides with the maximum shear-force distribution in the equivalent elastic undamped shear strut. The equation proposed in [12] approximated the “exact” curve that is obtained by solving the partial differential equation of the elastic undamped distributed parameter system. The new optimum distribution proposed in this paper is obtained representing the building by a discrete model with the mass lumped at several points and the lateral stiffness represented by discrete springs, under the two assumptions motioned above. The new optimum distribution applies modal analysis formulation without resorting to approximations. The application of the proposed optimum distribution to buildings with underground stories, for which the soil-structure interaction could be significant, is beyond the scope of this study. The proposed optimum distribution can be used in buildings with uneven height of the story.

2. Background

2.1. Idealization of a Structure with Metallic Dampers

Structures with metallic dampers consist of two parts working in parallel. The “main structure” is entrusted with sustaining the gravity loads while the structure moves laterally, remaining basically elastic. The other part is the “energy dissipation system”, entrusted with dissipating most of the energy input by the earthquake. The latter comprises the dampers and the auxiliary elements needed to connect them with the “main structure”. Figure 1 shows in bold lines the idealized curve that represents the i -th story shear force versus inter-story drift, Q_i - δ_i , of a structure with metallic dampers subjected to monotonic (Figure 1a) and cyclic (Figure 1b) loading. This curve is the sum of two curves. One represents the main structure, fQ_i - δ_i , and is characterized by the elastic stiffness $f k_i$. The other represents the energy dissipation system, sQ_i - δ_i , and is defined by the initial stiffness $s k_i$, the yield strength, $s Q_{y,i}$, and the yield displacement $s \delta_{y,i}$. The stiffness ratio K_i is defined by $K_i = s k_i / f k_i$. The “energy dissipation system” is typically made stiffer laterally than the “main structure”. Accordingly, hereafter, the “energy dissipation system” will be designated with the subindex “s” (stiff part) and the “main structure” with the subindex “f” (flexible part). The flexible part is assumed elastic and the stiff part is assumed to exhibit elastic-perfectly plastic restoring force characteristics.

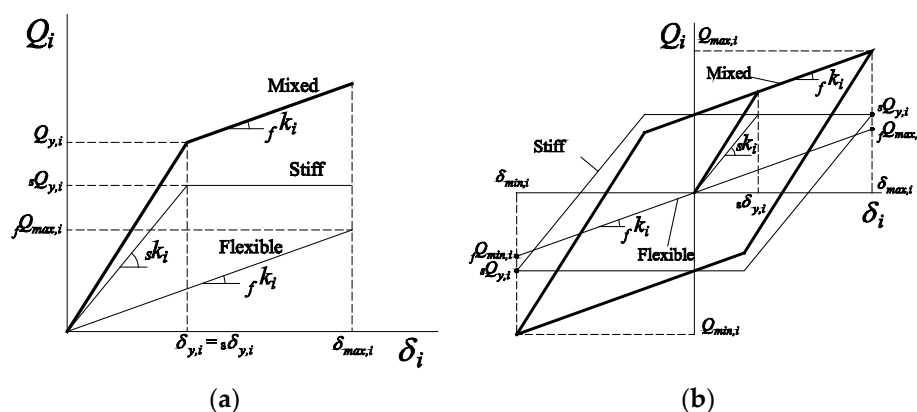


Figure 1. Story shear vs. inter-story drift relationship of a structure with metallic dampers subjected to (a) monotonic and (b) cyclic lateral loads.

The main structure must have a large enough elastic deformation capacity to achieve the maximum lateral displacement expected under seismic loads. As for the strength, past research [7] recommended the following value for the ratio $r_{q,i}$ between the averaged maximum shear force attained by the main

structure in the positive and negative domains, ${}_f\bar{Q}_{max,i} (= [{}_fQ_{max,i} + {}_fQ_{min,i}]/2)$, and the yield-shear force of the energy dissipation system, ${}_sQ_{y,i}$ at each story level:

$$r_{q,i} = \frac{{}_f\bar{Q}_{max,i}}{{}_fQ_{y,i}} \geq 0.8 \quad (1)$$

2.2. Energy-Based Design Framework

The dynamic response of an N -degree of freedom system subjected to a seismic ground motion is governed by the following force-equilibrium equation:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{F}(t) = -\mathbf{M}\mathbf{r}\ddot{u}_g(t) \quad (2)$$

where $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices, $\mathbf{F}(t)$ is the restoring force vector, $\mathbf{r}$ is the directivity ground motion vector, $\ddot{u}_g(t)$ is the ground acceleration, $\mathbf{u}$ is the relative displacement vector respect to the foundation ground and t the time. Pre-multiplying Equation (2) by $d\mathbf{u}(t)^T = \dot{\mathbf{u}}(t)^T dt$ and integrating over the duration of the earthquake, $t = t_d$, the energy balance equation is obtained:

$$\int_0^{t_d} \dot{\mathbf{u}}(t)^T \mathbf{M} \ddot{\mathbf{u}}(t) dt + \int_0^{t_d} \dot{\mathbf{u}}(t)^T \mathbf{C} \dot{\mathbf{u}}(t) dt + \int_0^{t_d} \dot{\mathbf{u}}(t)^T \mathbf{F} dt = - \int_0^{t_d} \dot{\mathbf{u}}(t)^T \mathbf{M} \mathbf{r} \ddot{u}_g(t) dt \quad (3)$$

where E_k is the kinetic energy, E_ξ is the damping energy, E_a is the absorbed energy and E_I is the relative input energy, (hereafter input energy). Further, E_I can also be expressed as an equivalent velocity, V_E , defined by $V_E = \sqrt{2E_I/M}$, where M is the total mass of the structure, and Equation (3) can be rewritten as:

$$E_k + E_\xi + E_a = E_I = \frac{1}{2} M V_E^2 \quad (4)$$

where E_a is the sum of the elastic strain energy, E_s , and the energy dissipated by plastic deformations or hysteretic energy, E_h , i.e., $E_a = E_s + E_h$. By definition, the sum of E_k and E_s is the so-called vibrational elastic energy, $E_e (= E_k + E_s)$. Housner [6] defined the sum of E_e and E_h as the energy that contributes to the damage $E_D (= E_e + E_h)$. E_D can also be expressed in the form of an equivalent velocity, V_D , by $V_D = \sqrt{2E_D/M}$ and thus Equation (4) can be rearranged as follows:

$$E_e + E_h = E_I - E_\xi = E_D = \frac{1}{2} M V_D^2 \quad (5)$$

Furthermore, several empirical expressions have been proposed in the literature to estimate V_D from V_E [7,13–18]. In this study, the one proposed by Akiyama [7] is used:

$$\frac{V_D}{V_E} = \frac{1}{1 + 3\xi + 1.2\sqrt{\xi}} \quad (6)$$

where ξ is the inherent fraction of damping of the structure. For elastic MDOF systems, $\mathbf{F}(t)$ can be expressed by $\mathbf{F}(t) = \mathbf{K}\mathbf{u}(t)$, where $\mathbf{K}$ is the elastic stiffness matrix. The damping matrix $\mathbf{C}$ can be in general non-proportional; yet, even in this case it can be approximated by a damping matrix that can be diagonalized by the natural vibration modes. This approximated damping matrix is obtained ignoring the off-diagonal coupling coefficients of the modal coordinate damping matrix. Then, the response vector $\mathbf{u}$ can be obtained by superimposing the contribution of each vibrational mode, i.e., $\mathbf{u} = \sum_{n=1}^N \varphi_n x_n(t)$, where $x_n(t)$ is the normal coordinate corresponding to mode n and φ_n the corresponding mode vector. Using a damping matrix $\mathbf{C}$ that can be diagonalized by the undamped vibration modes, taking into account the orthogonality properties, and pre-multiplying Equation (2) by φ_n^T , gives the following set of N -uncoupled equations in normal coordinates:

$$m_n \ddot{x}_n(t) + c \dot{x}_n(t) + k_n x_n(t) = -m_n \Gamma_n \ddot{u}_g(t) \quad (7)$$

where $m_n = \boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\varphi}_n$, $c_n = \boldsymbol{\varphi}_n^T \mathbf{C} \boldsymbol{\varphi}_n$ and $k_n = \boldsymbol{\varphi}_n^T \mathbf{K} \boldsymbol{\varphi}_n$ are the n -th generalized mass, damping and stiffness, respectively, and $\Gamma_n = \boldsymbol{\varphi}_n^T \mathbf{M} \mathbf{r} / m_n$ is the n -th modal participation factor. The displacements $x_n(t)$ obtained from Equation (7) are Γ_n times larger than those of an equivalent SDOF system, $x_{SDOF,n}(t)$, with mass, damping and stiffness given by m_n , c_n and k_n , respectively, subjected to the ground acceleration $\ddot{u}_g$. That is, $x_n(t) = \Gamma_n x_{SDOF,n}(t)$. Then, multiplying both sides of the Equation (7) by $dx_n = \dot{x}_n dt$ and integrating between $t = 0$ and $t \leq t_d$, the energy balance equation corresponding to the uncoupled n -th vibration mode is obtained as follows:

$$\int_0^t m_n \ddot{x}_n(t) \dot{x}_n(t) dt + \int_0^t c_n \dot{x}_n(t) \dot{x}_n(t) dt + \int_0^t k_n x_n(t) \dot{x}_n(t) dt = - \int_0^t m_n \dot{x}_n(t) \Gamma_n \ddot{u}_g(t) dt \quad (8)$$

The first three terms of the left-hand side are the kinetic energy $E_{k,n}$, the damping energy $E_{\xi,n}$ and the absorbed energy $E_{a,n}$ corresponding to vibration mode n , respectively. It is worth noting that, given the linear nature of Equation (8), the third term on the left-hand side should be, strictly speaking, denoted as the elastic strain energy, which coincides with the absorbed energy in this case. The term on the right-hand side is the input energy corresponding to vibration mode n , $E_{I,n}$. Each term of Equation (8) is Γ_n^2 times the corresponding value in the n -th equivalent SDOF system; in particular, $E_{a,n} = \Gamma_n^2 E_{an}^{SDOF}$, where E_{an}^{SDOF} is the absorbed energy of the n -th equivalent SDOF system. Moreover, substituting $\mathbf{u} = \sum_{n=1}^N \boldsymbol{\varphi}_n x_n(t)$ in Equation (3) and taking into account that $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ are orthogonal, the energy balance equation of the elastic MDOF systems at $t \leq t_d$ is:

$$\sum_{n=1}^N \left[\int_0^t m_n \ddot{x}_n(t) \dot{x}_n(t) dt \right] + \sum_{n=1}^N \left[\int_0^t c_n \dot{x}_n(t) \dot{x}_n(t) dt \right] + \sum_{n=1}^N \left[\int_0^t k_n x_n(t) \dot{x}_n(t) dt \right] = \sum_{n=1}^N \left[- \int_0^t m_n \dot{x}_n(t) \Gamma_n \ddot{u}_g(t) dt \right] \quad (9)$$

From Equations (3), (8), and (9), it can be observed that E_k , E_{ξ} , E_a and E_I in the elastic MDOF systems are obtained summing up the contribution of all n -th vibration mode energies, $E_{k,n}$, $E_{\xi,n}$, $E_{a,n}$ and $E_{I,n}$. In particular, $E_a = \sum_{n=1}^N E_{a,n}$ can also be expressed as follows:

$$E_a = \sum_{n=1}^N \Gamma_n^2 E_{an}^{SDOF} \quad (10)$$

Similar expressions can also be obtained for E_k , E_{ξ} and E_I . Equation (10) shows that the absorbed energy of an elastic MDOF system can be obtained as the superposition of the absorbed energy of N equivalent SDOF systems, each one associated with a vibration mode n , subjected to the ground acceleration $\ddot{u}_g$ and scaled by Γ_n^2 . Chou and Uang [19] proposed applying the superposition of $E_{a,n}$ likewise for structures that undergo plastic deformations to estimate the total absorbed energy at $t = t_d$. Further, Akiyama [7] showed that as the structure enters the nonlinear range, $E_e (=E_k + E_s)$ becomes negligible compared with E_h at $t \geq t_d$ and can be disregarded, i.e., $E_h = E_a$. Therefore, the following Equation (10), E_h can be estimated by:

$$E_h = \sum_{n=1}^N E_{hn} = \sum_{n=1}^N \Gamma_n^2 E_{hn}^{SDOF} \quad (11)$$

where E_{hn}^{SDOF} is the hysteretic energy dissipated by the n -th equivalent SDOF system. If the structure is classically damped, it suffices to add the contribution of the first r vibrational modes that mobilize 90% of the total mass of the structure. In the more general case that the structure is non-classically damped, it has been shown in recent research [20] that the modal mass ratio alone is not sufficient to ascertain the accuracy of a truncated model. A measure that is related to the damping matrix, called modal dissipation ratio of each mode in a complex modal analysis framework, should necessary be considered when constructing a reduced-order model.

2.3. Hysteretic Energy Accumulated under Cyclic and under Monotonic Loads

Under the cyclic reversals caused by the ground motion, each story i dissipates a portion of hysteretic energy $E_{h,i}$ of the total hysteretic energy $E_h (= \sum_{j=1}^N E_{h,j})$ dissipated by the overall structure. The quotient $\psi_i = E_{h,i}/E_h$ is defined hereafter as the i -th story hysteretic energy ratio. After the ground motion fades away, i.e., at $t \geq t_d$, $E_h = E_a$ because the elastic strain energy is totally recovered and $E_e = 0$, so that $\psi_i = E_{a,i}/E_a$. A modal hysteretic energy ratio $\psi_{n,i}$ can also be defined using the hysteretic energy dissipated at the i -th story in the mode n , $E_{hn,i}$, and the total hysteretic energy dissipated in mode n , $E_{hn} (= \sum_{j=1}^N E_{hn,j})$, i.e., $\psi_{n,i} = E_{hn,i}/E_{hn}$, and the remark made above for ψ_i at $t \geq t_d$ is also applicable to $\psi_{n,i}$. Similarly, if the structure is subjected to monotonically applied lateral loads following the n -th vibration mode instead of cyclic loads, the ratio between the hysteretic energy dissipated at the i -th story, $E_{hmn,i}$, and the total dissipated energy, $E_{hmn} (= \sum_{j=1}^N E_{hmn,j})$, define a new ratio $\psi_{mn,i}$ as follows:

$$\psi_{mn,i} = \frac{E_{hmn,i}}{E_{hmn}} = \frac{E_{hmn,i}}{\sum_{j=1}^N E_{hmn,j}} \quad (12)$$

Throughout numerical simulations conducted on MDOF systems subjected to several ground motions, Chou and Uang [19] showed that $\psi_{n,i}$ and $\psi_{mn,i}$ have similar values. Therefore, $E_{hn,i}$ can be estimated by $E_{hn,i} = \psi_{mn,i} E_{hn}$. Recalling that $E_{hn} = \Gamma_n^2 E_{hn}^{SDOF}$ and $E_{h,i} = \sum_{n=1}^N E_{hn,i}$ it follows that:

$$E_{h,i} = \sum_{n=1}^N \psi_{mn,i} \Gamma_n^2 E_{hn}^{SDOF} \quad (13)$$

In a general MDOF system subjected to a given ground motion characterized by its hysteretic energy spectrum $E_h^{SDOF}-T$, the hysteretic energy accumulated in the i -th story $E_{h,i}$ can be estimated through Equation (13), first, calculating $\psi_{mn,i}$ from a pushover analysis with the n -th vibrational mode loading pattern and, second, taking as E_{hn}^{SDOF} the ordinate of the $E_h^{SDOF}-T$ spectrum at period T_n .

3. Proposal of New Optimum Yield-Shear Force Coefficient Distribution

The main purpose of installing dampers is to avoid damage in the main structure and concentrate the plastic deformations in the energy dissipation system. Accordingly, in the following discussion the main structure is assumed to remain elastic and the cumulative plastic strain energy ratio η_i for this type of structure is defined as $\eta_i = {}_sE_{h,i}/({}_sQ_{y,i} \times {}_s\delta_{y,i})$, where ${}_sE_{h,i}$ denotes the hysteretic energy dissipated by the energy dissipation devices (dampers) installed at the i -th story. For convenience, noting that ${}_sQ_{y,i} = \sum_{j=i}^N (m_j g) {}_s\alpha_j$, ${}_s\delta_{y,i} = {}_sQ_{y,i}/{}_sk_i = \sum_{j=i}^N (m_j g) {}_s\alpha_j / (K_i f k_i)$ and using Equation (13), the plastic strain energy ratio η_i can be rewritten as follows:

$$\eta_i = \frac{K_i f k_i \sum_{n=1}^r (\psi_{mn,i} \Gamma_n^2 E_{hn}^{SDOF})}{\left[\sum_{j=i}^N (m_j g) \right]^2 {}_s\alpha_i^2} \quad (14)$$

where the sum in the numerator is extended to r vibration modes. For a given ground motion, the ${}_s\alpha_i$'s that make η_i equal in all stories, i.e., $\eta_i = \eta$, are defined here as the optimum yield-shear force coefficients of the energy dissipation system, ${}_s\alpha_{opt,i}$. The corresponding distribution, ${}_s\alpha_{opt,i}/{}_s\alpha_1$, is referred to hereafter as optimum yield shear force distribution ${}_s\bar{\alpha}_i (= {}_s\alpha_{opt,i}/{}_s\alpha_1)$, where ${}_s\alpha_1$ is the yield-shear force coefficient of the energy dissipation system at the first (ground) story. Using Equation (14) and noting that $\eta_i = \eta$, then ${}_s\bar{\alpha}_i$ is given by:

$${}_s\bar{\alpha}_i = \frac{M}{\sum_{j=i}^N m_j} \sqrt{\frac{K_i f k_i \sum_{n=1}^r (\psi_{mn,i} \Gamma_n^2 E_{hn}^{SDOF})}{K_1 f k_1 \sum_{n=1}^r (\psi_{mn,1} \Gamma_n^2 E_{hn}^{SDOF})}} \quad (15)$$

Taking into account that $s\delta_{y,i} = sQ_{y,i}/sk_i$, recalling Figure 1 and noting that $Q_{y,i} = sQ_{y,i} + f k_i s\delta_{y,i} = sQ_{y,i} + (s k_i / K_i) s\delta_{y,i} = sQ_{y,i} + sQ_{y,i}(1/K_i) = sQ_{y,i}(K_i + 1)/K_i$, the yield shear force coefficient of the whole structure (including main structure and energy dissipation system) at the i -th story α_i is related to $s\alpha_i$ by:

$$\alpha_i = s\alpha_i \left(\frac{K_i + 1}{K_i} \right) \quad (16)$$

Therefore, the distribution of the optimum total yield shear force coefficient, $\bar{\alpha}_i = \alpha_{opt,i}/\alpha_1$, is simply $\bar{\alpha}_i = \alpha_{opt,i}/\alpha_1 = [s\alpha_{opt,i}(K_i + 1)/K_i] / [s\alpha_1(K_1 + 1)/K_1] = s\bar{\alpha}_i[K_1(K_i + 1)] / [K_i(K_1 + 1)]$. If K_i is the same in all stories, it is obvious that $\bar{\alpha}_i$ coincides with $s\bar{\alpha}_i$. Using Equation (15) $\bar{\alpha}_i$ gives, then:

$$\bar{\alpha}_i = \frac{M}{\sum_{j=1}^N m_j} \sqrt{\frac{K_i f k_i \sum_{n=1}^r (\psi_{mn,i} \Gamma_n^2 E_{ln}^{SDOF})}{K_1 f k_1 \sum_{n=1}^r (\psi_{mn,1} \Gamma_n^2 E_{ln}^{SDOF})} \frac{K_1 (K_i + 1)}{K_i (K_1 + 1)}} \quad (17)$$

Past research [7] shows that in an elastic-perfectly plastic MDOF system designed with the optimum distribution associated with a given ground motion, damage spreads out evenly among the stories essentially regardless of the level of plastic deformation. This allows to assume that if the ground motion is scaled down by a factor s_y so that the level of plastic deformation in the dampers is zero, i.e., $\eta_i = \eta \approx 0$, the maximum total elastic force developed by the dampers at each story i in each mode n , $sQ_{maxn,i}$, coincides with the lateral strength required on the dampers in each mode n , $sQ_{yn,i}$, to carry all stories to the brink of yielding, that is $sQ_{maxn,i} = sQ_{yn,i}$. These forces can be expressed in terms of shear force coefficients, $s\alpha_{max,i}$, and their distribution, i.e., $s\bar{\alpha}_{max,i} = s\alpha_{max,i}/s\alpha_{max,1}$, would coincide with the optimum distribution $s\bar{\alpha}_i$, i.e., $s\bar{\alpha}_{max,i} = s\bar{\alpha}_i$. The vector of maximum elastic forces on the whole structure (i.e., main structure and energy dissipation system) at the n -th vibration mode, $\mathbf{F}_n$, of an elastic MDOF system subjected to a given ground motion characterized by its elastic response spectrum S_a-T scaled by factor s_y can be easily obtained from modal analysis as follows:

$$\mathbf{F}_n = \mathbf{M}\boldsymbol{\varphi}_n \Gamma_n s_y S_{an} \quad (18)$$

where S_{an} is the ordinate of the S_a-T spectrum at the period of vibration mode n . The corresponding maximum total shear force on the i -th story in mode n , $Q_{maxn,i}$, is simply obtained by summing up the components $F_{n,i}$ of vector $\mathbf{F}_n$ above the i -th story as follows:

$$Q_{maxn,i} = s_y \Gamma_n S_{an} \sum_{j=i}^N (m_j \varphi_{n,j}) \quad (19)$$

where $\varphi_{n,j}$ are the components of mode vector $\boldsymbol{\varphi}_n$. The maximum force in the energy dissipation system at the i -th story in mode n , $sQ_{maxn,i}$, can be related with $Q_{maxn,i}$ by $Q_{maxn,i} = sQ_{maxn,i} + (sQ_{maxn,i}/sk_i) f k_i = sQ_{maxn,i}(K_i + 1)/K_i$. Solving for $sQ_{maxn,i}$ in this expression and recalling that it has been assumed that $sQ_{maxn,i} = sQ_{yn,i}$, the following expression is obtained:

$$sQ_{yn,i} = \frac{K_i}{(1 + K_i)} s_y \Gamma_n S_{an} \sum_{j=i}^N (m_j \varphi_{n,j}) \quad (20)$$

In turn, for a design earthquake that causes plastic deformations in the energy dissipation system (i.e., a ground motion not scaled down by s_y), the maximum and minimum normalized plastic deformations experienced by the energy dissipation system of each story i in each vibration mode n are defined as follows: $\mu_{maxn,i} = (\delta_{maxn,i} - s\delta_{yn,i})/s\delta_{yn,i}$ and $\mu_{minn,i} = |(\delta_{minn,i} - s\delta_{yn,i})|/s\delta_{yn,i}$. Here $\delta_{maxn,i}$ and $\delta_{minn,i}$ are the maximum and minimum inter-story drifts attained at the i -th story in mode n , and $s\delta_{yn,i}$ is defined as $s\delta_{yn,i} = sQ_{yn,i}/sk_i$. The mean value is $\bar{\mu}_{n,i} = (\mu_{maxn,i} + \mu_{minn,i})/2$. It has been shown [7] that,

in mixed systems combining in each story an elastic part with an elastic-perfectly plastic part, as shown in Figure 1, the maximum deformations are roughly the same in the positive and negative domains and thus it can be assumed $\delta_{maxn,i} \approx |\delta_{minn,i}|$, or $\mu_{maxn,i} \approx \mu_{minn,i}$. Moreover, it has been shown [7] that if the structure is designed with the optimum distribution, then $\bar{\mu}_{n,i}$ is roughly the same in all stories and thus it can be assumed that $\bar{\mu}_{n,i} = {}_s\mu_n$. Consequently $(\delta_{maxn,i} - {}_s\delta_{yn,i}) = {}_s\mu_n {}_s\delta_{yn,i}$. The hysteretic energy accumulated at the i -th story, if pushed monotonically beyond the elastic range following the distribution of lateral forces of the n -th mode given by Equation (18) up to $\delta_{maxn,i}$, is:

$$E_{hmn,i} = {}_sQ_{yn,i}(\delta_{maxn,i} - {}_s\delta_{yn,i}) = {}_sQ_{yn,i}{}_s\mu_n{}_s\delta_{yn,i} \quad (21)$$

Recalling that ${}_s\delta_{yn,i} = {}_sQ_{yn,i}/{}_sk_i$ and $K_i = {}_fk_i/{}_sk_i$, using Equations (12), (20), and (21) $\psi_{mn,i}$ can be estimated with the following expression:

$$\psi_{mn,i} = \frac{\frac{K_i}{{}_fk_i(1+K_i)^2} \left(\sum_{j=i}^N m_j \varphi_{n,j} \right)^2}{\sum_{k=1}^N \left[\frac{K_k}{{}_fk_k(1+K_k)^2} \left(\sum_{j=k}^N m_j \varphi_{n,j} \right)^2 \right]} \quad (22)$$

Equation (22) indicates that when the distribution of lateral strength of the structure, expressed in terms of the yield-shear force coefficient, follows the optimum distribution, then $\psi_{mn,i}$ can be calculated from the mass and the elastic properties of the structures, with no need to perform a pushover analysis. In this study, Equation (15) with $\psi_{mn,i}$ calculated using Equation (22) is proposed as the optimum yield-shear force coefficient distribution for the energy dissipation system and will be denoted as ${}_s\bar{\alpha}_{prop,i}$ herein. Similarly, Equation (17) with $\psi_{mn,i}$ calculated using Equation (22) is proposed as the optimum yield-shear force coefficient distribution for the whole structure and will be denoted as $\bar{\alpha}_{prop,i}$ hereafter.

4. Numerical Validation

4.1. Prototype Structures, Modelization and Ground Motions

In order to validate the optimum yield shear-force coefficient distribution $\bar{\alpha}_{prop,i}$ proposed in this study, several prototype reinforced concrete (RC) structures consisting of waffle-flat plates supported on RC columns having 3, 6, and 9-stories (main structure) equipped with metallic dampers (energy dissipation system) were investigated. To model the prototype structures, it was assumed that the mass of each plate is concentrated in one point of mass m_i located at the centre of mass of the plate (lumped-mass model). Torsional and rocking effects are not considered and therefore a single degree of freedom is assigned to each mass: the horizontal displacement in the direction of the ground motion. The rest of parameters that define the numerical model are the i -th story lateral stiffness of the main structure ${}_fk_i$; and the lateral stiffness ${}_sk_i$ and lateral strength ${}_sQ_{yi}$ provided by the dampers. ${}_fk_i$ is defined as the quotient between the shear force endured by the main structure at the i -th story and the corresponding interstory drift in elastic conditions. ${}_sk_i$ is defined as the quotient between the shear force endured by the metallic dampers installed at the i -th story and the corresponding interstory drift in elastic conditions. ${}_sQ_{yi}$ is the maximum shear force that can be endured by the metallic dampers installed at the i -th story. The main structure is assumed to remain elastic and the metallic dampers exhibit an elastic-perfectly plastic hysteretic behaviour, as shown in Figure 1. The inherent damping fraction $\xi = 0.05$. The main structures are designed to sustain only the gravity loads specified under Spanish standards [21,22]. Therefore, ${}_fk_i$ and m_i for the prototypes with the same number of stories are identical. ${}_fk_i$ was determined from a static analysis in which the main structure was subjected to a distribution of lateral forces that followed the pattern of the first vibration mode. Table 1 shows ${}_fk_i$, m_i , the height of the story i h_i and the fundamental period of the main structure ${}_fT_1$. To validate the proposed optimum distribution in a large number of different cases, a variety of levels of plastic deformation for the dampers—expressed in terms of coefficient ${}_s\mu$ —, and a variety of

ground motions—characterized by the seismological parameters V_D , T_G , T_{NH} , I_d and the proximity to the source (i.e., near-fault or far-field earthquakes)—were considered. T_G is the predominant period of the ground motion, T_{NH} is the corner period of the pseudo-velocity Newmark-Hall spectrum, and I_D is a seismological parameter proposed by Cosenza and Manfredi [23] defined by $I_D = 2gI_A/(\pi PGA PGV)$ where I_A is the Arias intensity, PGA is the peak ground acceleration and PGV the peak ground velocity. Appendix A presents the original (unscaled) ground motions selected, together with T_G , T_{NH} , I_D , PGA and PGV . The lateral stiffness provided by the dampers at each story i , $s k_i$, was made proportional to the lateral stiffness of the main frame $f k_i$, i.e., $K_i = f k_i / s k_i = K$. Each ground motion was scaled in acceleration by applying a factor SF . The values of SF were kept in the range $0.3 \leq SF \leq 3$ to distort as little as possible the characteristics of the original signal, and were determined to keep the maximum allowed inter-story drift IDI_i below 0.75%, while providing a K smaller than 20. The limit $IDI_i \leq 0.75\%$ was fixed on the basis of code recommendations [8,24] and experimental results obtained by the authors in previous studies [25–27]. Also, values of K larger than 20 are not realistic because they lead to very small values of $s \delta_{yi}$ that can hardly be applied in practice. For a given set of values of parameters $s \mu$, SF , V_D , T_G , T_{NH} , I_d and the proximity to the source, the required K , $s Q_{y1}$ —expressed in terms of the base shear force coefficient $s \alpha_1 (=s Q_{y1}/Mg)$ —, and the normalized energy dissipation demand of the dampers η were calculated applying the method developed by Benavent-Climent [12], but using the optimum yield shear force distribution proposed in this study, with Equations (15) and (22) instead of the optimum distribution proposed in [12]. Tables 2–4 summarize the combinations of values for SF , K , V_D , $s \alpha_1$, η , $s \mu$ investigated, together with the resulting fundamental period of the whole structure in the elastic range T_1 . In applying Equation (15), the input energy E_{lm}^{SDOF} corresponding to each mode n was determined from the inelastic hysteretic energy spectrum E_h-T of the scaled ground motion.

The inelastic hysteretic energy spectrum E_h-T was calculated by following Akiyama's approach [7], as explained next. First, the elastic input energy spectrum E_I-T of the ground motion was obtained with the expression of the right-hand side of Equation (3). Second, the inelastic input energy spectrum, $\bar{E}_I - T$, was determined by averaging the elastic spectrum E_I-T in the range $T \leq T \leq T_{max}$; that is, the $\bar{E}_I$ corresponding to a given period T of the spectrum was obtained by $\bar{E}_I = \int_T^{T_{max}} E_I dT / (T_{max} - T)$. Here, T_{max} is a period longer than T , influenced by the level of plastic deformations experienced by the system characterized by $s \mu$. The calculation of T_{max} is explained in Appendix B as an extension of the formulation proposed by Akiyama [7] for conventional structures to systems with metallic dampers. Third, recalling that the elastic vibrational energy E_e can be neglected as explained in Section 2.2. (i.e., $E_h = E_D$), E_h is readily obtained from $\bar{E}_I$ by means of Equation (6) as $E_h = \bar{E}_I / (1 + 3\xi + 1.2 \sqrt{\xi})^2$.

Table 1. Lateral stiffness, mass and height of the main structure.

Prototype	3-Story			6-Story			9-Story		
Story	$f k_i$ (kN/cm)	m_i (kNs ² /cm)	h_i (m)	$f k_i$ (kN/cm)	m_i (kNs ² /cm)	h_i (m)	$f k_i$ (kN/cm)	m_i (kNs ² /cm)	h_i (m)
9	-	-	-	-	-	-	1523	5.24	3.10
8	-	-	-	-	-	-	1572	5.68	3.10
7	-	-	-	-	-	-	1618	5.68	3.10
6	-	-	-	1071	3.51	3.10	2149	5.68	3.10
5	-	-	-	1120	3.89	3.10	2191	5.68	3.10
4	-	-	-	1126	3.89	3.10	2206	5.68	3.10
3	571	2.22	3.10	1141	3.89	3.10	2707	5.68	3.10
2	569	2.56	3.10	1518	3.89	3.10	2803	5.68	3.10
1	510	2.56	3.50	1636	3.89	3.50	3259	5.68	3.50
$f T_1$ (s)		0.94			1.38			1.81	

Table 2. Properties of the designed metallic in three-story buildings.

Records	SF	K	T_1 (s)	V_D (cm/s)	$s\alpha_1$	η	$s\mu$
Near-Field	-	-	-	-	-	-	-
El Centro	0.90	6.6	0.34	65	0.18	15.7	5.7
Kobe	0.35	8.1	0.31	66	0.18	19.0	7.1
Lorca	1.00	8.5	0.31	53	0.22	7.5	5.9
Tolmezzo	1.25	8.3	0.31	60	0.18	16.1	7.4
Korinthos	1.00	4.1	0.42	60	0.16	10.3	3.8
Duzce (Duzce)	0.75	11.0	0.27	79	0.18	42.3	10.4
Kalamata	1.00	4.7	0.39	49	0.16	7.0	4.6
Far-Field	-	-	-	-	-	-	-
Duzce (Izmit)	1.00	9.6	0.29	55	0.18	14.4	8.6
Montebello	1.75	12.2	0.26	65	0.17	30.8	12.0
Petrovac	0.50	7.7	0.32	80	0.16	37.3	7.8
Hachinoe	0.95	7.7	0.32	53	0.18	10.7	6.9
Taft	1.00	1.8	0.56	52	0.07	16.3	3.8
Calitri	1.00	1.6	0.58	60	0.05	38.1	4.8
Tabas	0.40	6.0	0.36	55	0.18	9.0	5.0

Table 3. Properties of the designed metallic in six-story buildings.

Records	SF	K	T_1 (s)	V_D (cm/s)	$s\alpha_1$	η	$s\mu$
Near-Field	-	-	-	-	-	-	-
El Centro	0.70	3.7	0.64	65	0.13	6.6	2.9
Kobe	0.30	5.9	0.52	69	0.14	11.3	4.9
Lorca	1.00	6.0	0.52	46	0.06	18.1	12.9
Tolmezzo	1.58	4.6	0.58	66	0.17	5.0	2.9
Korinthos	1.20	4.5	0.59	69	0.15	6.9	3.1
Duzce (Duzce)	0.75	7.9	0.46	83	0.14	23.3	6.9
Kalamata	1.30	17.7	0.32	65	0.17	18.9	13.3
Far-Field	-	-	-	-	-	-	-
Duzce (Izmit)	1.00	12.2	0.38	65	0.17	12.9	8.8
Montebello	2.30	7.5	0.47	72	0.18	9.4	5.0
Petrovac	0.62	2.0	0.80	64	0.07	13.5	3.3
Hachinoe	1.10	3.4	0.66	53	0.14	3.2	2.4
Taft	1.70	8.2	0.45	81	0.14	22.6	7.3
Calitri	1.00	2.9	0.70	76	0.06	32.3	5.3
Tabas	0.40	6.8	0.49	63	0.12	12.8	7.1

Table 4. Properties of the designed metallic in nine-story buildings.

Records	SF	K	T_1 (s)	V_D (cm/s)	$s\alpha_1$	η	$s\mu$
Near-Field	-	-	-	-	-	-	-
El Centro	0.90	7.8	0.61	85	0.15	10.9	5.2
Kobe	0.36	10.7	0.53	86	0.15	15.7	7.6
Lorca	1.60	6.0	0.68	62	0.10	9.3	7.3
Tolmezzo	1.80	5.0	0.74	76	0.15	5.3	3.2
Korinthos	1.50	16.7	0.43	90	0.15	28.1	12.5
Duzce (Duzce)	0.90	2.9	0.92	79	0.10	7.8	2.8
Kalamata	1.20	6.0	0.68	53	0.05	25.4	15.7
Far-Field	-	-	-	-	-	-	-
Duzce (Izmit)	1.10	17.6	0.42	75	0.15	18.9	13.2
Montebello	3.30	5.0	0.74	74	0.12	8.2	4.4
Petrovac	0.80	4.2	0.79	87	0.10	14.3	4.3
Hachinoe	1.20	7.1	0.64	67	0.12	8.9	6.3
Taft	1.85	2.1	1.03	69	0.06	9.6	3.2
Calitri	1.00	4.1	0.80	92	0.08	20.8	4.7
Tabas	0.35	8.8	0.58	75	0.15	9.5	6.3

4.2. Optimum Distributions $\bar{\alpha}_{prop,i}$ Obtained with the Proposed Expression for Different Ground Motions

Figures 2 and 3 show the proposed optimum yield shear force coefficient distributions $\bar{\alpha}_{prop,i}$ —Equations (17) and (22)—obtained for the near-fault (Figure 2) and far-field (Figure 3) ground motions. In all cases the grow is approximately linear in the lower two thirds of the building, and increases exponentially in the upper stories, especially for taller buildings. This is due to the influence of the higher modes of vibration. It is also worth noting that in the lower two thirds part of the building $\bar{\alpha}_{prop,i}$ is basically the same for all records, whereas clear differences are observed in the upper one third depending on the ground motion considered. These tendencies and features have been identified in previous studies [7,12].

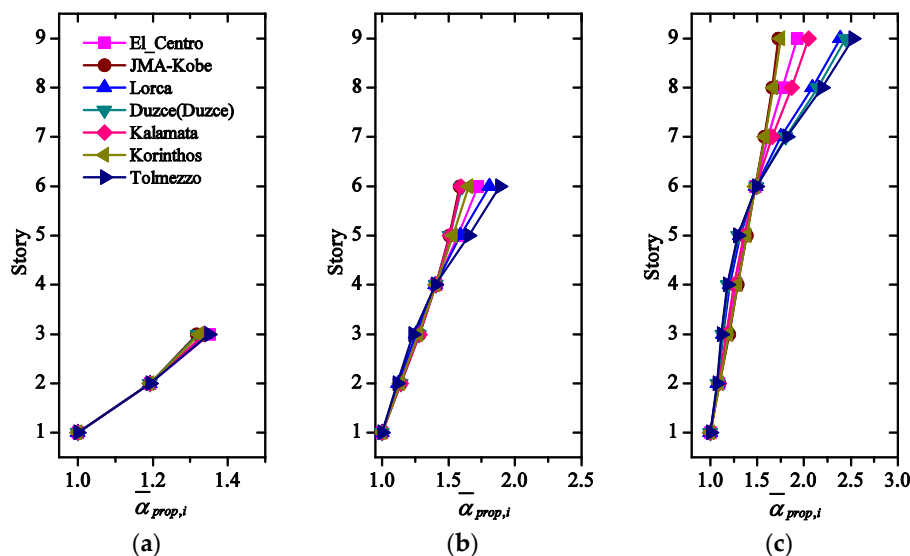


Figure 2. Optimum yield shear force distributions obtained with proposed distribution $\bar{\alpha}_{prop,i}$ for 3 (a), 6 (b) and 9 (c) story buildings subjected to seven near-field earthquakes.

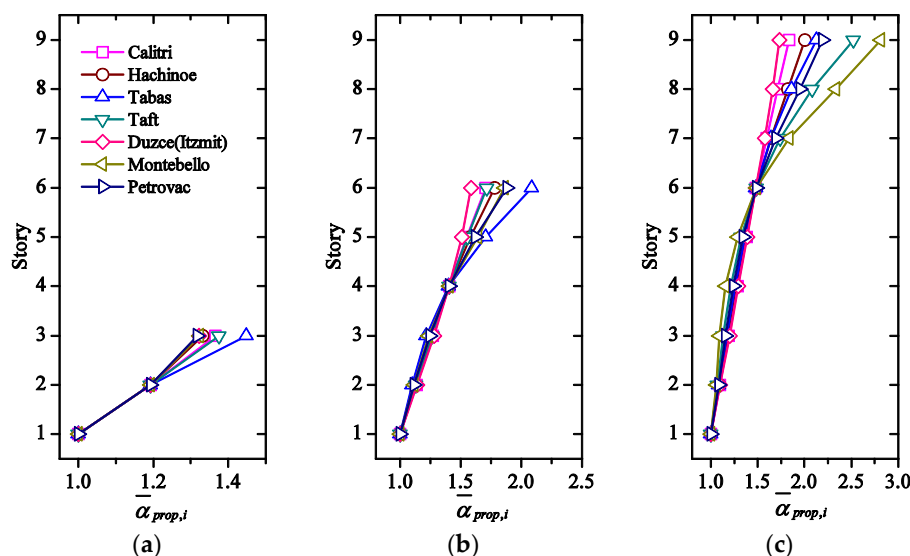


Figure 3. Optimum yield shear force distributions obtained with proposed expression $\bar{\alpha}_{prop,i}$ for 3 (a), 6 (b) and 9 (c) story buildings subjected to seven far-field earthquakes.

4.3. Comparison between Proposed and “Exact” Distributions

Similarly to the optimum yield shear force coefficient distributions proposed in previous studies [7,11,12] the new distribution proposed in this study provides an approximation to the

“exact” distribution, referred to as $\bar{\alpha}_{exact,i}$ hereafter, that would make η_i exactly the same in all stories. $\bar{\alpha}_{exact,i}$ is different for each ground motion and can be obtained only through an iterative trial-and-error procedure of nonlinear time history analyses. A number of multi-purpose algorithms existing in the literature may be used to efficiently conduct such an iterative process [28,29]. In this study, $\bar{\alpha}_{exact,i}$ was calculated using the pattern search method (PSM), which belongs to the direct search methodology for optimization of n-dimensional functions [30]. PSM has been used in previous studies on optimization of buildings with dampers [31] and on damage control of buildings [32,33]. There are other approaches such as the gradient-based method or genetic algorithms used successfully in structural optimization problems [28,29] that could potentially be more efficient to reach the “exact” optimum distribution. Nevertheless, it is out of the scope of this study to determine the most efficient method. In short, PSM is an iterative procedure to minimize an objective function. Some advantages of the PSM are: (i) it is a derivative-free method and (ii) it is simple. In this case, the objective function is the standard deviation of η_i , while the independent n-dimensional variable is the lateral strength of the structure. Through non-linear time history analyses with a given accelerogram, PSM varies the lateral strength distribution of the structure until the standard deviation of η_i is zero. The main drawback of using PSM in this study has been the significant increase of the calculation time with the increase of the number of stories of the buildings. PSM took about fifteen minutes to reach the optimum distribution for a three-story building subjected to a single ground motion. This time increased up to four hours for a six-story building and up to ten hours for a nine-story building.

Using the PSM, the exact distribution $\bar{\alpha}_{exact,i}$ was calculated for each prototype structure and for each ground motion, and then was compared with the counterpart approximate distribution proposed in this study, $\bar{\alpha}_{prop,i}$ in Figures 4 and 5. The figures show in bold lines the mean curves obtained averaging $\bar{\alpha}_{prop,i}/\bar{\alpha}_{exact,i}$ by means of the seven accelerograms applied to each prototype structure at each story level. It can be seen that the mean curves of $\bar{\alpha}_{prop,i}/\bar{\alpha}_{exact,i}$ are very close to the vertical line passing through the abscissa 1.0 (ranging from 0.9 to 1.2); this indicates that the proposed distribution is very close to the exact distribution. For comparison purposes, Figures 4 and 5 also show the mean curves obtained using the optimum distribution proposed by Benavent-Climent [12], $\bar{\alpha}_{Benavent,i}$ normalized by $\bar{\alpha}_{exact,i}$. It is clear in the figures that while both distributions $\bar{\alpha}_{prop,i}$ and $\bar{\alpha}_{Benavent,i}$ approximate the exact solution in the lower part of the structure, in the upper part the distribution proposed by Benavent-Climent deviates up to about 35% from the exact value.

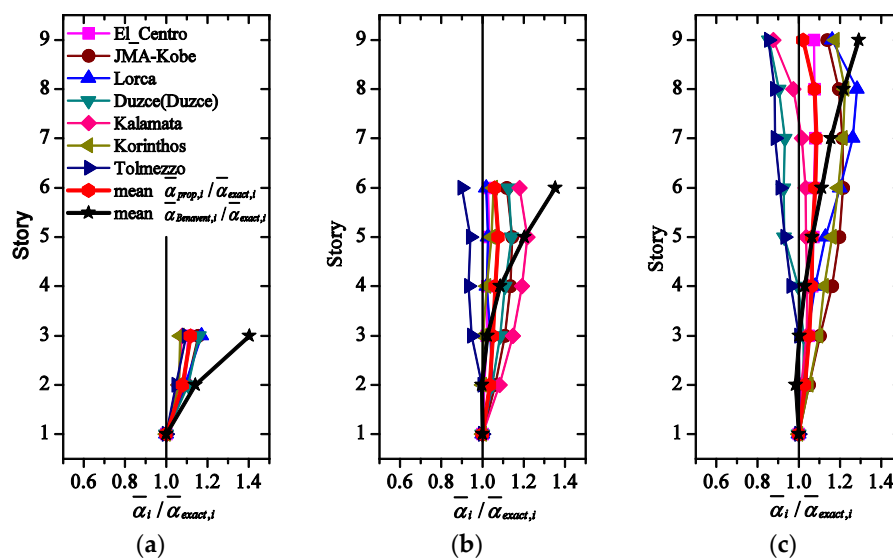


Figure 4. Ratios $\bar{\alpha}_{prop,i}/\bar{\alpha}_{exact,i}$ (record to record and mean) and $\bar{\alpha}_{Benavent,i}/\bar{\alpha}_{exact,i}$ (mean) obtained for 3 (a), 6 (b), and 9 (c) story buildings subjected to near-field earthquakes.

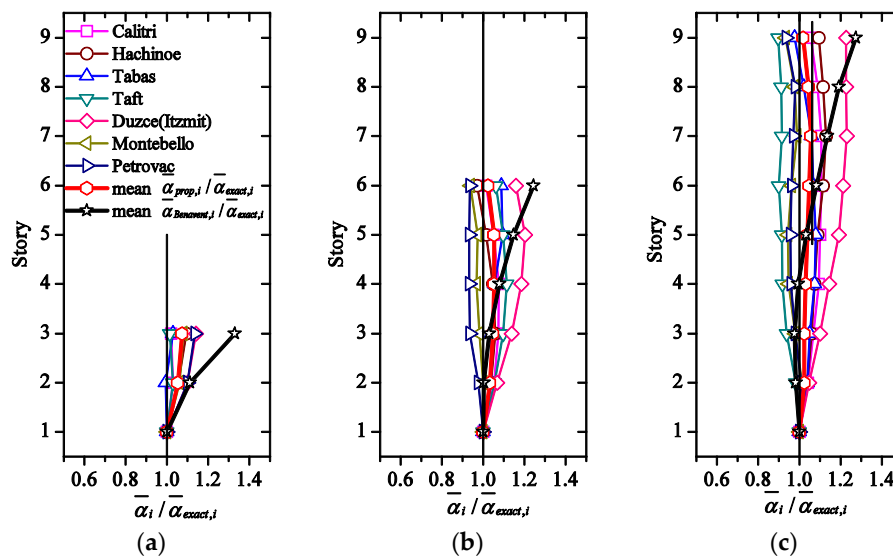


Figure 5. Ratios $\bar{\alpha}_{prop,i}/\bar{\alpha}_{exact,i}$ (record to record and mean) and $\bar{\alpha}_{Benavent,i}/\bar{\alpha}_{exact,i}$ (mean) obtained for 3 (a), 6 (b) and 9 (c) story buildings subjected to far-field earthquakes.

Under near-field ground motions, the coefficients of variation (COV) of $\bar{\alpha}_{prop,i}/\bar{\alpha}_{exact,i}$ calculated for all stories are 0.05, 0.07 and 0.10 for the three-, six-, and nine-story prototype structures, respectively. The counterpart COVs for far-field ground motions are 0.05, 0.06 and 0.08, respectively. Noticeably, there are not significant differences between the two types of earthquakes, the COV of $\bar{\alpha}_{prop,i}/\bar{\alpha}_{exact,i}$ being less than or equal than 0.10, independently the height of the building. The optimum distribution proposed by Benavent-Climent [12] provides a good approximation to the “exact” distribution in the lower part of the building but tends to over-estimate the required strength in the upper stories. The COV of $\bar{\alpha}_{Benavent,i}/\bar{\alpha}_{exact,i}$ under near-field records are 0.14, 0.14, 0.17 for three, six and nine-story prototype structures respectively, and 0.13, 0.13, 0.17 under far-field records. Therefore, the proposed optimum distribution $\bar{\alpha}_{prop,i}$ gives a better approximation to the “exact” one than to that obtained with $\bar{\alpha}_{Benavent,i}$. It is worth noting that, from a practical point of view, the main differences between $\bar{\alpha}_{Benavent,i}$ and $\bar{\alpha}_{prop,i}$, are: (i) $\bar{\alpha}_{Benavent,i}$ is based only on properties of the main structure while $\bar{\alpha}_{prop,i}$ requires also to know the ratio between the lateral stiffness of the dampers and the lateral stiffness of the main frame at each story; (ii) $\bar{\alpha}_{Benavent,i}$ depends on the predominant period of the ground motion T_G while $\bar{\alpha}_{prop,i}$ requires to know the inelastic input energy spectrum. Therefore, the time required to obtain the optimum distribution with $\bar{\alpha}_{prop,i}$ is larger than with $\bar{\alpha}_{Benavent,i}$, and the analytical expression of the former is more complex than the later. However, from a practical point of view, since these calculations are made usually with a computer, the differences in time and in programming complexity are negligible.

4.4. Damage Distribution among Stories

As explained in previous sections, the damage in a given story i can be characterized by the ratio η_i . It has likewise been explained in previous sections that the optimum distribution of the shear force coefficients is the one that makes η_i equal in all stories. This strictly occurs only when the distribution $\bar{\alpha}_{exact,i}$ is used, and the common value of η_i in this case will be named η_{exact} ($=\eta_i$) hereafter. When the approximate distributions $\bar{\alpha}_{prop,i}$ or $\bar{\alpha}_{Benavent,i}$ are used, in general the η_i 's do not have exactly the same value in all stories. In order to evaluate such deviations, the η_i obtained at each story through nonlinear time story analyses conducted for each individual ground motion and for each prototype structure designed with $\bar{\alpha}_{prop,i}$, referred to as $\eta_{prop,i}$ hereafter, are plotted in Figures 6 and 7 normalized by η_{exact} . Also shown in the figures with bold lines are the mean curves of $\eta_{prop,i}/\eta_{exact}$ obtained by averaging the response at each story level throughout the seven ground motions applied to each prototype structure. As can be seen, the mean curve $\eta_{prop,i}/\eta_{exact}$ is close to the vertical line passing through the abscissa 1.0.

This indicates that the proposed distribution provides a reasonably uniform distribution of damage among stories. More precisely, the COV of $\eta_{prop,i}/\eta_{exact}$ calculated for all stories and all the near-field earthquakes are 0.40, 0.33 and 0.42, for the three-, six-, and nine-story prototype structures, respectively; the counterpart values for far-field earthquakes are 0.33, 0.33, and 0.37.

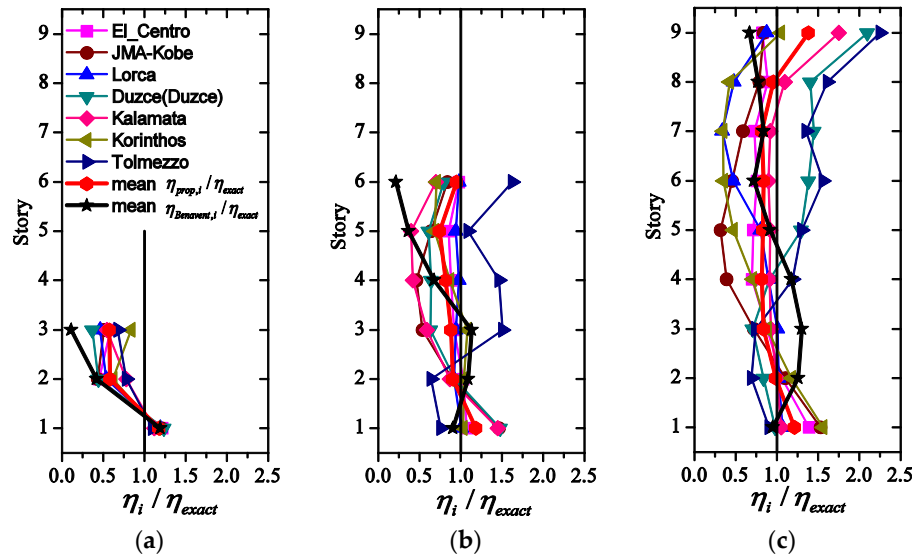


Figure 6. Ratios $\eta_{prop,i}/\eta_{exact}$ (record to record and mean) and $\eta_{Benavent,i}/\eta_{exact}$ (mean) obtained for 3 (a), 6 (b) and 9 (c) story buildings subjected to near-field earthquakes.

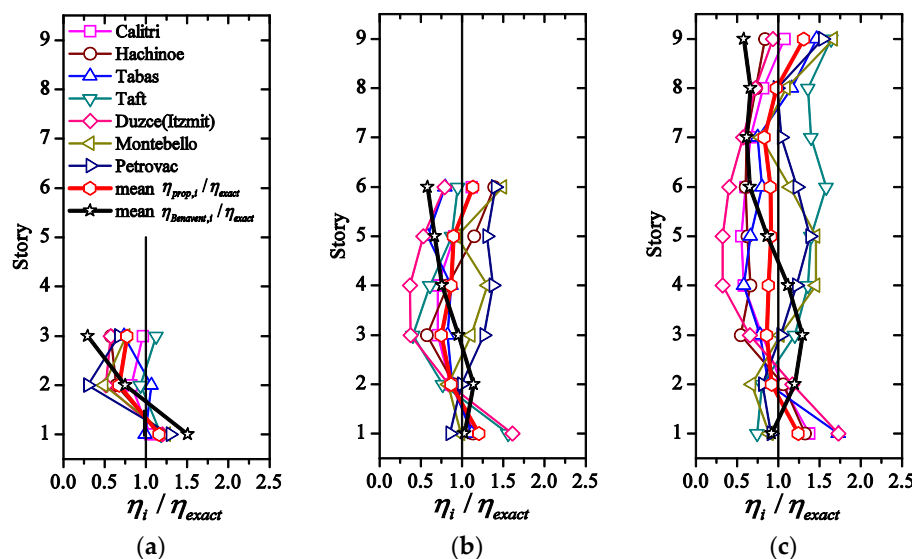


Figure 7. Ratios $\eta_{prop,i}/\eta_{exact}$ (record to record and mean) and $\eta_{Benavent,i}/\eta_{exact}$ (mean) obtained for 3 (a), 6 (b) and 9 (c) story buildings subjected to far-field earthquakes.

For comparison, the η_i obtained at each story through nonlinear time story analyses conducted for each individual ground motion and for each prototype structure designed with $\bar{\alpha}_{Benavent,i}$ was also calculated and it is referred to as $\eta_{Benavent,i}$ hereafter. Figures 6 and 7 show in bold lines the mean curves of $\eta_{Benavent,i}/\eta_{exact}$ obtained by averaging the response at each story level through the seven ground motions applied to each prototype structure. When $\bar{\alpha}_{Benavent,i}$ is used the COV of $\eta_{Benavent,i}/\eta_{exact}$ are 0.81, 0.61, and 0.63 for near-field earthquakes and 0.80, 0.59, and 0.52 for far-field earthquakes, respectively. It follows from these results that the optimum yield shear force distribution proposed in this study provides a much more uniform distribution of damage among the stories than the one proposed by

Benavent-Climent [12]. The reason lies in the bias of Benavent-Climent's equation, overestimating the required strength at the higher stories, which leads to lower values of $\eta_{Benavent,i}$ for the high stories, and higher values for the low stories.

Furthermore, Figure 8 compares the histories of energy E_h dissipated by the 6-story building designed with $\bar{\alpha}_{prop,i}$ and with $\bar{\alpha}_{Benavent,i}$ for two ground motions: Calitri (Figure 8a) and Montebello (Figure 8b). It can be seen that there are not significant differences between them.

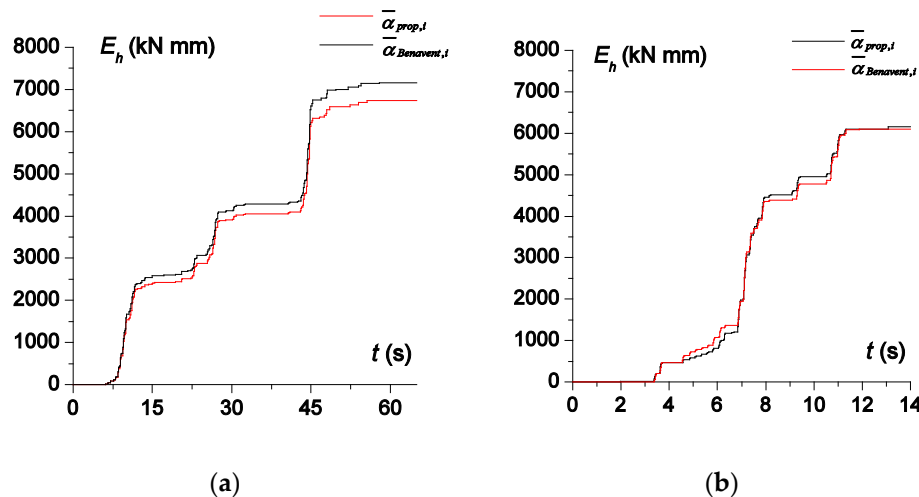


Figure 8. History of dissipated energy for the 6-story building designed with $\bar{\alpha}_{prop,i}$ and $\bar{\alpha}_{Benavent,i}$ subjected to Calitri (a) and Montebello (b) ground motions.

4.5. Maximum Interstory Drifts

Figures 9 and 10 show the maximum inter-story drifts normalized by the story height, IDI_i , obtained from the time history analyses conducted on the prototype structures designed with $\bar{\alpha}_{prop,i}$, denoted by $IDI_{prop,i}$ (record-to-record, mean and the envelope of the maximum values “maxenv”), and for those designed with $\bar{\alpha}_{exact,i}$, denoted by $IDI_{exact,i}$ (the mean and the envelope of the maximum values “maxenv”). Overall, comparing the mean curves it is seen that the IDI_i 's obtained with the proposed distribution (i.e., “mean $IDI_{prop,i}$ ” in the figures) are very close, almost coinciding with the exact values (i.e., “mean $IDI_{exact,i}$ ” in the figures). Similar observations can be made with the maximum envelope curves (i.e., “maxenv $IDI_{prop,i}$ ” versus “maxenv $IDI_{exact,i}$ ”). Moreover, the target IDI_i ($=0.75\%$) is not exceeded in any case. It is also noteworthy that despite the differences observed in the distribution of η_i for the structures designed with $\bar{\alpha}_{prop,i}$ and with $\bar{\alpha}_{exact,i}$ (Figures 6 and 7), the response in terms of displacement are in very good agreement. In other words, the small deviations of η_i among stories scarcely affect the maximum lateral inter-story drifts.

Deviations of the yield shear force coefficient distribution with respect to the exact value $\bar{\alpha}_{exact,i}$ inevitably cause deviations from the aspired even distribution of damage (i.e., η_i equal in all stories). Nonetheless, when these deviations are limited (to about 20% as occurs with the proposed distribution $\bar{\alpha}_{prop,i}$ according to the results of this study), then the damage concentration (i.e., values of η_i in specific stories markedly larger than in the rest of stories) is prevented and the response of the structure in terms of displacements $IDI_{prop,i}$ scarcely deviates from that obtained for a structure designed $\bar{\alpha}_{exact,i}$, i.e. from $IDI_{exact,i}$. These results emphasize the key importance of designing structures with a strength distribution close to that prescribed by the optimum yield-shear force distribution so as to prevent damage concentration and to control the response of the structure in terms of displacement. The results of this study also underline the improvements of the proposed optimum distribution with respect to the one proposed by Benavent-Climent [12] in past studies.

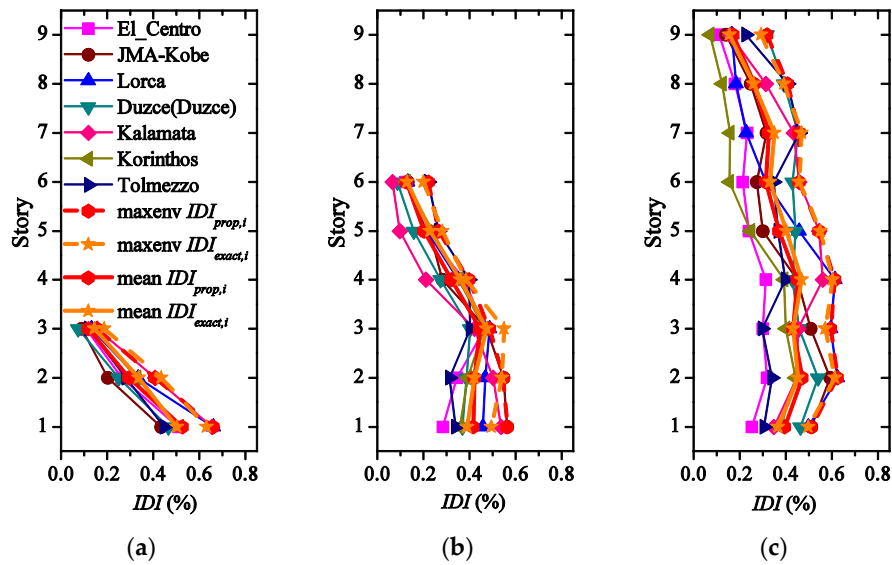


Figure 9. Interstory drift ratios $IDI_{prop,i}$ (record-to-record, mean and maximum envelope) and $IDI_{exact,i}$ (mean and maximum envelope) obtained for 3 (a), 6 (b), and 9 (c) story buildings subjected to near-field earthquakes.

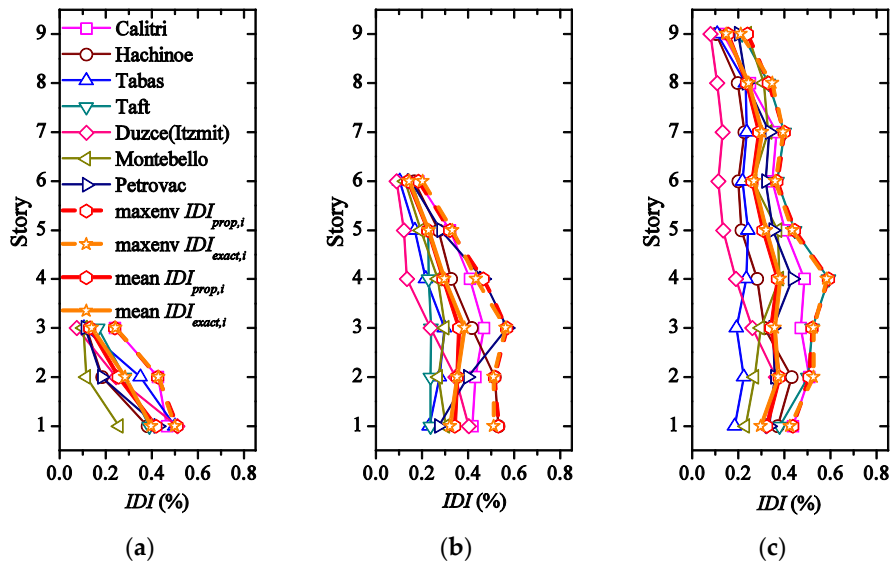


Figure 10. Interstory drift ratios $IDI_{prop,i}$ (record-to-record, mean and maximum envelope) and $IDI_{exact,i}$ (mean and maximum envelope) obtained for 3 (a), 6 (b) and 9 (c) story buildings subjected to far-field earthquakes.

Finally, a comparison was carried out between the IDI_i 's of the prototype structures designed using $\bar{\alpha}_{prop,i}$ or with $\bar{\alpha}_{Benavent,i}$, respectively $IDI_{prop,i}$ and $IDI_{Benavent,i}$. The goodness of the agreement of IDI_i or $IDI_{Benavent,i}$ with respect to $IDI_{exact,i}$, is quantified in terms of the normalized square mean error, NMSE, defined as follows:

$$NMSE = 1 - \frac{\sum_{i=1}^N (IDI_i - IDI_{exact,i})^2}{\sum_{i=1}^N (IDI_{exact,i} - \text{mean}(IDI_{exact,i}))^2} \quad (23)$$

NMSE ranges between $-\infty$ (very bad fit) and 1 (perfect fit). Values above 0.70 are considered a reasonably good fit. The results are shown in Figure 11. It is seen that the structures designed using $\bar{\alpha}_{prop,i}$ lead to NMSE values greater than 0.70 (Figure 11a) and 0.50 (Figure 11b) in the case of near-field

and far-field records, respectively. Nonetheless, for the structures designed with $\bar{\alpha}_{Benavent,i}$, even though the target IDI_i is not exceeded and most analyses show $NSME$ greater than 0.50, there are results giving poor values for $NSME$, especially with near-field records (Figure 11a).

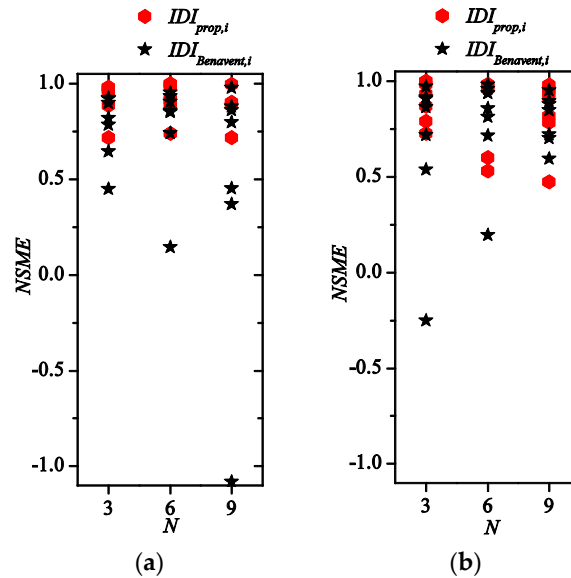


Figure 11. Normalized mean square error of $IDI_{prop,i}$ and $IDI_{Benavent,i}$ with respect to $IDI_{exact,i}$ for (a) near-field earthquakes and (b) far-field earthquakes.

4.6. Ductility Distribution among Stories

Following past studies [7], it was assumed in Section 3 that when the structure is designed with the optimum yield-force coefficient distribution the ductility demand $\bar{\mu}_i$ is roughly the same in all stories (i.e., $\bar{\mu}_i = {}_s\mu$). This assumption is further examined in this subsection. Using the $IDI_{prop,i}$'s of Section 4.5, the corresponding ductility factor $\bar{\mu}_i$ at each story i , referred to as $\bar{\mu}_{prop,i}$ herein, was calculated for the structures designed with the proposed optimum distribution $\bar{\alpha}_{prop,i}$ by:

$$\bar{\mu}_{prop,i} = [(h_i IDI_{prop,i} / 100) - {}_s\delta_{yi}] / {}_s\delta_{yi} \quad (24)$$

where h_i is given in Table 1. The yield displacement of the dampers is obtained with:

$${}_s\delta_{yi} = {}_sQ_{yi} / {}_sK_i = [\bar{\alpha}_{prop,i} \alpha_1 \sum_{j=i}^N m_j g] / [K_i f k_i] \quad (25)$$

where m_i , $f k_i$ are given in Table 1 and ${}_s\alpha_1$, $K_i = K$ in Tables 2–4. $\bar{\mu}_{prop,i}$ was compared with the ductility factor ${}_s\mu$ assumed for designing the prototype structures shown in Tables 2–4. The quotient $\bar{\mu}_{prop,i} / {}_s\mu$ calculated for each ground motion and the mean curve “mean $\bar{\mu}_{prop,i} / {}_s\mu$ ” are plotted in Figures 12 and 13. Also plotted in the figures are the mean curves “mean $\bar{\mu}_{Benavent,i} / {}_s\mu$ ” and “mean $\bar{\mu}_{exact,i} / {}_s\mu$ ” obtained using $IDI_{Benavent,i}$, $\bar{\alpha}_{Benavent,i}$ and $IDI_{exact,i}$, $\bar{\alpha}_{exact,i}$ instead of $IDI_{prop,i}$ and $\bar{\alpha}_{prop,i}$ in Equations (24) and (25). The curves “mean $\bar{\mu}_{prop,i} / {}_s\mu$ ”, “mean $\bar{\mu}_{Benavent,i} / {}_s\mu$ ” and “mean $\bar{\mu}_{exact,i} / {}_s\mu$ ” are found to lie approximately in a vertical line. Noting that the denominator ${}_s\mu$ is the same in all cases and has the same value for all stories, this means that the ductility demand $\bar{\mu}_i$ is approximately the same in all stories. The variation of “mean $\bar{\mu}_{prop,i} / {}_s\mu$ ”, “mean $\bar{\mu}_{Benavent,i} / {}_s\mu$ ” or “mean $\bar{\mu}_{exact,i} / {}_s\mu$ ” throughout the stories was further quantified by calculating the corresponding COV. The results are shown in Table 5. It can be seen that the structures designed with $\bar{\alpha}_{exact,i}$ so that η is exactly the same in all stories also exhibit a very uniform distribution of $\bar{\mu}_i$ among stories, the COVs ranging from 0.05 to 0.12. This confirms Akiyama's [7] results and further supports one of the assumptions made in this

study. For structures designed with the proposed approximate optimum distribution $\bar{\alpha}_{prop,i}$, the COV slightly increases with respect to $\bar{\alpha}_{exact,i}$, in particular for the three story structure, but is still small. The distribution proposed by Benavent-Climent [12] presents the largest COVs.

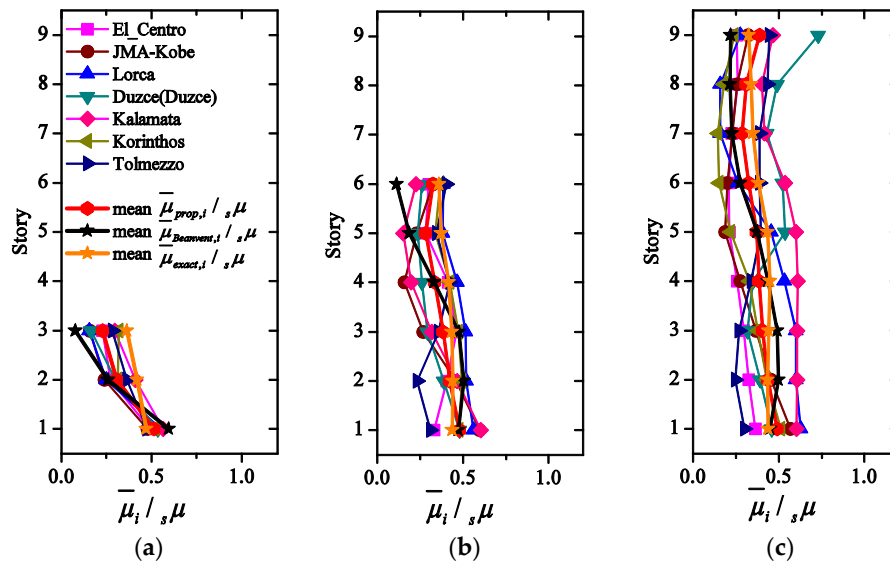


Figure 12. Ratios $\bar{\mu}_{prop,i}/s\mu$ (record to record and mean), $\bar{\mu}_{Benavent,i}/s\mu$ (mean) and $\bar{\mu}_{exact,i}/s\mu$ (mean) obtained for 3 (a), 6 (b) and 9 (c) story structures subjected to near-field earthquakes.

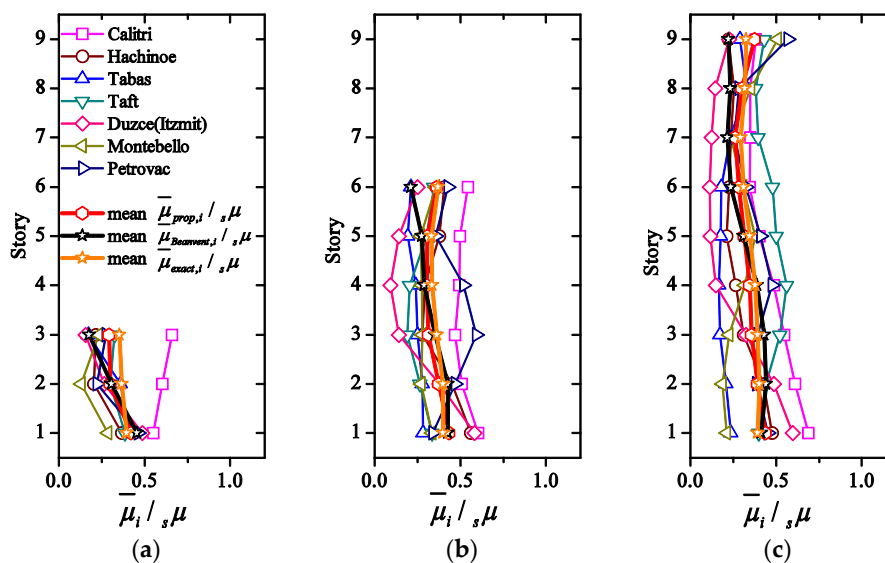


Figure 13. Ratios $\bar{\mu}_{prop,i}/s\mu$ (record to record and mean), $\bar{\mu}_{Benavent,i}/s\mu$ (mean) and $\bar{\mu}_{exact,i}/s\mu$ (mean) obtained for 3 (a), 6 (b) and 9 (c) story structures subjected to far-field earthquakes.

Table 5. COV of “mean $\bar{\mu}_{exact,i}/s\mu$ ”, “mean $\bar{\mu}_{prop,i}/s\mu$ ” and “mean $\bar{\mu}_{Benavent,i}/s\mu$ ”.

Type of Records	Near-Field			Far-Field		
Prototypes		N			N	
	3	6	9	3	6	9
COV of “mean $\bar{\mu}_{exact,i}/s\mu$ ”	0.10	0.08	0.12	0.05	0.07	0.11
COV of “mean $\bar{\mu}_{prop,i}/s\mu$ ”	0.35	0.18	0.16	0.18	0.14	0.16
COV of “mean $\bar{\mu}_{Benavent,i}/s\mu$ ”	0.69	0.43	0.32	0.36	0.24	0.28

Finally, it is likewise observed in Figures 12 and 13 that “mean $\bar{\mu}_{prop,i}/s\mu$ ”, “mean $\bar{\mu}_{Benavent,i}/s\mu$ ” and “mean $\bar{\mu}_{exact,i}/s\mu$ ” are in general below 1. The reason is that the expression used in reference [12] to relate $\bar{\mu}_i$ with η_i , developed by Cosenza and Manfredi [23], is somewhat conservative and provides in general a safe-side estimation of the maximum displacements.

5. Experimental Validation

5.1. Test Model

A waffle-flat slab RC structure supported on RC columns and equipped with metallic dampers (test model) was built in the Laboratory of Structures of the University of Granada (Spain), and subjected to unidirectional dynamic loadings with a shaking table. The test model is representative of a prototype two story building at a scale of two fifths. The overall dimensions and characteristics of the test model are shown in Figure 14. Due to space limitations in the laboratory, the test model does not represent the whole buildings but rather a portion of it. More precisely, to reduce its height it was assumed that under lateral loadings the sections of zero bending moment on the columns of the second story are located at mid-height. With this assumption, the columns of the second story were replaced by half columns with a pin connection at the top. The weight that the prototype building supports above mid-height of these columns was replaced by steel blocks put on the top (indicated as “added weight” in Figure 14). Similarly, to reduce the horizontal dimensions of the test model it was assumed that under lateral loading the mid-span section of the RC slab moves horizontally (i.e., there is no vertical movement) and that the bending moment is zero. To simulate these boundary conditions, the mid-span section of the RC slab was connected to the steel blocks put on the top of the half-columns of the second story with pin-ended steel bars. The very large flexural stiffness of these steel blocks prevented any possible vertical displacement. Steel blocks were also attached to the slab to represent the gravity loads acting on the floor. The mass m_1 of the RC floor slab with the added weight was $m_1 = 4079$ kg. The mass m_2 of the added weight put on top of the half columns of the second story was $m_2 = 7058$ kg. The total mass of the test model was 11137 kg.

One metallic damper was installed in each story along the direction of shaking. The type of metallic damper used was developed by the authors in past studies [34] and is named web plastification damper (WPD). The details of the WPDs are shown in Figure 15. The WPD is constructed by assembling several short-length segments of I-shaped steel sections and two U-shaped steel bars. The WPD is installed in the structure as a diagonal bar connecting two consecutive floor levels. When the damper is subjected to axial deformations, the webs of the I-sections are forced to undergo out of plane plastic deformations. The two U-shaped steel bars function as auxiliary elements aimed to transfer the horizontal forces from the slabs to the I-shaped steel sections and are designed to remain elastic.

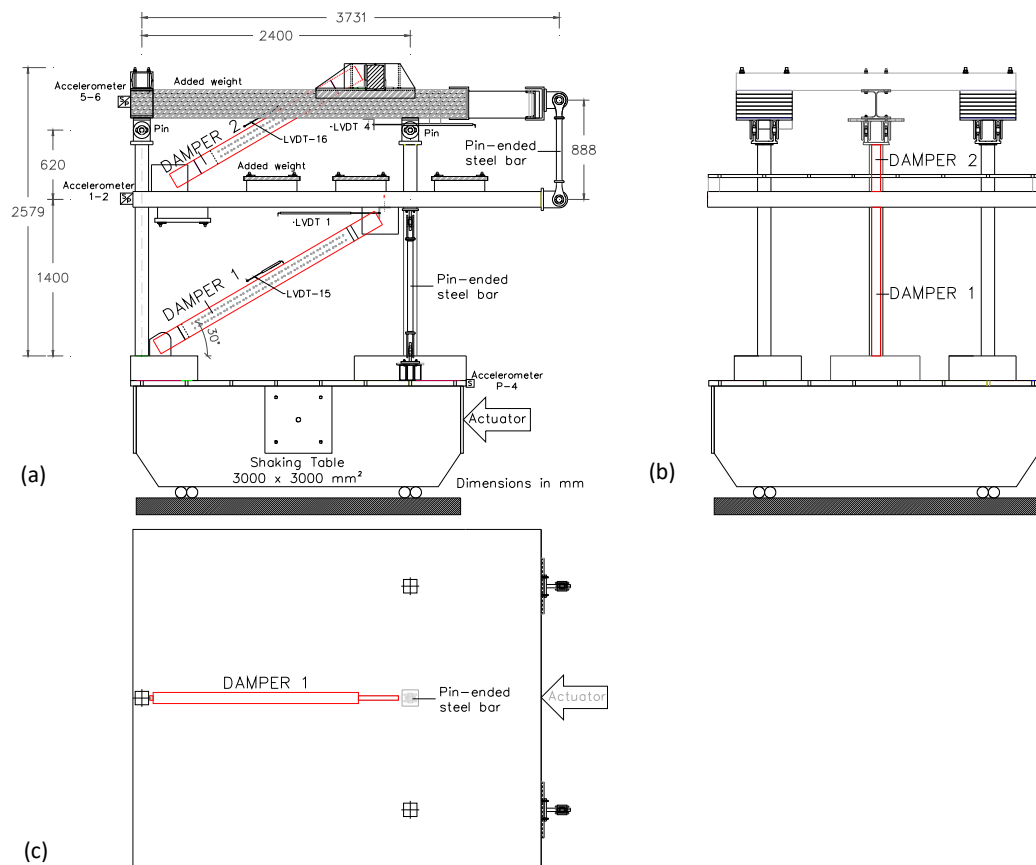


Figure 14. Test model and experimental setup: (a) elevation in the direction of lateral loading; (b) elevation perpendicular to the direction of loading; (c) plan. Dimensions in mm.

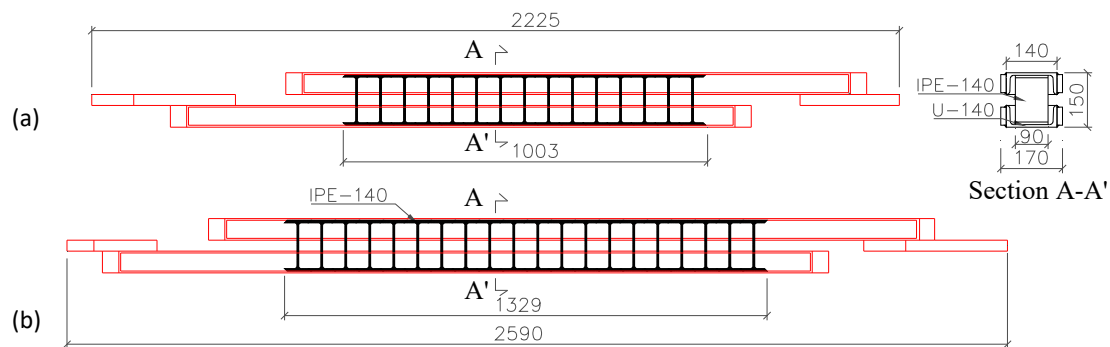


Figure 15. Metallic dampers tested: (a) damper 2 (second story); (b) damper 1 (first story) (dimensions in mm).

5.2. Test Set-Up and Instrumentation

The RC elements of the test model were instrumented with strain gages attached to the longitudinal reinforcement, uniaxial accelerometers and displacement transducers (linear variable differential transformers, LVDT). The ends of the WPDs were also instrumented with strain gages that measured the axial forces acting on them. The accelerometers and the LVDT's provided the horizontal accelerations and displacements of the masses. Additional LVDTs installed in the WPDs measured their axial displacements. Data were acquired continuously with a scan frequency of 200 Hz. Figure 16 offers a general view of the experimental set up.



Figure 16. Test set-up.

5.3. Seismic Simulations

The test model was subjected to three consecutive seismic simulations. In each simulation, the shaking table reproduced the accelerogram recorded at Calitri during the Campano-Lucano earthquake (Italy, 1980) scaled in time by the factor 0.63. In the first seismic simulation, C100, the original accelerogram was applied without scaling in amplitude. In the second simulation, C200, the amplitude of the original accelerogram was multiplied by two, and in the third simulation C300 by three. The resulting peak accelerations applied to the shaking table were 0.16, 0.31, and 0.47 g, respectively (here g is the acceleration of gravity). Figure 17a shows the accelerogram (scaled only in time by 0.63) and Figure 17b the corresponding elastic spectrum E_I - T corresponding to a damping fraction of $\xi = 1.8\%$ (i.e., the damping fraction measured during the test as discussed later).

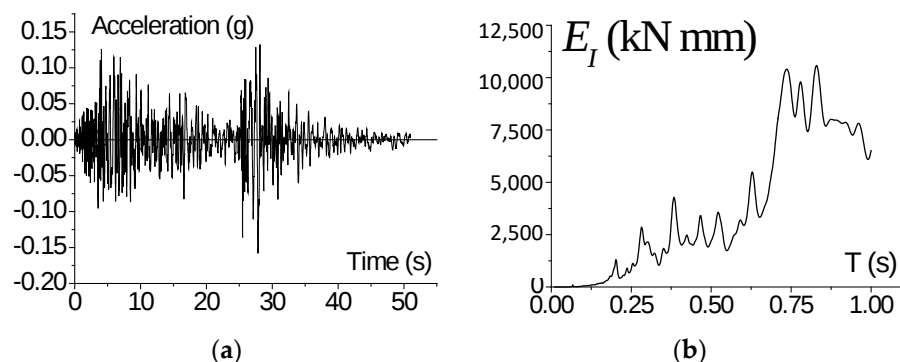


Figure 17. Accelerogram (a) and input energy spectrum (b) used in the shaking table tests.

5.4. Test Results

The readings provided by the gauges attached to the steel reinforcement of the RC elements indicate that the strains remained below or very close to the yield strain, meaning that the RC main structure remained elastic during the tests. This is corroborated by the fact that the damping fraction ξ obtained experimentally from free vibration tests conducted at the end of each seismic simulation

remained constant ($\xi = 1.8\%$). The maximum inter-story drifts IDI were 0.74% in the first story and 0.86% in the second one. These IDI s are slightly below and above, respectively, the value assumed in codes for the elastic limit of RC waffle-flat slab structures (0.75%). The corresponding ductility factors $s\mu_i$ were $s\mu_1 = 3.9$ and $s\mu_2 = 4.0$ for the first and second stories respectively.

In contrast to the main structure, the WPDs underwent plastic deformations. Figures 18 and 19 show the axial force versus axial displacement curves, N - d , measured on the metallic dampers installed in the first (Figure 18) and second story (Figure 19) in each seismic simulation. The energy dissipated by each damper at the end of the tests is obtained by integrating these N - d curves. Specifically, at the end of the tests the hysteretic energy dissipated by the damper of the first story was $E_{h1} = 4885$ kNmm and that of the damper of the second story $E_{h2} = 3073$ kNmm. The corresponding normalized ratios $\eta_i = E_{hi}/(sQ_{yi} \times s\delta_{yi})$ are $\eta_1 = 33.4$ and $\eta_2 = 27.2$ for the first and second stories. The values of sQ_{yi} and $s\delta_{yi}$ used for obtaining η_i were calculated with the equations developed in [34], as discussed in the next subsection.

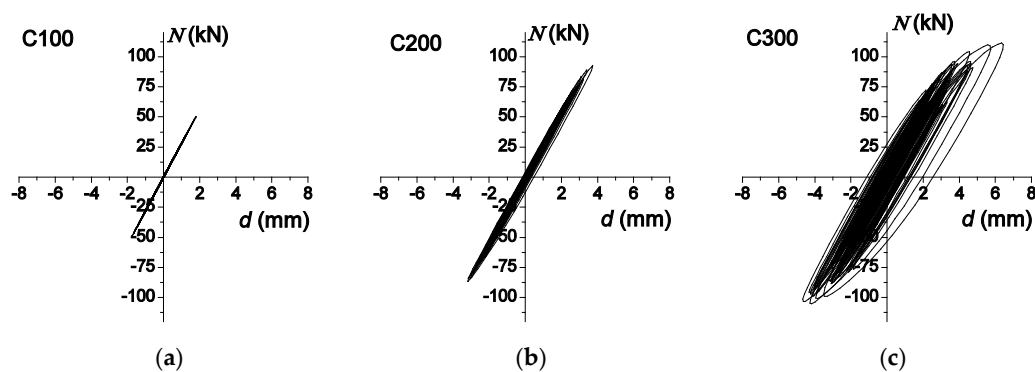


Figure 18. Axial force vs. axial displacement in the metallic damper of the first story for the seismic simulations C100 (a), C200 (b) and C300 (c).

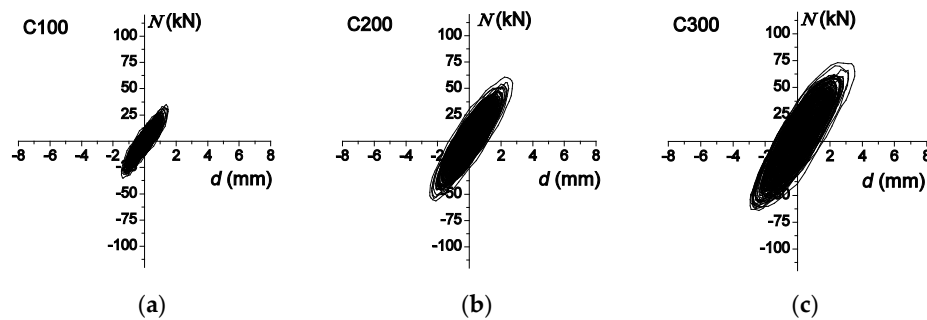


Figure 19. Axial force vs. axial displacement in the metallic damper of the second story for the seismic simulations C100 (a), C200 (b) and C300 (c).

5.5. Discussion

A nonlinear finite element model representing the test model without the dampers was developed using Engineer's Studio software (V.1.07.02, Forum-8, London, UK) [35]. The numerical model was subjected to nonlinear static pushover analysis by applying an inverted triangle distribution of lateral forces. From this analysis, the lateral stiffness $f k_i$ and yield strength $f Q_{yi}$ of the main RC structure (without dampers) was obtained, giving $f k_1 = 3.6$ kN/mm and $f Q_{y1} = 20.6$ kN in the first story, and $f k_2 = 4.1$ kN/mm and $f Q_{y2} = 22.4$ kN in the second story. The lateral stiffness $s k_i$ and yield strength $s Q_{yi}$ provided by the dampers was calculated with the equations developed in [34] giving $s k_1 = 31.3$ kN/mm and $s Q_{y1} = 68.7$ kN in the first story, and $s k_2 = 24.8$ kN/mm and $s Q_{y2} = 53$ kN in the second story. Given these $s Q_{yi}$'s, the actual yield shear force coefficients of the energy dissipation system $s\alpha_i (= s Q_{yi} / \sum_{j=i}^N m_j g)$ corresponding to the lateral strengths $s Q_{yi}$ indicated above are $s\alpha_1 = 0.63$

in the first story and $s\alpha_2 = 0.77$ in the second story, and the distribution $s\bar{\alpha}_i (= s\alpha_i/s\alpha_1)$ is $s\bar{\alpha}_1 = 1$ and $s\bar{\alpha}_2 = 1.22$. Using Equation (16), the total yield shear force coefficients of the tested structure are $\alpha_1 = 0.70$ in the first story and $\alpha_2 = 0.9$ in the second story, and the distribution $\bar{\alpha}_i (= \alpha_i/\alpha_1)$ is $\bar{\alpha}_1 = 1$ and $\bar{\alpha}_2 = 1.28$ for the first and second stories, respectively.

Meanwhile, idealizing the test model as a lumped mass system with two degrees of freedom (the horizontal displacement of each mass m_1 and m_2), and using the m_i and f_k indicated above, the vibration periods T_i and the corresponding vibration modes vectors φ_i of the RC main structure in elastic conditions were determined, giving $T_1 = 0.15$ s, $\varphi_1 = [1, 0.518]$ for the first mode and $T_2 = 0.05$ s, $\varphi_2 = [-0.3, 1]$ for the second one. With the above information, the $\psi_{mn,i}$ parameters were calculated with Equation (22), giving $\psi_{m1,1} = 0.594$ and $\psi_{m1,2} = 0.406$ in the first mode, and $\psi_{m2,1} = 0.428$ and $\psi_{m1,2} = 0.572$ in the second mode. Further, using the input energy spectrum E_I - T of the accelerogram applied to the shaking table (Figure 17b) and applying the procedure explained at the end of Section 4.1, the inelastic hysteretic energy spectra E_h - T of the scaled accelerograms applied to the shaking table were calculated. Substituting the E_{hm}^{SDOF} 's provided by these spectra in Equation (17) and using the $\psi_{mn,i}$'s and other dynamic properties indicated above, the values of the optimum yield shear force coefficient distribution proposed in this study, Equations (17) and (22), were calculated giving $\bar{\alpha}_{prop,1} = 1$ and $\bar{\alpha}_{prop,2} = 1.22$. The values are very close to the actual yield-shear force coefficient distribution of the tested structure (i.e., $\bar{\alpha}_1 = 1$ and $\bar{\alpha}_2 = 1.28$). This explains why the cumulative plastic deformation ratios were very similar in the two stories (i.e., $\eta_1 = 33.4$ and $\eta_2 = 27.2$), as well as the ductility factors $s\mu_i$ (i.e., $s\mu_1 = 3.9$ and $s\mu_2 = 4.0$), as shown in Section 5.4. This finding serves to validate, experimentally, the optimum yield-shear force coefficient distribution proposed in this study.

6. Conclusions

Past earthquakes have accentuated the importance of preventing a concentration of damage in multi-storey buildings, since it is a prevalent cause of collapse. Such an objective can be attained by using an appropriate distribution of lateral strength, one that can be expressed in terms of yield shear force coefficients. The present study focusses on structures with metallic dampers, proposing a new approach to obtain an optimum distribution of the yield shear force coefficients among stories, $\bar{\alpha}_{prop,i}$, that makes the damage—expressed in terms of cumulative plastic strain energy normalized by the product of yield strength and yield inter-story drift—approximately equal in all stories. The proposed optimum distribution is derived by applying modal analysis formulation and adopting two basic assumptions. One is that the damage spreads out evenly among the stories regardless of the level of plastic deformation. The second is that the ductility demand is approximately the same in all stories. These assumptions are supported by the results of past studies; the second one is further assessed in this study both numerically and experimentally. The proposed optimum distribution is validated through numerical simulations and shaking table tests. The new distribution if compared with the “exact” distribution $\bar{\alpha}_{exact,i}$ obtained from non-linear time history analyses, and with the distribution proposed by Benavent-Climent in previous studies, $\bar{\alpha}_{Benavent,i}$, leads us to these main conclusions:

- (1) $\bar{\alpha}_{prop,i}$ is in good agreement with $\bar{\alpha}_{exact,i}$; the ratio $\bar{\alpha}_{prop,i}/\bar{\alpha}_{exact,i}$ ranges between 0.9 to 1.2 and the coefficient of variation (COV) is less than 0.10;
- (2) $\bar{\alpha}_{prop,i}$ improves significantly upon the one proposed by Benavent-Climent. $\bar{\alpha}_{Benavent,i}$, particularly for the upper 1/3 of the height of the structure.
- (3) The interstory-drifts of the structures designed with $\bar{\alpha}_{prop,i}$ or with $\bar{\alpha}_{exact,i}$ are very similar.
- (4) Deviations of the yield-shear force coefficient distribution with respect to $\bar{\alpha}_{exact,i}$, inevitably causes deviations of η_i from the aspired even distribution of damage (i.e., $\eta_i = \eta$ equal in all stories). Nonetheless, if such deviations of the yield-shear force coefficient distribution are less than about 20%: (i) the variations of η_i among stories are small, thus damage concentration becomes a minor problem, and (ii) the inter-story drifts scarcely deviate from those obtained with $\bar{\alpha}_{exact,i}$.

- (5) In structures having the optimum yield-shear force distribution, the ductility demand $\bar{\mu}_i$ is approximately the same for all stories.

Author Contributions: A.B.-C. conceived the overall idea of the methodology. J.D.-Á. developed the idea, carried out the numerical modelling, the corresponding non-linear calculations, postprocessed the data and wrote a first draft. Both authors reviewed and supervised the final version of the manuscript. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Near-field ground motion records used in the analyses.

Earthquake	Record	Country	Year	T_{NH} (s)	T_G (s)	PGA (cm/s ²)	PGV (cm/s)	I_D
El Centro	El Centro	EE.UU	1940	0.60	0.46	341.31	37.23	8.95
Hyogo-ken Nanbu	Kobe	Japan	1995	0.84	0.34	820.32	90.24	7.11
Lorca	Lorca	Spain	2011	0.43	0.46	325.84	35.40	2.40
Friuli	Tolmezzo	Italy	1976	0.51	0.27	349.85	20.96	6.80
Alkion	Korinthos	Greece	1981	0.80	0.54	225.66	22.43	7.89
Duzce	Duzce	Turkey	1999	0.70	0.43	369.88	35.72	12.17
Kalamata	Kalamata	Greece	1986	0.50	0.56	327.50	25.97	3.01

Table A2. Far-field ground motion records used in the analyses

Earthquake	Record	Country	Year	T_{NH} (s)	T_G (s)	PGA (cm/s ²)	PGV (cm/s)	I_D
Izmit	Duzce	Turkey	1999	0.60	0.35	303.77	41.35	5.11
Northridge	Montebello	EE.UU.	1994	0.39	0.30	163.34	11.02	11.22
Montenegro	Petrovac	Montenegro	1979	0.64	0.46	445.30	38.37	16.54
Tokachi-oki	Hachinoe	Japan	1968	0.35	0.49	224.39	43.19	5.82
Kern County	Taft	EE.UU.	1952	0.70	0.35	152.90	17.15	12.97
Campano Lucano	Calitri	Italy	1980	1.00	1.20	155.00	26.16	16.51
Tabas	Tabas	Iran	1978	0.25	0.30	908.35	84.34	9.75

Appendix B

Akiyama [7] proposed a simple method to obtain the lengthened period, T_{max} , as opposed to the elastic one T , of a SDOF with elastic-perfectly plastic restoring force characteristics that undergoes plastic deformations characterized by the maximum deformation ratio, μ . T_{max} is obtained by calculating the time it takes the system to complete a cycle of deformation in free vibration (path O–A–B–C–D–E–O in Figure A1a). The procedure proposed by Akiyama is extended here to structures with metallic dampers that are characterized by a post elastic stiffness (A–B and DE, Figure A1b) exerted by the flexible part.

The differential equation that governs the movement of the SDOF system along paths O–A and B–C in free vibration (Figure A1b) is:

$$\ddot{y} + \omega_m^2 y = 0 \quad (\text{A1})$$

where $y(t)$ is the displacement, t the time, $\omega_m = \sqrt{m k / M}$, M is the mass of the system and $m k$ is the lateral stiffness of the system obtained by adding $f k$ and $s k$. Along paths O–A and B–C, the solutions are the following:

- O–A:

Boundary conditions: $y(0) = 0$; $\dot{y}(0) = v_0$

$$y_{A-B}(t) = \frac{v_0}{\omega_m} \sin(\omega_m t) \quad (\text{A2})$$

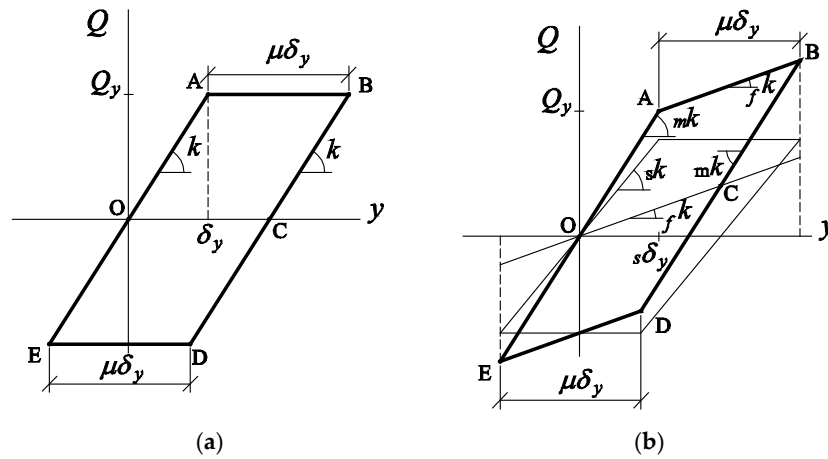


Figure A1. Deformation cycle of a SDOF system corresponding to μ : (a) Elastic-Perfectly plastic structure; (b) structure with metallic dampers.

Taking into account that $v_A = dy/dt > 0$, the time taken by the SDOF systems from O to A, t_{OA} , is the following:

$$t_{OA} = \frac{\alpha_0 \pi}{2\omega_m} \quad 0 < \alpha_0 < 1 \quad (\text{A3})$$

• B–C:

Boundary conditions: $\dot{y}_B(t) = 0$; $v_c = \dot{y}_C(t) > 0$

$$y_{B-C}(t) = y_B - \frac{v_C}{\omega_m} \sin(\omega_m(t - t_B)) \quad (\text{A4})$$

Since $\dot{y}_B(t) = 0$, the time taken by SDOF systems from B to C, t_{BC} , is obtained as follows:

$$t_{BC} = \frac{\pi}{2\omega_m} \quad (\text{A5})$$

For SDOF systems in free vibration having undergone a plastic deformation represented by μ , the differential equation for the stretch A–B (Figure A1b) is:

$$\ddot{y} + \omega_f^2 y + (\omega_m^2 - \omega_f^2) y_A = 0 \quad (\text{A6})$$

where $\omega_f = \sqrt{f k / M}$.

The solution is obtained as follows:

• Segment A–B:

Boundary conditions: $y_A(t_{OA}) = v_O / \omega_m \sin(\alpha_0 \pi / 2)$; $\dot{y}_A(t_{OA}) = v_O \cos(\alpha_0 \pi / 2)$ using Equations (A2) and (A3).

$$y_{A-B}(t) = \frac{v_O \omega_m}{\omega_f^2} \sin\left(\frac{\alpha_0 \pi}{2}\right) \cos(\omega_f(t - t_{OA})) + \frac{v_O}{\omega_f} \cos\left(\frac{\alpha_0 \pi}{2}\right) \sin(\omega_f(t - t_{OA})) + \left(1 - \frac{\omega_m^2}{\omega_f^2}\right) y_A \quad (\text{A7})$$

Taking into account that $\dot{y}_B(t) = 0$ in Equation (A7), the time taken by SDOF systems from A to B, t_{AB} , is obtained as follows:

$$t_{AB} = \frac{1}{\omega_f} \tan^{-1} \left[\frac{\omega_f}{\omega_m} \cot\left(\frac{\alpha_0 \pi}{2}\right) \right] \quad (\text{A8})$$

Eventually, the time taken by the SDOF system to complete the path from O to C, t_{OC} , is $T_{max}/2$ [7] and it is obtained by adding t_{OA} (Equation (A3)), t_{AB} (Equation (A8)), and t_{BC} (Equation (A5)). Therefore, T_{max} can be expressed as:

$$T_{max} = \frac{\alpha_0 \pi}{\omega_m} + \frac{2}{\omega_f} \tan^{-1} \left[\frac{\omega_f}{\omega_m} \cot \left(\frac{\alpha_0 \pi}{2} \right) \right] + \frac{\pi}{\omega_m} \quad 0 < \alpha_0 < 1 \quad (A9)$$

Furthermore, the ductility μ can be calculated by means of the quotient $(y_B - y_A)/y_A$. Taking into account Equations (A2), (A3), (A7), and (A8) the following expression is obtained for μ :

$$\mu = \left(\frac{\omega_m}{\omega_f} \right)^2 \left[\cos(\omega_f t_{AB}) - 1 \right] + \frac{\omega_m}{\omega_f} \cot \left(\frac{\alpha_0 \pi}{2} \right) \sin(\omega_f t_{AB}) \quad (A10)$$

It is important to note that μ obtained from Equation (A10) decreases monotonically with respect to α_0 . Furthermore, T_{max} relies on the value of α_0 . For this reason, an iterative procedure is necessary to obtain T_{max} for a given μ_{target} as follows:

- (1) A tentative value is adopted for α_0 taking into account that $0 < \alpha_0 < 1$.
- (2) t_{AB} is obtained from Equation (A8) and the result, together with α_0 , is used in Equation (A10) to obtain μ_{iter} .
- (3) If $\mu_{iter} = \mu_{target}$ (with a fixed tolerance) the valid value for α_0 is found, used in Equation (A9) to obtain T_{max} . If not, the procedure is repeated from step 1 with a new value for α_0 .

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