

Comment

## Comment on the Paper Entitled "A New Cumulative Fatigue Damage Rule Based on Dynamic Residual S-N Curve and Material Memory Concept" by Peng Z., Huang H., Zhou J. and Li Y. Published in Metals (2018; 8 (6): 456)

Kris Hectors <sup>1,2,\*</sup> and Wim De Waele <sup>1</sup>

- <sup>1</sup> Department of EMSME, Laboratory Soete, Faculty of Engineering and Architecture, Ghent University, Technologiepark 46, BE-9052 Zwijnaarde, Belgium; Wim.DeWaele@UGent.be
- <sup>2</sup> SIM vzw, Technologiepark 48, BE-9052 Zwijnaarde, Belgium
- \* Correspondence: kris.hectors@ugent.be

Received: 18 October 2019; Accepted: 22 October 2019; Published: 19 December 2019



Dear Editors,

We are writing to draw attention to what we consider to be a consequential error in the research article entitled "A new cumulative fatigue damage rule based on dynamic residual S-N curve and material memory concept" by Peng Z., Huang H., Zhou J. and Li Y. published in *Metals* (2018; 8 (6): 456).

On page 9 of 17, the authors write that the total fatigue life can be calculated by their Equation (34), i.e.,  $N_{pre} = \sum_{i=1}^{k} n_i / \sum_{i=1}^{k} D_i$ . This formula states that the lifetime ( $N_{pre}$ ) of a component subjected to a sequence of load blocks can be predicted by the ratio of the total number of applied cycles ( $\sum_{i=1}^{k} n_i$ ) and the corresponding accumulated damage ( $\sum_{i=1}^{k} D_i$ ). Equation (34) implicitly assumes that the number of cycles of each load block can be scaled with the same scalar up to failure, corresponding to  $\sum D_i = 1$  and  $\sum n_i = N_{pre}$ . This is however only true if the damage accumulation is described as a linear function of cycle ratios. The use of Equation (34) for non-linear damage accumulation models is inherently wrong which can easily be shown by using a two-level load block sequence as an example:

• Piece-wise linear damage model, Kwofie's model

$$N_{pre} = \frac{\sum n_i}{\sum D_i} = \frac{n_1 + n_2}{\frac{n_1}{N_1} + \frac{n_2}{N_2} \frac{\ln(N_2)}{\ln(N_1)}} = \frac{Cn_1 + Cn_2}{C\frac{n_1}{N_1} + C\frac{n_2}{N_2} \frac{\ln(N_2)}{\ln(N_1)}}$$
(1)

• Non-linear damage model, model proposed by Peng et al.

$$N_{pre} = \frac{\sum n_i}{\sum D_i} = \frac{n_1 + n_2}{\frac{n_1}{N_1} + \frac{n_2}{N_2} \left(\frac{N_1}{N_2}\right)^{\frac{e^{-\frac{n_1}{N_1}} - e^{-1}}{1 - e^{-1}}}} \neq \frac{Cn_1 + Cn_2}{C\frac{n_1}{N_1} + C\frac{n_2}{N_2} \left(\frac{N_1}{N_2}\right)^{\frac{e^{-C\frac{n_1}{N_1}} - e^{-1}}{1 - e^{-1}}}$$
(2)

For Equation (1) an unscaled load history results in the same predicted lifetime as when using a load history where the number of cycles of each block is scaled with a scalar *C*, owing to the linearity of the damage model. In the case of a non-linear damage model this is not true anymore as illustrated by Equation (2). The power in the denominator of Equation (2) is called the decay coefficient and is defined in Equation (7) in the paper of Peng et al. Since this coefficient is a function of  $n_1$ , linear scaling of the load block cycles  $\sum n_i$  (numerator of Equation (2)) results in a non-linear scaling of the accumulated damage  $\sum D_i$  (denominator of Equation (2)). This means that Equation (34) produces erroneous  $N_{pre}$ 



for the proposed non-linear damage model of Peng et al. Therefore part of the results reported in the paper of Peng et al., more specifically in the Tables 2–4, 6, 8 and in Figures 3–5 are wrong.

$$\alpha_1 = \frac{e^{-\frac{n_1}{N^1}} - e^{-1}}{1 - e^{-1}} \tag{3}$$

To illustrate the significance of this error, Table 1 is included at the back of this letter. Row (a) of each load scenario (e.g., high–low) shows the lifetime prediction as calculated by Peng et al. Row (b) shows the accumulated damage when the number of applied cycles is equal to  $N_{pre}$ . Evidently, the damage sum should be equal to unity in the second case but as shown  $\sum D_i \neq 1$ . Finally, row (c) shows the lifetime prediction based on a correct implementation of the damage model of Peng et al. i.e., the number of cycles corresponding to  $\sum D_i = 1$  is calculated. An error of approximately 5000 cycles is demonstrated; for multi-level block loading this error could become even more significant.

High–Low Loading Sequence $\sigma_1 = 485MPa - \sigma_2 = 400MPa$							
	$n_1/n_2$	n1	<i>n</i> 2	$\sum n_i$	$\alpha_1$	$\sum D_i$	Npre
(a) $N_{pre}$ Peng et al.	0.268	13,749	51,304	65,053	0.6501	0.7450	87,314
(b) Corresponding damage	0.268	18,454	68,860	87,314	0.5491	1.0687	
(c) Correct N <sub>pre</sub>	0.268	17,497	65,290	82,787	0.5689	1.0000	82,786
Low–High Loading Sequence $\sigma_1 = 400MPa - \sigma_2 = 485MPa$							
(a) $N_{pre}$ Peng et al.	2.341	109,310	46,693	156,003	0.1653	1.1264	138,502
(b) Corresponding damage	2.341	97,047	41,455	138,502	0.2309	1.0221	
(c) Correct $N_{pre}$	2.341	94,482	40,359	134,841	0.2453	1.0000	134,847

**Table 1.** Predicted lifetime calculated by Peng et al. for their non-linear damage model and two block loading sequences (a), with the corresponding damage (b) and the corrected lifetime prediction (c).



© 2019 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).