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Article

On the Relationship between Structural Quality Index and Fatigue Life Distributions in Aluminum Aerospace Castings †

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Abstract: Tensile and fatigue testing results of D357 and B201 aluminum alloy aerospace castings reported in the literature have been reanalyzed. Yield strength–elongation bivariate data have been used as a measure of the structural quality of castings, and converted into quality index. These results as well as fatigue data have been analyzed by using Weibull statistics. A distinct relationship has been observed between expected fatigue life and quality index. Moreover, probability of survival in fatigue life was found to be directly linked to the proportions of the quality index distributions in two different regions, providing further evidence about the strong relationship between elongation, *i.e.*, structural quality and fatigue performance.

Keywords: structural quality; metal fatigue; weakest link; elongation

1. Introduction

There are a number of tests used by engineers to determine the mechanical properties of structural parts used in aerospace and automotive applications. Among those tests, the most widely used one is the tensile test which is required in many industrial standards and specifications, such as MIL-A-21180D [1]. Although the root cause in 90% of all in-service failures in metallic components is fatigue [2,3], to the authors' knowledge, there is no requirement or specification for fatigue performance in any industrial or military specification.

In castings, the probability of premature failure under stress increases with increasing number density and size of structural defects such as pores and inclusions. Hence, the degradation in mechanical properties including tensile strength [4,5], elongation (e_F) [6–9], fracture toughness as well as fatigue life (N_f) [10] is directly related to the structural quality of castings.

To quantify structural quality by using tensile data, a new quality index, Q_T, has been introduced by one of the authors and his coworkers [11–13].

$$Q_T = \frac{e_F}{e_{F(\max)}} = \frac{e_F}{\beta_0 - \beta_1 \sigma_Y} \tag{1}$$

where, $e_{F(\max)}$ is the maximum elongation, alternatively referred to as the "ductility potential" of the alloy representing the defect-free condition, σ_Y is the yield strength, β_0 and β_1 are alloy dependent constants which were determined from the maximum ductility values over a wide range of yield strength, by analyzing hundreds of data from the aerospace and premium casting literature. Tiryakioğlu and Campbell [13,14] divided the Q_T space into three distinct regions and provided recommendations for quality improvement for each region. When tensile data are in Region 1 ($0 \leqslant Q_T < 0.25$), the premature failure is primarily due to "old" oxides which was the surface of re-melted castings, foundry returns and/or ingot. In this region, tensile specimens do not neck and fatigue failure starts from defects on or close to the specimen surface. Region 2 ($0.25 \leqslant Q_T < 0.70$) represents castings that are free from major "old" oxides but there is still a considerable density of "young" oxides, entrained into the casting during melt transfers and/or filling of the mold. Tensile specimens may show some necking and there will be occasional fatigue failures initiating from internal defects with facets around them [15]. In Region 3 ($0.70 \leqslant Q_T \leqslant 1.0$), tensile specimens are expected to neck and deform significantly beyond ultimate tensile strength [16]. Moreover, fatigue fracture is predominantly due to internal defects, exhibiting facets on fracture surfaces.

Recently, the tensile elongation requirement for castings in industrial and military specifications was interpreted as a *de facto* fatigue life specification [17]. Furthermore, it was shown that there is a distinct relationship between the Q_T and N_f distributions in A206-T7 castings. A similar approach is followed in this study and data from aerospace literature are reanalyzed for a potential relationship between the quality index and fatigue life in aerospace castings.

2. Materials and Methods

2.1. Experiments by Ozelton et al.

Four datasets reported by Ozelton *et al.* [18] who investigated the durability and damage tolerance for D357-T6 and B201-T7 cast aluminum alloys were reanalyzed in this study. For both alloys, two solidification rates based on the pour temperature and the chill material were used. The experimental details for "slow" and "fast" cooled specimens are given in Table 1.

Table 1. Experimental design used by Ozelton *et al.* for D357 and B201 aerospace castings.

	D357-T6		B201-T7	
Solidification Rate	Slow	Fast	Slow	Fast
Pouring *T* (°C)	782	748	787	732
Chill Material	Iron	Copper	Iron	Copper

2.2. Statistical Analysis

Because mechanical properties that involve fracture can be directly linked to casting defects, the Weibull distribution [19–21] based on the "weakest link" theory [22], has been used to characterize these properties. For the Weibull distribution, the cumulative probability function is expressed as:

$$P = 1 - \exp\left[-\left(\frac{\sigma - \sigma_T}{\sigma_0}\right)^m\right] \tag{2}$$

where, P is the probability of failure at a given stress (or fatigue life) at or lower, σ_T is the threshold value below which no failure is expected, σ_0 is the scale parameter and m is the shape parameter, alternatively known as the Weibull modulus. Note that when $\sigma_T = 0$, Equation (2) reduces to the 2-parameter Weibull distribution. The mean of the Weibull distribution is found by:

$$\overline{\sigma} = \sigma_T + \sigma_0 \Gamma\left(1 + \frac{1}{m}\right) \tag{3}$$

where, Γ represents the gamma function. The probability density function, f, for the Weibull distribution is expressed as;

$$f = \frac{m}{\sigma_0}\left(\frac{\sigma - \sigma_T}{\sigma_0}\right)^{m-1} \exp\left[-\left(\frac{\sigma - \sigma_T}{\sigma_0}\right)^m\right] \tag{4}$$

Tensile data were transformed to Q_T by using the β_0 and β_1 of 36 and 0.064 MPa^{-1} for D357 [12] and 34.5 and 0.047 MPa^{-1} for B201 [23], respectively. Weibull distributions with both two and three parameters have been fitted to the tensile and fatigue life data.

3. Results

Ozelton *et al.* performed tensile and fatigue tests in accordance with the ASTM B557 and ASTM E466, respectively. The geometry of the fatigue specimen was carefully selected to mimic aircraft components with holes where fatigue cracks are usually initiated due to stress concentrations. In total, 170 fatigue life and 165 tensile test results obtained by Ozelton *et al.* have been re-evaluated in the present investigation. It is significant that there were no fatigue run-outs in the datasets.

The dot-plot for elongation data for D357-T6 and B201-T7 castings is presented in Figure 1. Note in Figure 1a that the highest data for "fast" solidification is higher than in "slow" solidification, although minimum data in both datasets are similar. Hence the scatter is higher in "fast" solidification D357 castings. In Figure 1b, the elongation data for "fast" B201-T7 castings are only slightly higher than in "slow" castings. Moreover, there is an apparent gap in both datasets, as indicated in Figure 1b.

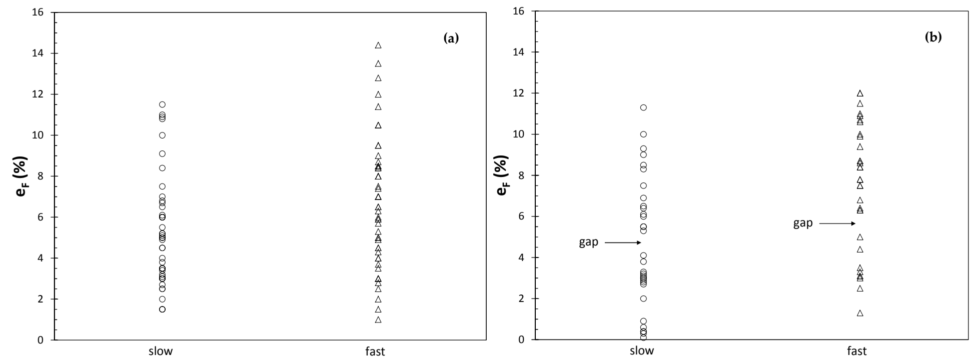

Figure 1. Dot-plot for elongation data of "slow" and "fast" specimens for (**a**) D357-T6 and (**b**) B201-T7 aluminum alloy castings.

The fatigue life data for the two aluminum alloy castings are presented in Figure 2. For D357, minimum data are almost identical for "slow" and "fast" castings, Figure 2a. However longest fatigue life is significantly higher for "fast" castings. As in elongation, Figure 1b, fatigue life data in B201 aluminum alloy castings have a significant gap, as shown in Figure 2b. The gaps in datasets are an indication that data have been collected from two distinct distributions. Hence, there is evidence that there is a mixture of at least two distributions in elongation and fatigue life data for B201-T7 aluminum alloy castings.

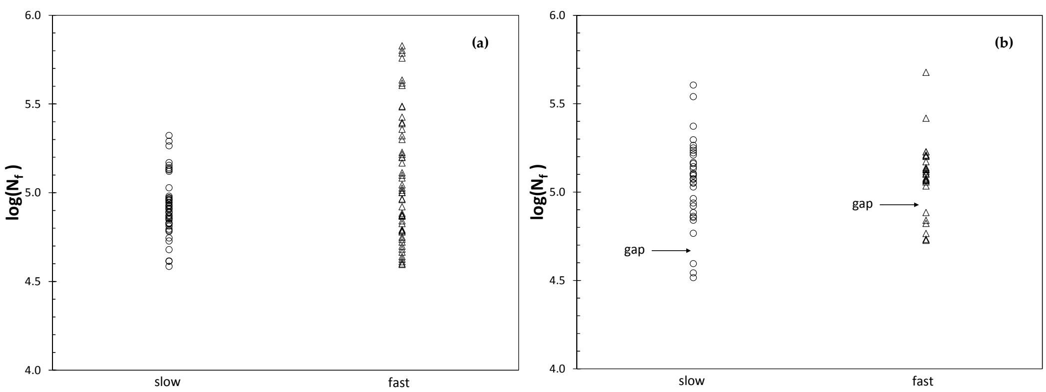

Figure 2. Dot-plot for fatigue life of "slow" and "fast" specimens for (**a**) D357-T6 and (**b**) B201-T7 aluminum alloy castings.

Weibull distributions were fitted to Q_T and N_f data by using the maximum likelihood method. The Weibull probability plots for Q_T and N_f for D357-T6 aluminum alloy castings are presented in Figure 3. Estimated Weibull parameters for each dataset are given in Table 2. Fits indicated by the two curves in Figure 3 are in close agreement with the data. The goodness-of-fit of the estimated parameters was tested by using the Anderson-Darling statistic [24]. In all cases, the hypothesis that the data come from the fitted Weibull distributions could not be rejected. Note in Figure 3a that the Q_T data for fast cooled castings fall on almost a straight line, which indicates that the threshold is close to zero (0.010), Table 2. The data for slow-cooled castings indicate a curve relationship which is indicative of a positive threshold. Moreover, both fatigue life distributions have almost the same threshold. Because lowest fatigue life in a distribution is determined by the largest defects possible in specimens [25], the size of the largest defects is almost the same in "fast" and "slow" datasets, regardless of how fast the metal solidified.

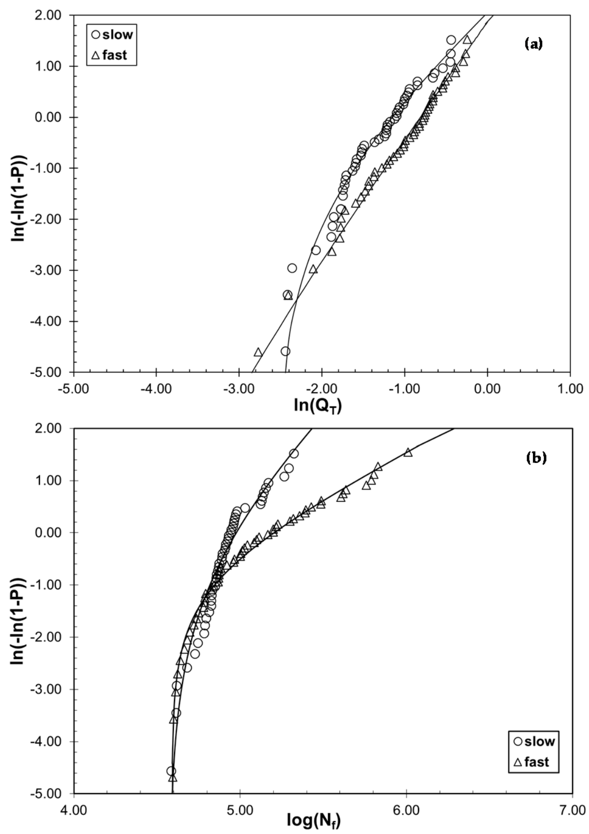

Figure 3. Weibull probability plots of (**a**) Q_T and (**b**) N_f in D357-T6 aluminum alloy castings.

Note that for B201, fatigue life and quality index data was found to have Weibull mixtures, as indicated in Table 2. In such cases, the cumulative probability is expressed as [26];

$$P = pP_L + (1-p)P_U \tag{5}$$

where, p is the fraction of the lower Weibull distribution in the mixture and subscripts L and U refer to the lower and upper Weibull distributions, respectively. The probability density function for a Weibull mixture is given as:

$$f = pf_L + (1-p)f_U \tag{6}$$

The Weibull probability plots for Q_T and N_f for B201-T7 aluminum alloy castings are provided in Figure 4. Note that for both Q_T and N_f, there are inflection points in the probability plots which are indicative of Weibull mixtures [25–27].

4. Discussion

Both Q_T and N_f data for B201 aluminum alloy castings showed Weibull mixtures is noteworthy. Analysis of fracture surfaces in A206-T7 aluminum alloy castings showed [10,17,28] that the lower distribution for elongation was attributed to the "old", coarse oxide bifilms that were generated during previous melt processing or were on the skin of the ingots. For fatigue life, the lower distribution is due to the fatigue crack initiation at surface defects. The two lower distributions are linked because the probability that a defect will be on the surface of the fatigue specimen increases with its size and number density [17]. Hence, premature fracture in fatigue has to be accompanied by low elongation, or alternatively, Q_T. As expected, increased solidification rate has a positive effect on both Q_T and N_f. It is also noteworthy that the improvement is most significant in the lower distributions. Moreover, the lower distributions remain significantly separated from the upper distributions, showing that chilling is a much less effective way to improve properties than eliminating structural defects, mainly bifilms and pores. It has been only recently understood [29,30] that the degradation of and variability in the mechanical properties of castings are related to these very defects that are incorporated into the bulk of the liquid by an entrainment process, in which the surface oxide folds over itself. In most steel castings, the oxide has a significantly lower density than the metal, and therefore floats to the surface quickly, leaving the metal relatively free of defects. In aluminum alloys, the folded oxide has practically neutral buoyancy, so that defects tend to remain in suspension. The layer of air in the folded oxide can: (i) grow into a pore as a result of the negative pressure due to contraction of the solidifying metal and/or rejection of gases, originally dissolved in liquid metal, upon solidification; or (ii) remain as an un-bonded surface, like a crack, in the solidified alloy, which usually serves as heterogeneous nucleation sites for intermetallics.

Table 2. Estimated Weibull parameters.

Alloy	Solidification Rate	Distribution Tag		Weibull parameters			
				p	σ_T	σ_0	m
D357-T6	Slow	Q_T			0.077	0.239	1.52
		N_f			37,291	57,340	1.43
	Fast	Q_T			0.010	0.433	2.27
		N_f			38,688	120,966	0.74
B201-T7	Slow	Q_T	Lower	0.250	0	0.066	1.13
			Upper		0.179	0.234	1.34
		N_f	Lower	0.114	26,351	16,963	1.62
			Upper		67,720	90,284	0.82
	Fast	Q_T	Lower	0.188	0	0.200	4.79
			Upper		0	0.623	6.26
		N_f	Lower	0.281	51,372	18,730	0.95
			Upper		113,610	58,081	0.57

Prior studies [5,31–35] have shown that there are multiple types of defects in castings, including bifilms and pores associated with bifilms. From a process viewpoint, it is not surprising to find Weibull plots for tensile data that reveal at least two populations of defects [25]:

1. the original rather fine scattering of defects remaining in suspension in the original poured liquid from the crucible or ladle (prior damage). These "old" bifilms have a typical minimum thickness of approximately 10 μm and show only coarse wrinkles.
2. the large new bifilms (new damage) that would have been produced during the melt transfer and/or pouring and filling if the filling system was not designed properly. These "young" oxides have a minimum thickness of tens of nanometers or less and show fine wrinkles on fracture surfaces of castings.

The probability density functions (Equations (4) and (6)) for Q_T and corresponding N_f distributions are presented in Figure 5. The different shapes of N_f distributions are a product of the use of the 3-parameter version of the Weibull distribution and the value of the shape parameter; when $m \leqslant 1$, the shape of the 3-parameter Weibull distribution resembles that of an exponential decay curve.

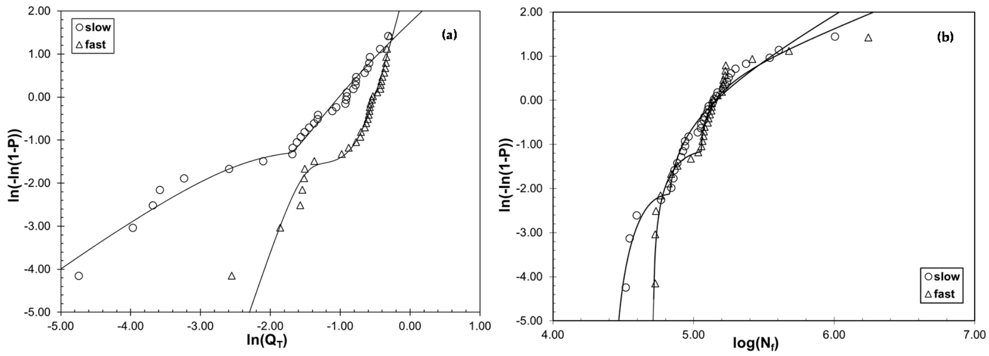

Figure 4. Weibull probability plots for (**a**) Q_T and (**b**) N_f in B201-T7 aluminum alloy castings.

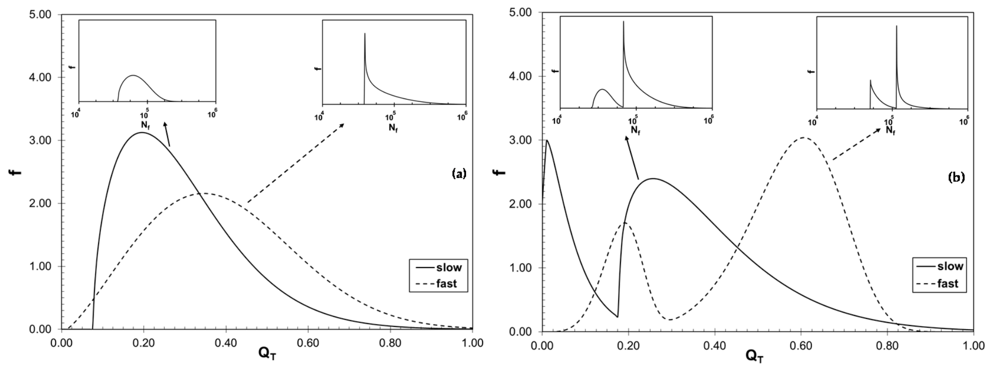

Figure 5. Probability density functions for Q_T and corresponding probability density functions for N_f for (**a**) D357 and (**b**) B201 aluminum alloy castings.

The expected (mean) values for all six distributions were calculated by using Equation (3) and the estimated Weibull parameters in Table 2. As stated above, the lower distributions in Q_T and N_f in B201 were associated with each other. The relationship between expected Q_T and N_f values is presented in Figure 6. Similar to what was reported for A206-T7 aluminum alloy castings [17], a linear relationship was observed between expected Q_T and the logarithm of expected fatigue life. The best fit line indicated with dashed lines in Figure 6 has the following equation:

$$\log(\overline{N}_f) = 4.57 + 1.46\overline{Q}_T \tag{7}$$

Note that the best fit equation estimates approximately 10^6 cycles when the specimen could reach the maximum quality (defect free condition). Therefore, fatigue life of aerospace castings can be extended by at least six times if structural defects are eliminated.

The proportions of the Q_T distributions in all three regions as well as the probability of survival after 10^5 cycles were calculated by using estimated Weibull parameters in Table 2. The results are presented in Table 3. The proportion of Region 1 ($Q_T \leqslant 0.25$) *versus* probability of $N_f > 10^5$ is plotted in Figure 7a. Clearly, probability of survival after 10^5 cycles decreases significantly with the proportion of castings in Region 1. A similarly strong relationship between the probability of survival after 10^5 cycles *versus* the proportion of castings in Region 3 ($Q_T > 0.70$) is presented in Figure 7b. As the casting quality is improved, probability of survival also increases, as can be expected. Hence there is strong evidence in Figure 7, as well as in Figure 6, that elongation (quality index) and fatigue performance are related; (i) there is a strong correlation between mean values, and (ii) the probability of survival is directly linked to the proportion of the elongation distribution in Region 1 and Region 3. Therefore, the statement that the elongation requirement in industrial specifications is a *de facto* fatigue life specification is justified. Research is underway to expand this understanding and develop predictive models for aerospace castings.

Table 3. Fraction of distributions for Q_T in each region and probability of survival at 10^5 cycles.

Distribution Tag			$P\ (Q_T \leqslant 0.25)$	$P\ (0.25 \leqslant Q_T < 0.70)$	$P\ (Q_T \geqslant 0.70)$	$P\ (N_f \geqslant 10^5)$
D357	Slow		0.458	0.529	0.014	0.321
	Fast		0.230	0.713	0.056	0.546
B201	Slow	*L*	0.989	0.011	0.000	0.000
		U	0.183	0.763	0.054	0.650
	Fast	*L*	0.946	0.054	0.000	0.084
		U	0.003	0.871	0.126	1.000

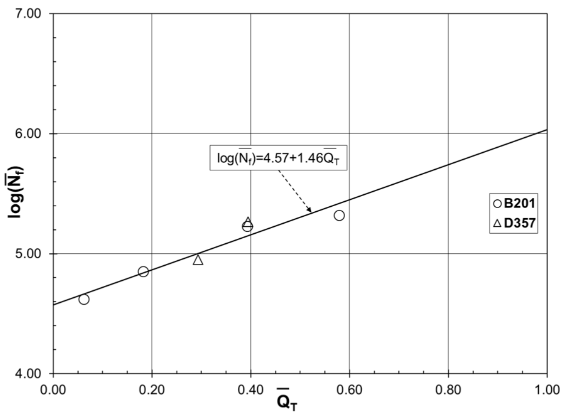

Figure 6. Relationship between the means of Q_T and N_f distributions.

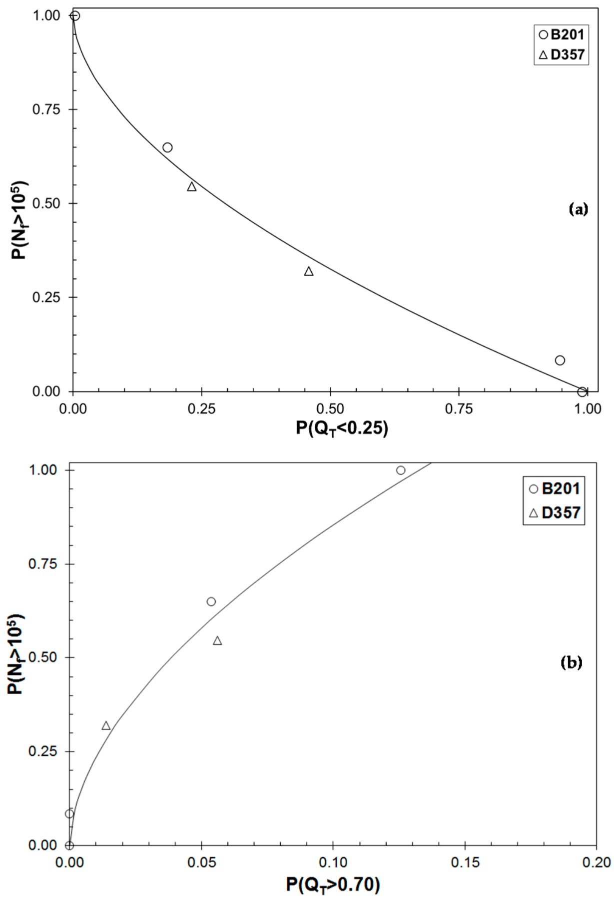

Figure 7. The change in the probability of survival after 10^5 cycles *versus* the estimated fraction of the Q_T distribution in (**a**) Region 1 and (**b**) Region 3.

5. Conclusions

- The quality index, Q_T, can be used to characterize the structural integrity of D357 and B201 aluminum alloy castings.
- Probability plots for both Q_T and N_f distributions for B201 showed strong indications of Weibull mixtures.
- There is a strong relationship between the mean Q_T and N_f values as calculated from estimated Weibull parameters.
- There is a strong negative correlation between the proportion of Q_T in Region 1 and probability of survival for 10^5 cycles. Similarly, a strong positive correlation exists between the proportion of Q_T in Region 3 and probability of survival for 10^5 cycles, providing further evidence for the strong link between elongation and fatigue performance.
- The statement that the elongation requirement in industrial specifications is a *de facto* fatigue life specification is justified.

Author Contributions: Hüseyin Özdeş and Murat Tiryakioğlu collaborated in all phases of this paper, namely the data collection, statistical analysis and interpretation of results.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

β_0, β_1	alloy dependent constants
f	Weibull probability density function
Γ	gamma function
m	shape parameter
σ	mean value of the Weibull distribution
N_f	fatigue life
σ_0	scale parameter
p	fraction of the lower distribution
σ_T	threshold value below which no failure is expected
P	probability of failure
σ_Y	yield strength (MPa)
P_L	probability from lower distribution
e_F	elongation
P_U	probability from upper distribution
$e_{F(\max)}$	maximum elongation, ductility potential
Q_T	quality index

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