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Effective Stiffness and Seismic Response Modification Models Recommended for Cantilever Circular Columns of RC Bridges, First Part: Serviceability Limit State

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Abstract: This paper's main aim was to explain the process of characterising the structural overstrength factor (R), seismic behaviour factor (Q), and the effective elastic stiffness, K_{eff}, of cantilever-reinforced concrete (RC) urban bridge columns with solid circular cross-sections for use in seismic design under the Serviceability Limit State (SLS). Similarly, mathematical models have been proposed to determine the average values of effective stiffness and seismic response modification factors suggested for cantilever-reinforced concrete bridge columns at SLS. This is because multiple design codes stipulate that cantilever RC bridge columns must meet the SLS requirements. Therefore, to comply, the lateral displacement ductility demand must not exceed unity after a moderate or small earthquake. While the behaviour of the materials remains in the elastic range, this performance criterion can be conservative. If the materials undergo small deformations, the slight damage can be quickly repaired to meet the SLS.

Keywords: response modification factors; seismic response; reinforced concrete bridges; serviceability limit state

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1. Introduction

For the seismic design of bridges, the current codes suggest using the Seismic Design Method Based on Forces (SDMBF). This method involves reducing the accelerations from the design elastic spectrum by using the response modification factors (RMFs). These factors consider the total ductile capacity of the lateral deformation of the bridge. This capacity is composed of two elements: the ductility (Q) of the structure and the structural overstrength (R). Uang [1] described two RMFs which are Q', associated with Q, and R', representing the structural overstrength, in the seismic design codes for bridges. In Figure 1, the current and bi-linear lateral load response curves of a structure, such as that of cantilever columns, are illustrated. The curves are defined by the ratio between the basal shear (Vb) and the lateral displacement at the column end (Δ). Additionally, the concepts of Q' and R' are presented. The purpose of Q' is to decrease the basal shear corresponding to the elastic behaviour of the structure (Vb_e) to the basal shear corresponding to a substantial global yield of the structure (Vb_y). Similarly, R' is employed to decrease Vb_y to the design basal shear of the structure, Vb_d.

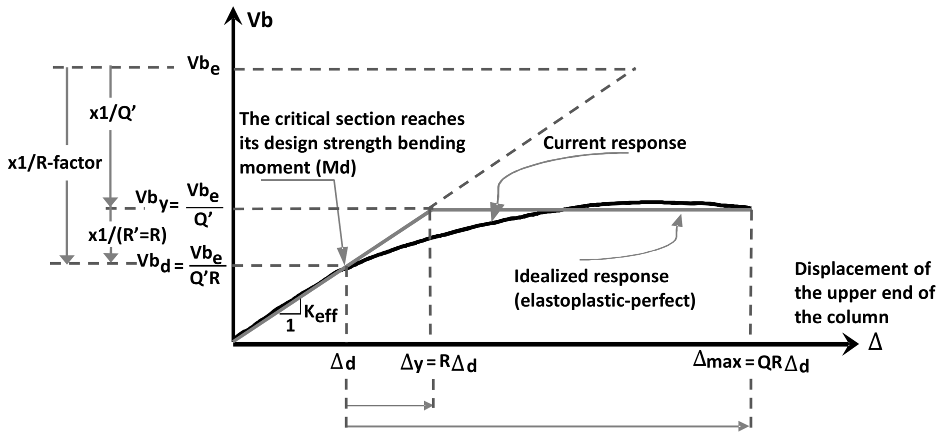


Figure 1. Basal shear force/lateral displacement ratio at the cantilever end of the column, adapted from Uang [1].

Seismic design codes (AASHTO [2], ATC-32 [3], and EUROCODE [4]) for bridges use the equal displacement and equal energy approximations, or their variants, proposed by Newmark and Hall [5] to establish the relationship between Q' and Q. In the medium and long-period spectral regions, the equal displacement approximation holds true, i.e., $Q' \cong Q$. Similarly, in the short-period spectral region, the equal energy approximation expressed by Equation (1) relates Q' and Q:

$$Q\prime = \sqrt{2Q - 1} \tag{1}$$

In bridge design codes such as AASHTO [2], ATC-32 [3], and EUROCODE [4], only a single FMR is established, which is determined by the product $Q'R'$, as shown in Figure 1. If the value given for this factor is constant, the equal displacements approximation is used, resulting in $Q{\cdot}R$. This product represents the overall lateral deformation ductility of the bridge. The factor in this paper is referred to as the R-factor. The CDS-MDOC [6] does not employ the equal displacements approximation to relate Q' to Q.

Figure 1 illustrates that, when the SDMBF is applied, a structure's effective elastic stiffness (K_{eff}) remains constant. Hence, K_{eff} is independent of the structure's strength (Vb_e, Vb_y, or Vb_e). Priestley et al. [7] ruled that deeming the K_{eff} of a reinforced concrete (RC) member to be independent of its strength is not adequate. As the flexural or flexural-compressive strength capacity changes, it modifies the required longitudinal reinforcement, causing a modification to the cracked rigidity. To estimate K_{eff} for RC members, it is necessary to consider the effect of cracking. Current bridge design codes (including [2–4,6]) consider this through an effective moment of inertia (I_{eff}) of the section, estimated as $I_{eff} = k_{eff}{\cdot}I_g$, where k_{eff} is a constant less than one, called the effective inertia factor in this study, and I_g is the geometric moment of inertia of the section. As a result, when k_{eff} is implemented in bridge design codes, the relation between the effective elastic stiffness and the strength of RC members is also neglected.

The values of Q and R, along with their respective RMFs, as suggested by bridge design codes, were based on engineering judgments [8]. The values are dependent on the structural system type, material, ductility as per specific design requirements, and the importance of the bridge. Therefore, if the bridge substructure is built using RC cantilevers, which are expected to behave ductile, they specify a constant value for these factors. However, Priestley et al. [7,8] mentioned that this is not adequate, since Q and R, and their RMFs, also depend on the geometry of the columns (length and depth of the cross-section), the magnitude of the acting axial load, the compressive strength of the concrete, and the longitudinal and transverse reinforcement ratios, among other parameters.

According to Priestley et al. [7], approximations such as equal displacements and equal energies are dependent on the structures' initial elastic period and damping. However, these do not apply when estimating the inelastic response of the structure. This is because the lateral stiffness and its distribution on structures, change when the structures exhibit inelastic behaviour. On the other hand, Priestley et al. [7] asserted that inelastic behaviour leads to increased structural damping as it is associated with the dissipation of hysteretic energy.

Because of the limitations of the SDMBF, certain bridge design codes such as Caltrans and Eurocode suggest that assessing the fulfilment of performance criteria can be carried out through a simplified inelastic analysis of the structure. In the early 2000s, the development of seismic evaluation and design procedures based on deformations and other response parameters, known as 'performance-based', began. Direct displacement-based (DDB) procedures are notable among these methods for the following reasons: (1) the maximum inelastic or design displacements, the yield displacements, and the corresponding ductility are calculated for the structure, so these parameters are not general; and (2) they characterise the structure by means of a secant stiffness at maximum displacement and equivalent viscous damping obtained as the sum of the elastic damping plus the equivalent of the hysteretic energy dissipated during the inelastic response of the structure. The DDB seismic design procedure for bridges can ensure the fulfillment of performance criteria with reasonable accuracy [7]. Likewise, using DDB seismic assessment methods for bridges permits the fair evaluation of their structural performance [9,10]. However, these procedures have not been implemented in several current seismic design codes for bridges, such as AASHTO [2], EUROCODE [4], ATC-32 [3], CDS-MDOC [6], Caltrans [11], etc. Therefore, further research is needed to overcome the limitations of the SDMBF.

Regarding the structural performance criteria outlined in the current international seismic design codes for bridges, most require that bridges be designed to meet the Serviceability Limit State (SLS) [4,6], and the Collapse Prevention Limit State (CPLS) [2,3,11]. In this research an attempt is made to adequately characterise the structural overstrength factor (R), the seismic behaviour factor or ductility (Q), and the effective inertia factor (k_{eff}) of RC bridge columns to be used in their seismic analysis and design in both limit states. However, due to the extensive of this topic, the research is divided into two parts. This paper presents the first part of the research which aims to characterise the R, Q and k_{eff} factors of columns in the SLS. The second part of the research concerning these factors for PCLS is presented in Vargas et al., [12].

The CDS-MDOC [6] establishes for the SLS that bridges of conventional importance must be functional and their structural components must remain elastic after a moderate or small earthquake. According to Figure 1, this indicates that $Q' = Q = R = 1$ and therefore also *R-factor* = 1, for all bridge components. Moreover, the handbook [6] suggests $k_{eff} = 0.5$ as an estimate of I_{eff} for RC bridge columns that are to be used in both limit states. To achieve SLS compliance in RC cantilever columns, as recommended in the CDS-MDOC [6], the tensile strain of the reinforcing steel (ε_s) needs to be limited at the point of yielding strain (ε_y). Reinforcing steels produced in Mexico have a yield strain (ε_y) of 0.0023 [13]. For this paper, an exhaustive search was performed in the technical literature to determine whether the performance criterion specified in the CDS-MDOC [6] for RC cantilever columns in this limit state is conservative, in terms of column response modification factors, column ductility, and overstrength, and allowable deformations in reinforcing steel and concrete, all corresponding to the SLS of columns forming the substructure of RC bridges. The search showed that the available information is very scarce, and the comparison of the performance criterion specified in CDS-MDOC [6] with other recommended criteria [2,11] for the same purpose suggests that the former is conservative.

Kowalsky [14] outlined two critical strains for the materials used in the solid cross-section RC cantilever bridge columns for the SLS. The first one limits the ultimate compressive strain of concrete at the extreme compression fiber (ε_c) to 0.004. This restriction corresponds to the initiation of unconfined concrete crushing. The second one is restricted to $\varepsilon_s = 0.015$ in the layer of tensile reinforcement closest to the section's surface to control any crack width to no more than 1 mm after an earthquake event. Any value exceeding this threshold requires that the column be rehabilitated [8]. Otherwise, the column may lose its functionality. However, Kowalsky [14] does not provide the values of k_{eff}, Q, and R for RC cantilever bridge columns that correspond to the material's limit deformations.

The bridge design recommendations of the Applied Technology Council of the United States of America [3] specify requirements analogous to those abovementioned. RC structural members that comprise bridges are classified as important to meet a Serviceability Limit State not exceeding ε_c = 0.004 and ε_s = 0.010. In ATC-32 [3], k_{eff} =0.5 is specified if the columns have approximately the same length and cross-section. If these conditions are not met, it is recommended to calculate k_{eff} as a function of the axial load acting on the column and the amount of longitudinal reinforcement. However, in ATC-32 [3] no values of Q and R for RC cantilever bridge columns are specified, since it is considered appropriate to assume that the amplitude of displacements for the design earthquake corresponding to the SLS is equal to the displacement calculated using an elastic analysis, without modification. This criterion also implies that $Q' = Q = R = 1$ for RC cantilever bridge columns.

Caltrans [11] specified that bridges classified as "Recovery" shall have a minimum damage condition in a so-called functional design earthquake, for which the entire structure shall remain essentially elastic. The specified qualitative performance criteria include flexural cracks, minor spalling, and possible shear cracks. Furthermore, the seismic analysis and design of the substructure members do not specify R, Q, and k_{eff} values. This performance criterion is similar to that specified by the CDS-MDOC [6].

According to AASHTO [2], bridges classified as essential are required to maintain their elastic behaviour even after a large design earthquake, which is known as the "immediate occupancy" damage state, in which all load-bearing members of the structure remain essentially elastic and minor spalling of concrete columns may occur. Cantilever substructure columns require an R-factor of 2.0, with no specified k_{eff} value.

Using Kowalsky's [14] or ATC-32 [3] guidelines instead of the ε_y mentioned by CDS-MDOC [6] leads to less conservative and more cost-effective designs. However, for the application of these limit strains in the SDMBF, it is necessary to estimate R, Q, and k_{eff} values that correspond to these strains for RC cantilever bridge columns. Additionally, the upcoming section of this manuscript demonstrates that the R, Q, and K_{eff} (k_{eff}) values of RC cantilever bridge columns are intrinsically related. Therefore, when estimating these factors, their correlation should be taken into consideration, which is currently not accounted for in the seismic design codes applicable to bridges. Moreover, as mentioned above, the fact that R, Q, and k_{eff} depend on the geometry of the columns: length (L) and depth of the cross-section (h for rectangular sections and D or diameter for circular sections), the magnitude of the axial load (P), the compressive strength of the concrete ($f'c$), and the amounts of longitudinal (ρ) and transverse (ρ_t) reinforcement, among other parameters [3,7,8], suggests that the most rational way to estimate the values of R, Q, and k_{eff} is by means of functions that depend on these parameters, which is also not considered in the current seismic design codes for bridges. It is expected that the implementation of these functions in the MDSBF will reasonably overcome the limitations of this method concerning the estimation of R, Q, and k_{eff}, but will not overcome those concerning the use of the equal displacements and equal energies approximations or other alternatives based on considering the initial elastic period and damping of the structure. Consequently, it is recommended to perform at least a static inelastic analysis of the designed column under monotonically increasing lateral load, known as "pushover" analysis, to verify that the limit deformations proposed for its SLS will not be exceeded under the design seismic demand. Alternatively, it is suggested, as in the AASHTO [2], to verify the expected performance of the designed column by applying a current method for seismic evaluation of bridges based on direct displacements [9].

In view of the above, the main objective of the first part of this research was to obtain functions characterising the structural overstrength factor (R), the seismic behavior factor or ductility (Q), and the effective inertia factor (k_{eff}) of cantilever RC urban bridge columns with solid circular cross-sections in the SLS, which form the substructure of simply supported straight-axis bridges (Figure 2). These functions can be used for the seismic analysis and design of the columns in the SLS and in the transverse direction of the bridges, since in this direction of bridges with these characteristics the columns can be idealised as independent single-degree of-freedom oscillators. This is possible thanks to the explicit influence of geometric and mechanical variables. For this purpose, mathematical models have been developed to estimate the mean values of R, Q, and k_{eff}. The models are obtained from the limit strains ε_c and ε_s recommended in ATC-32 [3]. This is carried out by considering the influence of the most relevant geometrical and structural properties of the columns.

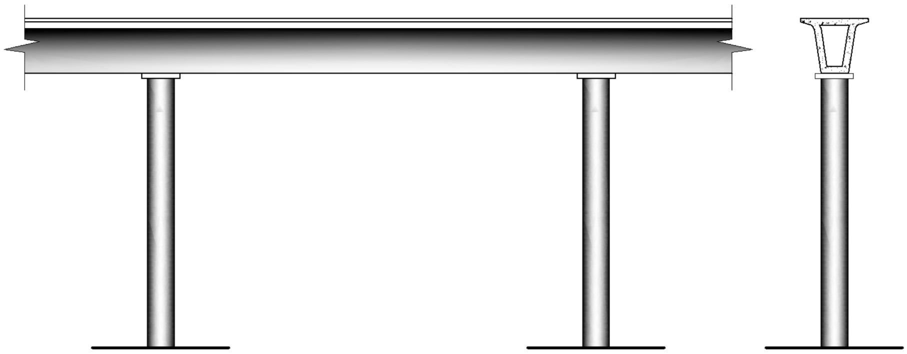


Figure 2. RC cantilever column of a single supported straight-axis urban bridge.

2. Definition of Factors *R*, *Q* y K_{eff}

Figure 1 shows the current and bi-linear lateral load response curves of a structure, such as a cantilever column, that can stably dissipate energy. This curve is defined by the ratio between the basal shear (Vb) and the lateral displacement at the end of the column (Δ). Three parameters can be obtained from the idealised curve: K_{eff}, its initial lateral stiffness, Q and R. These parameters, together with the reactive mass of the column and a critical fraction of the viscous damping, provide the necessary properties to characterise a one-degree-of-freedom oscillator. With this oscillator and the reduced elastic design spectrum by the product $Q'(Q)\cdot R'$, where Q' is the RMF related to ductility, the design strength of the column is determined for the SLS.

In Figure 1, the slope of the first branch of the idealised curve defines the K_{eff} for the column. This slope is a secant line starting from the origin and intersecting the current response curve at a point whose ordinate is the design basal shear (Vb_d) corresponding to the design bending moment (M_d) of the critical section of the column. The lateral displacement of the top of the column corresponding to Vb_d is Δ_d. K_{eff} is therefore calculated using Equation (2).

$$K_{eff} = \frac{Vb_d}{\Delta_d} \tag{2}$$

If the actual response curve shows no significant change in slope, representing a global yielding of the structure, the bilinear curve is usually idealised as perfect elastoplastic [15], and on this understanding, the R and Q factors are established. Q' is defined by Equation (3).

$$Q\prime = \frac{Vb_e}{Vb_y} \tag{3}$$

Vb_e represents the basal shear that is necessary for the column to exhibit elastic response during the design earthquake associated with the Serviceability Limit State (SLS), and Vb_y denotes the point where the column reaches yield on the idealised curve (Figure 1). Equation (4) is used to estimate the factor R:

$$R = \frac{Vb_y}{Vb_d} \tag{4}$$

As shown in Figure 1, the ratio of $\Delta_y/\Delta_d = R$ is analogous to that expressed in Equation (4), where Δ_y is the lateral displacement of the column where the yielding starts, which belongs to Vb_y. Equation (5) defines Δ_y:

$$\Delta_y = R\Delta_d \tag{5}$$

Q, provided by Equation (6), is a measure of the ductile lateral deformation capacity of the column, where Δ_{max} denotes its lateral displacement capacity.

$$Q = \frac{\Delta_{max}}{R\Delta_d} \tag{6}$$

Equations (2), (4), and (6) show the correlation between the factors R, Q, and K_{eff}.

3. Developed Methodology to Obtain R, Q, and k_{eff}

3.1. Geometric and Structural Parameters

This paper investigates the inelastic responses of RC concrete cantilever columns represented by basal shear versus lateral displacement through a parametric study. These columns are the substructures used for simply supported straight-axis urban bridges. The study aims to establish predictive models that could help estimate the average values of the factors R, Q, and k_{eff} for their analysis in the SLS, see [16]. The aspect ratio L/D, axial load/strength ratio $P/(A_g{\cdot}f'c)$, concrete's compressive strength ($f'c$), and the longitudinal reinforcement steel ratio (ρ) are the columns' geometrical and mechanical properties with the most significant impact on factors R, Q, and k_{eff}. The axial load/strength ratio is determined by three parameters: the axial load P, the gross cross-sectional area A_g, and $f'c$.

Solid cross-section columns with a 1500 mm diameter were used for parametric analyses, which were evaluated based on multiple values of $f'c$, L/D, and $P/(A_g{\cdot}f'c)$. Refer to Tables 1 and 2 for details.

Table 1. Properties of the columns in terms of their geometry and mechanics.

Parameter	Value
$f'c$ (MPa)	24.51, 29.42, 34.32
f_y nominal (MPa)	412
r (mm)	50
L/D (dimensionless)	3, 5, 7, 9
$P/(A_g{\cdot}f'c)$ (dimensionless)	0.10, 0.15, 0.20, 0.25, 0.30
ρ (%)	1, 2, 3, 4

Table 2. Concrete parameters.

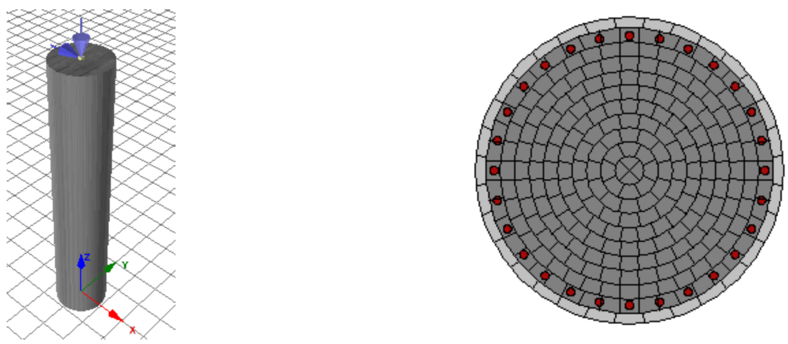
$f'c$ (MPa)	f_f (MPa)	E_c (MPa)
24.51	3.12	21,783.34
29.42	3.42	23,865.70
34.32	3.69	25,776.64

The size of the cross-section, the L/D and $P/(A_g{\cdot}f'c)$ ratios, the cover (r), and the $f'c$ used are typical for cantilever columns of RC urban bridges in Mexico [17–19], meeting the limits set in the NTC-DCEC [20]. The f_y value and the spacing between ρ used in this analysis conform to the requirements set by NTC-DCEC [20] for ductile structures (with Q values of 3 and 4).

3.2. Numerical Model Description

Two hundred and forty models were created (Table 1) to consider all the geometrical and mechanical properties of the columns. The columns were fixed at their base. According to NTC-DCEC [20] guidelines, the design bending moment (M_d) and design shear force on the critical cross-section were estimated for each model. To prevent shear failure, the shear force required for design was computed from the maximum nominal bending moment, obtained via an inelastic analysis, with due consideration given to the overstrength of the critical section. The design shear force was calculated according to the specifications for ductile structures (Q = 3 and Q = 4).

The finite element method was used to perform the inelastic analysis, using SeismoStruct v7.0 [21]. A force-based, three-dimensional inelastic frame type was used to model the columns. The columns were divided into six integration sections along their lengths and their cross-section consisted of 300 fibers (refer to Figure 3).



(**a**) (**b**)

Figure 3. Numerical models of the RC cantilever column are used in urban bridges: (**a**) global model, and (**b**) column cross-section discretisation.

Mander et al. [22] established the model for the stress–strain relation of free (cover) and confined (core) concrete, which is a non-linear uniaxial constant confinement model. NTC-DCEC recommendations [20] were used to define the concrete properties such as elasticity modulus (E_c = $4400\sqrt{f'c}$), bending tensile strength (f_f = $4400\sqrt{f'c}$), and strain corresponding to $f'c$ (ε_c = 0.002). The stress–strain relation of the confined concrete was calculated using SeismoStruct. The elasticity modulus of steel (Es) is equal to 200,000 MPa.

Dodd and Restrepo-Posada [23] proposed the stress–strain relation of reinforcing steel. To determine Es and the stress–strain relation, the values reported by Rodriguez and Botero [13] for reinforcing steel produced in Mexico were used. Second-order effects are considered in the analysis of the columns.

3.3. Criteria to Define SLS

The SLS is defined based on two performance criteria, which are defined by the limit strain of the materials recommended in ATC-32 [3]:

1. The longitudinal tensile reinforcement that lies closest to the critical cross-section surface must have ε_s = 0.010 to limit the crack width;

2. The concrete in the outer compression fiber of the critical cross-section must have ε_c = 0.004 to prevent damage and the need for rehabilitation.

3.4. Calculations for Factors *R*, *Q*, and k_{eff}

The method for obtaining the *R*, *Q*, and k_{eff} factors is described in the literature [1,15]. To achieve this, the columns underwent an inelastic analysis and were exposed to two static loads. Initially, a vertical force was applied at the top of the columns, representing the self-weight and tributary weight of the superstructure (Figure 3). After that, a series of steadily increasing horizontal displacements was applied to the free end, representing the seismic force.

The capacity curve (CC) represents the inelastic response of the columns (refer to Figure 4), where CC is limited when the lowest of the performance criteria occurs.

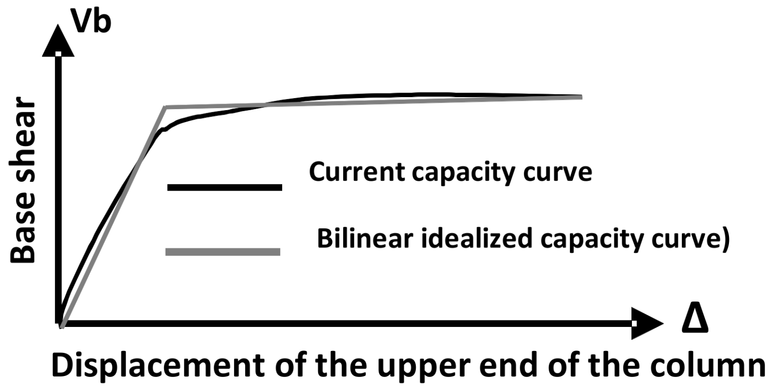


Figure 4. The column capacity curve and its idealised form.

From the origin, a secant line was drawn to intersect the CC at a point where the abscissa represents column Vb_d and the ordinate represents Δ_d. The straight line in Figure 1 represents the elastic range of the idealised bi-linear CC. Its slope, known as K_{eff}, is used to calculate the I_{eff} using Equation (7).

$$I_{eff} = \frac{K_{eff}L^3}{3E_c} = \frac{Vb_d L^3}{3E_c \Delta_d} \tag{7}$$

Following this, k_{eff} was estimated using the cross-section ratio of I_{eff}/I_g. Afterward, the equal energy dissipation criterion (equal areas under both curves) was applied, imposing the condition that the slope of the second branch of the idealised CC is approximately equal to zero, where the yielding is located. Therefore, the idealised CC can be characterised in this way. Equation (4) is used to calculate R and Equation (6) is used to calculate Q.

For each column, the properties of $f'c$, L/D, $P/(A_g \cdot f'c)$, and ρ (independent variables) were used to calculate R, Q, and k_{eff} values (dependent variables). Multiple linear regression analysis was used to obtain the dependent variable models. The proposed models defining the average values of R, Q, and k_{eff} are shown in Equations (8)–(10).

$$R = 0.953 - 0.00024f'c - 0.00784\frac{L}{D} + 2.590\frac{P}{A_g f'c} + 0.4470\rho \tag{8}$$

$$Q = 0.931 + 0.00033f'c + 0.00853\frac{L}{D} + 2.060\frac{P}{A_g f'c} - 9.90\rho \tag{9}$$

$$k_{eff} = 0.202 - 0.000005f'c + 0.00462\frac{L}{D} + 1.58\frac{P}{A_g f'c} + 7.05\rho \tag{10}$$

The R^2 multiple determination coefficients of the proposed models, as provided by Equations (8)–(10), were 93.5%, 83.7%, and 89.9%, respectively. These R^2 values are considered acceptable.

4. Results

The proposed models were compared with the recommended values given in the design codes and those obtained by the idealised capacity curve (CC). For instance, the values of R (Figure 5), Q (Figure 6), and k_{eff} (Figure 7) are compared for RC columns ($f'c$ = 24.51 MPa). The values are a function of $P/(A_g \cdot f'c)$ and ρ, given a range of values of L/D.

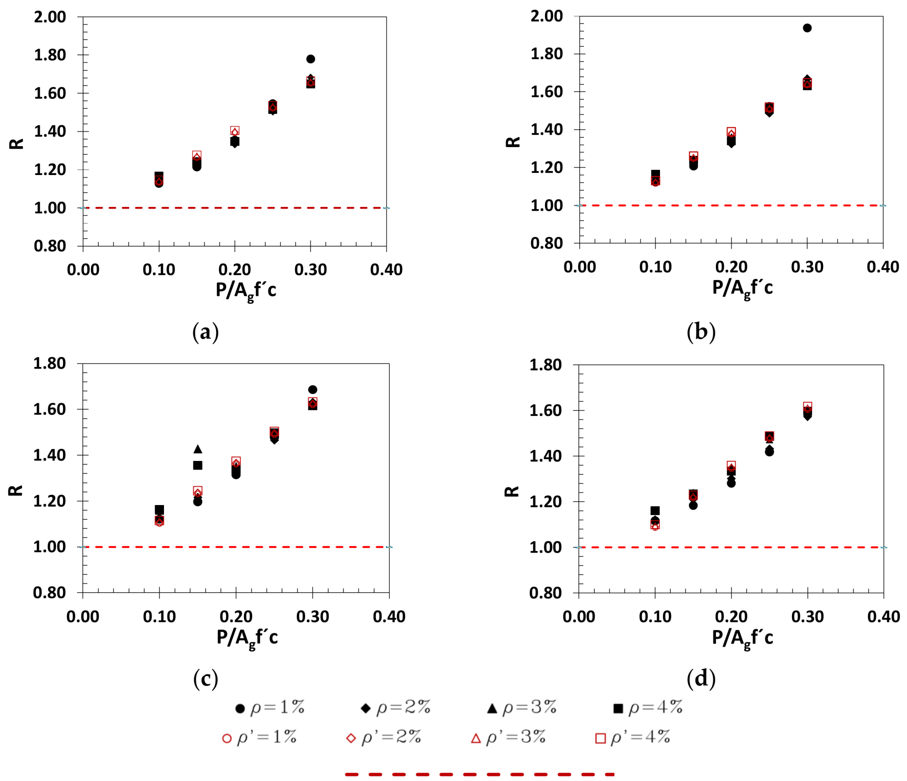


Figure 5. *R*-values comparison: (**a**) *L/D* = 3; (**b**) *L/D* = 5; (**c**) *L/D* = 7; (**d**) *L/D* = 9. [R values are shown by a red dotted line specified by CDS-MDOC [6], ρ' is estimated by Equation (7) in red tags and ρ is determined by CC in black labels].

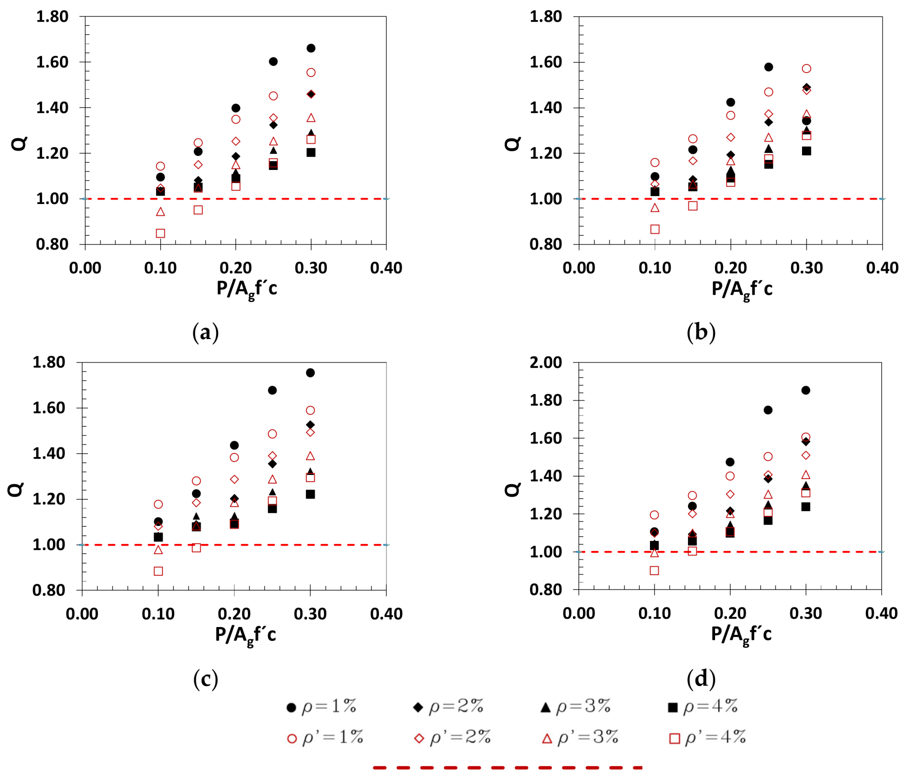


Figure 6. *Q*-values comparison: (**a**) *L/D* = 3; (**b**) *L/D* = 5; (**c**) *L/D* = 7; (**d**) *L/D* = 9. [*Q* values are shown by a red dotted line specified by CDS-MDOC [6], ρ' is estimated by Equation (8) in red tags and ρ is determined by CC in black labels].

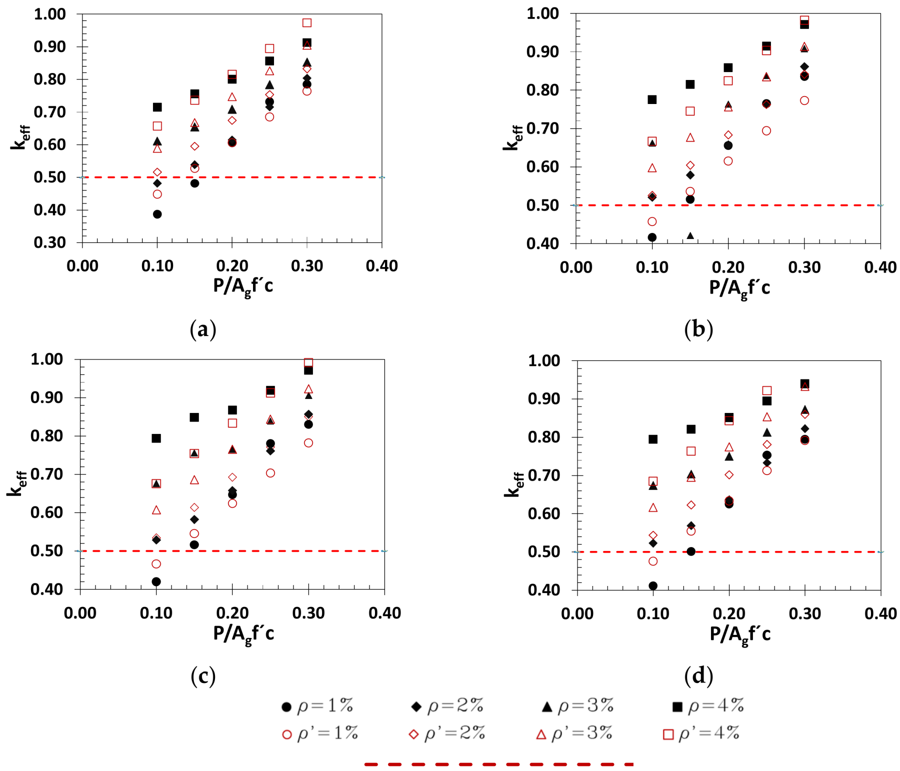


Figure 7. k_{eff} values comparison: (**a**) L/D = 3; (**b**) L/D = 5; (**c**) L/D = 7; (**d**) L/D = 9. [k_{eff} values are shown by a red dotted line specified by CDS-MDOC [6], ρ' is estimated by Equation (9) in red tags and ρ is determined by CC in black labels].

The R-values obtained from the CC were compared with those obtained from the proposed model (Figure 5). The mean R-values were suitably adjusted to those obtained from the CC, which is in agreement with the R^2 = 93.5% of this model. The R-value to be considered for the SLS is not explicitly specified in the CDS-MDOC [6]. However, as Q is unitary, it can be seen from Equation (6) that R is also unitary. The mean R-values were between 115% and 200% higher than the values recommended by CDS-MDOC [6].

The Q-values in the CC are compared to those in the model (Figure 6). The mean Q-values fit satisfactorily with those obtained from the CC. This consistency is supported by an R^2 of 83.7% of this model. Additionally, only when the values of $P/(A_g{\cdot}f'c)$ fell within the 0.10 to 0.15 range were the mean Q-values less than unity (up to −17%). In such cases, Q must be assigned a unit value because $Q \geq 1$. However, the average Q-values were higher than those from CDS-MDOC [6], ranging from 110% to 190%.

The k_{eff} values in the CC were compared to the model (see Figure 7). The mean k_{eff} values fit well with those obtained from the CC, and this consistency is supported by an R^2 value of 89.9% of the model. Figure 7 demonstrates that only when ρ = 1% and $P/(A_g{\cdot}f'c)$ = 0.1 and 0.15, and any values of L/D combination are used, were mean values for k_{eff} underestimated (up to −20%) compared to those considered by CDS-MDOC [6]. For other combinations, the mean k_{eff} values range from 20% (ρ = 2% and $P/(A_g{\cdot}f'c)$ = 0.15) to 100% (ρ = 4% and $P/(A_g{\cdot}f'c)$ = 0.30) higher than the considered value (k_{eff} = 0.5) Cantilever columns generally underestimate the k_{eff} (by up to −20%) or overestimate it (by up to 100%). In contrast, the proposed model estimated similar values to those obtained in CC (±20%).

In Figures 5–7, the trend of the values of R, Q, and k_{eff} obtained from the idealised CC are well predicted by the respective models. Therefore, in this section the influence of the independent variables, called $f'c$, L/D, $P/(A_g{\cdot}f'c)$, and ρ, on the variables R, Q, and k_{eff} is discussed. The coefficient of ρ in Equation (8) is positive, which means that more longitudinal reinforcement in the column will increase its R, holding other variables constant. Equation (8) shows a positive coefficient of $P/(A_g{\cdot}f'c)$, indicating that increasing only the axial load on a column will increase its *R*. Equation (8) shows a negative value, indicating that *R* will decrease if this column's parameter is increased. Equation (8) shows that the coefficient of $f'c$ is negative. Therefore, increasing the concrete's strength will decrease its *R*.

On the other hand, it can be observed that the coefficient of ρ in Equation (9) is negative, indicating that if only the amount of longitudinal reinforcement in the column is increased, its *Q* will decrease. Further to this, the positive coefficient of $P/(A_g \cdot f'c)$ in Equation (9) can also be observed, which means that if only the axial load in a column is increased, its *Q* will be increased. Additionally, the positive coefficient of *L/D* in Equation (9) implies that if only this parameter of the column is increased, its *Q* will increase. Moreover, the positive coefficient of $f'c$ in Equation (9) means that if only the strength of the concrete is increased, its *Q* will increase.

On the other hand, it can be seen from Equation (10) that the coefficient of ρ is positive, which means that if only the amount of longitudinal reinforcement in the column is increased, its k_{eff} will be increased. Similarly, the coefficient of $P/(A_g \cdot f'c)$ in Equation (10) is positive, which means that if only the axial load in a column is increased, its k_{eff} will be increased. Moreover, in Equation (10) it can be observed that the coefficient *L/D* is positive, indicating that if only this column parameter is increased, its k_{eff} will increase. Likewise, in Equation (10) it can be established that the coefficient of $f'c$ is negative, indicating that if only the strength of the concrete is increased, its k_{eff} will decrease.

The *R-factors* of the CC, the proposed models, and those specified by CDS-MDOC [6] and AASHTO [2] were calculated and compared (Figure 8). The *R-factors* values obtained from the proposed model fit satisfactorily with those obtained from the CC. In contrast, comparing the *R-factor* values obtained by the proposed models with those obtained by CDS-MDOC [6] shows a similarity (differences < 15%), only when the variables $P/(A_g \cdot f'c) = 0.10$ and $\rho = 3$ and 4% are combined. In the remaining instances, the *R-factor* of the proposed models exceeded that specified in the CDS-MDOC by 20% to 260% [6]. In the comparison between the proposed model and AASHTO [2], the *R-factors* were similar (differences < 10%) only when $P/(A_g \cdot f'c) = 0.25$ $\rho = 2$ and 3% were combined. For the other combinations, the proposed model showed differences between −50% and 35% with respect to AASHTO [2]. The *R-factors* of the proposed model became lower when $P/(A_g \cdot f'c) = 0.10$, 0.15, and 0.20. Combining $P/(A_g \cdot f'c) = 0.25$ and $\rho = 1$ and 3% resulted in higher proposed *R-factors.*

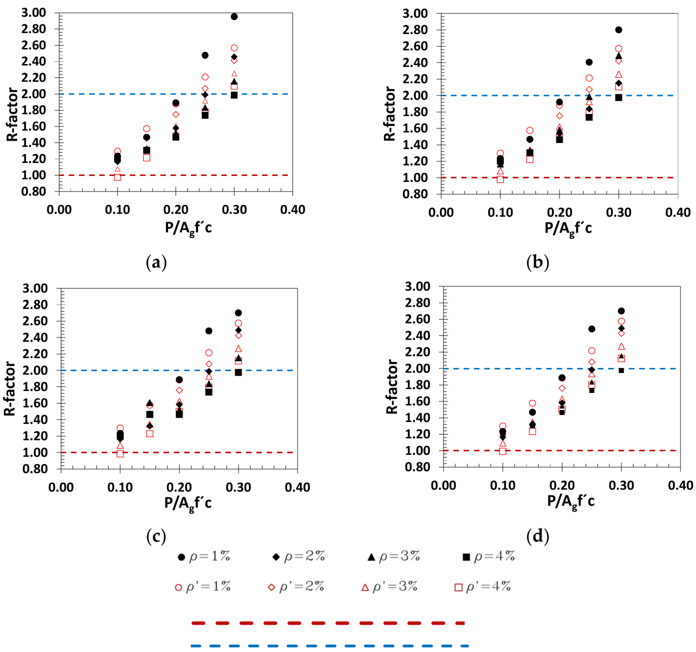


Figure 8. *R-Factors* values comparison: (**a**) *L/D* = 3; (**b**) *L/D* = 5; (**c**) *L/D* = 7; (**d**) *L/D* = 9. [R-Factors values are shown by a red dotted line specified by CDS-MDOC [6] and blue dotted line are given by AASHTO [2], ρ' are estimated by Equations (8) and (9) in red tags and ρ is determined by CC in black labels].

5. Design Method Using the Proposed Models

Herein, we provide a description of the proposed process for designing RC cantilever columns with a solid circular cross-section for seismic conditions. The process uses the proposed models to estimate *R*, *Q*, and k_{eff} factors. Only the procedure for the direction transverse to the longitudinal axis of the bridge and for the Serviceability Limit State (SLS) is described since the proposed models are only valid for these conditions. Furthermore, for simplicity, only the main steps of the procedure are shown, there are no code requirements and only the two main load combinations are considered.

1. To determine the dimensions of the bridge superstructure and estimate the maximum weight that each column will support, design the superstructure to carry the combination of "maximum gravity" loads (dead load plus maximum live load);

2. From the column length *(L)* already known, and its maximum aspect ratio *L/D*, specified in the code used, where *D* is the cross-section diameter of the column. Thus, calculate the minimum diameter (D_{min}), or propose a larger one according to another criterion. After that, the corresponding cross-sectional area, A_g, must also be calculated;

3. Design the column bents, the width must be equal to or greater than *D*;

4. Calculate the axial load (*P*) stand-in on the column as the sum of the maximum weight of the superstructure acting on it plus the weight of the bent;

5. Estimate the minimum $f'c$ of the concrete ($f'c_{min}$), from the maximum $P/A_g f'c$ ratio specified in the code used or propose a higher value according to other criteria. If $f'c_{min}$ is considered excessive, increase *D*, and repeat the procedure from step 2. Once $f'c$ has been determined, calculate the modulus of elasticity of the concrete (*Ec*) using the equation given in the applied code, which is frequently a function of $f'c$ and the weight of the concrete;

6. Carry out the flexural compression design of the column for the "maximum gravity" load combination to obtain a provisional value for the longitudinal reinforcement ratio (ρ). This ratio is called ρ_{grav} and must be within the range given by the minimum ρ (ρ_{min}), and the maximum ρ (ρ_{max}) specified by the used code. If $\rho_{grav} < \rho_{min}$, ρ_{min} must be set, if $\rho_{grav} > \rho_{max}$ it is an unacceptable value, and *D* must be increased and the procedure repeated from step 2;

7. Assess the weight of the superstructure contributing to each column from the gravity temporal loads involved in the combination of "gravity plus earthquake loads".

8. Calculate the axial load acting on the column, which is given by the sum of the previously estimated weight of the contributing superstructure plus the weight of the bent. This axial load is called $P_{temporal}$;

9. Determine the reactive mass of the column, m_r, considered to be concentrated at the center of gravity of the superstructure. m_r is the sum of $P_{temporal}/g$, where *g* is the acceleration of gravity, plus the weight of the upper half of the column divided by *g*;

10. With the values of $f'c$, *L/D*, $P_{temporal}/A_g f'c$ and a proposed value of ρ_{prop}, similar to ρ_{grav}, use the models to calculate the *R*, *Q*, and k_{eff} values of the column for seismic analysis;

11. Using the value of k_{eff}, calculate the effective moment of inertia, I_{eff}, of the column cross-section as follows $I_{eff} = k_{eff}\, I_g$ where I_g is the moment of inertia of the cross-section;

12. Develop the numerical model for the earthquake analysis using the values for m_r, $P_{temporal}$, I_{eff}, $f'c$, and *Ec* and the geometry and support conditions of the column;

13. From the elastic spectrum for the SLS specified by the applicable design codes, calculate the corresponding inelastic design spectrum by following the codes' recommendations for this purpose and the previously estimated *R* and *Q* values;

14. Carry out a spectral modal seismic analysis of the column to obtain the internal design forces: the axial load, P_u, and the bending moment, M_u;

15. Determine the required longitudinal reinforcement ratio (ρ_{req}) to resist the combined action of P_u and M_u. If $\rho_{req} < \rho_{min}$, ρ_{min} should be placed, and if $\rho_{req} > \rho_{max}$ then it is an unacceptable ratio, *D* should be increased, and the procedure from step 2 should be repeated;

16. Once ρ_{req} has been defined in step 15, it needs to be compared with ρ_{prop} in step 10. Therefore, ρ_{req} is considered acceptable for the combination of "gravity plus temporal"

loads if the relative difference between both is within an acceptable range, e.g., ± 2%. The amount of longitudinal reinforcement finally placed in the column is the greater of ρ_{req} and ρ_{grav}, the diameter for the longitudinal reinforcement bars and a bar arrangement that meets the design requirements, and the column bending design is completed. If the relative difference between ρ_{req} and ρ_{prop} is not within ± 2%, ρ_{req} is taken as the new ρ_{prop} and the procedure is repeated from step 10 onwards;

17. Once the flexo-compresive design of the column has been completed, the minimum transverse reinforcement ratio (ρ_{tMin}) is calculated. This is specified by the design code used. This transverse reinforcement ratio is determined by proposing a diameter for the transverse reinforcement bars and a bar arrangement that meets the design requirements. The transverse reinforcement ratio required (ρ_{tReq}) is then estimated to resist the shear force associated with the bending moment assessed from ρ_{req} and the expected longitudinal, transverse, and concrete strengths. The greater of ρ_{tMin} and ρ_{tReq} will be used. If the latter is greater, it may be necessary to propose a different diameter for the bars of the transverse reinforcement and/or a different arrangement of these bars that meets the design requirements. Therefore, the column design has been completed.

6. Conclusions

This research provides several conclusions:

1. The model proposes that the estimated values of R and Q can be up to 200% higher than those given by CDS-MDOC [6], depending on the combination of geometrical and mechanical properties of the RC columns. Therefore, it is expected that designing the columns in accordance with the proposed models will be more cost-effective for the SLS than designing them using CDS-MDOC [6];
2. k_{eff} considered by CDS-MDOC [6] is subestimated (up to −20%, when $P/(A_g \cdot f'c) = 0.1$ and 0.15) and overestimated (100% when $\rho = 4\%$ and $P/(A_g \cdot f'c) = 0.30$). On the other hand, the proposed model estimates values comparable to those obtained by CC (within ±20%). The CDS-MDOC [6] approach overestimates R values by up to 35% compared to the proposed model. Compared to the proposed models, the CDS-MDOC [6] approach underestimates the ductile capacity of lateral displacement of RC cantilever columns by up to 450%;
3. The main contribution of this research was to demonstrate the relation between R, Q, and k_{eff} (dependent variables) and $f'c$, L/D, $P/(A_g \cdot f'c)$, and ρ (independent variables). By doing so, the presented models evaluate the mean values of effective stiffness and seismic response modification. These models provide a practical recommendation for structural design in the serviceability limit state of cantilevered circular columns of RC bridges;
4. In this work, it was shown that the R-factor should not be considered constant, since, as can be seen in Figure 8, the CDS-MDOC [6] underestimates it for almost all the cases analysed, being most critical for high axial load ratios, reaching up to 260%. With respect to AASHTO [2], it is observed that it can be overestimated by up to 100% for low-axial-load ratios and underestimated by up to 40% for high-axial-load ratios;
5. Finally, current codes are conservative, giving values of R, Q, and k_{eff}; in contrast, the proposed models give more economical results with the recommended minimum safety levels.

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