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Review

A Review of Nonlinear Mechanisms for Frequency Up-Conversion in Energy Harvesting

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Abstract: Vibration-based energy harvesting has garnered considerable attention from researchers over the past two decades, using different transduction mechanisms. In this context, the utilization of piezoelectric materials has proven to be highly successful, due to their power density, across a broad range of voltages. A primary challenge in environmental vibration harvesting lies in the frequency mismatch between the devices, which typically exhibit optimal performance at hundreds or thousands of hertz due to their small size (centimeter or millimeter) and the environmental vibration. The latter has considerable energy density around tens of hertz. For this reason, over the last 15 years, the scientific community has concentrated on exploring techniques for band broadening or frequency up-conversion by intentionally introduced (or designed) nonlinearities. This review, following an introduction to the topic of vibration energy harvesting, provides a description of the primarily developed mechanisms, presenting a chronological development for each, from the initial works to the most recent advancements. Additionally, the review touches upon implementation efforts at the micro-electromechanical systems (MEMS) scale for each described technique. Finally, the incorporation of nonlinearities through electronic circuits to enhance performance is briefly discussed.

Keywords: vibration energy harvesting; piezoelectric materials; frequency up-conversion; nonlinear mechanisms

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1. Introduction

Over the past decade, considerable strides have been made in the advancement of ultra-low-power sensors, primarily within the realm of micro-electromechanical systems (MEMS) [1]. The term "Energy harvesting" denotes the process of capturing and converting wasted environmental energy into a usable form for human consumption, notably electricity. Energy-harvesting (EH) processes, by their nature, exhibit low levels of environmental pollution. Historically, humans have sought various means of energy recovery, such as through wind or water mills. Energy manifests in diverse forms in our surroundings such as solar, thermal, electromagnetic (in proximity to electrical circuits, radio or television signals), and, notably, mechanical energy in the form of vibrations.

This paper delves into vibration energy harvesting, also known as vibration-based energy harvesting (VEH), which is an area that has garnered significant attention from researchers over the past three decades. The primary aim of EH technology is to power small electronic devices, aiming to mitigate or eliminate the reliance on electrochemical batteries. This article will show that research in VEH extends beyond very small devices and encompasses objects at the mesoscale (typical dimensions in the centimeter range). Notably, due to advancements in low-power electronics, even minuscule devices capable of recovering power in the range of 10–100 microwatts are of keen interest to researchers [2].

The early work on vibration energy harvesting, particularly within microsystems, is credited to Williams and Yates in their 1996 paper, "Analysis of micro-electric generator for microsystems" [3]. Their prototype, employing bulk micromachining technology, converted energy through an electromagnetic transducer. However, it is important to note that the principle is applicable to other conversion mechanisms. Energy transduction in VEH primarily relies on three mechanisms: electromagnetic (inductive), electrostatic (capacitive), and piezoelectric. Electromagnetic transduction stems from the movement of an electrical conductor in a magnetic field, while the capacitive mechanism arises from the relative motions of two conductors separated by a dielectric material. The piezoelectric effect results from specific crystal structures exhibiting spontaneous charge separation under the right conditions, producing an electric dipole [4]. This effect allows these materials to deform physically in the presence of an electric field or, conversely, generate an electrical potential due to mechanical deformation.

These mechanisms are complemented by magnetostriction [5–7] and the utilization of electroactive polymers [8,9]. While all three transduction mechanisms have been extensively explored, piezoelectric transduction has received more attention over the past twenty-five years, as extensively described by Erturk and Inman [10]. The emphasis on piezoelectric transduction is underscored by the famous power density plot published by Cook-Chennault in 2008 [11] (see Figure 1). This plot highlights the electrical power generated per unit volume of the device versus the operating voltage. Notably, piezoelectric materials intersect with the operational domain of lithium-ion devices and exhibit a robust power output across a wide range of electrical voltages. Despite its vintage (2008), this plot retains its significance.

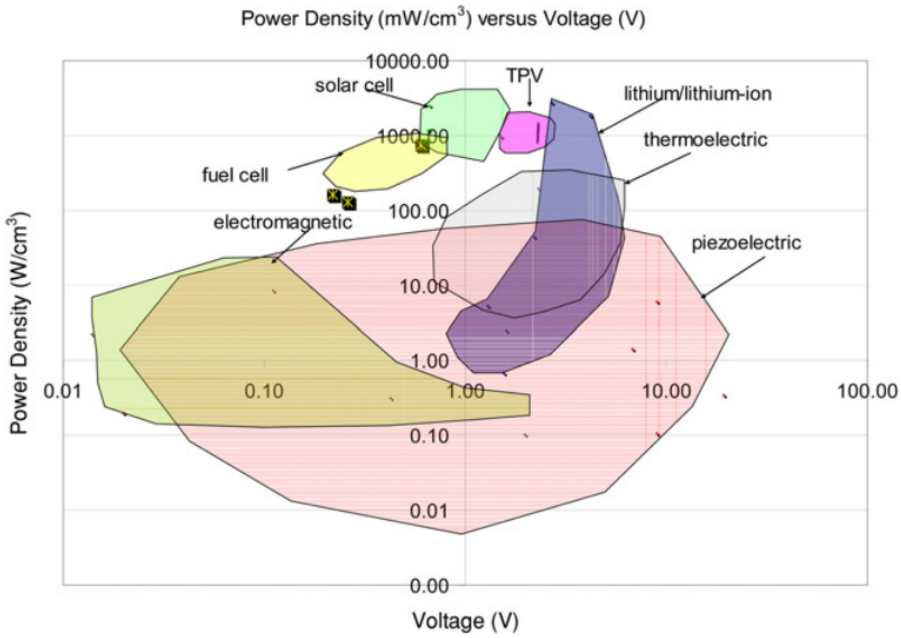

Figure 1. Plot of power density versus voltage for different power supply strategies, from [11] with the permission of IOP.

Piezoelectric generators inherently possess robust transduction capacity, maintaining efficiency even at small scales [12]. In contrast, electromagnetic solutions prove more intricate to implement, often involving complexities such as coil assembly in confined spaces within wafer-scale systems, and typically operate at low voltages (typically under 1V [13]). Electrostatic solutions, however, are comparatively less complex to realize, even at the MEMS scale [14–17]. Nevertheless, they necessitate an initial polarizing voltage or charge, as highlighted by Beeby et al. [13]. In practical applications where a generator charges the battery, this initial excitation level can be derived from the battery itself. Electret materials can be employed in electrostatic generators to hold an initial charge for extended periods. Despite their promise, electrostatic generators often exhibit high output impedance, rendering them less suitable as direct power supplies. Additionally, their relatively high output voltage (>100 V) can result in limited current supply, necessitating more intricate electronic circuitry for management. While piezoelectric solutions offer considerable promise, they are not without drawbacks. These crystals can deteriorate with age, and are notably brittle [18,19], prone to degradation [20], and susceptible to charge leakage [21,22].

2. Piezoelectric Vibration Energy Harvesting (PVEH)

A common structure for piezoelectric vibration energy harvesters (PVEH) involves a layered cantilever beam or plate. Typically, it comprises one or two layers of active (piezoelectric) material affixed onto a structural layer composed of semiconductor materials such as silicon or metal. When a single piezoelectric layer is employed, the harvester is referred to as a "unimorph" (see Figure 2, left), whereas the use of two piezoelectric layers characterizes it as a "bimorph" (see Figure 2, right). In the case of bimorph harvesters, these layers can be electrically connected either in series or in parallel.

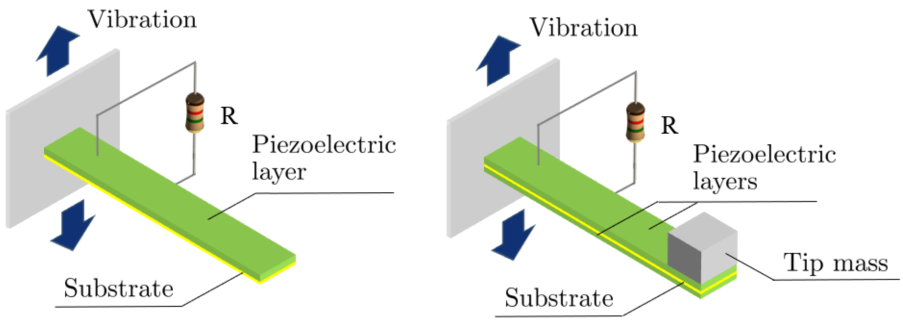

Figure 2. Schematic of typical piezoelectric cantilever energy harvester with resistive electric load. Left: monomorph cantilever without a tip mass, right: bimorph with the addition of a tip mass, as a possible alternative configuration, to match a specific external frequency.

As depicted in Figure 2, this structure illustrates the basic working principle: the clamp of the system, when excited by external acceleration (environmental vibrations), initiates vibrations within the structure. While the cantilever scheme remains a widely employed design, alternative configurations have gained traction, especially in various applications. For instance, Anton et al. integrated piezoelectric patches onto the wings of unmanned aerial vehicles [23–25], as well as in the biomedical field for in vivo monitoring of knee prostheses [26,27]. Given the multiphysics nature of the problem, the strain deformation energy induced by vibration can be converted into electrical energy using piezoelectric materials. An electrical circuit, illustrated as resistive in Figure 2, is connected to the structure. This system demonstrates effective energy scavenging, suitable for low-power electronics, particularly under conditions of high dynamic amplification, observed with harmonic forcing functions near the resonance condition, where the input frequency closely matches the harvester frequency. An added tip mass can be used to match a specific resonant frequency, as indicated in Figure 2 (right), with reference, for example, to the bimorph case. Another possibility for the frequency tuning is the use of electronic strategies [28]. However, the environment in which EHs operate predominantly holds energy distributed over frequencies within 100 Hz ([4,29–31]). When considering powering wearable electronics, human motion frequencies typically range between 1 and 5 Hz ([32–34]). As explained earlier, EHs must be minimally invasive and as small as possible, resulting in high natural frequencies (hundreds or thousands of hertz). Consequently, due to frequency mismatch, the harvester often operates in a static regime. To address this limitation, researchers are exploring strategies to widen the operational bandwidth of energy harvesters, seeking good amplification even at out-of-resonance conditions. This can be achieved through linear techniques or by leveraging nonlinear phenomena, an approach that has been emerging since the early 2000s ([35,36]).

Energy harvesters can be classified in numerous ways—by transduction mechanism, field of application, size, or operational frequency. Here, within the context of piezoelectric transduction, harvesters are categorized based on the governing physics: linear or nonlinear. This division aligns with the historical evolution of research in this field.

3. Nonlinear Systems

The advent of nonlinear dynamics in mechanical systems traces back to Huygens' investigations on the pendulum [37] and its applications in celestial mechanics. Subsequently, a wealth of studies emerged, focusing on mathematical modeling, computational methodologies, and the interpretation of complex phenomena.

In accordance with Lacarbonara and Rega [38,39] with the work (and the legacy) of Professor Ali H. Nayfeh, *"a new paradigm in higher education and engineering called nonlinear dynamics for design has begun in our time, which involves advantageously exploiting nonlinear phenomena and principles to improve the performance of engineering systems"*.

Over the past 15 years, remarkable strides have been achieved in the domain of vibration-based energy harvesting, capitalizing on various nonlinear phenomena or sources of nonlinearities, whether they stem from geometric or physical attributes in mechanical systems [40]. The subsequent discussion delves into key works and associated phenomena, with a primary focus on magnetic interaction, a central theme of this study. It is important to note that regardless of their origin, nonlinearities can be categorized as intentional, when deliberately introduced following the principle outlined above, to enhance the performance of the energy harvester as explained in detail in the review, or inherent, arising from internal sources within the operational regime of the mechanical system (e.g., softening [41–43]). These nonlinearities can intermingle, coexisting within the system. This review is focused on the classification of intentional mechanical nonlinearities, with only a brief mention given later to electronic nonlinearities. In the following, various intentional mechanisms to enhance performance are described in different subsections.

Furthermore, in Table 1, such mechanisms are classified and some example works are listed, with evidence of authors, year, frequency band in which the device was analyzed, and transduction principle. Indeed, although this review is more concerned with harvesters with piezoelectric materials, some interesting performance-enhancing mechanisms have also been implemented for devices with other types of transduction (e.g., electromagnetic or mixed).

3.1. Snap-Through Buckling (STB) Mechanisms

A promising solution to tackle the frequency match problem involves employing multistable systems utilizing snap-through buckling (STB) in mechanical structures. These systems, extensively studied for broader applications [44–49], function by having more than one stable configuration, exploiting impulsive excitation to transition between configurations. The advantage lies in their ability to activate the mechanism without requiring the external perturbation to match the frequency of the system. Instead, an energy threshold, often provided by external acceleration, induces migration between configurations based on total potential energy (TPE). This impulsive migration has been harnessed in various applications, such as generating impulses in cantilever clamps as made by Jung et al. [50], or more recently by Speciale et al. [51] (see the schematic in Figure 3), incorporating piezo patches directly onto bistable mechanisms [52–55], and even exploring electromagnetic transduction versions [56].

Researchers such as Masana and Daqaq further studied these systems under random vibrations, introducing axial loads to manipulate the potential function, achieving voltage outputs roughly ten times higher in the bistable case compared to the monostable [57–59]. Scaling down, Xu et al. implemented a MEMS-scale bistable piezoelectric beam in 2019 [60]. Others, such as Andò et al., used STB systems with a central proof mass to induce impacts on piezoelectric beams during configuration changes [61–63]. Arrieta et al. extended the STB concept to piezoelectric plates instead of beams [64] in 2010.

Table 1. Classification of nonlinear mechanisms and related examples in chronological order.

Mechanism	Authors	Year	Frequency or Velocity Range	Transduction
Snap-through buckling	Masana and Daqaq [57]	2011	10–100 Hz	Piezoelectric
	Cottone et al. [56]	2013	20–106 Hz	Electromagnetic
	Xu et al. [60]	2019	10–100 Hz	Piezoelectric
	Speciale et al. [51]	2020	20–30 Hz	Piezoelectric
Impact-based	Umeda et al. [65]	1996	N.A. *	Piezoelectric
	Alghisi et al. [66]	2015	10–100 Hz	Piezoelectric
	Halim et et al. [67]	2019	5.2 Hz	Piezo-electromagnetic
	Nastro et al. [68]	2022	around 4 Hz	Piezoelectric
Mechanical plucking	Pozzi and Zhu [69]	2012	0.1–1 Hz	Piezoelectric
	Janphuang et al. [70]	2015	3–19 Hz	Piezoelectric
	Kathpalia et al. [71]	2018	N.A. *	Piezoelectric
	Fu and Liao [72]	2019	0.1–2 m/s	Piezoelectric
Magnetic interaction broadband and plucking	Stanton et al. [73]	2009	10–16 Hz	Piezoelectric
	Erturk and Inman [74]	2011	5–7 Hz (broadband)	Piezoelectric
	Fu and Yeatman [75]	2019	1–11 Hz	Piezoelectric
	Lo et al. [76]	2021	5–15 Hz	Piezoelectric
	Miao et al. [77]	2022	0.5–5 Hz	Electromagnetic
Others	Leadenham and Erturk [78]	2014	10–20 Hz (broadband)	Piezo-electromagnetic
	Nie et al. [79]	2019	7.5–17.5 Hz	Piezoelectric
	Wang et al. [80]	2021	1–20 Hz	Piezoelectric
	Rosso et al. [32]	2022	1–6 Hz	Piezoelectric

* Not applicable.

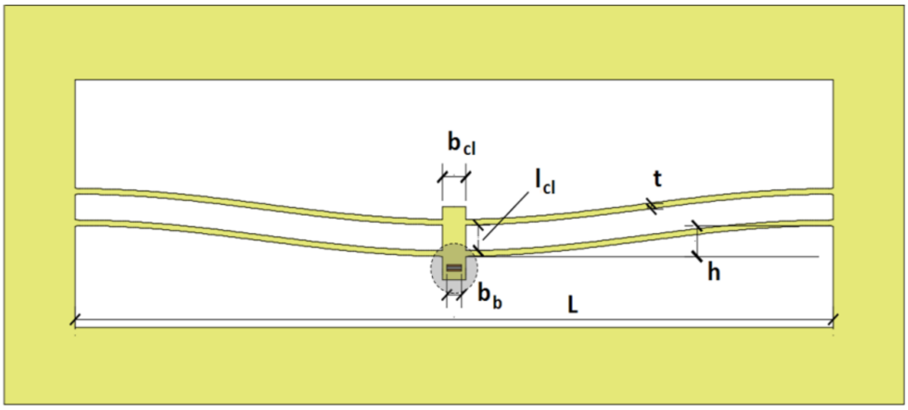

Figure 3. Schematic of the bistable mechanism design by Speciale et al. from [51].

3.2. Impact-Based Mechanisms

Frequency up-conversion (FuC) finds its roots in electronics and telecommunications as a process that translates the frequency energy spectrum of a signal without altering the distances between components or the bandwidth of the original signal [81,82]. Within this framework, a frequency converter is termed an up-converter if it outputs higher frequencies than the input and a down-converter in the opposite scenario. In applications such as aerospace and telecommunications, mixers, often nonlinear circuits, facilitate frequency conversion for AC power, motor control, and electrical systems with varying frequencies [83,84]. Over the last decade, the concept of FuC has been integrated into vibration energy harvesting. It aims to dynamically amplify high-frequency harvesting despite low-frequency input signals. Two primary methods have been explored: direct impacts on the harvester or indirect impacts on elastic supports interacting with the harvester, leading to piecewise linear systems [40]. Early studies, such as that of Umeda et al. [65] (1996), involved boundary-clamped piezoelectric plates stimulated by free-fall impacts, causing a nonlinearity in the system, allowing vibrations at different frequencies before and after impact (see Figure 4, left). Aceti et al. [85] (2023) analyzed the performance of a piezoelectric cantilever harvester by varying the position of a mass impacting the transducer and the tip mass value. The study revealed that the most effective configuration, in terms of power output, is an impact at the cantilever tip without a tip mass. Recent approaches have utilized indirect impacts to safeguard the piezoelectric material. Nastro et al. [68] (2022) developed a wearable energy harvester using indirect impacts via a wristband and watchband. Alghisi et al. [66] (2015) designed a triaxial impact-based energy harvester using piezoelectric disks, while others such as Ferrari et al. [86] (2012) and Gu et al. [87] (2011) explored similar techniques. Indirect impact techniques have been implemented in various ways. Ju and Ji [88] (2018) used vibrations from impacts of a metal sphere movement in a closed compartment to activate a piezoelectric harvester, as depicted in Figure 4 (right). Other researchers designed systems where impacts occur at the seismic or tip mass of the harvester [89,90]. Rosso et al. [32] proposed enhancing a cantilever piezoelectric energy harvester through indirect impacts on a moving mass and nonlinear magnetic interaction, demonstrating that magnetic interaction efficacy depends on motion energy levels. Additionally, techniques involving mixed transduction mechanisms have been explored. Halim et al. [67] (2019) developed a doubly clamped piezoelectric beam with central magnetic and iron masses, impacting a moving mass to activate vibration and collecting energy changes using a coil from the resulting magnetic field alterations.

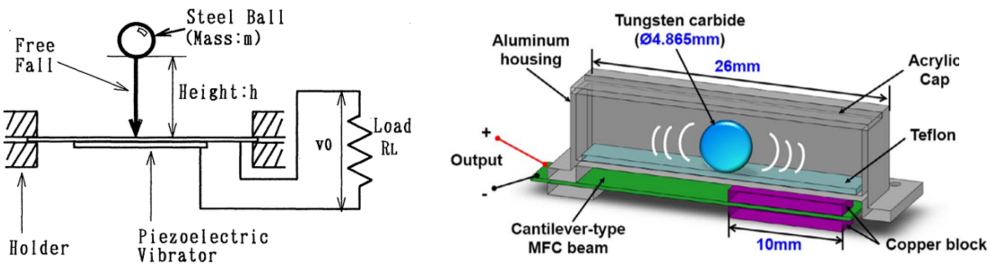

Figure 4. (**Left**): schematic from early investigations on PVEH from impacts by Umeda et al., from [65] with the permission of IOP. (**Right**): schematic of the prototype by Ju and Ji, from [88] with the permission of Elsevier.

3.3. Mechanical Plucking-Based Frequency Up-Conversion Mechanisms

This innovative approach involves inducing an initial quasi-static displacement via a flexible plectrum at the tip of the piezoelectric transducer, followed by a sudden release of this condition. The dynamic consequence leads to a free vibration regime in the piezoelectric beam, operating at its natural frequency and facilitating the recovery of electrical energy from voltage oscillations. Pioneered by Pozzi et al. in 2011 [69,91,92], the concept, termed the "pizzicato" harvester, introduced a knee joint wearable device (depicted in Figure 5) utilizing relative rotation between the shank and thigh to pluck piezoelectric bimorphs clamped on a hub via distributed plectra along an outer ring. Similarly, at Georgia Tech, Kathpalia et al. [71,93] developed a theoretical model and validated it experimentally using a system where a nonlinear flexible plectrum plucks the piezoelectric beam along a straight path. They also observed a correlation between the performance of the harvester and the geometry of the plectrum, as illustrated in their experimental setup (Figure 5, bottom).

Fu and Liao [72], in 2019, proposed a model for plucking based on Hertzian contact theory, demonstrating good agreement between the theory and experimental results. Similarly, mechanical plucking was successfully achieved at the MEMS scale by Janphuang et al. [70,94]. Their design involved a rotating gear plucking a MEMS cantilever using its teeth, driven by an oscillating mass. However, a significant drawback of these systems lies in the inherent brittleness of piezoelectric crystals, raising concerns about potential damage caused by mechanical contact, which could compromise the integrity of the harvester. To address this issue, Pozzi proposed a contactless version of the "pizzicato" approach [95].

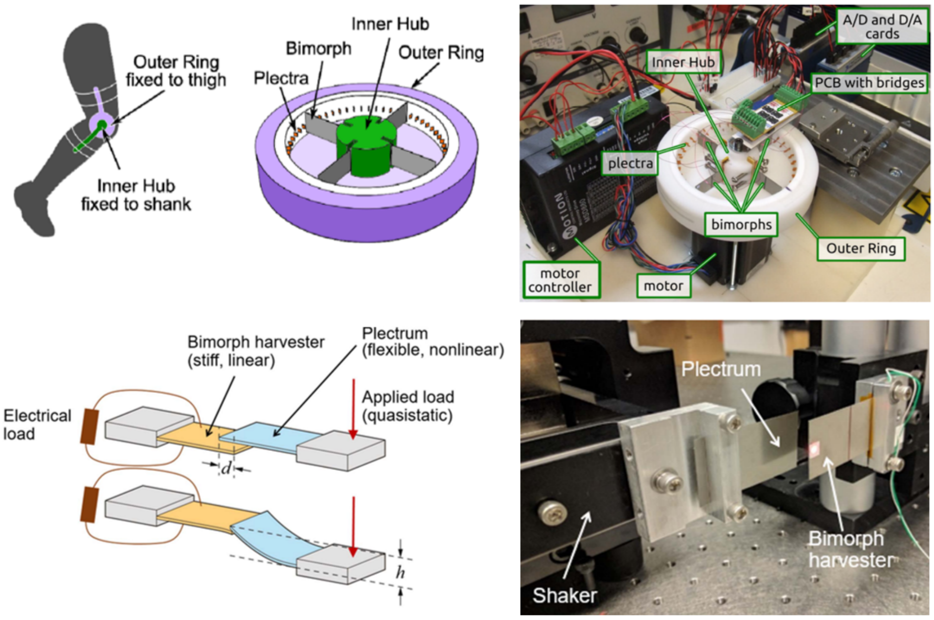

Figure 5. (**Top, from left to right**): schematic of pizzicato harvester and prototype by Pozzi et al., from [92] with the permission of IOP. (**Bottom, from left to right**): schematic mechanism of the plucking and experimental setup proposed by Kathpalia et al., from [71] with the permission of IOP.

3.4. Magnetic Interaction Broadband and Plucking Mechanisms

The strategy behind widening the bandwidth using magnetic interaction is to deliberately introduce nonlinearities for system potential manipulation. These systems can generally exist as monostable or multistable configurations. The magnetic interaction serves two primary purposes: enhancing the response of the system by increasing oscillation amplitudes beyond the resonance condition of the linear cantilever or plucking the piezoelectric cantilever, often engaging with additional systems such as oscillators or rotors. In the first case, as will be seen later, the magnetic force is provided by the interaction between a magnet attached to the tip of the harvester and magnets fixed to the same frame of the harvester. In this way, due to a vibration imposed on the frame, the same oscillations of the piezoelectric beam activate a magnetic restoring force that can amplify the dynamics. In the second case, on the other hand, the interaction of the magnet on the harvester tip occurs with magnets on a movable external system, which typically moves at externally driven velocities in the case of demonstrators (rotors) or is a low-frequency vibrating system [96]. If the interaction is sufficiently fast [42], the magnetic force acts as an impulsive load with a kind of plucking, activating the vibration frequency of the beam (which is why it is referred to as up-conversion). In 2009, Erturk et al. integrated Moon and Holmes' chaotic strange attractor concept [97,98] with piezoelectric layers, establishing the "piezomagnetoelastic energy harvester" [35,99]. The experimental setup, featuring a 145 mm long, 26 mm wide, and 0.26 mm thick ferromagnetic beam, exhibited a 200% increase in open circuit voltage compared to its linear counterpart. The ferromagnetic cantilever, in conjunction with magnets (Figure 6), induced multistability in the system, with the number of equilibrium points contingent on the spacing of the magnets. Erturk's design, utilizing three equilibrium points (two stable), facilitated the creation of the Duffing oscillator, offering greater amplitudes compared to the linear system, courtesy of the unique shape of the magnetoelastic potential. Subsequent studies within the same research group extensively investigated the dynamic response of this system [74,100,101]. Similarly, Cottone et al. [36], in 2009, proposed a device with a similar approach, emphasizing the significance of magnet spacing for the potential energy of the system.

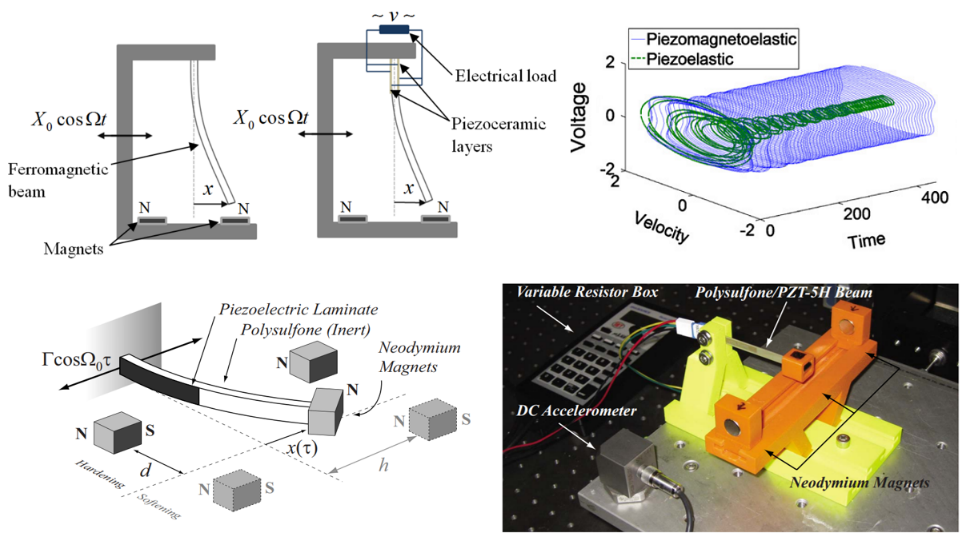

Figure 6. (**Top, from left to right**): schematic of the strange attractor studied by Moon and Holmes, and same concept with piezoelectric layers proposed by Erturk et al., from [35] with the permission of AIP. Comparison between linear and nonlinear response for harmonic excitation, from [74] with the permission of Elsevier. (**Bottom, from left to right**): schematic and prototype of the reversible hysteretic energy harvester by Stanton, from [73] with the permission of AIP.

In that same year, Stanton et al., at Duke University, introduced a method to induce softening or hardening responses by strategically tuning the distance between a tip magnet affixed to the harvester and a pair of magnets mounted on an elastic support [73]. This adjustment widens the bandwidth by incorporating hysteretic behavior, enhancing the performance of the system outside the resonance condition. These endeavors marked the onset of a productive era in piezoelectric vibration energy harvesting through magnetic interaction. Ferrari and colleagues [102,103] conducted a study in 2010 involving a ferromagnetic cantilever with a piezoelectric film, resembling Erturk's work, although with a single magnet positioned at the base. Subsequently, these systems have garnered significant attention, leading to extensive theoretical investigations and modeling [104–111]. Further explorations have delved into configurations with a higher number of stable states [112] and combinations with impact-based frequency up-conversion [113]. Recently, Tao-Hai et al. [114] (2022) scrutinized the performance of a bistable system under both wind-induced galloping and base excitation. Kim et al. [115] introduced an intriguing method to leverage bistability by placing two adjacent cantilevers, each equipped with a tip magnet, on the same elastic support. The interaction between these magnets under external vibration amplified the dynamics of both oscillators, resulting in a broader bandwidth and superior performance compared to a standard bistable system without magnetic coupling. Similar concepts have been explored by other researchers such as Koszewnki [116], Yang et al. [117] focusing on internal resonance, and Sun [118] analyzing galloping excitation. In 2020, Chandwani et al. [119] utilized a magnetic interaction scheme akin to Ferrari's work [102] to combine two distinct wide frequency operational bands, incorporating additional nonlinearities from a tip mass with a position-varying center of gravity through rolling cylinders. Notably, the literature showcases the successful implementation of multistable solutions at the MEMS scale. Wu et al. [120] proposed a MEMS energy harvester featuring magnetically coupled oscillators, generating 41.6 μW of power with an impedance of 0.3 MΩ. This device demonstrated impressive voltage values (in the order of volts) below 50 Hz. Other studies exploring coupled systems with a central spring-magnetic mass interacting with piezoelectric beams can be found in works such as [121,122]. The application of magnetic interaction extends to mixed energy conversion mechanisms. Li et al. [123] devised an energy harvester combining piezoelectric and electromagnetic transduction. This hybrid system utilizes magnetic force to amplify the vibration of a drive beam with an embedded magnet. The resulting spatial variation in magnetic flux, caused by the oscillation, is then captured using a coil around the magnet tip. Additionally, the drive beam's impact on a piezoelectric transducer induces impact-based frequency up-conversion. The concept of frequency up-conversion with magnets slightly differs from the previously detailed broadband magnetic mechanisms, as elucidated by Tang et al. [104]. Unlike the collective application of piezoelectric cantilevers and magnets experiencing base excitation, the frequency up-conversion mechanism involves a stationary piezoelectric cantilever excited solely by a magnetic force at its free end. This mechanism typically involves placing one magnet at the cantilever tip, while the other is secured to a "seismic mass", a mechanical system vibrating at significantly lower frequencies compared to the piezoelectric transducer. This arrangement has two common configurations in the literature: one with a mechanical system vibrating at low frequencies, and the other with a rotating mechanism, often observed in the literature. Tang et al. [124], a different researcher than that previously mentioned, introduced a system comprising two opposing piezoelectric cantilevers with a magnetic tip mass and a central compartment housing a moving magnet for external input acceleration. This prototype, built at a centimeter scale, generated an average power of over 10 µW within a broad frequency range of 10–22 Hz under 1 g acceleration. In a similar vein, Pillatsch et al., in [125,126], explored magnetic plucking mechanisms based on a rotating proof mass (see Figure 7, bottom left). Their studies, conducted in 2014, included extensive modeling, utilizing the electromechanical model of Erturk and Inman [10] for piezoelectric beams and employing both an inverse square method and finite element simulations for magnetic interaction. They demonstrated the viability of inverse square modeling to glean system behavior, despite its inability to match the precision of a full finite element simulation. Another work by Pillatsch et al. in 2012 saw them experiment with a prototype using the same principle, achieving a power output of 2.1 mW at an excitation frequency of 2 Hz and an acceleration of 2.72 m/s^2. Since these works, much attention has been directed towards rotational mechanisms in the implementation of harvesters. Ramezanpour et al. [127,128] utilized a rotating pendulum with a gear, observing increased output voltage with a higher number of magnets. Similarly, Xue and Roundy [129], in 2017, employed a concept akin to Pillatsch's design with a rotating proof mass externally driven to optimize the configuration of the magnetic poles for improved PVEH performance. They highlighted the predominance of the first mode of vibration for in-plane plucking, generating power in the mW range below an 8 Hz rotor frequency. Shu et al. [130], in 2018, studied a similar system, leveraging a dipole–dipole approach for modeling magnetic interaction and employing standard interface circuits. Additionally, Dauksevicius et al. [131], in the same year, delved extensively into the physical aspects of plucking dynamics, conducting finite element studies on the magnetic force's dependence on the gap size between magnets. This study identified three operation phases, denoted as "quasi-static phenomenon", "transition regime", and a "dynamic phase" dominated by the first mode. However, despite the focus on interaction velocity, there was no validation of the magnetic phenomenon or consideration of material nonlinearities resulting from changes in magnetic force. Kleiva and Dauksevicius also explored multimagnet excitation and devised a prototype for wearable applications. Various works have investigated similar mechanisms, such as those made by Fu and Yeatman [75] (see Figure 7, top), or others [132,133] with combinations of snap-through mechanisms [134]. More recently, Rosso et al., after conducting investigations on the magnetic interaction itself [96], studied the dynamics of the magnetic plucking by means of FFTs by also observing inherent material nonlinearities (piezo softening) in the plucking phenomenon [42,43,135]. Lo et al. [76], in 2021, introduced a magnetic plucking mechanism via a rotor in two distinct points of a piezoelectric cantilever simultaneously, activating two vibrational modes and achieving both frequency up-conversion and broadening (see Figure 7, bottom right). The magnetic plucking principle has also been applied in electromagnetic [77] and mixed piezo-magnetic transduction [136], leading to patented ideas as well [137,138]. Additionally, Bodduluri et al. [139] implemented magnetic plucking at the MEMS scale with an integrated process of the magnetic material deposition.

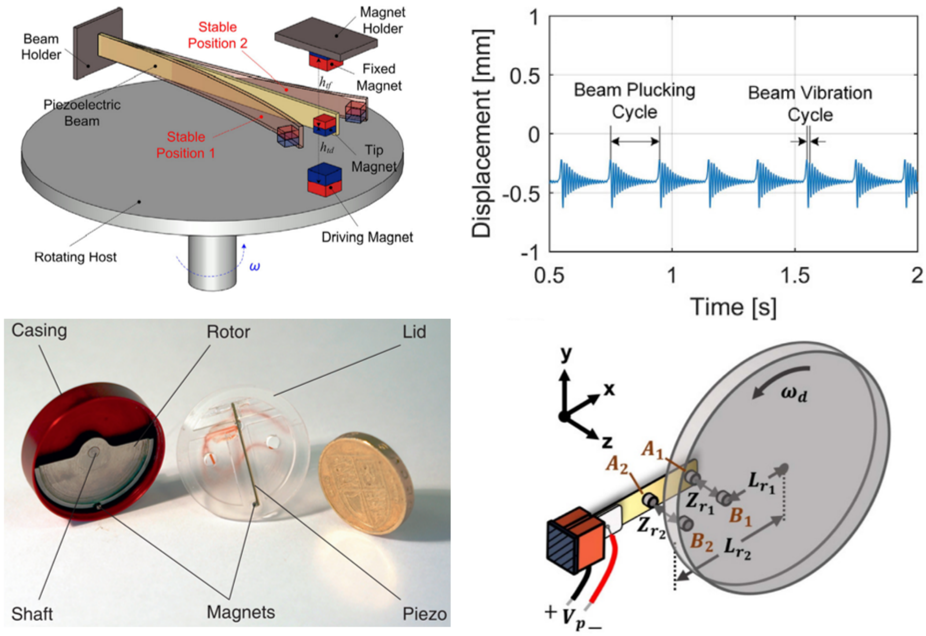

Figure 7. (**Top, from left to right**): schematic of the energy harvester using magnetic frequency up-conversion and example of beam tip displacement under the magnetic plucking, by Fu and Yeatman, from [75] with the permission of Elsevier. (**Bottom, from left to right**): prototype of energy harvester based on rotating proof mass designed by Pillatsch et al., from [125] with the permission of Elsevier. Schematic of the mixed modes energy harvester proposed by Lo et al. [76] with the permission of IOP.

3.5. Other Types of Nonlinearities

Many forms of intentional mechanical nonlinearities can be found in the literature, offering ways to enhance harvester performance. These are nonlinearities introduced through design features, distinct from intrinsic material nonlinearities that are extensively explored later. Some works focus on the nonlinear effect in device modeling. In 2014, Cammarano et al. [140] conducted an analytical study on a nonlinear oscillator employing cubic stiffness. Their study investigated various parameters, such as input acceleration, damping, and nonlinear stiffness, and their impact on the bandwidth of the system, offering optimal operational conditions. Zou et al. [141] (2021) introduced a nonlinear interaction force through a precompressed spring and a miniature bearing, inducing a load at the end of a cantilever piezoelectric beam. This method facilitated the creation of mono/bi/tri-stable energy harvesters based on the spring layout, allowing manipulation of desirable nonlinearities in the mechanical system. Other designs utilize pendulum systems [142], combined with magnetic interaction to create broadband systems [143,144]. Broadening the frequency band of vibrational energy harvesters is also achievable by taking advantage of systems that easily enter a nonlinear regime, as demonstrated by Hajati and Kim [145]. Gafforelli et al. exhibited nonlinear deformations by broadening the frequency band in a double clamped beam [146,147] (see Figure 8—bottom left). Wang et al. utilized the double clamped scheme to create a guided beam in a MEMS device [148]. However, a clamped–clamped beam without axial prestress faces limitations due to the relatively large axial stiffness, which restricts power generation by limiting maximum deflections. To overcome this, Leadenham and Erturk designed the M-shaped energy harvester (top of Figure 8), a prebent monostable beam with substantial stretching capabilities. They proposed purely piezoelectric and electromagnetic versions, and a mixed version [78,149]. Recent works have presented structural design features that facilitate entering the nonlinear regime [150–154]. The L-shaped beam axis design has been exploited in some works, including current ones, to realize modal interaction phenomena for broadening frequencies. These phenomena have been extensively studied in the past by Nayfeh, Mook, and collaborators [155–159]. Abdelkefi proposed coupling bending and torsional modes to broaden the band of piezoelectric energy harvesters [160] (bottom right of Figure 8). In 2015, Erturk and coworkers introduced an L-shaped beam-mass structure with quadratic nonlinearity, piezoelectric coupling, and an electrical load to explore primary resonance behaviors. This structure was aimed at bandwidth enhancement around the first and second linear natural frequencies [161]. Similar works exploiting the 1:2 internal resonance have been published by Nie et al. [79] (2019), and Li et al. [162] in 2021. These designs were coupled with magnetic interaction, as demonstrated by Wang et al. [80] (2021) who proposed an L-shaped beam actuated via rotating magnetic excitation on the free branch of the L-shape.

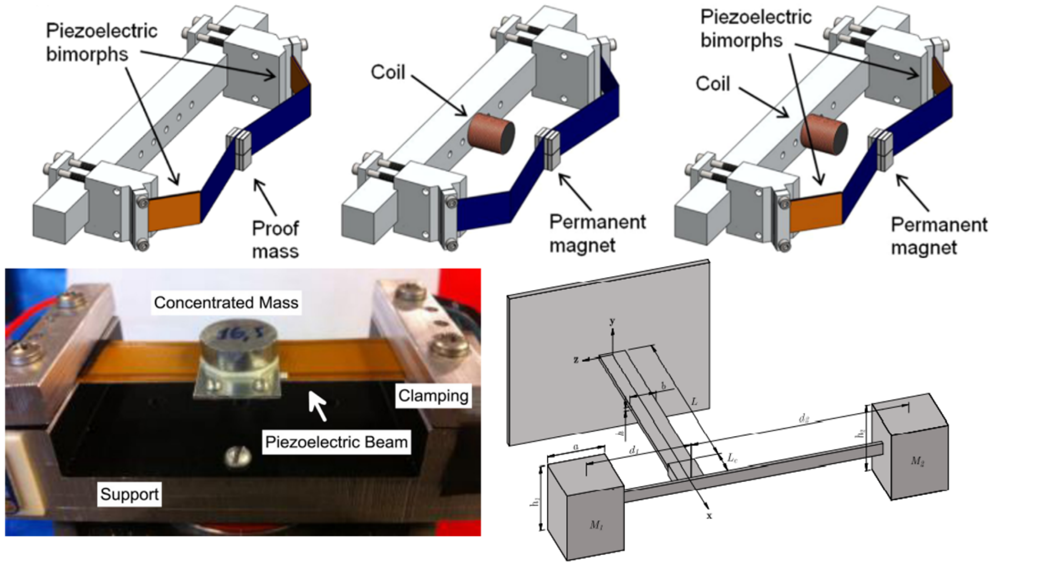

Figure 8. (**Top, from left to right**): schematic of the M-shaped energy harvester by Leadenham and Erturk [78] (with the permission of Elsevier) employing piezoelectric patches attached to the high strain energy regions, with permanent magnet and coil in the high kinetic energy region and combination of the previous two to realize a hybrid mechanism. (**Bottom, from left to right**): bridge-shaped energy harvester realized by Gafforelli et al., from [146] with the permission of AIP, and schematic of the bending–torsional vibration energy harvester designed by Abdelkefi et al., from [160] with the permission of Elsevier.

3.6. Nonlinearities through Electronics Circuit

This review paper primarily focuses on the mechanical aspects of PVEH, while the electrical facet is simplified, represented by a resistor, which is a common approach in analogous research [10,163]. However, various electronic circuits can be employed to manage the electrical energy harvested. Ottman et al. introduced an adaptive control technique for the dc-dc converter, continuously adjusting the optimal load [164]. They later presented an optimized method using a step-down dc-dc converter for energy harvesting from a piezoelectric beam [165]. Studies by the group of Guyomar proposed the "synchronized switch harvesting on inductor" (SSHI) nonlinear technique, which is efficient in nonresonant systems [166–168].

Silva et al. employed nonlinear switching circuits on the M-shaped energy harvester designed by Leadenham [78] in 2019. Leadenham and Erturk explored the nonlinear process of AC-DC conversion using nonideal diodes [169]. Bonnin et al. proposed an impedance matching network that amplifies the dynamic response of the harvester at a frequency chosen by the designer [170]. Yan et al. [171], and Yu and Zhou [172] used an inductor–resistor combination to broaden the frequency band. Semi-active control techniques have also been the subject of interest, both from a theoretical [173–175] and experimental point of view, and also very recently [176]. In the latter work, Pepe et al., (2023) developed an energy harvester for cars with a semi-active control technique, manipulating the resistor using an operational amplifier circuitry that can control the electrical impedance at any time. These studies showcase the extensive investigation into nonlinear electronics in energy-harvesting mechanisms.

4. Conclusions

As described, vibration energy harvesting technology with piezoelectric materials is a research field of intense and growing interest. The well-known problem is the frequency mismatch between devices and input ambient frequency. For this reason, current efforts are focused on the development of intentionally designed nonlinearities to have good performance out-of-resonance conditions. Our aim in this review article is to provide a concise summary of the main nonlinear mechanisms that have been explored over the past 15 years. Introduced nonlinearities may be due to structural design, as in the case of bistable arches and special shapes (e.g., L- or M-shaped), or to interactions between physical systems, as in the case of magnetic forces or impacts. Additional nonlinearities can be introduced by electronic circuitry. Possible future developments in the use of nonlinear techniques to improve the performance of harvesters involve combining multiple nonlinearities such as particular shapes with nonlinear interactions. A further development, more theoretical but important, could be to investigate in more detail the effect of designed nonlinearities on the actual structural behavior.

Often, harvesters are modeled with linear physics, but under certain mechanical or electrical conditions, this assumption may be wrong. Many of the techniques here presented are applied on piezoelectric devices, but the concepts can also be appropriately adapted to devices with hybrid or different transduction mechanisms (e.g., electromagnetic or electrostatic). Finally, it is our hope that this review article can be a useful reference for those present and future researchers focusing on the topic of frequency up-conversion techniques for energy harvesting from vibrations.

The exploration of vibration energy harvesting technology, particularly with piezoelectric materials, is a research field experiencing intense and growing interest. A well-recognized challenge in this domain is the frequency mismatch between devices and the ambient input frequency. Consequently, ongoing efforts are directed towards deliberately incorporating designed nonlinearities to ensure optimal performance under out-of-resonance conditions. In this review article, our objective is to provide a succinct overview of the primary nonlinear mechanisms that have been investigated over the past 15 years. Introduced nonlinearities can stem from structural design, exemplified by bistable arches and particular shapes (e.g., L- or M-shaped), or from interactions between physical systems, as observed in cases involving magnetic forces or impacts. Electronic circuitry also plays a role in introducing additional nonlinearities, and, historically speaking, they arrived later [40]. Future developments in nonlinear techniques to enhance harvester performance may involve the combination of multiple nonlinearities, such as specific shapes coupled with nonlinear interactions. Furthermore, a more theoretical yet crucial avenue for exploration could delve into a detailed examination of the impact of designed nonlinearities on the actual structural behavior. Despite harvesters often being modeled with linear physics, it is important to note that under certain mechanical or electrical conditions, this assumption may not hold true. While many of the techniques presented here are applied to piezoelectric devices, the underlying concepts can be suitably adapted to devices employing hybrid or different transduction mechanisms, such as electromagnetic or electrostatic. In conclusion, we aspire for this review article to serve as a valuable reference for both current and future researchers engaged in the exploration of frequency up-conversion techniques for energy harvesting from vibrations.

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