Supplementary material:

I. Quasi static analytical model

For an incompressible elastomer, the state of the membrane is fully described by λ_1, λ_2 and D_0, where D_0 is the nominal electric displacement. Then the free energy function can be written as $W = W(\lambda_1, \lambda_2, D_0)$. Partial derivatives of the free energy function give

$$s_1 = \frac{\partial W(\lambda_1,\lambda_2,D_0)}{\partial \lambda_1} \ , \qquad \text{(S1)}$$

$$s_2 = \frac{\partial W(\lambda_1,\lambda_2,D_0)}{\partial \lambda_2}, \qquad \text{(S2)}$$

$$E_0 = \frac{\partial W(\lambda_1,\lambda_2,D_0)}{\partial D_0}, \qquad \text{(S3)}$$

where s_1 and s_2 are the nominal radial and circumferential stress and E_0 is the nominal electric field.

The force balance of the external force F and the reaction force exerted by the deformed membrane gives

$$2\pi \frac{T}{\lambda_1 \lambda_2} r \sigma_1 \sin\theta = F, \qquad \text{(S4)}$$

where $\sigma_1 = \lambda_1 s_1$.

Considering the inner disk moves an infinitesimal distance δu, thermodynamic equilibrium states that the increase in free energy should be equal to the total work done by the external loads (neglecting the viscosity of the membrane for quasi-static actuation), that is

$$\int_A^B 2\pi \frac{T}{\lambda_1 \lambda_2} \delta W R dR = F\delta u + V\delta Q, \qquad \text{(S5)}$$

where δW is the change in free energy density of the membrane and δQ is the change in the charge on the electrodes.

The following relationship can be obtained from Equation (S5):

$$\frac{d(Rs_1 \cos\theta)}{dR} = s_2. \qquad \text{(S6)}$$

By combining Equation (S5) and (S6) one can get

$$\frac{d\theta}{dR} = -\frac{s_2}{Rs_1}\sin\theta. \qquad \text{(S7)}$$

To describe the free energy density W as well as s_1 and s_2, The Ogden model [31] was adopted in this work, which is given as

$$W = \sum_{n=1}^{N} \frac{\mu_n}{\alpha_n}\left(\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \frac{1}{\lambda_1^{\alpha_n}\lambda_2^{\alpha_n}} - 3\right) + \frac{1}{2\varepsilon_0\varepsilon_r}\frac{D_0^2}{\lambda_1^2\lambda_1^2}, \qquad \text{(S8)}$$

where μ and α are material parameters, N is the number of terms in this model, ε_r is the relative dielectric constant of the material and $\varepsilon_{r0} = 8.85 \times 10^{-12} F/m$ is the permittivity of free space.

Substituting Equation (S8) into (S1), (S2), (S3), we can obtain the expressions for s_1, s_2, E_0 as

$$s_1 = \sum_{n=1}^{N} \mu_n\left(\lambda_1^{\alpha_n-1} - \lambda_1^{-\alpha_n-1}\lambda_2^{-\alpha_n}\right) - \lambda_1\lambda_2^2\varepsilon_0\varepsilon_r(V/T)^2, \qquad \text{(S9)}$$

$$s_2 = \sum_{n=1}^{N} \mu_n\left(\lambda_2^{\alpha_n-1} - \lambda_1^{-\alpha_n}\lambda_2^{-a_n-1}\right) - \lambda_1^2\lambda_2\varepsilon_0\varepsilon_r(V/T)^2, \qquad \text{(S10)}$$

$$E_0 = V/T. \qquad \text{(S11)}$$

II. DEA fabrication and experimental setup

Experiments were conducted to validate the quasi-static model. The experimental setup is illustrated in Figure S1 where a conical DEA is mechanically deformed out-of-plane, a voltage is applied to the DEA membrane and the force on the membrane is measured. To fabricate the DEA, an off-the-shelf 40 µm thick silicone elastomer (Parker Hannifin. Co) was adopted. First, a circular piece of elastomer was pre-stretched by a stretch ratio of λ_p = 1.2 × 1.2. An acrylic ring with a radius of b = 20 mm was bonded to the membrane by silicone adhesive (Sil-Poxy, Smooth-On, Inc). Inner disk was bonded to the centre of the membrane with the same method. Carbon conductive grease (MG chemicals) was hand brushed onto the silicone elastomer. Two samples were prepared, one with a disk radius of a = 4 mm and the other has a radius of a = 6 mm. The conical DEA was fixed to the test rig and a linear actuator (MOTECH MOTOR, 170106) was used to deform the DEA from h = 0 to 10 mm with a low velocity of 0.06 mm/s to eliminate the effect of viscosity in the elastomer response. A load cell (TEDEA, No.1004) was utilized to measure the reaction force exerted by the DEA and a laser displacement sensor measured the displacement of the disk. An UltraVolt high voltage amplifier was used to apply voltage (V = 1500 V) across the membrane.

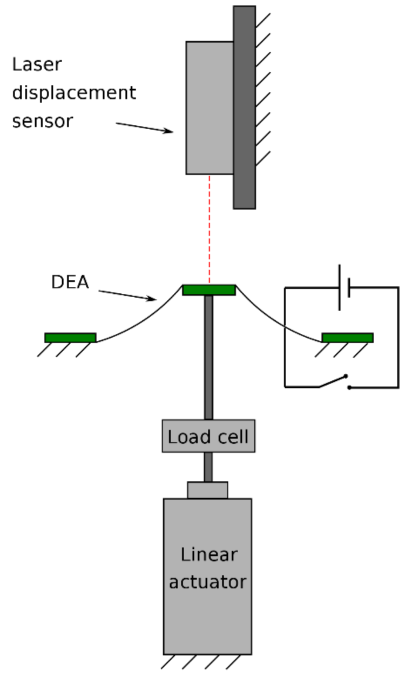

Figure S1. Schematic diagram of the model verification experiment setup. A conical DEA is mounted and deformed out-of-plane by a linear actuator, a load cell is used to measure the protrusion force and a laser displacement sensor is used to measure the displacement.

III. Optimization approach specification

To optimize the conical DEA, the characteristic DEA geometry, determined by the inner disk to outer ring radius ratio a/b, and the pre-stretch ratio have to be tuned. Three conical DEA configurations are considered: (I) biasing spring; (II) biasing mass; (III) antagonistic conical DEA. For each case, configuration-specific parameters can also affect its performance. For case I, the specific parameters are the compression spring stiffness and initial force caused by pre-deflection F_0. For case II, it is the weight of the mass Mg, and for case III, it is the length of the spacer L. In order to avoid overcomplicating the optimization with too many variables, in the following studies we choose to vary the general parameters that affect all three cases, which are the radius ratio a/b and pre-stretch ratio while leaving all configuration-specific parameters for each case fixed throughout the study. We also choose to keep the outer ring radius b constant at b = 20 mm while varying a from 2 mm to 10 mm (resulting a/b from 1/10 to 1/2) with an increment of 1 mm and pre-stretch ratio from 1 to 1.3 with an increment of 0.1. The following list will report the specific parameters chosen in this study for the three cases and the reason why these values are chosen.

- Case I: single cone DEA with a biasing compression spring

As have been shown by [19], the properties of the linear compression spring, namely the stiffness K and the initial force caused by pre-deflection F_0, can affect the maximum stroke output of a conical DEA. Specifically, a single cone DEA with a spring that has a lower stiffness can output a larger stroke, but the effect of pre-deflection is less clear. Hence in this case, we choose a spring stiffness K = 0.05 N/mm, as it was found to be one of the lowest stiffness springs available off-the-shelf. The initial force is set to be F_0 = 0.8 N, which will ensure a suitable out-of-plane deformation for most of the DEA samples to be studied.

- Case 2: single cone DEA with a biasing mass

In this case, a mass weight of 0.25 N is chosen to ensure a suitable out-of-plane deformation for most of the DEA samples.

- Case 3: antagonistic double cone DEA

L = 20 mm is chosen based on previous experience of the authors [16] [17] as it gives the best overall performance.

The stroke d generated by a DEA is related to the electric field applied to it, and to have a larger stroke, a higher electric field is desirable. The maximum electric field used in this study is chosen to be 80 V/μm, which is the dielectric strength reported by the manufacturer for this silicone elastomer [33]. Due to the inhomogeneous distribution of electric field on the conical DEA, the highest electric field occurs near the boundary between the membrane and central disk. In order to take the dielectric strength in to account in the optimization, the DEA is mechanically deformed out-of-plane with a constant actuation voltage applied (no biasing element is included at this stage). As the membrane is stretched more, the thickness becomes thinner which causes the electric field to increase (recall that $E = V/T$). Once the maximum electric field on the membrane exceeds the threshold of $E_{max} = 80\ V/\mu m$, we mark the state of (h, F) as the dielectric breakdown point for this DEA at this voltage V, where h is the displacement of the DEA membrane at dielectric breakdown and F is the force exerted by the membrane at dielectric breakdown. By applying different actuation voltages, different dielectric breakdown points can be obtained and finally a safe actuation boundary can be drawn. Figure S2 shows an example of this procedure. The maximum stroke that can be achieved from a specific conical DEA is simply the distance between the intersection of the protrusion force with the force exerted by the DEA when it is passive at point I and with the safe actuation boundary at point II (as illustrated in Figure S2). The useful work from this DEA W_{out} is simply the integration of the force difference between the protrusion force and F_{DEA} over point I to II, as is shown in the zoomed-in section in Figure S2. Another failure mode to be considered is mechanical failure, i.e. the membrane experiences a significantly large load thus the membrane ruptures. Here the threshold is set at $\lambda_1\lambda_2 = 2.4 \times 2.4$ as recommended by the manufacturer.

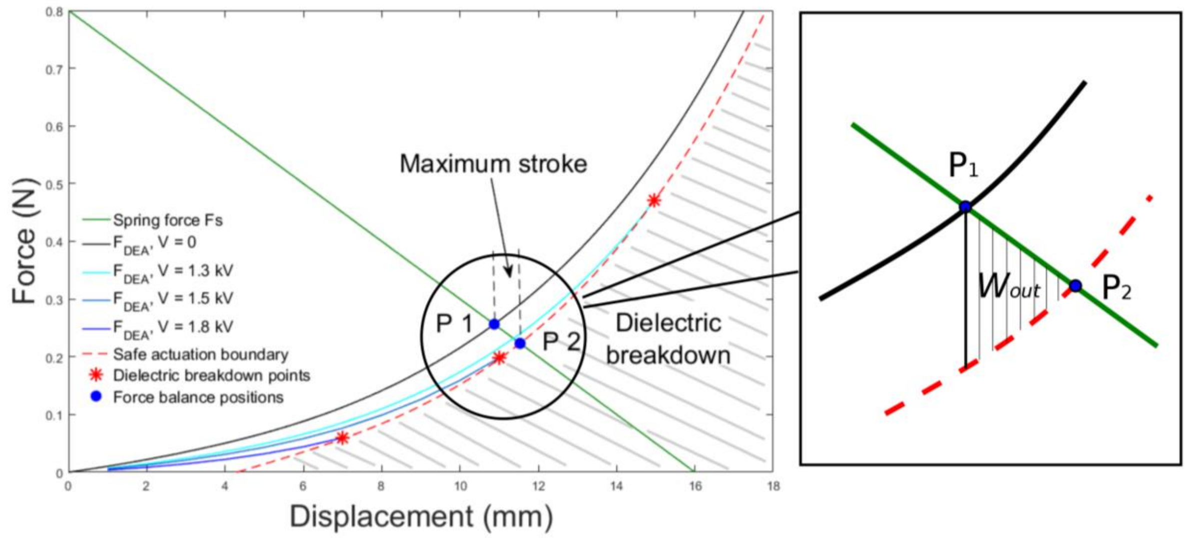

Figure S2. Example of the spring force and DEA exerted force as function of displacement. Red star points indicate the maximum electric field on the DEA exceeds the threshold E_{max} and can cause dielectric breakdown. The intersection between spring force line (green) and the boundary curve (red dash) formed by these breakdown points is the largest displacement of the DEA without causing dielectric breakdown. No mechanical failure occurs in this example.