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Correction

Correction: Bennati et al. An Elastic Interface Model for the Delamination of Bending-Extension Coupled Laminates. *Appl. Sci.* 2019, *9*, 3560

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check for updates

We, the authors, wish to make the following corrections to our paper [1].

Equations (26), (27), (28), and (41) were affected by some typos and should be substituted by the following ones (corrections are colored in red):

$$
\begin{aligned}
u_1(x) =& -(\mathsf{a}_1 + \mathsf{b}_1 h_1)\left[\frac{f_5}{\lambda_5^2}\exp(-\lambda_5 x) + \frac{f_6}{\lambda_6^2}\exp(-\lambda_6 x)\right] + \mathsf{b}_1 \sum_{i=1}^{4} \frac{f_i}{\lambda_i^3}\exp(-\lambda_i x) + \\
& - [(\mathsf{a}_1 + \mathsf{b}_1 h_1) f_7 + \mathsf{b}_1 f_{10}]\frac{x^2}{2} - (\mathsf{a}_1 f_8 + \mathsf{b}_1 f_{12})\, x + f_{14}, \\
w_1(x) =& \sum_{i=1}^{4}\left(\frac{\mathsf{d}_1}{\lambda_i^2} - \mathsf{c}_1\right)\frac{f_i}{\lambda_i^2}\exp(-\lambda_i x) - (\mathsf{d}_1 h_1 + \mathsf{b}_1)\left[\frac{f_5}{\lambda_5^3}\exp(-\lambda_5 x) + \frac{f_6}{\lambda_6^3}\exp(-\lambda_6 x)\right] + \\
& + [(\mathsf{d}_1 h_1 + \mathsf{b}_1) f_7 + \mathsf{d}_1 f_{10}]\frac{x^3}{6} + (\mathsf{b}_1 f_8 + \mathsf{d}_1 f_{12})\frac{x^2}{2} - (f_{16} + \mathsf{c}_1 f_{10})\, x + f_{15}, \quad \text{and} \\
\varphi_1(x) =& \mathsf{d}_1 \sum_{i=1}^{4} \frac{f_i}{\lambda_i^3}\exp(-\lambda_i x) - (\mathsf{d}_1 h_1 + \mathsf{b}_1)\left[\frac{f_5}{\lambda_5^2}\exp(-\lambda_5 x) + \frac{f_6}{\lambda_6^2}\exp(-\lambda_6 x)\right] + \\
& - [(\mathsf{d}_1 h_1 + \mathsf{b}_1) f_7 + \mathsf{d}_1 f_{10}]\frac{x^2}{2} - (\mathsf{b}_1 f_8 + \mathsf{d}_1 f_{12})\, x + f_{16},
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
u_2(x) =& (\mathsf{a}_2 - \mathsf{b}_2 h_2)\left[\frac{f_5}{\lambda_5^2}\exp(-\lambda_5 x) + \frac{f_6}{\lambda_6^2}\exp(-\lambda_6 x)\right] - \mathsf{b}_2 \sum_{i=1}^{4} \frac{f_i}{\lambda_i^3}\exp(-\lambda_i x) + \\
& + [(\mathsf{a}_2 - \mathsf{b}_2 h_2) f_7 + \mathsf{b}_2 f_{11}]\frac{x^2}{2} + (\mathsf{a}_2 f_9 + \mathsf{b}_2 f_{13})\, x + f_{17}, \\
w_2(x) =& -\sum_{i=1}^{4}\left(\frac{\mathsf{d}_2}{\lambda_i^2} - \mathsf{c}_2\right)\frac{f_i}{\lambda_i^2}\exp(-\lambda_i x) - (\mathsf{d}_2 h_2 - \mathsf{b}_2)\left[\frac{f_5}{\lambda_5^3}\exp(-\lambda_5 x) + \frac{f_6}{\lambda_6^3}\exp(-\lambda_6 x)\right] + \\
& + [(\mathsf{d}_2 h_2 - \mathsf{b}_2) f_7 - \mathsf{d}_2 f_{11}]\frac{x^3}{6} - (\mathsf{b}_2 f_9 + \mathsf{d}_2 f_{13})\frac{x^2}{2} - (f_{19} - \mathsf{c}_2 f_{11})\, x + f_{18}, \quad \text{and} \\
\varphi_2(x) =& -\mathsf{d}_2 \sum_{i=1}^{4} \frac{f_i}{\lambda_i^3}\exp(-\lambda_i x) - (\mathsf{d}_2 h_2 - \mathsf{b}_2)\left[\frac{f_5}{\lambda_5^2}\exp(-\lambda_5 x) + \frac{f_6}{\lambda_6^2}\exp(-\lambda_6 x)\right] + \\
& - [(\mathsf{d}_2 h_2 - \mathsf{b}_2) f_7 - \mathsf{d}_2 f_{11}]\frac{x^2}{2} + (\mathsf{b}_2 f_9 + \mathsf{d}_2 f_{13})\, x + f_{19},
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
f_{10} &= -\frac{\alpha_3}{B k_x \alpha_4} \mathsf{d}_2 f_7, \quad f_{11} = \frac{\alpha_3}{B k_x \alpha_4} \mathsf{d}_1 f_7, \\
f_{12} &= -\frac{(\mathsf{a}_1 + \mathsf{b}_1 h_1)\,\mathsf{d}_2 - (\mathsf{b}_2 - \mathsf{d}_2 h_2)\,\mathsf{b}_1}{\alpha_4} f_8 - \frac{\mathsf{a}_2 \mathsf{d}_2 - \mathsf{b}_2^2}{\alpha_4} f_9, \\
f_{13} &= \frac{\mathsf{a}_1 \mathsf{d}_1 - \mathsf{b}_1^2}{\alpha_4} f_8 + \frac{(\mathsf{a}_2 - \mathsf{b}_2 h_2)\,\mathsf{d}_1 - (\mathsf{b}_1 + \mathsf{d}_1 h_1)\,\mathsf{b}_2}{\alpha_4} f_9, \\
f_{14} &= f_{17} - (h_1 + h_2)\, f_{19} - \frac{1}{k_x}\left[\frac{1}{B} + \frac{\alpha_3}{B\alpha_4}(\mathsf{c}_1\mathsf{d}_2 - \mathsf{c}_2\mathsf{d}_1)\, h_1\right] f_7, \\
f_{15} &= f_{18}, \quad \text{and} \quad f_{16} = f_{19} + \frac{\alpha_3}{B k_x \alpha_4}(\mathsf{c}_1\mathsf{d}_2 - \mathsf{c}_2\mathsf{d}_1)\, f_7,
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
g_{10} &= \left(-\tfrac{\alpha_3}{B k_x \alpha_4}\mathsf{d}_2 + \beta_0 \tfrac{\mathsf{b}_2 - \mathsf{d}_2 h_2}{B\alpha_4}\right) g_7, \quad g_{11} = \left(\tfrac{\alpha_3}{B k_x \alpha_4}\mathsf{d}_1 - \beta_0 \tfrac{\mathsf{b}_1 + \mathsf{d}_1 h_1}{B\alpha_4}\right) g_7, \\
g_{12} &= -\tfrac{(\mathsf{a}_1 + \mathsf{b}_1 h_1)\mathsf{d}_2 - (\mathsf{b}_2 - \mathsf{d}_2 h_2)\mathsf{b}_1}{\alpha_4} g_8 - \tfrac{\mathsf{a}_2\mathsf{d}_2 - \mathsf{b}_2^2}{\alpha_4} g_9, \\
g_{13} &= \tfrac{\mathsf{a}_1\mathsf{d}_1 - \mathsf{b}_1^2}{\alpha_4} g_8 + \tfrac{(\mathsf{a}_2 - \mathsf{b}_2 h_2)\mathsf{d}_1 - (\mathsf{b}_1 + \mathsf{d}_1 h_1)\mathsf{b}_2}{\alpha_4} g_9, \\
g_{14} &= g_{17} - (h_1 + h_2)\, g_{19} - \left[\tfrac{1}{B k_x} + \tfrac{\alpha_3}{B k_x \alpha_4}(\mathsf{c}_1\mathsf{d}_2 - \mathsf{c}_2\mathsf{d}_1)\, h_1 + \beta_0 \tfrac{\mathsf{b}_1\mathsf{c}_2 - \mathsf{c}_1\mathsf{b}_2 + \mathsf{c}_1\mathsf{d}_2 h_2 + \mathsf{c}_2\mathsf{d}_1 h_1}{B\alpha_4} h_1\right] g_7, \\
g_{15} &= g_{18}, \quad \text{and} \quad g_{16} = g_{19} + \left[\tfrac{\alpha_3}{B k_x \alpha_4}(\mathsf{c}_1\mathsf{d}_2 - \mathsf{c}_2\mathsf{d}_1) + \beta_0 \tfrac{\mathsf{b}_1\mathsf{c}_2 - \mathsf{c}_1\mathsf{b}_2 + \mathsf{c}_1\mathsf{d}_2 h_2 + \mathsf{c}_2\mathsf{d}_1 h_1}{B\alpha_4}\right] g_7,
\end{aligned}
\tag{41}
$$

Furthermore, we observe that the constant terms in the shear stress expressions (18) and (32) (corresponding to Jourawski's solution for an unbroken beam) should not contribute to the Mode II energy release rate $\mathcal{G}_{\mathrm{II}}$. Thus, the peak values of the shear interfacial stress entering Equation (44) should be computed as $\tau_0 = \tau(0) - f_7/B$ and $\tau_0 = \tau(0) - g_7/B$ in the balanced and unbalanced cases, respectively. As a consequence, Equations (45) and (46) should be replaced by the following ones:

$$
\mathcal{G}_{\mathrm{I}} = \frac{\mathcal{H}(\sigma_0)}{2k_z B^2}\left(\sum_{i=1}^{4} f_i\right)^2 \quad \text{and} \quad \mathcal{G}_{\mathrm{II}} = \frac{1}{2k_x B^2}(f_5 + f_6)^2
\tag{45}
$$

and

$$
\mathcal{G}_{\mathrm{I}} = \frac{\mathcal{H}(\sigma_0)}{2k_z B^2}\left(\sum_{i=1}^{6} g_i\right)^2 \quad \text{and} \quad \mathcal{G}_{\mathrm{II}} = \frac{1}{2k_x B^2}\left(k_x \beta_0 \sum_{i=1}^{6} \frac{g_i}{\mu_i\,(\mu_i^2 - \alpha_3)}\right)^2 .
\tag{46}
$$

The corrections do not affect the results and scientific conclusions of the paper. We apologize for any inconvenience caused.

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References

1. Bennati, S.; Fisicaro, P.; Taglialegne, L.; Valvo, P.S. An Elastic Interface Model for the Delamination of Bending-Extension Coupled Laminates. *Appl. Sci.* **2019**, *9*, 3560. [CrossRef]