SUPPLEMENTARY MATERIAL of the Article:

Development and evaluation of a Fluctuating Plume Model for Odour Impact Assessment

The model developed, although apparently rather complex mainly for the complexity of its analytical expressions, is very simple in its implementation, quite similar to a normal Gaussian Plume model. It should be noted, however, that the equations obtained refer to a single point source emitting at a z_s height without buoyancy.

Even in the case of a single active source, the introduction in the model of a gradual plume elevation is not trivial. Roughly, one can operate as in the normal Gaussian plume models assuming that in the relations for the mean concentration and for the concentration variance simply replaces at z_s with $z_s + \Delta h(x)$ where $\Delta h(x)$ is the plume rise that varies downwind according to the relations normally used in Gaussian plume models [1].

The problem connected to the combined effect of the presence of several active sources, in practice the overlapping of effects, dealt with in a very limited number of experimental and theoretical works [2–6], has no clear solution to date. In this regard, consider the case of two sources located in distinct points of the spatial domain and a receptor point: the instantaneous C_1 and C_2 concentrations at the receptor, contributions from the two distinct sources, will have to be described, according to Fluidynamic, as two distinct stochastic processes given the stochastic nature of PBL. Moreover, both processes will be completely characterized by their respective probability density function (or by the infinite central moments) and the two probability densities (PDF) a priori may be different both for typology and for parameters that distinguish them. Moreover, it is certain that at the point in question they will give rise to an instantaneous concentration C_3 sum of the two. This derives, among other things, from the need to preserve the mass (it has been assumed, of course, that there are no removal processes and in particular chemical reactions). The sum of two distinct stochastic processes (even if *a priori* not independent) will also be a stochastic process, characterised by its own function of probability density, probably different from that of the two processes that generated it both in the type of distribution and in the parameters that characterise it. For generality, let's consider the fact that C_1 and C_2 are concentrations relative to the same chemical species and, moreover, let's not make any hypothesis on whether they are correlated or not. These instantaneous concentrations (C_1, C_2 and C_3) can be seen, for Reynolds' hypothesis, as the sum of an average value and a zero-mean fluctuation (c_1, c_2, c_3) that is how:

$$C_1 = \overline{C_1} + c_2 \qquad C_2 = \overline{C_2} + c_2 \qquad C_3 = \overline{C_3} + c_3 \qquad [A1]$$

We are, of course, interested in determining what the average value of C_3 can be, and this immediately descends from the given definitions and the hypothesis that there are no chemical reactions in place:

$$\overline{C_3} = \overline{C_1} + \overline{C_2} \qquad [A2]$$

Therefore, in the case of the average concentration, the effects overlapping used in the normal models for Air Pollution is obviously correct. But we are also interested in the knowledge of the variance σ^2_3 defined as:

$$\sigma_3^2 = \overline{(C_3 - \overline{C})^2} = \overline{C_3^2} - \overline{C}^2 = \overline{c_3^2} \qquad \text{[A3a]}$$

Remembering the general properties of a stochastic variable and the definition of mean value and fluctuation derives from Reynolds' Hypothesis, it is easy to show that:

$$\sigma_3^2 = \sigma_1^2 + \sigma_2^2 + 2 \cdot Cov(C_1, C_2) \qquad \text{[A3b]}$$

Where the covariance between C_1 and C_2 is defined as:

$$Cov(C_1, C_2) = \overline{(C_1 - c_1)(C_2 - c_2)} = \rho \cdot \sigma_1^2 \sigma_2^2 \qquad \text{[A3c]}$$

where ρ is the correlation coefficient. How ρ varies according to the downwind distance x, the crosswind distance separating the two sources is not clear even in light of the few measurements available. What seems to emerge is that it is zero at very short downwind distances (in fact the plumes are independent of each other), while it seems positive at long distances from the emission points. The theoretical work of Sawford (1985) partly clarifies the question by pointing out how at long distances it actually tends to a positive but not zero value, while at intermediate distances there can be negative values. All this derives from the complex interaction of plumes with turbulent eddies. Having no other information about it, we can simply put $\rho = 0$ and, therefore, consider valid the overlapping of the effects not only for the average value but also for the variance. Alternatively, taking as reference Fig. 7 of Sawford (1985)'s work, for nearby sources the following semi-empirical relation could be used:

$$\rho(x) = 0.7 \cdot \left(1 - exp\left[-\frac{\tau}{a}\right]\right) \rho(x) = 0.7 \cdot \left(1 - exp\left[-\frac{\tau}{a}\right]\right) \qquad \text{[A4]}$$

where $a = 0.05$ e $\tau = x/(U \cdot T_{Lv})$ or on could assume complete mixing of two plumes with $\rho = 1$.

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