

Article

Intelligent Collision Avoidance Method for Ships Based on COLREGs and Improved Velocity Obstacle Algorithm

Xingya Zhao ^{1,2}, Yixiong He ^{1,2}, Liwen Huang ^{1,2}, Junmin Mou ^{1,2}, Ke Zhang ^{1,2,*}  and Xiao Liu ^{1,2,*}

¹ School of Navigation, Wuhan University of Technology, Wuhan 430063, China

² Hubei Key Laboratory of Inland Shipping Technology, Wuhan 430063, China

* Correspondence: kezhang_rq@whut.edu.cn (K.Z.); lxiao@whut.edu.cn (X.L.)

Abstract: Collision prevention is critical for navigational safety at sea, which has developed rapidly in the past decade and attracted a lot of attention. In this article, an improved velocity obstacle (IVO) algorithm for intelligent collision avoidance of ocean-going ships is proposed in various operating conditions, taking into account both a ship's manoeuvrability and Convention on the International Regulations for Preventing Collisions at Sea (COLREGs). An integrated model combines a three-degree-of-freedom manoeuvring model with ship propeller characteristics to provide a precise prediction of ships in various manoeuvring circumstances. In the given case, what is different to present studies, this improved algorithm allows for decision-making in two ways: altering course and changing speed. The proposed technique is demonstrated in a variety of scenarios through simulation. The findings reveal that collision-avoidance decision-making can intelligently avoid collisions with the target ships (TSs) in multi-ship situations.



Citation: Zhao, X.; He, Y.; Huang, L.; Mou, J.; Zhang, K.; Liu, X. Intelligent Collision Avoidance Method for Ships Based on COLREGs and Improved Velocity Obstacle Algorithm. *Appl. Sci.* **2022**, *12*, 8926. <https://doi.org/10.3390/app12188926>

Academic Editors: Dimitrios S. Paraforos, Giovanni Randazzo, Anselme Muzirafuti and Stefania Lanza

Received: 20 July 2022

Accepted: 26 August 2022

Published: 6 September 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

Keywords: intelligent collision avoidance; improved velocity obstacle; COLREGs; variable speed; improved MMG

1. Introduction

Maritime transactions, due to the large capacity and low cost, which play a vital part in the global economy have become a larger and more important means of international and domestic transportation. Marine accidents, on the other hand, always represent a significant risk to societies and the environment in terms of human life and property loss, environmental damage, and transportation safety [1]. Furthermore, among all sorts of Marine catastrophes, ship collisions as one of the key contributors would have a negative and unanticipated influence on the stakeholders' reputation and property [2].

According to studies, human error accounts for more than 80% of ship collisions [3], while violations of International Regulations for Preventing Collisions (COLREGs) by mariners account for more than 56% of major marine accidents [4]. Human error is frequently caused by crews failing to observe COLREGs and good seamanship, such as failing to take steps timely, wrong avoidance action, insufficient risk assessment, and failing to continue at a safe pace [5]. On the other hand, the COLREGs guidelines do not provide precise operational instructions [6]. How to assist or perhaps replace the officer of the watch (OOV) in making decisions to avoid collision is an intriguing subject that more and more researchers are addressing. Many techniques have been presented in recent years to handle this subject, and in 2018, the International Maritime Organization (IMO) formulate the standard of the degree level for Maritime Autonomous Surface Ship (MASS). Although the MASS concept has been introduced in recent years, there is still a significant gap in autonomous navigation. Due to the limitations of present ship navigation equipment and shortcoming of the algorithm, it is not feasible to completely replace human operators with machines in a short period [7]. Thus, the decision-making for ship-intelligent collision avoidance is a complex process, especially in the multi-ship situation. At

present, much research and technologies have been developed for the intelligent collision avoidance problem.

For decades, the geometric analysis approach, which includes the Distance to the Closest Point of Approaching (DCPA) and the Time to Closest Point of Approaching (TCPA), has advised OOWs and ship captains to conduct effective collision-prevention procedures in sea practice [8]. Then, in the early 1970s the ship domain which refers to the area around a ship that prevents other ships from entering to maintain navigational safety, was introduced. Many researchers have presented multiple definitions of ship domain over the years, with varied forms and sizes [9,10]. Some researchers evaluated the ship domain of the target ship (TS) [10], while others consider the ship domain of their ship [11]. The velocity obstacle algorithm was presented to give greater support for the OOW in collision prevention [12], which described a set of velocities to determine whether their own velocity fell into the velocity obstacle zone, hence reducing the danger of collision. As a result, a growing number of researchers were focusing on the use of velocity obstacle (VO) algorithms for collision avoidance in a variety of settings, including confined waters [13], and considering rules [14].

Furthermore, nonlinear VO based on ship motion was proposed in [15], and Chen [16] adopted an improved time-discretized non-linear velocity obstacle technique for multi-ship encounter detection. With the advancement of computer technology, the study of vessel collision avoidance has progressed to a new level, and deep reinforcement learning based on COLREGs has been proposed for use in multi-ship collision avoidance [17]. Simultaneously, genetic algorithms for ship collision avoidance and path planning were made public [18,19]. Ni [20], established a ship manoeuvrability-based simulation for ship navigation in collisions, and Wang [21], presented a ship manoeuvrability-based collision avoidance dynamic support system in close-quarters situations. Zhang adopted linear extension algorithms for a multi-ship anti-collision decision in the manner of course alternation and speed reduction [22]. To decide the path planning for autonomous ships, Zhang used a modified artificial potential field and a modified velocity obstacle algorithm method [23]. Singh suggested a system that combines offline optimum path planning with a constrained A* algorithm which employs an artificial potential field to increase online adaptive weighting based on USV manoeuvring reaction time [24,25]. For the study of collision avoidance in the multi-ship situation, many algorithms have been applied in recent years. The paper [26], proposes a unique maximum-course and minimum-speed change approach for decision-making on the basis of a novel Fuzzy-logic algorithm. Furthermore, VO algorithms are provided for collision avoidance based on the target ships whose motions are non-linear and probabilistically predictable [27]. Moreover, the article [28], presents a generalised VO algorithm for ship collision avoidance and designs a collision, which is more reliable and suitable for close-range ship collision avoidance. Considering the ship maneuverability, multi-ship, COLREGs, off-course, and seamanship, Wang [29], designed the multistage optimisation decision model based on the modified VO method.

Despite a lot of research work and achievements being completed on ship collision avoidance, there are still some unignorable shortcomings in the available studies. Most of the current research on decision-making for collision avoidance in the multi-ship situation considers the ship motion model insufficiently, especially in the process of the changing speed of ship motion. Little research has been focused on the model in which the ship's speed and rotation of the propeller are not matched. Moreover, the decision-making of ship-intelligent collision avoidance should be expected to follow COLREGs and good seamanship, which are guidelines for all vessels. Although humans are good at interpreting navigational regulation, negligence will lead to marine accidents. Therefore, a model combining the advantages of human intelligence and machine intelligence is necessary for ship collision avoidance. The advantages of machine intelligence in computing power are extremely complementary to human intelligence. In other words, this model will not only provide real-time decision-making for collision avoidance but will also take into account the COLREGs and good seamanship.

Considering the characteristics of ship movement, an accurate manoeuvring model is required to deduce future ship motion trends and states for collision avoidance, especially when large ships are involved in adjustable speed operations.

In conclusion, this paper aims to design an intelligent collision avoidance model for ships, which combines the COLREGs and good seamanship mentioned above, as well as ship manoeuvrability, based on the advantages of both machine intelligence and human intelligence. Furthermore, the primary research topic for this project is: how can the ship combine machine intelligence and human intelligence to make intelligent collision avoidance decisions? The main contributions of this article are as follows:

1. This paper provides the improved mathematical model group (MMG) based on ship manoeuvrability to deduce the process of own ship (OS) motion, which is also appropriate for the situation when a ship's speed and rotation of propeller are not matched.
2. An integrated algorithm for collision avoidance decision-making based on improved VO algorithm and MMG is presented, which comprehensively considers COLREGS, sea practice, and good seamanship.
3. The decision-making of collision avoidance following the manoeuvrability habit of manned ships is provided by combining a dynamic feasible manoeuvring decision interval and an optimisation evaluation function, which will gap reduce the difference of awareness about manoeuvrability in the process of collision avoidance between the intelligent and manned ship.

The contents of the research are arranged as follows: The quantitative analysis of COLREGs, improved MMG, and improved Velocity Obstacle (IVO) algorithm are covered in Section 2. The intelligent collision avoidance method based on the collision risk index (CRI) model is covered in Section 3, which introduces dynamically feasible manoeuvring intervals calculated by IVO algorithm and the collision decision determined by optimisation functions. The case study in Section 4 demonstrates the accuracy of MMG and the practicality of the proposed strategy. In Section 5, there is an analysis and discussion on future research. Finally, in Section 6, the conclusions are outlined.

2. Methodology

2.1. Methodological Overview of the Research

In this research, the IVO algorithm based on the improved MMG and COLREGs is presented to obtain the collision avoidance decision. Given that COLREGs are the guidelines for ship collisions, quantitative analysis of the rules is required for intelligent collision avoidance, which is described in Section 2.2. Considering the definition of collision avoidance, the problem is divided into two sub-problems: "collision detection" and "collision resolution". "Collision detection" utilises the collision risk index (CRI) model to establish the avoidance timing and avoidance sequence for the TS in the multi-ship circumstance, which is illustrated in Section 3.1. The solution to the collision problem is to obtain the collision avoidance decision-making by the improved MMG illustrated in Section 2.3 and IVO illustrated in Section 2.4, and the details are elaborated in Section 3. The framework of the methodology can be found in Figure 1.

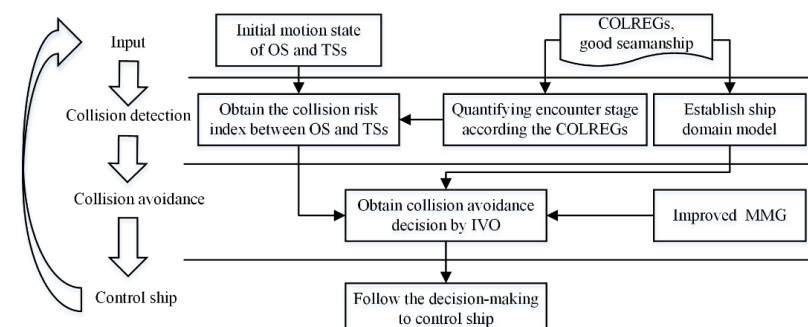


Figure 1. The flowchart of the research methodology.

2.2. Quantitative Analyses of COLREGs

COLREGs are guidelines for avoiding ship collisions, and every decision made by the OOW in the face of approaching and outgoing ships is governed by rules permitted. Therefore, understanding and following the rules is the key precondition for preventing ship collisions. Although the rules outline some provisions and divisions of the ship's movements, there is no explanation of the exact decision-making for avoidance actions. The quantitative analysis of the collision avoidance rules must take precedence, and the ship's actions should conform to the rules' constraints. Therefore, for the decision-making of intelligent collision avoidance, the construction of strategies for quantitative analysis of the rules is also essential.

Much research was presented about the quantitative analysis of the rules, including fruitful research fields like the ship domain, encounter situation, narrow channel, and so on. This paper focuses on two main components: One is determining the meeting stage of various encounter scenarios, and the other is the adaptation of a ship domain model to one's vessel. Sections 2.2.1 and 2.2.2, respectively, detail the four-stage theory and ship domain by quantitative analyses of rules.

2.2.1. Four-Stage Theory

According to COLREGs, two power-driven vessels in a hazardous conflict approach one another in four stages if they maintain their speed and course [30]. The four-stage theory is established to expound on the timing to avoid the collision, which involves "no risk of collision (No CR)", "risk of collision (CR)", "close-quarter situation (CS)", and "immediate danger stage (ID)" as shown in Figure 2. For stand-on vessels and give-way vessels, the timing of taking measures to prevent collisions is distinct in the same encountering situation. Furthermore, actions taken by the vessel, which include altering courses and changing speed, are varying with the development of the stage and two vessels approaching each other [31].

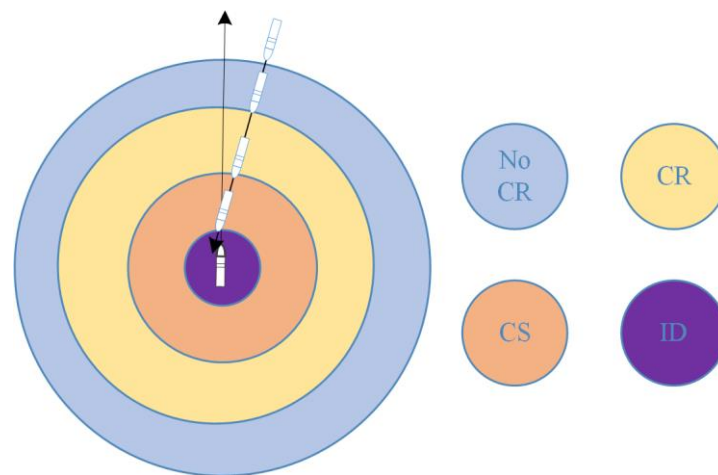


Figure 2. The diagram of the four-Stage theory.

In stage 1: the vessel is too far to cause a collision, while both vessels are free to move during this time up until the first point of the CR. Instead of a series of minute changes in course and speed as required by Rule 8 and good seamanship in practice, any necessary course and speed changes must be large enough to be easily apparent to another vessel visually or by radar if the circumstances permit. Being able to observe another vessel's purpose clearly, ensures that the concerned vessel avoids the potential of misunderstandings. Furthermore, if the TS eventually invades the OS's ship domain on the basis that both ships keep their initial course and speed, the potential collision risk (PCR) exists between the ships. The first time-in-point of collision risk (FTCR) indicates the conclusion of this stage and also leads to the formation of stage 2.

In stage 2: The conflict is increasing, and the giving-way vessel shall take action as soon as feasible to avoid the collision, as per COLREGs. Meanwhile, the stand-on vessel should maintain its speed and course in the first stages, but use all available means to monitor the success of the acts made as necessary by the other vessel. For the giving-way vessel, this is the “golden time” to take precautions. On the one hand, to act early is not only in the spirit of COLREGs, but also gives the ship more time to deal with the urgency. On the other hand, it provides the stand-on vessel more time to observe and check whether the conflict occurred or not after the collision-avoidance action. With the two vessels coming close together, the risk of a collision is high, and the stand-on vessel is permitted to take whatever actions necessary to avoid a collision after it is determined that the give-way ship is not acting correctly and effectively in accordance with COLREGs. This timing is determined by the situation and ship’s manoeuvrability, and it is best defined by the ship captain’s evaluation. CR that is the initial point exists at the confined sea only when $TCPA \leq 20$ mi in this study [30].

In stage 3: CS is created when both ships are unable to pass each other at a safe distance whichever collision avoidance action by each ship is taken.

In stage 4: This is the most perilous stage when facing a situation where a disagreement has escalated to the point where it cannot be resolved just by the action of the give-way ship. Both vessels must execute collision-avoidance measures; otherwise, a collision will occur in a short period [31].

In this paper, the model of CRI is utilised to quantify the encounter stage of ships, whose larger value indicates a higher risk, and the detail is shown in Section 3.1.

2.2.2. Ship Domain

Ship domain is the region around a ship that the OOW or Captain wants to keep free of obstructions and other vessels, and it is thought to be inviolable. According to good seamanship in practice and COLREGs, the ship must keep a safe distance when approaching various types of power-driven ships or static obstructions [32]. In terms of collision avoidance, the safety distance is equivalent to the ship domain. In this work, two different ship domains are used which include the left eccentric ellipse model (Figure 3a) for the head-on or overtaking situation and the left eccentric circle model (Figure 3b) for the crossing situation.

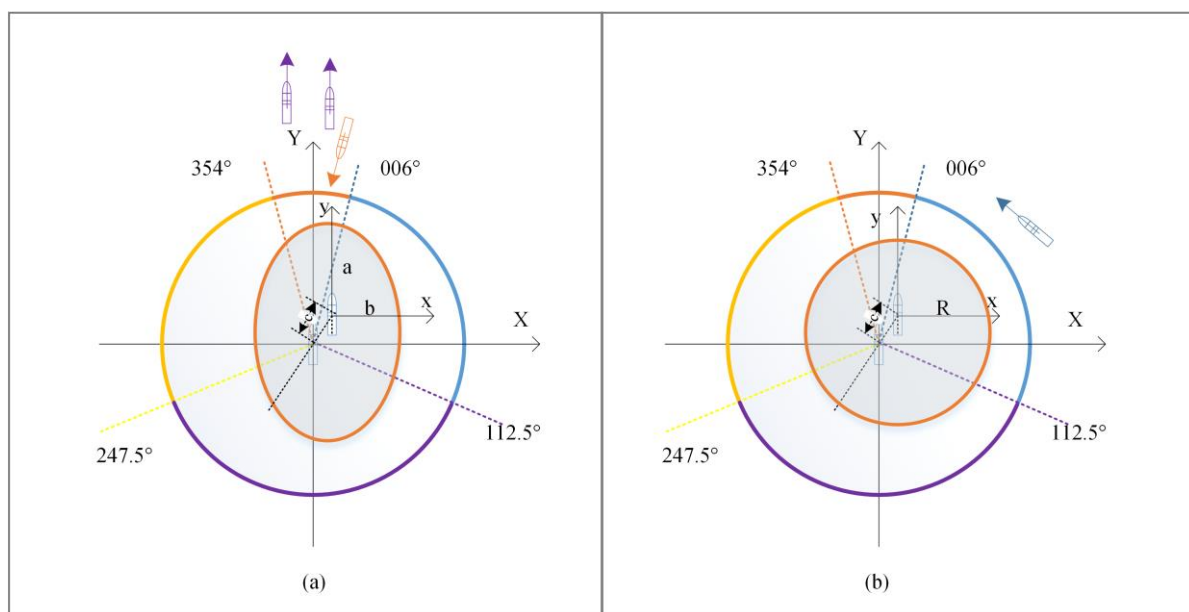


Figure 3. Ship domain model in different situations: (a) Head-on & overtaking situation; (b) cross situation.

An imaginary ship, which is located on the starboard and in front of its ship, is set up in the centre of the ship domain of both the left eccentric ellipse and eccentric circle model. More safety distance should be kept from the front of the vessel and the starboard side based on the consideration of COLREGs rules and good seamanship. In practice, the suitable range of the ship domain is established by the OOW or captain depending on the situation and the specifics of both the OS and the TS. It is critical for collision avoidance since an incorrect ship domain would increase the probability of collision [33].

In Figure 3, a and b are the major and short axial lengths of the ellipse respectively; c is the distance from the imaginary ship to the fact ship; R is the radius of the circle in the cross situation; and L_o and L_T are the lengths of the OS and TS respectively, where

$$\begin{cases} a = 2(L_o + L_T) \\ b = 1.5(L_o + L_T) \\ c = 0.5(L_o + L_T) \\ R = 2(L_o + L_T) \end{cases} \quad (1)$$

2.3. Improved MMG Model

For intelligent collision avoidance decision-making in the form of altering course or changing speed by a computer system, it is clear that an accurate and exact motion model for ship navigation is required. The focus of this research is on decision-making for collision avoidance in the confined sea when visibility is good. Considering rolling, pitching, and heaving motions have little impact on ship collision, only sway, surge, and yaw are considered in the calm sea. Therefore a mathematical model called MMG with three degrees of freedom is used to properly deduce OS's movements [34]:

$$\begin{cases} (m + m_y)\ddot{v} + (m + m_x)ur = Y_H + Y_P + Y_R \\ (m + m_x)\ddot{u} - (m + m_y)vr = X_H + X_P + X_R \\ (I_{zz} + i_{zz})\dot{r} = N_H + N_R + N_P \end{cases} \quad (2)$$

where m is the total mass of the vessel, m_x and m_y , respectively, indicate the additional vertical and horizontal mass. Furthermore, u and v denote the lateral and longitudinal speed; $\dot{u}$ and $\dot{v}$ denote acceleration in their direction; r and $\dot{r}$ are the angular velocity and angular acceleration; I_{zz} and i_{zz} are the moment of inertia and additional moment of inertia respectively; X and Y represent the force or moment in vertical axis directions and in horizontal axis directions, respectively; and N respects the turning moment. H , P and R are the external forces acting on the hull, propeller, and rudder respectively.

The propeller's power must be sufficient to overcome both the hull's inherent resistance and the additional resistance brought on by the propeller's operation, which is expressed as follows:

$$T = R + \Delta T \quad (3)$$

where T and ΔT are the forces of propeller and propeller derating respectively. R is the resistance of the ship hull.

In the past, a lot of research was aimed at establishing an accurate ship motion model with the precondition of fixed rotation of the ship propeller, in which the ship's speed and rotation of propeller are matched. In this state of the ship moving, the thrust is only influenced by the rotation of the propeller when additional elements are known. However, the research in this paper focuses on the ship motion and collision avoidance decision method under variable speed. In this situation, the rotation of the propeller and the actual speed of the ship are unmatched, which is more complicated than the ship's motion in a fixed rotation. Thrust is influenced by more factors besides the rotation of the propeller, such as ship longitudinal and lateral speeds, thrust deduction, coordinate of buoyancy, and so on. Therefore, a new ship thrust model is needed for the MMG model.

In practice, the propeller's effective thrust is often expressed using the thrust derating factor as follows:

$$X_p = (1 - t_{p0})T \quad (4)$$

where t_{p0} is the thrust derating coefficient, which is impacted by the ship's model, as well as the propeller's load and size. Furthermore, for a particular ship, t_{p0} is a variable for alternation of revolution and ship movement. The alternation of revolution would affect the ship's resistance by causing a shift in the inflow and outflow of the propellers. the thrust derating coefficient t_{p0} under the impact of propeller load is illustrated in Equation (5):

$$t_{p0} = \begin{cases} 0.04 + (t'_{p0} - 0.04)u/u_0, & u \leq u_0 \\ t'_{p0}, & u > u_0 \end{cases} \quad (5)$$

where u_0 is the designated speed and u is the real-time speed. Further, t'_{p0} is the propeller derating factor at u_0 . It is also vital to evaluate the impact of lateral and rotational motion on the ship motion model in order to increase the accuracy of the ship motion prediction model. The function of the thrust derating coefficient by the ship moving can be expressed as follows:

$$(1 - t_p) = (1 - t_{p0}) + f \quad (6)$$

where the f is the function of the effect of ship motion, which is determined as follows:

$$\begin{cases} f = k_t \beta_R \\ k_t = 0.00023(\gamma_A \cdot L/D_\rho) - 0.028 \\ \gamma_A = (B/d)\{1.3(1 - C_b) - 3.1l_{cb}\} \\ l_{cb} = x_c/L \cdot 100 \\ \beta_R = \beta - l_R \frac{r}{V} \end{cases} \quad (7)$$

where x_c is the vertical coordinate of buoyancy. β_R denotes the drift angle at the rudder, and β is the drift angle at the center of the ship, which can be determined by $\beta = \text{atan}(-u/v)$. B and d denote the ship's width and the ship draft, respectively. L denotes the ship's length and D_ρ is the diameter of the propeller. γ_A and l_{cb} are the rectification coefficients of the hull and floating centre coefficient, respectively. C_b and l_R denote the block coefficient of the ship and drift angle coefficient.

The force of the propeller, mainly from the thrust of the propeller, is expressed as follows:

$$T = \rho(NP)^2 D_p^2 K_T \quad (8)$$

where K_T is the factor of thrust; ρ is the density of water; D_p represents the diameter of the propeller; and NP denotes the real-time rotation rate of the propeller, which uses to keep and control the ship speed.

There are two different styles of ship manoeuvre at sea: first is manoeuvring in the manner of only altering course while maintaining a constant revolution per minute (RPM), which nearly invariably occurs in open water. The second is manoeuvring in the manner of changing speed by reducing or increasing the revolution of the main engine, which almost always occurs in narrow water or high-density traffic seas. In previous studies, the ship's speed was rarely taken into account in the ship motion model, with just the ship's course being taken into account. Moreover, the study using ship models that are affected by changes in direction and speed almost exclusively focuses on small boats. The difference of characteristics of propellers between tiny ships and large ships is huge.

Furthermore, the ship in the variable-speed process has more influencing components than the ship in the variable-course motion process, and the interaction between each factor is more sophisticated. Furthermore, minor speed changes have little effect on the ship's collision avoidance in practice, which is why captains and pilots prefer to manage the telegraph rather than using a small variation of the main engine revolution to modify the speed. Telegraphs are used to control the ship's speed, which includes ten different

presupposed specific NPs. Propulsion particulars about a bulk carrier called “Huayang dream” are illustrated in Table 1, where the gap of RPM between different telegraphs is relatively large in order to accommodate a large vessel speed change.

Table 1. Propulsion particulars about RPM and speed of ship “Huayang dream”.

Item	Telegraph	RPM (r/min)	Speed (knot)
Ahead	Nav. ahead	92	14
	Full ahead	85	12
	Half ahead	70	10.5
	Slow ahead	48	5
	Dead slow ahead	35	3
	Stop	0	0
Astern	Dead slow astern	−35	−3
	Slow astern	−48	−5
	Half astern	−70	−10.5
	Full astern	−85	−12

Those telegraphs are applied to adjust the speed to manoeuvre the ship to avoid collision by OOW and pilots in practice. If NP_1 is the initial rotation rate of the propeller of a telegraph at the time t_1 , and NP_2 is the final rotation rate of the propeller of a telegraph at the time t_2 , the rotation rate of the propeller at a time t can be determined by the following:

$$NP^t = \begin{cases} NP_2, & t > t_2 \\ NP_1, & t < t_1 \\ NP_1 - (t - t_1) \times k, & t_1 < t < t_2, NP_1 > NP_2 \\ NP_1 + (t - t_1) \times k, & t_1 < t < t_2, NP_1 < NP_2 \end{cases} \quad (9)$$

where k is the meaning of a change of rotation rate, which was deduced to be around 0.25 rotation per second based on the researcher’s experience as the ship’s captain/officers and the model projection.

The Runge–Kutta method and improved MMG should be used to deduce the future state of the ship by combining the initial state of OS, such as speed, course, and position. The details of simulation results about improved MMG are provided in Section 4.1.

2.4. Improved Velocity Obstacle

The VO method defines a collection of velocity sets in which a collision occurs if the velocity falls inside one of them. Assuming that two ships, V_O and V_T , represent the velocity of the OS and the TS, respectively. V_{OT} is the relative velocity of OS caused by TS, then this can be expressed in the following way:

$$\vec{V}_{OT} = \vec{V}_O - \vec{V}_T \quad (10)$$

As displayed in Figure 4, the white circle region, the TS’s ship domain, defines all of the TS’s possible positions when a collision occurs [28]. Once V_{OT} falls into the Relative collision cone (RCC), the collision between ships exists. On the contrary, there is no risk of collision existing between ships. The absolute collision cone (ACC, purple region) is presented by RCC along V_T , which implies that the V_O dropping into ACC is equivalent to V_{OT} falling into RCC. In other words, the collision between ships exists when V_O falls into the ACC, which can be shown as Equation (11) considering that the VO algorithm is time-varying:

$$\vec{V}_O^{(t)} \in VO^{(t)} = RCC \otimes \vec{V}_T^{(t)} \quad (11)$$

where operation $\otimes$ is the Minkowski addition. However, since it ignores the characteristics of ship manoeuvring and the particular navigation environment, typical VO cannot be

directly applied to ship collision avoidance even though it is very effective and simple in identifying the presence of a collision. Therefore, an IVO is presented, which combines typical VO, MMG, and COLREGs.

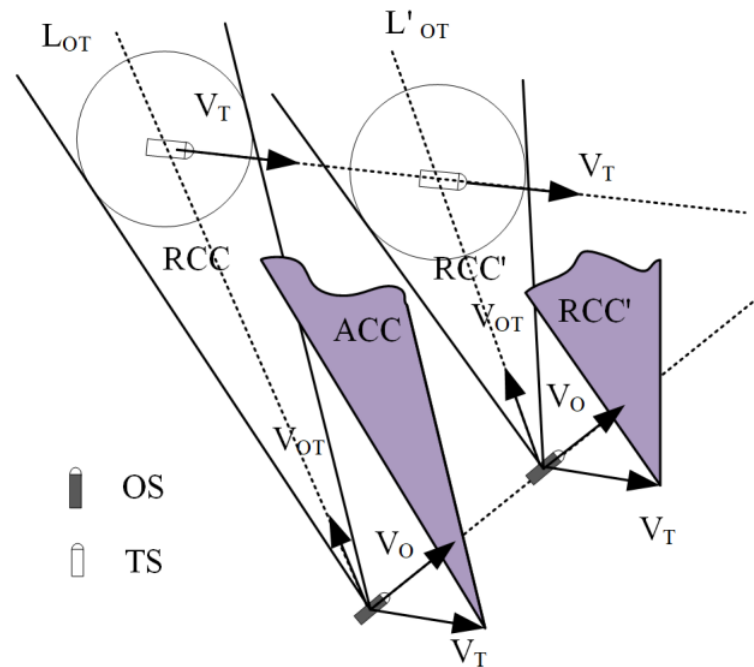


Figure 4. The diagram of the VO method.

The ship's movement is non-linear due to the large inertia of its properties, especially during significant changes in course and speed [34]. Therefore, an IVO model based on the consideration of the ship-maneuvring motion characteristics is proposed to quantify the VO set in the encounter situation, which includes telegraph and course being able to be manoeuvred by the ship.

The typical VO set is shown as the red region in Figure 5a, which is linear and appropriate for dynamic collision avoidance methods under ideal conditions. The outer and inner of the circle denote the maximum and minimum speeds of the ship, respectively, which are based on the manoeuvrability parameter of the propeller. In practice, the maximum and minimum speeds are maintained by the telegraph of "Full. ahead" (F) of the ship and "Dead slow ahead" (D), respectively, and the other speed for collision avoidance can be controlled by the "Half. ahead" (H) and "Slow. ahead" (S). Except for these, other telegraphs are seldom used to avoid collisions based on good seamanship since their slow speeds make it impossible to maintain the course. Therefore, an IVO algorithm shown in Figure 5b is provided combined with the four telegraphs and a typical VO, which can be used to avoid collision based on the ship's manoeuvrability. Four sectors denote the VO set of the different telegraphs, which can be expressed as followed:

$$\begin{cases} VO_F = ACC \oplus T_F \\ VO_H = ACC \oplus T_H \\ VO_S = ACC \oplus T_S \\ VO_D = ACC \oplus T_D \end{cases} \quad (12)$$

where $\oplus$ shows mathematical expression. T_F , T_H , T_S , and T_D denote telegraph F , H , S , and D , respectively. VO_F (purple area), VO_H (yellow area), VO_S (blue area), and VO_D (red area) denote the VO sets of telegraph F , H , S , and D , respectively.

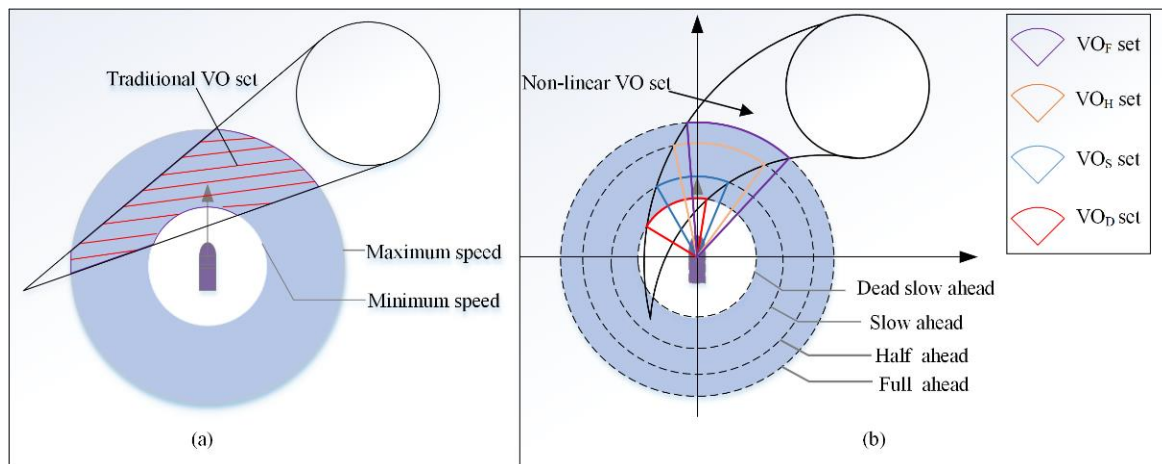


Figure 5. The diagram of the IVO method: (a) Traditional VO set; (b) Non-linear VO set.

3. Autonomous Collision Avoidance Method

3.1. Collision Risk Index

The CRI [32], is a physical quantity for evaluating the risk of a ship collision, used to determine the severity of the collision threat between ships, especially in the multi-ship encounter scenario. The two elements of the CRI are the time of collision risk index (TCRI) and the space of collision risk index (SCRI). The SCRI is the likelihood of two ships colliding in space, whereas the TCRI is the urgency of collision avoidance measures which must be taken by the two ships at risk of collision. The range of CRI is from 0 to 1, where the risk of collision is increasing with the value. The SCRI can be determined as follows:

$$u_{st} = \begin{cases} 1, \exists (x, y)^t \in \text{Domain}^t \cap t \in [0, \text{TCPA}] \\ 0, \forall (x, y)^t \notin \text{Domain}^t \cap t \in [0, \text{TCPA}] \end{cases} \quad (13)$$

where u_{st} denotes the function of SCRI and $(x, y)^t$ denotes the position of TS at t . The Domain^t denotes the ship domain of OS at t . The expression of TCRI is given as follows:

$$u_{tT} = \begin{cases} 1, & TCS \leq 0 \\ (1 - \frac{TCS}{t_0})^{3.03}, & 0 < TCS < t_0 \\ 0, & TCS \geq t_0 \end{cases} \quad (14)$$

where u_{tT} means the function of TCRI. Time to the close-quarters situation (TCS) is the duration from the current moment to the first time-in-point of the close-quarters situation (FTCS), whereas the t_0 is the duration from the first time-in-point of collision risk (FTCR) to FTCS. When the OS takes the most effective collision avoidance action to make other ships merely pass on the edge of the ship domain is referred to as the FTCS. Regardless of the OS's actions, if this time elapses, the TS will enter the ship domain of the OS. Thus, the CRI model can be established by combining the TCRI and SCRI as follows:

$$u_T = u_{st} \cdot u_{tT} \quad (15)$$

3.2. Dynamic Feasible Manoeuvring Interval

The feasible manoeuvring decision interval, which contains all the decision-making for collision avoidance of ships, can be provided by the IVO algorithm based on the improved MMG. If both ships maintain their original course and speed, as shown in Figure 6, a risk of collision between ships is possible. The C_0 and C_1 are the minimal course changes of the feasible manoeuvring decision to the port and starboard side respectively at the present

timing. The moving target will enter (not enter) the OS's domain if the angle is lower (greater) than this.

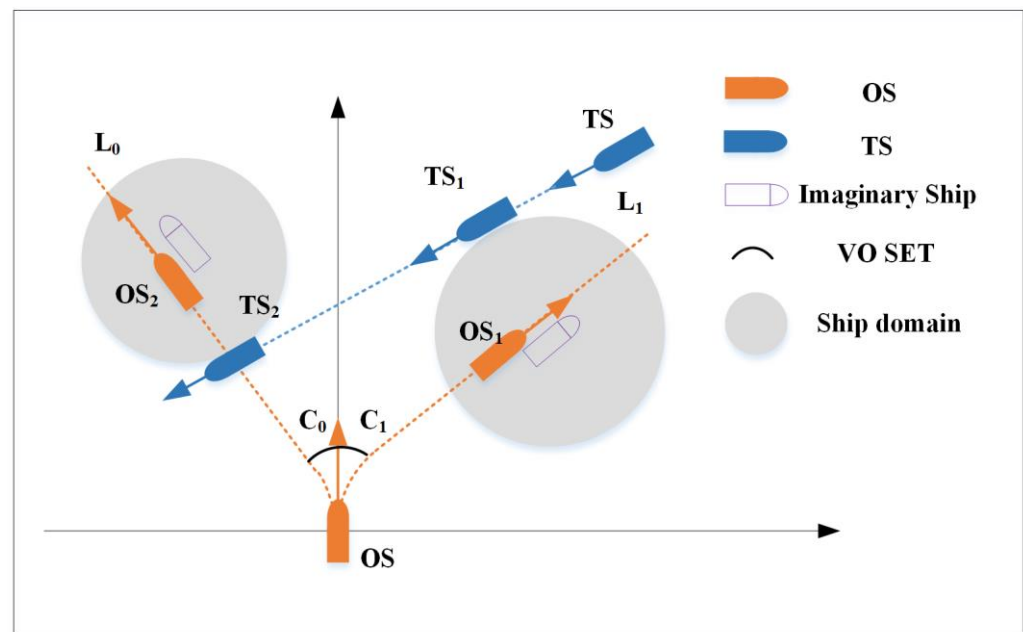


Figure 6. Feasible manoeuvring decision interval.

However, under the constraint of COLREGs, some decisions should be discarded in marine practice. Figure 7 illustrates collision avoidance decision-making under the COLREGs constraint in a variety of encounter scenarios. The dotted line indicates the side of the course followed by the COLREGs, with Figure 7a indicating an overtaking situation, Figure 7b indicating a head-on situation, and Figure 7c indicating a cross situation. The course is followed by the rules illustrated by the dotted line. Thus, the dynamically feasible manoeuvring interval of OS is offered for preventing collision action based on the MMG model, COLREGs, and IVO algorithm. The interval made at the time t is described as follows, which includes all feasible ship-manoevring decisions:

$$D^t = \begin{bmatrix} (C_1^t, T_F^t), \dots, (C_m^t, T_F^t) \\ (C_1^t, T_H^t), \dots, (C_n^t, T_H^t) \\ (C_1^t, T_S^t), \dots, (C_p^t, T_S^t) \\ (C_1^t, T_D^t), \dots, (C_q^t, T_S^t) \end{bmatrix} \quad (16)$$

where C means the altering course which $C \in (-90, 90)$ in the overtaking situation and $\Delta C \in (0, 90)$ in the head-on and cross situation. D^t denotes OS's dynamic feasible manoeuvring interval at a time t . m, n, p and q denotes the total number of the sets at the telegraph of F, H, S and D , respectively.

Furthermore, combined with the model of CRI, the dynamic feasible manoeuvring intervals are still appropriate for collision avoidance in the multi-ship scenario. There are generally two types of collision avoidance for TSs in the multi-ship scenario. One is that OS should take measures to avoid collision for one TS where the value of CRI is greater than the threshold. Although, a variety of TSs are in danger for OS, where the value of CRI of some TSs is too low to take collision avoidance at this moment. Therefore, in this situation, OS just avoids collision for the one TS at the same time. The other is that OS should take measures to avoid collision for two or more ships at once where their values of CRI are all

greater than the threshold. Therefore, the dynamically feasible manoeuvring interval of TSs is the intersection of the manoeuvring interval of each TS, which is shown as Equation (17):

$$D^t = D_1^t \cap D_2^t \cdots D_i^t \cdots D_{m-1}^t \cap D_m^t \quad (17)$$

where m denotes the total number of TSs with the higher value of CRI and D_i^m denotes the dynamically feasible manoeuvring interval of TS_i .

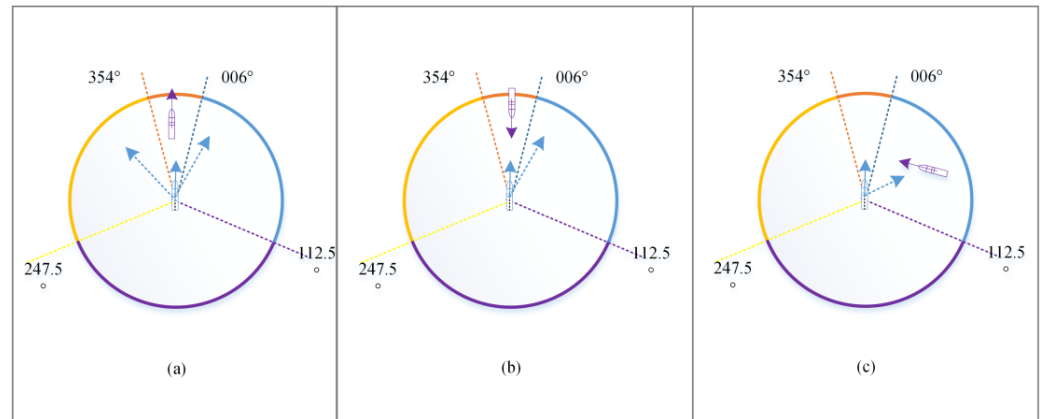


Figure 7. Decisions in different encountering situations: (a) Overtaking situation; (b) Head-on situation; (c) cross situation.

3.3. Optimisation Evaluation Function

According to the provided algorithm, a variety of decisions in the dynamic feasible manoeuvring interval should be made at any given time, which can safely avoid obstructions and ships. The two criteria of economy and timeliness are also taken into consideration when choosing collision avoidance decision options for ships as evaluation indicators on the basis of safety.

The ship's timeliness can be measured by how long it takes to complete the collision avoidance action. There is less risk of collision during an encounter stage when the time value is short. The function of timeliness can be shown as follows:

$$f_1 = \frac{\Delta t_{(i)}}{\max(\Delta t)} \quad (18)$$

where $\Delta t_{(i)}$ means the period consumed by the collision avoidance action of i and f_1 denotes the timeliness.

The economy mostly pertains to the ship's fuel usage during collision avoidance, which is primarily shown in the steering amplitude, which can be determined as follows:

$$f_2 = \Delta c \quad (19)$$

where Δc means the amplitude of altering course, and the lower the amplitude, the less energy the ship avoidance operation costs.

In conclusion, the optimisation evaluation function is calculated decision-making in terms of ship timeliness and economy, which can be expressed as follows:

$$F = af_1 + bf_2 \quad (20)$$

where a and b represent the weights of timeliness and economy indicators, respectively, and $a + b = 1$.

The smaller the evaluation function F , the better the decision option. When the weight a of timeliness is greater, the ship takes less time in avoidance decision, which means that the potential danger in the whole process of collision avoidance is decreased. When

the weight b of the economy is greater, the steering amplitude is greater, which means the safety distance between ships is larger. But the time of ship avoidance increases, and the potential danger also increases. Therefore, it satisfies different situation requirements to adjust the corresponding weights of timeliness and economy indicators according to different scenarios.

3.4. Collision Avoidance Decision

A model is proposed for the challenging subject of multi-ship collision avoidance, which integrates the improved MMG model and COLREGs. These decisions encompass altering course and changing speed, and the flowchart of decision-making for collision avoidance is shown in Figure 8.

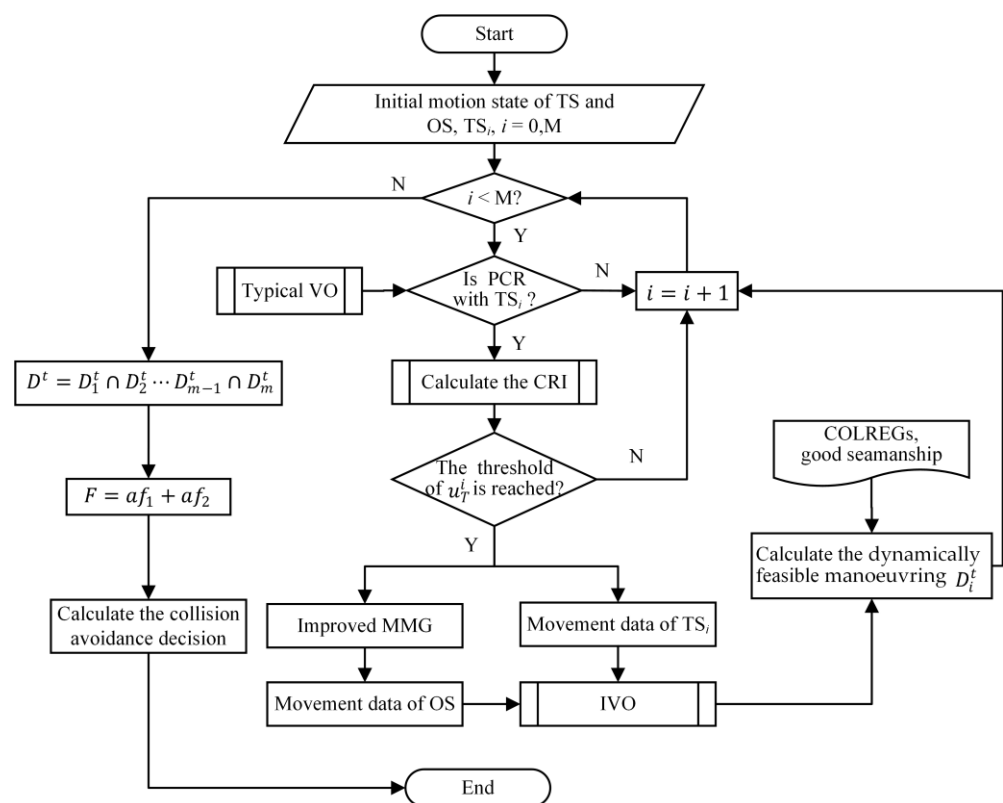


Figure 8. Decision-making of collision avoidance.

Step 1 is the input module which is used to collect the data of initial ship motion by the ship's current navigation equipment, the M means the number of TS.

Step 2 is the risk detection module, which uses a typical VO algorithm to judge whether or not the collision risk exists by the typical VO algorithm.

Step 3 is the CRI module, which is used to determine the timing and priority of the TSs avoidance, based on the value of CRI between OS and TS.

Step 4 is to obtain the dynamically feasible manoeuvring decisions, D^t , based on the IVO, COLREGs, and MMG.

Step 5 is to identify the ship's final manoeuvring decision by the optimisation evaluation function.

4. Case Study

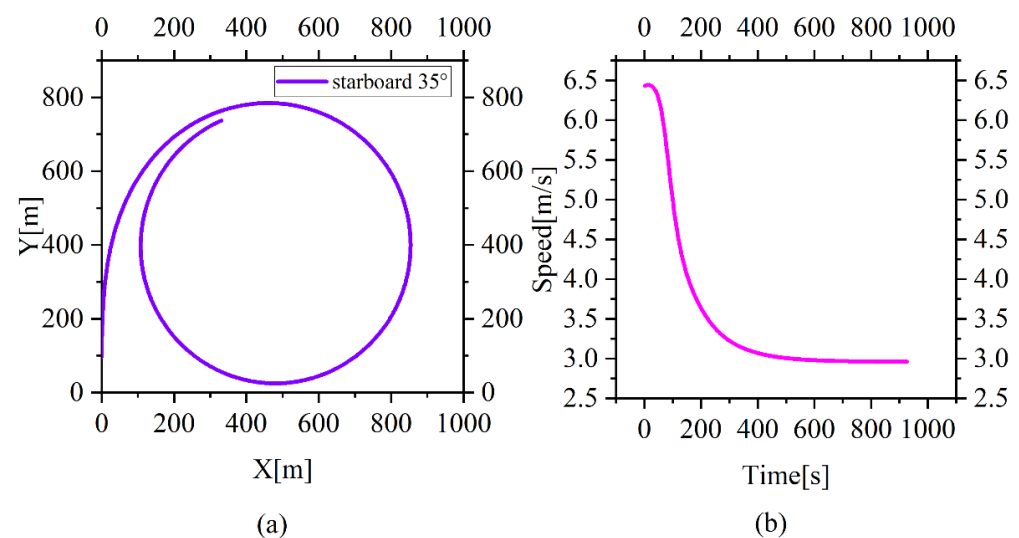
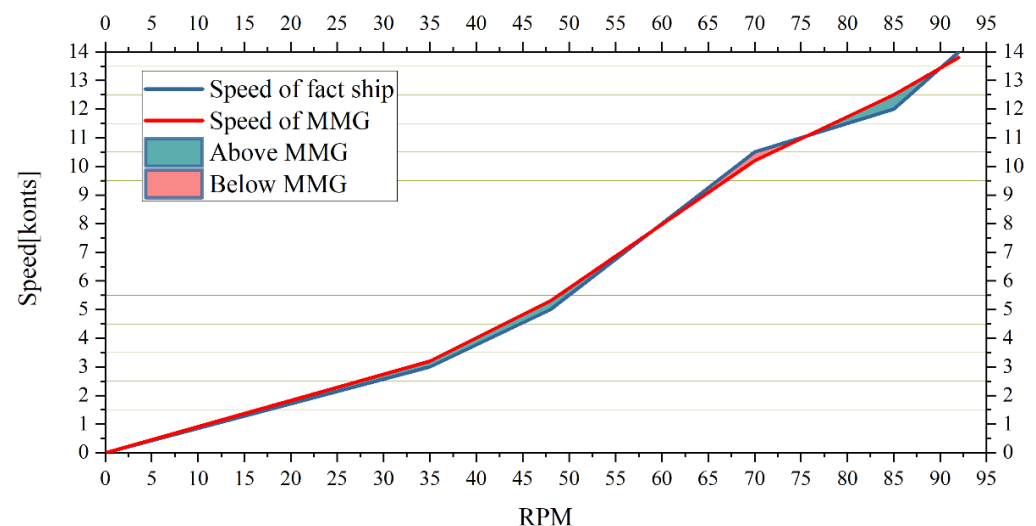
4.1. Simulation of Improved MMG Model

After finishing the MMG factor modification, a panama maximum size bulk carrier is designated as the simulated ship, with detailed information as indicated in Table 2. Initially, the speed and course are 12.5 knots and 000°, respectively.

Table 2. Principal particulars and manoeuvrability characteristics of OS.

Parameters	Values	Parameters	Values
Length Overall/m	225	Propeller number	1
Breadth/m	32.5	Propeller advance/m	4.738
Draft/m	14.5	Acreage of Rudder/m ²	56.88
Block coefficient	0.8715	Propeller blade number	4
Propeller diameter/m	6.8	Propeller type	Fixed pitch
Propeller pitch/m	2.81	blade ratio	0.91

As illustrated in Figure 9a, turning one circle takes 927 s, and the final speed is 5.75 knots (2.96 m/s), a 54 percent drop in speed. The advance and transfer of the ship are, respectively, 785 m (3.48 L) and 430 m (1.91 L), where L represents the length overall (LOA) of OS. Figure 9b shows that the first decrease in speed is rapid and that the speed at the last trend is constant. When comparing the decrease in speed, transfer, advance, and other factors under the turning circle in the real world, the precision meets the collision avoidance criterion at sea. Figure 10 shows that under different RPMs of the propeller, the real ship speed and the simulation speed in MMG are closer, and the parameter setting of the MMG model has fulfilled the requirements of ship collision avoidance.

**Figure 9.** Simulation of turning circle. (a) ship trajectory at starboard 35°; (b) tendency of speed.**Figure 10.** Difference between real ship and simulation ship.

Four turning circles are illustrated in Figure 11a, which is taken by different telegraphs. The gap between different turning circles is small because the ship's speed has minimal influence on the advance of the ship's turn in actuality. When the speed tends to be steady, a ship controlled by telegraph from D to F at 1200 s. The tendency of speed change is in alignment with the practice of navigation in reality. Therefore, the accuracy of improved MMG conforms to the requirement of intelligent collision avoidance.

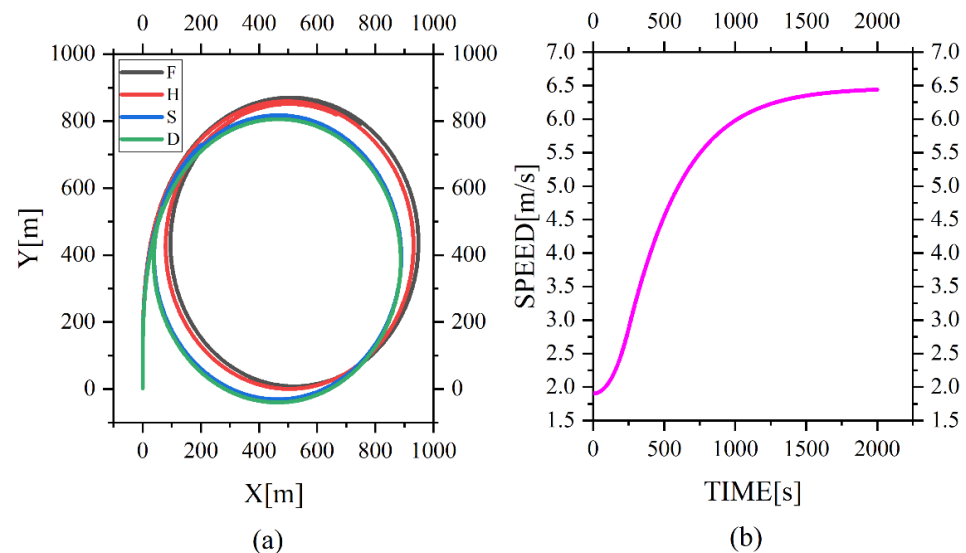


Figure 11. Simulation of the different telegraphs. (a) Turing circle at different telegraph; (b) Speed change from D to F.

4.2. Simulation Result

Two groups of case studies are conducted to demonstrate the feasibility of the proposed method. Every case study includes four TSs, which are in different encountering situations.

4.2.1. Scenario 1

Initial information of relative ship about the multi-ships scenario case 1 is illustrated in Table 3. In this case, according to COLREGs, OS has a responsibility to move out of the way with TS₁ in a crossing position and TS₂ in a head-on situation, and TS₃ in the overtaking situation. The dynamic feasible manoeuvring interval is shown in Figure 12, where the black colour, red colour, green colour, and blue colour denote the D telegraph, S telegraph, H telegraph, and F telegraph, respectively.

Table 3. Initial information of the ships for simulation in scenario 1.

Ship List	Position (mile)	Course (°)	Speed (knot)	LOA (m)	TCS	CRI
OS	(0, 0)	000	12.5	225	-	-
TS1	(2, 2)	300	8.0	225	725	0.202
TS2	(0, 4.4)	181	6.0	299	623	0.035
TS3	(0.2, 2.0)	000	5.5	225	743	0.393
TS4	(2, 0)	000	6.5	245	0	0

OS takes measures to avoid collision by altering course 37° to the starboard side at the “H” telegraph, and Figure 13a shows the ship's path to avoid the current collision for multi-ships, which is shown in different colours. However, OS will introduce additional collision risk with TS₄ while avoiding collision for TS₁, TS₂, and TS₃. Thus, OS should take caution to alter course to the starboard side for the collision avoidance of TS₄, and the relative distance between the OS and the TS is shown in Figure 13b, in which P₁, P₂, P₃, and P₄ are represented as the minimum distance of relative TS, respectively. Figure 13d

represents the OS speed change during alternation operation. Figure 13c demonstrates the general tendency of DCPA between OS and TS. Under the guidance of the telegraph, the ship's speed gradually decelerates, initially rapidly, then slowly and steadily, as seen in Figure 13d.

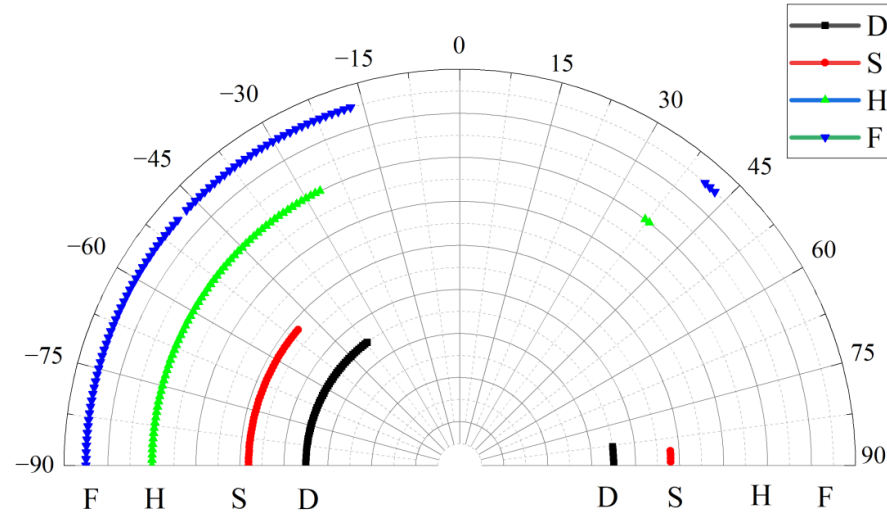


Figure 12. Dynamic feasible manoeuvring interval of scenario 1.

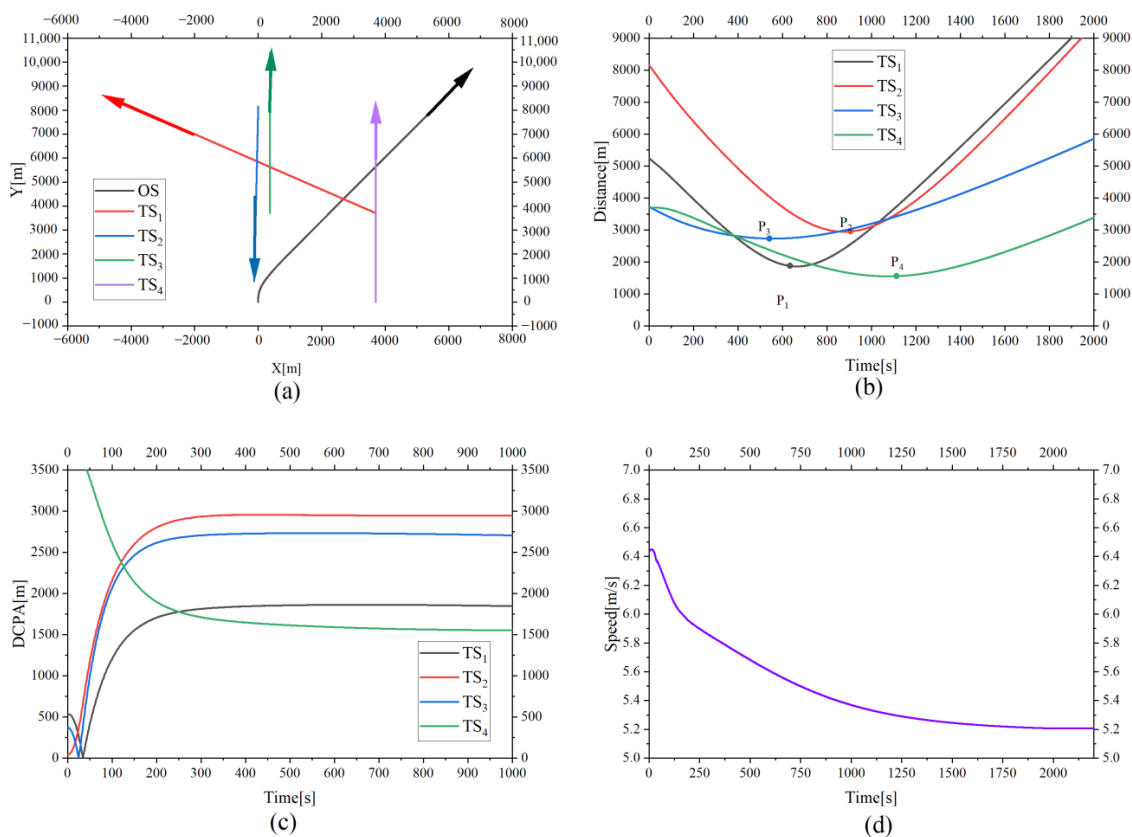


Figure 13. Simulation results of scenario 1: (a) Trajectory of OS and TSs; (b) Distance between OS and different TSs; (c) DCPA between OS and different TSs; (d) Speed of OS.

4.2.2. Scenario 2

Table 4 illustrates the ship's initial information about the multi-ship scenario case 2. For the large ship of TS₁, TS₂, and TS₄, OS is the give-way vessel taking manoeuvres to avoid the collision. However, OS is the stand-on vessel in the encountering situation

with TS₃. The dynamic feasible manoeuvring decision interval calculated by the IVO algorithm is illustrated in Figure 14, where the “D” telegraph, “S” telegraph, “H” telegraph, and “F” telegraph are represented by the black colour, red colour, green colour, and blue colour, respectively.

Table 4. Initial information of the ships for simulation in scenario 2.

Ship list	Position (mile)	Course (°)	Speed (knot)	LOA (m)	TCS (s)	CRI
OS	(0, 0)	000	12.5	225	-	-
TS1	(1, 5.0)	200	14.5	225	451	0.009
TS2	(0, 5.5)	175	14.5	299	569	0.001
TS3	(−1.0, 3.5)	145	9.0	225	410	0.124
TS4	(0.3, 1.5)	000	8.0	245	852	0.525

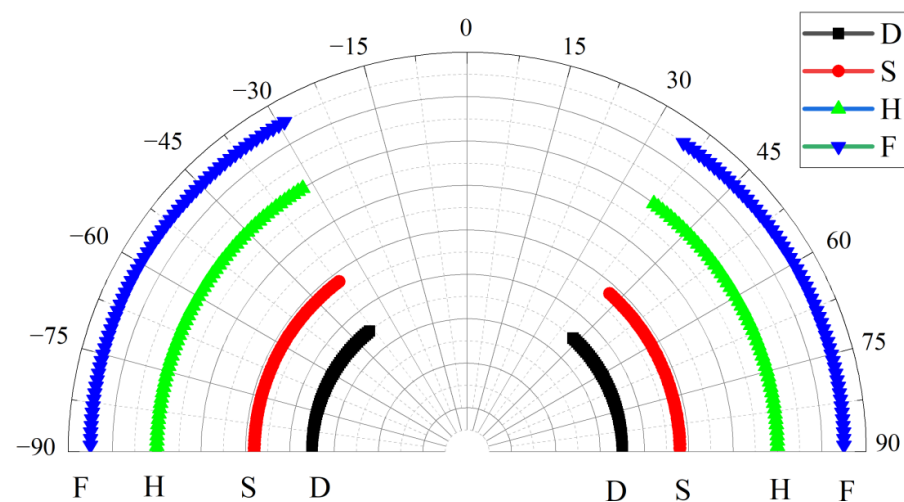


Figure 14. Dynamic feasible manoeuvring interval of scenario 2.

Figure 15a shows the ship’s motion trajectory when the OS avoids collision by altering course 35° to the starboard side. The ship’s relative distance curves between TS₁, TS₂, TS₃, and TS₄ are shown in Figure 15b, which reach the lowest point in P₁, P₂, P₃, and P₄. The minimum relative distance is greater than the safety value, which means the TS is not invading the ship domain. Figure 15c shows the differences in DCPA between this ship and the TS, which shrinks at first and then increases when the course is altered. As shown in Figure 15d, the tendency of OS speed is variable in the whole procedure of altering course and then tends to be stable when finishing the altering action.

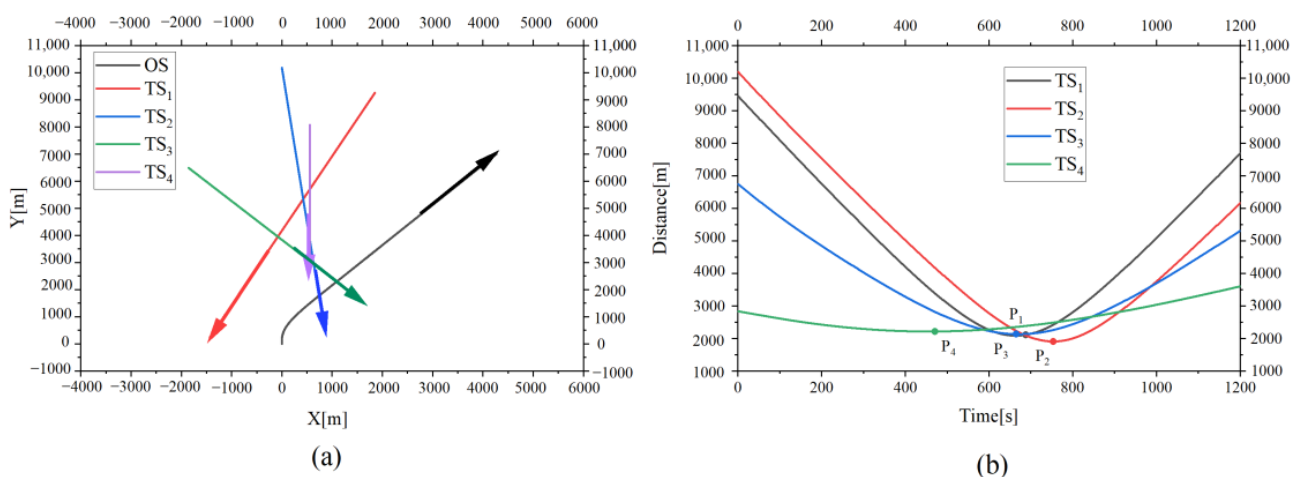


Figure 15. Cont.

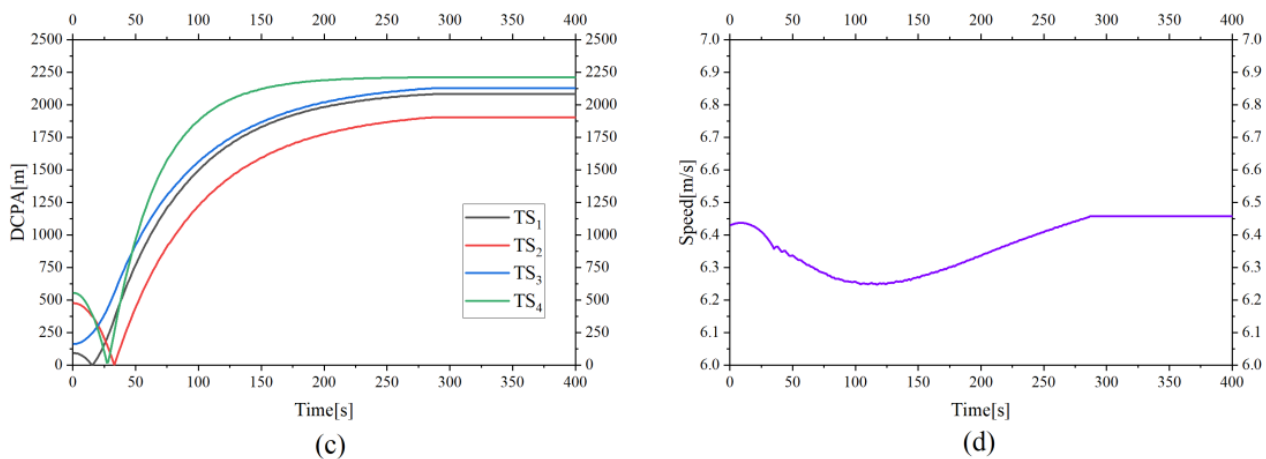


Figure 15. Simulation results of scenario 2: (a) Trajectory of OS and TSs; (b) Distance between OS and different TSs; (c) DCPA between OS and different TSs; (d) Speed of OS.

5. Analysis and Discussion

5.1. Results and Discussion

The core algorithm for avoiding collisions, which combines the improved MMG and VO algorithms, was proposed in the preceding section. Two groups of case studies are offered to demonstrate the effectiveness of the basic algorithm in various encounter circumstances in this paper. The results and discussion are as follows.

In the situation of a multi-vessel encounter, the ship's avoidance action should not only evaluate the vessels with whom the ship is at risk of collision with, but should also take into account whether this action will generate a new danger of collision with other TS. In Scenario 1, the action taken by OS to avoid collision with TS₁, TS₂, and TS₃, will impede the safe movement of TS₄; In Scenario 2, when taking measures to present the TS in danger, the ships must take into account the TS₃, although the risk of collision does not exist.

In some circumstances, changing speed can successfully lessen the amplitude of the alternation of course, which is highly advantageous for confined seas. In Scenario 1, only three decisions by altering course to the starboard side to avoid collision for all the TSs, and the minimum degree (41°) in the manner of only altering course are greater than the minimum degree (37°) in the manner of both alterations of course and speed. However, in Scenario 2, only altering course is the optimal decision for collision avoidance. Thus, alternation of speed is a benefit for collision avoidance in some special circumstances.

The dynamic feasible manoeuvring interval is shown in Figure 12. The outcome demonstrates that decision-making at distinct telegraphs may be used to avoid collision in common scenarios. The evaluation function is used to determine the optimal decision based on timeliness and economy. From Figures 13b and 15b, the relative distance between OS and TS shows that TS is not invaded its ship domain.

In all case investigations, the minimum distance between OS and TS at various decision-making points is more than $2(L_1 + L_2)$, indicating that TS does not invade OS's ship domain. Furthermore, the resulting DCPA is significantly bigger than the safety threshold. In light of the aforementioned case studies, it is apparent that the presented collision avoidance algorithm is highly adaptable and credible in dealing with ship collision risk in a variety of typical encounter situations. Moreover, changing the speed of the telegraph can be suitable for the most complicated situation. As a result, the fundamental algorithm is very advantageous for the ship in confined waters.

The main contributions of this paper are drawn in two aspects based on simulation results: On the one hand, the improved MMG is more feasible and valuable when considering the influence of ship manoeuvrability by changing speed. On the other hand, an intelligent collision avoidance model for ships is provided based on the IVO algorithm and improved MMG. In this paper, the decision-making including altering course and changing

speed takes into account COLREGs, ship manoeuvrability, and sea practice, which are reliable and useful for collision avoidance in multi-ship collision.

5.2. Comparison Analysis

In this research, decision-making for intelligent collision avoidance of ships based on ship manoeuvring process deduction is proposed. This method can efficiently resolve ship collisions in a variety of multi-ship encounter scenarios. In the past, many methodologies and techniques for the collision-free problem were presented by various relevant scholars [16,20,25,28,31,35,36], however, this paper offers unique diversity and superiority compared to the previous studies.

A lot of related studies pay more attention to the collision avoidance for a single ship or open sea, and ignore the multi-ship scenario and confined sea [31,36,37]. Compared to a single ship, collision avoidance is quite complicated in multi-ship circumstances. Although some studies [26–29] have provided some algorithms and models for the multi-ship situation which consider the ship motion model insufficiently, most of current research pays less attention on the model in which the ship's speed and rotation of propeller are not matched. The core model provides two benefits for decision-making of collision avoidance. One is using the CRI model to identify the most dangerous ship out of a group of ships with varying degrees of collision risk. Another is that the decision-making by the proposed algorithm will not create additional collision risk, which will not impede the safe navigation of other TSs.

In addition, few studies consider the characteristics of ship motion in the variable speed of the ship. Therefore, it is highly necessary to deduce the next state of OS for collision avoidance. In this research, an improved MMG model is introduced to derive the ships' manoeuvre motion process, which reduces the gap between the collision-free algorithm and practical applications. Furthermore, the COLREGs and good seamanship are also taken into account for the proposed method for intelligent collision avoidance.

In summary, this paper presents an intelligent collision avoidance method based on an IVO algorithm, which utilises alternation of course and changing of speed to present collision for the ship. Many factors are taken into account, including the improved MMG model, COLREGs, and good seamanship. Moreover, this model can resolve the conflict between ships in complicated circumstances, which is also effective for multi-ship encounter situations.

6. Conclusions

This research offers an intelligent ship collision avoidance model based on IVO and improved MMG, which take full account of COLREGs and good seamanship. In order to avoid multi-ship collision, a CRI model is established on the basis of SCRI and TCRI. The dynamic feasible manoeuvring interval is determined by combining with IVO and COLREGs, which include all the decision-making for collision avoidance of ships. On this basis, the decision of optimisation is determined by the optimisation evaluation function, which includes altering course and changing speed. In addition, a series of scenarios including various encounter circumstances with multi-ships are shown to demonstrate the efficacy and viability of this methodology.

Although the proposed methodology has some achievements, it also has some limitations. The telegraph about stern telegraphs and "Stop" is not considered in the speed alternation since ship manoeuvres at low speeds are difficult to predict and the lowest speed of the vessel should be maintained to keep the course in the navigation for safety. Furthermore, collision avoidance only considers changing both speed and course at the same moment, but in the real-world situation, additional judgments must be made.

In order to improve the dependability and efficacy of the proposed strategy in a real-world environment, we plan to consider the ship motion model at low speeds to avoid collision in future work. Furthermore, adjusting the speed and course at different times and changing the course more than once is considered for future development.

Author Contributions: Conceptualization, L.H. and Y.H.; Methodology, X.Z. and Y.H.; formal analysis, X.Z. and X.L.; investigation X.Z. and Y.H.; Writing original draft, X.Z. and Y.H.; Writing review & editing, X.L. and K.Z.; Visualization, X.Z.; Supervision, Y.H. and J.M.; funding acquisition, L.H. and Y.H. All authors have read and agreed to the published version of the manuscript.

Funding: This work is partially supported by the National Key Research and Development Program (Grant number: 2019YFB1600603) and the National Natural Science Foundation of China (Grant number: 52071249). This research was financially supported by “Study on Decision-making Method of Ship Autonomous Navigation in Inland Typical Waterways”(Item Number B2021519).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of Interest.

Nomenclature

Symbols	Definitions	Symbols	Definitions
L_o	length of the OS	I_{zz}	moment of inertia
L_T	length of the TS	i_{zz}	additional moment of inertia
a	major axial lengths of the ellipse	X	vertical axis directions
b	short axial lengths of the ellipse	Y	horizontal axis directions
c	distance from the imaginary ship to the fact ship elative course	N	turning moment
m	total mass of the vessel	H	hull
m_x	additional vertical mass	P	propeller
m_y	additional horizontal mass	R	rudder
u	lateral speed	T	force of propeller
$\dot{u}$	lateral acceleration	ΔT	force of propeller derating
v	longitudinal speed	t_{p0}	thrust derating coefficient
$\dot{v}$	longitudinal acceleration	u_0	designated speed
r	angular velocity	u	real-time speed
$\dot{r}$	angular acceleration	t'_{p0}	propeller derating factor at u_0
B	ship's width	f	function of the effect of ship motion
d	ship draft	x_c	vertical coordinate of buoyancy
L	ship's length	β_R	drift angle at the rudder
D_p	diameter of propeller	β	drift angle at the ship's center
γ_A	rectification coefficient of the hull	$\otimes$	Minkowski addition
l_{cb}	floating centre coefficient	F	Full. ahead
C_b	block coefficient	H	Half. ahead
l_R	drift angle coefficient.	S	Slow. ahead
K_T	factor of thrust	D	Dead slow ahead
ρ	density of water;	T_F	Full. ahead
D_p	diameter of the propeller	T_H	Half. ahead
NP	real-time rotation rate of the propeller	T_S	Slow. ahead
V_O	velocity of the OS	T_D	Dead slow ahead
V_T	velocity of the TS	$(x, y)^t$	position of TS at t
u_{st}	function of SCRI	$Domain^t$	ship domain of OS at t
u_{tT}	function of TCRI.	C	altering course

References

- Chen, P.; Huang, Y.; Mou, J.; van Gelder, P.H.A.J.M. Ship Collision Candidate Detection Method: A Velocity Obstacle Approach. *Ocean Eng.* **2018**, *170*, 186–198. [\[CrossRef\]](#)
- Li, J.; Wang, H.; Zhao, W.; Xue, Y. Ship's Trajectory Planning Based on Improved Multiobjective Algorithm for Collision Avoidance. *J. Adv. Transp.* **2019**, *2019*, 4068783. [\[CrossRef\]](#)
- Eleftheria, E.; Apostolos, P.; Markos, V. Statistical Analysis of Ship Accidents and Review of Safety Level. *Saf. Sci.* **2016**, *85*, 282–292. [\[CrossRef\]](#)
- Statheros, T.; Howells, G.; McDonald-Maier, K. Autonomous Ship Collision Avoidance Navigation Concepts, Technologies and Techniques. *J. Navig.* **2008**, *61*, 129–142. [\[CrossRef\]](#)

5. Hänninen, M.; Kujala, P. Influences of Variables on Ship Collision Probability in a Bayesian Belief Network Model. *Reliab. Eng. Syst. Saf.* **2012**, *102*, 27–40. [\[CrossRef\]](#)
6. Du, L.; Goerlandt, F.; Kujala, P. Review and Analysis of Methods for Assessing Maritime Waterway Risk Based on Non-Accident Critical Events Detected from AIS Data. *Reliab. Eng. Syst. Saf.* **2020**, *200*, 106933. [\[CrossRef\]](#)
7. Huang, Y.; Chen, L.; Negenborn, R.R.; van Gelder, P.H.A.J.M. A Ship Collision Avoidance System for Human-Machine Cooperation during Collision Avoidance. *Ocean Eng.* **2020**, *217*, 107913. [\[CrossRef\]](#)
8. Mitrofanov, O. An Anti-Collision Indicator. *J. Navig.* **1968**, *21*, 163–170. [\[CrossRef\]](#)
9. Wang, N. An Intelligent Spatial Collision Risk Based on the Quaternion Ship Domain. *J. Navig.* **2010**, *63*, 733–749. [\[CrossRef\]](#)
10. Weng, J.; Meng, Q.; Qu, X. Vessel Collision Frequency Estimation in the Singapore Strait. *J. Navig.* **2012**, *65*, 207–221. [\[CrossRef\]](#)
11. Mou, J.; Li, M.; Hu, W.; Zhang, X.; Gong, S.; Chen, P.; He, Y. Mechanism of Dynamic Automatic Collision Avoidance and the Optimal Route in Multi-Ship Encounter Situations. *J. Mar. Sci. Technol.* **2021**, *26*, 141–158. [\[CrossRef\]](#)
12. Pedersen, E.; Inoue, K.; Tsugane, M. Simulator Studies on a Collision Avoidance Display That Facilitates Efficient and Precise Assessment of Evasive Manoeuvres in Congested Waterways. *J. Navig.* **2003**, *56*, 411–427. [\[CrossRef\]](#)
13. Szlapczynski, R.; Szlapczynska, J. A Method of Determining and Visualizing Safe Motion Parameters of a Ship Navigating in Restricted Waters. *Ocean Eng.* **2017**, *129*, 363–373. [\[CrossRef\]](#)
14. Zhao, Y.; Li, W.; Shi, P. A Real-Time Collision Avoidance Learning System for Unmanned Surface Vessels. *Neurocomputing* **2016**, *182*, 255–266. [\[CrossRef\]](#)
15. Large, F.; Laugier, C.; Shiller, Z. Navigation among Moving Obstacles Using the NLVO: Principles and Applications to Intelligent Vehicles. *Auton. Robots* **2005**, *19*, 159–171. [\[CrossRef\]](#)
16. Chen, P.; Huang, Y.; Papadimitriou, E.; Mou, J.; van Gelder, P.H.A.J.M. An Improved Time Discretized Non-Linear Velocity Obstacle Method for Multi-Ship Encounter Detection. *Ocean Eng.* **2020**, *196*, 106718. [\[CrossRef\]](#)
17. Zhao, L.; Roh, M. II COLREGs-Compliant Multiship Collision Avoidance Based on Deep Reinforcement Learning. *Ocean Eng.* **2019**, *191*, 106436. [\[CrossRef\]](#)
18. Gupta, D.K.; Vasudev, K.L.; Bhattacharyya, S.K. Genetic Algorithm Optimization Based Nonlinear Ship Maneuvering Control. *Appl. Ocean Res.* **2018**, *74*, 142–153. [\[CrossRef\]](#)
19. Tsou, M.C.; Kao, S.L.; Su, C.M. Decision Support from Genetic Algorithms for Ship Collision Avoidance Route Planning and Alerts. *J. Navig.* **2010**, *63*, 167–182. [\[CrossRef\]](#)
20. Ni, S.; Liu, Z.; Cai, Y. Ship Manoeuvrability-Based Simulation for Ship Navigation in Collision Situations. *J. Mar. Sci. Eng.* **2019**, *7*, 90. [\[CrossRef\]](#)
21. Wang, X.; Liu, Z.; Cai, Y. The Ship Maneuverability Based Collision Avoidance Dynamic Support System in Close-Quarters Situation. *Ocean Eng.* **2017**, *146*, 486–497. [\[CrossRef\]](#)
22. Zhang, J.; Zhang, D.; Yan, X.; Haugen, S.; Guedes Soares, C. A Distributed Anti-Collision Decision Support Formulation in Multi-Ship Encounter Situations under COLREGs. *Ocean Eng.* **2015**, *105*, 336–348. [\[CrossRef\]](#)
23. Zhang, L.; Mou, J.; Chen, P.; Li, M. Path Planning for Autonomous Ships: A Hybrid Approach Based on Improved Apf and Modified vo Methods. *J. Mar. Sci. Eng.* **2021**, *9*, 761. [\[CrossRef\]](#)
24. Singh, Y.; Bibuli, M.; Zereik, E.; Sharma, S.; Khan, A.; Sutto, R. A Novel Double Layered Hybrid Multi-Robot Framework for Guidance and Navigation of Unmanned Surface Vehicles in a Practical Maritime Environment. *J. Mar. Sci. Eng.* **2020**, *8*, 624. [\[CrossRef\]](#)
25. Mina, T.; Singh, Y.; Min, B.C. A Novel Double Layered Weighted Potential Field Framework for Multi-USV Navigation towards Dynamic Obstacle Avoidance in a Constrained Maritime Environment. In Proceedings of the Oceans 2019 Mts/Ieee Seattle, Seattle, WA, USA, 27–31 October 2019. [\[CrossRef\]](#)
26. Ahmed, Y.A.; Hannan, M.A.; Oraby, M.Y.; Maimun, A. COLREGs Compliant Fuzzy-Based Collision Avoidance System for Multiple Ship Encounters. *J. Mar. Sci. Eng.* **2021**, *9*, 790. [\[CrossRef\]](#)
27. Huang, Y.; van Gelder, P.H.A.J.M.; Wen, Y. Velocity Obstacle Algorithms for Collision Prevention at Sea. *Ocean Eng.* **2018**, *151*, 308–321. [\[CrossRef\]](#)
28. Huang, Y.; Chen, L.; van Gelder, P.H.A.J.M. Generalized Velocity Obstacle Algorithm for Preventing Ship Collisions at Sea. *Ocean Eng.* **2019**, *173*, 142–156. [\[CrossRef\]](#)
29. Shaobo, W.; Yingjun, Z.; Lianbo, L. A Collision Avoidance Decision-Making System for Autonomous Ship Based on Modified Velocity Obstacle Method. *Ocean Eng.* **2020**, *215*, 107910. [\[CrossRef\]](#)
30. He, Y.; Jin, Y.; Huang, L.; Xiong, Y.; Chen, P.; Mou, J. Quantitative Analysis of COLREG Rules and Seamanship for Autonomous Collision Avoidance at Open Sea. *Ocean Eng.* **2017**, *140*, 281–291. [\[CrossRef\]](#)
31. Wang, B.; He, Y.; Hu, W.; Mou, J.; Li, L.; Zhang, K.; Huang, L. A Decision-Making Method for Autonomous Collision Avoidance for the Stand-on Vessel Based on Motion Process and Colregs. *J. Mar. Sci. Eng.* **2021**, *9*, 584. [\[CrossRef\]](#)
32. He, Y.; Huang, L.; Xiong, Y.; Hu, W. The Research of Ship Aca Actions at Different Stages on Head-on Situation Based on Cri and Colregs. *J. Coast. Res.* **2015**, *73*, 735–740. [\[CrossRef\]](#)
33. Chen, P.; Huang, Y.; Mou, J.; van Gelder, P.H.A.J.M. Probabilistic Risk Analysis for Ship-Ship Collision: State-of-the-Art. *Saf. Sci.* **2019**, *117*, 108–122. [\[CrossRef\]](#)
34. Yasukawa, H.; Yoshimura, Y. Introduction of MMG Standard Method for Ship Maneuvering Predictions. *J. Mar. Sci. Technol.* **2015**, *20*, 37–52. [\[CrossRef\]](#)

-
35. Guo, S.; Zhang, X.; Zheng, Y.; Du, Y. An Autonomous Path Planning Model for Unmanned Ships Based on Deep Reinforcement Learning. *Sensors* **2020**, *20*, 426. [[CrossRef](#)]
 36. He, Y.; Liu, X.; Zhang, K.; Mou, J.; Liang, Y.; Zhao, X.; Wang, B.; Huang, L. Dynamic Adaptive Intelligent Navigation Decision Making Method for Multi-Object Situation in Open Water. *Ocean Eng.* **2022**, *253*, 111238. [[CrossRef](#)]
 37. Zhang, K.; Huang, L.; Liu, X.; Chen, J.; Zhao, X.; Huang, W.; He, Y. A Novel Decision Support Methodology for Autonomous Collision Avoidance Based on Deduction of Manoeuvring Process. *J. Mar. Sci. Eng.* **2022**, *10*, 765. [[CrossRef](#)]