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Investigation of the Energy Characteristics of a Circuit under the Charge of a Supercapacitor and an Equivalent Linear Capacitor

Oleh Biletsky [1], Vitalii Kotovskyi [2], Nikolaj Višniakov [3,*] and Andžela Šešok [4]

[1] Department of Theoretical Electrical Engineering, National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Peremogy Ave. 37, 03056 Kiev, Ukraine
[2] Department of General Physics, National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Peremogy Ave. 37, 03056 Kiev, Ukraine
[3] Department of Mechanics and Material Engineering, Vilnius Gediminas Technical University, J. Basanavičiaus Str. 28, 03221 Vilnius, Lithuania
[4] Department of Biomechanical Engineering, Vilnius Gediminas Technical University, J. Basanavičiaus Str. 28, 03221 Vilnius, Lithuania
* Correspondence: nikolaj.visniakov@vilniustech.lt; Tel.: +370-682-22-866

Abstract: In this paper, the energy characteristics of the charge circuits of a supercapacitor (nonlinear capacitor) and an equivalent linear capacitor from a DC voltage source, which is a lithium-ion battery for such energy storage devices, are analyzed. It is established under what conditions the losses of electricity in the charge circuits of linear and equivalent nonlinear capacitors from the DC voltage source are reduced. The influence of final and initial voltages on similar terminals and capacitance terminals on similar energy losses is analyzed. The regularities of increasing the energy transfer coefficient in the circuits of the aperiodic charge of supercapacitors and equivalent linear capacitors from a DC voltage source (battery) with increasing initial voltages at the capacitor terminals are determined.

Keywords: battery; capacitance; charge; electricity losses; energy processes; internal resistance; nonlinearity; supercapacitor

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1. Introduction

The progressive development of energy-intensive technologies requires the development and improvement of devices capable of storing electricity and quickly providing load. Strict requirements are placed on the ability of devices to operate hundreds of thousands of cycles without deteriorating energy performance. Supercapacitors (SC) are capacitive energy storage devices that are successfully used in switching electrical systems and electric vehicles. SC are widely known as ultracapacitors, ionizers or two-layer electrochemical capacitors [1–17]. These elements occupy an intermediate area between conventional batteries and linear capacitors. The specific power of the SC is 23 times higher than the specific power of modern lithium-ion batteries [11], due to relatively low internal resistance. The capacitance of the SC can be from 1 to 10 kF, at nominal voltage (2.7 to 4 V). The voltage applied to the SC is limited by the voltage of electrochemical decay of the electrolyte, at which the electrolyte begins to decompose more intensely and the supercapacitor fails. For use at higher voltages, the SCs are connected in series and receive rated voltages of hundreds of volts.

SCs can withstand up to a million charge-discharge cycles without destroying their structure (Table 1), in contrast to lithium-ion batteries, which are capable of operating at a maximum of several thousand such cycles [11,16,17]. Rechargeable batteries are best used for steady-state power consumption. Therefore, in the combined power sources of electric vehicles, supercapacitors provide pulse modes of high power consumption in the load, and the battery provides long-term operation of the electromechanical system in steady state [3,5,8–12].

Table 1. Characteristics of supercapacitor, capacitor and lithium-ion battery.

Device Characteristics	Supercapacitor	Capacitor	Lithium-Ion Battery
Charge time	1–30 s	$10^{-3} < t < 10^{-6}$, s	$1 < t < 5$, h.
Discharge time	1–30 s	$10^{-3} < t < 10^{-6}$, s	$t > 0.3$, h.
Number of cycles	1,000,000	1,000,000	>500
Specific energy, W h/kg	<30	<0.1	<200
Specific power, W/kg	<91,000	>1,000,000	<3900

A detailed review of the results of the analysis of the energy characteristics of supercapacitors [2,8–21] showed that research has been conducted without taking into account the loss of electricity in the circuits of charge, recharging and internal redistribution of supercapacitor charges. There was also no analysis of electricity losses in the charge circuits of supercapacitors from the battery under non-zero initial voltage conditions.

The aim of this work is to compare the energy characteristics (energy transfer coefficient and energy loss) of a linear capacitor and supercapacitor, when charged by a battery, both at zero and non-zero initial voltage conditions, as well as to establish the influence of initial and final voltages on power losses in the charge circuits of linear capacitors and supercapacitors.

2. Methods

2.1. Energy Processes in the Aperiodic Charge Circuit of a Supercapacitor and an Equivalent Linear Capacitor

The dependence of the capacitance of supercapacitors on the voltage at its terminals was taken into account during the research. In studies conducted in [2,3,13–23] it was noted that the physics of the SC involves the presence of part of the capacitance, which depends on the applied voltage to its terminals. Experimental measurements of charge from a DC voltage source were performed for SCs with a capacitance of 470, 1500 and 2600 F [18,20,21].

The total capacitance of the SC consists of a series connection of many capacitors, and nanoelectrodes of primary capacitors, which are due to a double electric layer of charges on the surface of the pores of activated carbon of the main electrodes of the SC. One of the possible explanations for the reason for the increase in the total capacitance of the SC with increasing voltage is connection with the increase in the dielectric constant of the electrolyte or with a decrease in the thickness of the electric double layer [18]. The total capacitance of the IC can be represented by the expression:

$$C = \frac{\varepsilon\varepsilon_0 S}{d}, \quad (1)$$

where ε is the dielectric constant of the electrolyte, ε_0 is the dielectric constant of vacuum, S is the total surface area of the developed surface of the nanoporous electrode, and d is the thickness of the electric double layer (dielectric between the charges of the electric double layer).

The explanation of the nonlinear increase in the capacitance of the SC with increasing voltage on it is that the area of accumulation of charges in the electrodes of the SC increases with the application of higher voltages. Under such conditions, the process occurs due to the penetration of charges into the developed nanopores of electrodes of smaller diameter [3,18]. The process causes an increase in the charge accumulated in the SC, nonlinear relative to the increase in voltage at the terminals of the SC.

Under the capacitance C between two bodies, which have equal and opposite in sign charges, the absolute value of the ratio of charge on one of the bodies to the voltage between the bodies is given by

$$C = Q/U. \tag{2}$$

From the definition of capacitance follows the unit of its dimension 1 C/V = 1 F. The capacitance is between two conductive bodies that are separated by a dielectric.

In an electrostatic field, the voltage between two bodies U can be linearly expressed by the charge Q; then the ratio Q/U is independent of neither the value of Q nor the value of U. Capacitance depends only on the configuration of bodies, their size, distance between bodies, and dielectric properties. This is also true for linear capacitors (LC). Exceptions are, for example, devices that use ferroelectrics, i.e., substances in which the dielectric constant is a function of U. For supercapacitors, the magnitude of the capacitance C is a function of the voltage U [2,3,8–20]. Expression (2) is valid for both LC and SC, because it establishes the relationship between the charge accumulated on each of the plates Q and the voltage between them U.

Under the conditions of the experiment, the SC was charged from a DC voltage source [15]. Small ripples with a value of 10 mV at a frequency of 1 MHz were superimposed on the DC voltage component. The capacitance of the SC increased nonlinearly in a given voltage range at its terminals. Given this dependence, the total capacitance of the SC can be approximated, with sufficient accuracy, as the sum of the fixed capacitance C_1 = const and capacitance $C_v(U) = k\,|U|$, which linearly depends on the voltage U. The coefficient k = const and has dimension [F/V].

According to the physics of the double electric layer [2,3,18,20], the total capacitance of supercapacitors is determined by the expression:

$$C(U) = C_1 + k\,|U|. \tag{3}$$

Taking into account expression (3) for branching the modes of operation of the SC, in accordance with works [2,3,8–20], it is necessary to use branched substitution schemes. In order to take into account the distributed electrical parameters of supercapacitors, their branched substitution schemes with different numbers of parallel branches containing concentrated linear and nonlinear elements C, R and L were proposed. The schemes can be improved taking into account the duration of energy processes.

This paper investigates the change in the energy and dynamic characteristics of the SC when the voltage at its terminals changes when charged by the battery. The optimal solution of this problem is to use an equivalent scheme of substitution of SC with a variable number of parallel branches with different values of time constants $\tau = RC$ [20]. With a duration of transient processes to $\tau \leq 30$ min, the scheme of replacement of SC is shown in Figure 1, which accurately reflects the energy processes in the supercapacitor. The parameters of this substitution scheme were determined experimentally [20].

The first branch (instantaneous branch) is represented by a capacitor, the capacitance of which depends on the voltage [6,18] according to expression (3). The branch consists of elements C_1 and R_1, the values of which do not change, and the element $C_v(U_1)$, the value of which depends on the applied voltage to the terminals of the SC. This branch has a low time constant and its capacitors are charged in the second time range. The second branch (delayed branch), with constant parameters C_2 and R_2, operates in the minute time range. The third (long-term) branch has the longest time constant and operates in the range of 10 min (parameters C_3 and R_3 are also constant). To take into account the self-discharge of the SC, a substitution circuit is added in parallel resistor R_4. The battery is represented by a DC voltage source E with internal resistance R_{AB}.

In refs. [3,18] it was experimentally confirmed that the equivalent circuit substitution scheme accurately reflects its energy and dynamic characteristics in charge-discharge processes.

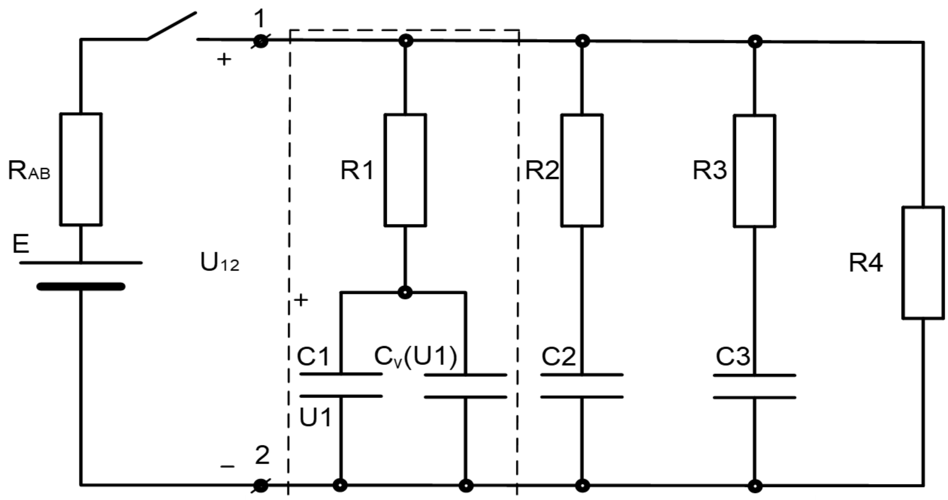

Figure 1. The scheme of replacement of SC.

Voltages $U_{R_{AB}}(t), U_{R_1}(t)$, respectively, on the resistors R_{AB}, R_1 of Figure 1, are expressed by the formulas:

$$U_{R_{AB}}(t) = R_{AB} \cdot i(t); \tag{4}$$

$$U_{R_1}(t) = R_1 \cdot i(t), \tag{5}$$

After combining the resistances of the charging circuit $R_\Sigma = R_{AB} + R_1$, according to Formula (3), we can write:

$$U_{R_\Sigma}(t) = R_\Sigma \cdot i(t) = R_\Sigma \cdot \left((C_1 + 2 \cdot k \cdot |U_{SC}(t)|) \cdot \left(\frac{dU_{SC}(t)}{dt} \right) \right) \tag{6}$$

Coefficient k = const and has the dimension (F/V).

According to Kirchhoff's second law for the scheme of Figure 1, the expression is:

$$E = U_{R\Sigma}(t) + U_{SC}(t) \tag{7}$$

Finally, the formula can be written as follows:

$$E = R_\Sigma \cdot \left((C_1 + 2 \cdot k \cdot |U_{SC}(t)|) \cdot \left(\frac{dU_{SC}(t)}{dt} \right) \right) + U_{SC}(t) \tag{8}$$

We obtain a nonlinear inhomogeneous differential equation of the first order:

$$2 \cdot k \cdot R_\Sigma \cdot |U_{SC}(t)| \cdot \left(\frac{dU_{SC}(t)}{dt} \right) + R_\Sigma \cdot C_1 \cdot \left(\frac{dU_{SC}(t)}{dt} \right) + U_{SC}(t) = E. \tag{9}$$

We introduce the following substitutions:

$$U_{SC}(t) = U, \ \frac{dU_{SC}(t)}{dt} = U'; \tag{10}$$

Differential Equation (9) can be represented as:

$$2 \cdot k \cdot R_\Sigma \cdot |U| \cdot U' + R_\Sigma \cdot C_1 \cdot U' + U = E. \tag{11}$$

This nonlinear inhomogeneous first-order differential equation can be solved to obtain an unknown function $U_{SC}(t)$. The solution can be found by direct or indirect (approximate) methods [5]. This differential equation is nonlinear, so the analytical form of the solution of such an equation must be sought using the elements of symbolic mathematics in the MATLAB application package. With complex functions of external perturbations (broken, non-integrated, discontinuous functions), it is sometimes extremely difficult to find an analytical solution. In such cases, approximate methods for solving differential equations (DE) are used.

The approximate solution of the nonlinear inhomogeneous differential equation of the 1st order in the range $t_i = 0$ to t_f will be written as the sum of two exponents:

$$U_{SC}(t) = a \cdot e^{bt} + c \cdot e^{dt}, \quad (12)$$

Coefficients (determined within 95%):

$$\mathrm{a} = 1.93(1.918 \ldots 1.942),$$

$$\mathrm{b} = 0.00281(0.002686 \ldots 0.002941),$$

$$\mathrm{c} = -1.858(-1.870 \ldots 1.845),$$

$$\mathrm{d} = -0.158(-0.1614 \ldots 0.1564).$$

Data approximation statistics:

R-square: 0.9973—the area of correlation between the initial values and approximate values. A value close to 1 indicates that the variance is negligible.

Adjusted R-square: 0.9973—this is the number of degrees of freedom of the approximated correlation area. A value close to 1 indicates a good approximation.

RMSE: 0.04401—the root of the standard error or standard error. A value close to 0 indicates that the approximation can be used successfully because, the standard deviation of the sample mean is within the norm.

2.2. Energy Characteristics of Charge of Equivalent Linear and Nonlinear Capacitors from the Storage Battery

The change in the magnitude of the charge of the capacitor in time dQ characterizes the electric current at the time of charge, based on which we have the expression:

$$i(t) = dQ/dt, \quad (13)$$

whence accordingly $dQ = i(t)\,dt$.

For SC, it should be borne in mind that the charge differential dQ is a linear part of an infinitesimal increment of a function equal to the product of $C(U)$ by U, and the formula for determining dQ has the form [3]:

$$dQ = d[C(U){\cdot}U] = (C_0 + 2\,k{\cdot}U){\cdot}dU. \quad (14)$$

After substituting expression (14) into expression (13), we obtain the formula for determining the change in the amount of current over time:

$$i(t) = (C_0 + 2\,k{\cdot}U){\cdot}(dU/dt). \quad (15)$$

The differential capacitance of the supercapacitor can be calculated by the formula:

$$C_{diff}(U) = C_0 + 2\,k{\cdot}U. \quad (16)$$

Experimental results confirm that the capacitance of the SC changes nonlinearly with changing voltage. This curve can be approximated with sufficient accuracy by the linear function $C(U)$ when the SC is used at a voltage of $0.5U_{\mathrm{n}}$ to U_{n} (where U_{n} is the nominal voltage of the SC). A number of studies [3,8–20] have confirmed that the supercapacitor stores up to 75% of energy in this voltage range.

The capacitance of ordinary linear capacitors C does not depend on the voltage of their charge U, and the energy accumulated in them under the condition of zero initial voltage is determined by the expression:

$$W_{LC} = CU^2/2. \quad (17)$$

If the SC charge started at zero initial conditions (t_i = 0 voltage U_i = 0 and the magnitude of the charge Q_i = 0), then taking into account expressions (13)–(14) we can obtain an expression for the energy W_{SC} accumulated in the SC at a certain voltage final voltage $U_f = U$:

$$W_{SC} = \int_{t_i}^{t_f} U(t) \cdot i(t)dt = \int_{Q_i}^{Q_f} UdQ = \int_{U_i}^{U_f} U(C_0 + 2kU)dU = C_0U^2/2 + 2k\,U^3 3 \quad (18)$$

The dose of energy taken from the battery (AB) W_{AB} can be found by the formula:

$$W_{AB} = \int_{t_i}^{t_f} U_{AB} i(t)dt, \quad (19)$$

where U_{AB}—the battery voltage.

The energy transfer coefficient η is equal to the ratio of the energy received in the LC or in the SC to the energy taken from the AB for the entire charge time:

$$\eta_{LC} = \left(W_{LC}\left(t_f\right) - W_{LC}(t_i)\right) / \left(W_{AB}(t_i) - W_{AB}\left(t_f\right)\right), \quad (20)$$

$$\eta_{SC} = \left(W_{SC}\left(t_f\right) - W_{SC}(t_i)\right) / \left(W_{AB}(t_i) - W_{AB}\left(t_f\right)\right), \quad (21)$$

where $W_{LC}(t_i)$, $W_{SC}(t_i)$, $W_{LC}\left(t_f\right)$, $W_{SC}\left(t_f\right)$ are energies that have been accumulated in the LC and SC in accordance with the switching and after the completion of the transient process of charge from the AB; $W_{AB} = W_{AB}(t_i) - W_{AB}\left(t_f\right)$ i.e., the energy given by AB during the transition process.

According to expressions (17)–(19) we obtain the energy of losses in the charge circuit of the LC and SC from the battery. The energy of losses is the difference between the energy given by the AB and the energy received during the charge of the LC or SC:

$$W_{loss1} = \left(W_{AB}(t_i) - W_{AB}\left(t_f\right)\right) - \left(W_{LC}\left(t_f\right) - W_{LC}(t_i)\right), \quad (22)$$

$$W_{loss2} = \left(W_{AB}(t_i) - W_{AB}\left(t_f\right)\right) - \left(W_{SC}\left(t_f\right) - W_{SC}(t_i)\right). \quad (23)$$

When modeling, the SC cannot be limited to one constant value of capacitance, as in the case of LC. If it is necessary to specify one value of capacitance, for SC it is necessary to calculate the equivalent capacitance, $C_{ekv}(U_1)$, for a specific value of voltage U_1. According to expressions (17) and (18), the SC accumulates more energy than the LC is charged to the same voltage. In a previous study [22], it was shown that when discharged and then charged to the same voltage for SC and LC with equal internal resistances ($R_{LC} = R_{SC}$), the supercapacitor has a load resistance R_n with more energy and with greater pulse power than LC.

The expression for the capacitance of the LC, C_W, which is equivalent to the capacitance of the SC, provided the equality of the accumulated energies ($W_{LC} = W_{SC}$) and the voltage on their plates ($U_{LC} = U_{SC} = U$), can be obtained from (18):

$$C_W = C_0 + 4\,kU/3 \quad (24)$$

To study and compare the energy characteristics of the circuit when charging the SC and LC from the battery, mathematical modeling was performed in the Matlab application package. The time constant was calculated by the formula $\tau = RC$. The parameters of the SC and lithium-ion battery [11,20] used in the study are given in Table 2.

Table 2. Parameters of the SC and lithium-ion battery.

Parameters of the SC Substitution Scheme								Parameters of the Lithium-Ion AB	
R_1, Ohm	C_1, F	k, F/V	R_2, Ohm	C_2, F	R_3, Ohm	C_3, F	R_4, Ohm	E, V	R_{AB}, Ohm
0.0025	270	190	0.9	100	5.2	220	9000	2.3	0.012

We assume that the initial voltage on the LC and SC was zero.

The studies were performed under the condition that the capacitance of the linear capacitor, C_W, is equivalent to the capacitance of the supercapacitor, provided that the energy stored in them is equal ($W_{LC} = W_{SC} = 2196$ (J) at $U_{LC} = U_{SC} = 0.99U_n$) calculated according to expression (24) and is equal to $C_W = 846.84$ F. The internal supports of the LC and the first (instantaneous) branch of the SC are the same: $R_{LC} = R_1 = 0.0025$ Ohms.

3. Results and Discussion

To determine the optimal charge modes from the lithium-ion battery of the supercapacitor and to compare this with similar modes of the equivalent linear capacitor, an analysis of energy characteristics was performed. An analysis of change of a dose of energy arriving in the SC and LC was carried out for energy losses in the circuit, the dose of energy taken from the battery, and the energy transfer coefficient from the battery depending on the voltage to which the SC and LC are charged. Five points were considered: point 1, in which the voltage on the SC and LC $U_{LC} = U_{SC} = 0.25U_n$; point 2, in which the voltage on the SC and LC $U_{LC} = U_{SC} = 0.5U_n$; point 3, in which the voltage on the SC and LC $U_{LC} = U_{SC} = 0.75U_n$; point 4, in which the voltage on the SC and LC $U_{LC} = U_{SC} = 0.9U_n$, and point 5, in which the voltage on the SC and LC $U_{LC} = U_{SC} = 0.99U_n$ while $(W_{LC} = W_{SC} = 2196$ (J).

The energy characteristics of SC and LC in the process of charging from AB at different voltage change ranges were considered as $U_{LC} = U_{SC} = (0.25 \ldots 0.5)U_n$; $(0.5 \ldots 0.75)U_n$; $(0.75 \ldots 0.9)U_n$; $(0.9 \ldots 0.99)U_n$.

Figure 2 shows the functional dependencies.

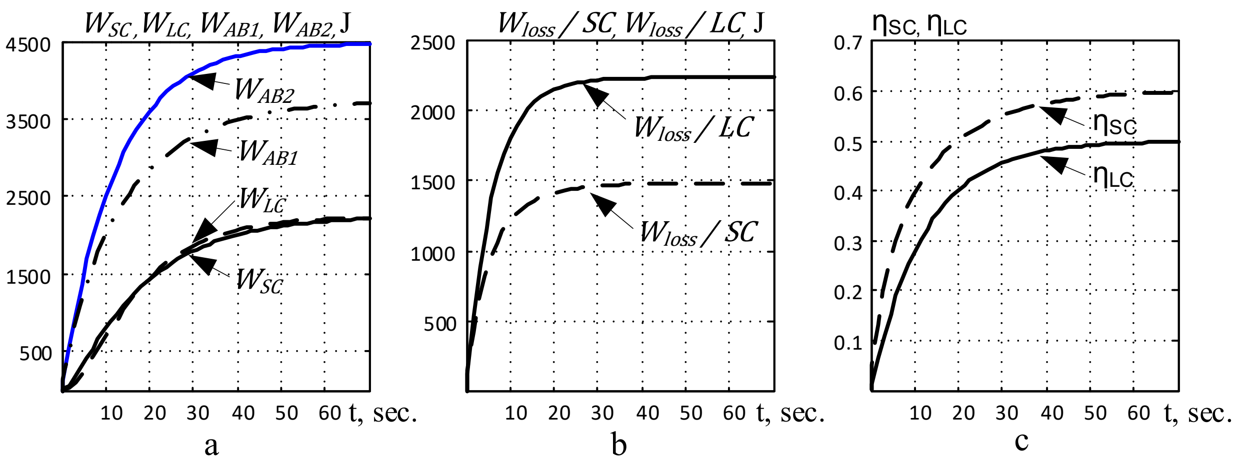

Figure 2. Functional dependences: (**a**) = the dose of energy W_{SC}, which enters the SC at each moment of its charge and the energy dose W_{AB1}, which is selected from the AB; the dose of energy W_{LC}, which enters the LC at each time of its charge and the dose of energy W_{AB2}, which is selected from the AB; (**b**) = energy losses W_{loss} in the charge circuit LC and SC; (**c**)—energy transfer coefficients from the AB when charging the linear capacitor η_{LC} and supercapacitor η_{SC}.

Table 3 shows the energy characteristics (energy doses received in the SC and LC; energy doses taken from the battery, energy losses in the circuit; energy transfer coefficient from the AB) and time t_{LC} and t_{SC} at points 1–5, when the voltages at SC and LC were equal to $0.25,\ 0.5,\ 0.75,\ 0.9$ and $0.99U_n$ under the condition of equality of the accumulated energies in them $(W_{LC} = W_{SC} = 2196$ (J) at $U_{LC} = U_{SC} = 0.99U_n)$.

Table 3. Energy characteristics and time t_{LC} and t_{SC} at points 1–5.

Points	1	2	3	4	5
U_{LC}, V	$0.25U_n$	$0.5U_n$	$0.75U_n$	$0.9U_n$	$0.99U_n$
W_{SC}, J	68.77	371.70	1052.00	1703.00	2196.00
W_{LC}, J	140.10	560.10	1260.00	1816.00	2195.00
W_{AB}, J/SC	501.50	1293.00	2372.00	3159.00	3680.00
W_{AB}, J/LC	1120.00	2240.00	3360.00	4034.00	4435.00
W_{loss}, J/SC	432.73	921.30	1320.00	1456.00	1484.00
W_{loss}, J/LC	979.90	1679.90	2100.00	2218.00	2240.00
η_{SC}, %	13.7	28.7	44.3	53.9	59.7
η_{LC}, %	12.5	25.0	37.5	45.0	49.5
t_{SC}, s	1.604	5.166	13.490	26.810	63.910
t_{LC}, s	3.533	8.511	17.020	28.330	56.560

The results of numerical simulations in the Matlab application package in Table 3 confirmed the correctness of the analytical dependences (3)–(24).

Table 3 shows that when the SC was charged from $U_{0SC} = 0$ to a voltage of $0.25U_n$, the energy transfer coefficient from AB η_{SC} increased 1.1 times compared to the energy transfer coefficient in LC η_{LC} under similar conditions. With a further increase in voltage to $0.5U_n$, the coefficient η_{SC} nonlinearly increases to 28.7%, which is 1.15 times greater than the coefficient η_{LC}. It is known that when a fully discharged LC is charged, its energy transfer coefficient is 50%. This study confirmed that the energy transfer coefficient of the SC when charged from the voltage $U_{0SC} = 0$ is 60%.

The dose of energy entering the LC when charged to a voltage of $0.25U_n$ is two times greater than the dose of energy entering the SC, while the energy of losses in the charge circuit of the LC is 2.26 times greater than the losses in the SC circuit. When the LC is charged to a voltage of $0.75U_n$, the dose of energy entering the LC is 1.2 times greater than the dose of energy entering the SC, while the losses are 1.59 times greater than the losses in the SC charge circuit. When the LC and SC are charged to a voltage of $0.99U_n$, the doses of accumulated energies in them are the same at $W_{LC} = W_{SC} = 2196$ (J), and the energy of losses in the LC charge circuit are 51% higher than the losses in the SC charge circuit.

The dose of energy taken from the battery when charging the LC to a voltage of $0.25U_n$ is 2.23 times greater than the dose of energy taken when charging the SC. When the LC is charged to voltages of $0.75U_n$ and $0.9U_n$, the dose of energy taken from the battery is 1.42 times higher.

Table 4 shows the energy characteristics when charging a linear capacitor and a supercapacitor in different ranges of voltage change: $(0.25\ldots0.5)U_n$; $(0.5\ldots0.75)U_n$; $(0.75\ldots0.9)U_n$; $(0.9\ldots0.99)U_n$ and time intervals t_{SCin} and t_{LCin}, during which the voltage changes in the specified ranges.

From Table 4 it follows that when recharging from the battery, the energy transfer coefficient for LC and SC is in the range of 37.5 to 95%. When the voltage changes within $(0.25$ to $0.5)U_n$, the energy transfer coefficients at the charge of LC and SC are 37.5 and 38.3%, respectively. As the voltage increases, the coefficients η_{SC} and η_{LC} increase almost equally ($\eta_{SC} > \eta_{LC}$ by 0.5 to 0.8% in all ranges). When the SC and LC charge in the range $(0.9$ to $0.99)U_n$, the energy transfer coefficient of the process increases to 95%.

Table 4. Energy characteristics when charging a linear capacitor and a supercapacitor in different ranges of voltage change.

The Range of Change Voltage, $(U_i - U_n)/U_n$, V	**0.25 . . . 0.5**	**0.5 . . . 0.75**	**0.75 . . . 0.9**	**0.9 . . . 0.99**
W_{SCin}, J	302.93	680.30	651.00	493.00
W_{LCin}, J	420.00	699.90	556.00	379.00
W_{ABin}, J/SC	791.50	1079.00	787.00	521.00
W_{ABin}, J/LC	1120.00	1120.00	674.00	401.00
$W_{loss\ in}$, J/SC	488.57	398.70	136.00	28.00
$W_{loss\ in}$, J/LC	700.00	420.10	118.00	22.00
η_{SCin}, %	38.3	63.0	82.7	95.0
η_{LCin}, %	37.5	62.5	82.5	95.0
$t_{SCin,}$ s	35.62	83.24	13.320	37.100
$t_{LCin,}$ s	49.78	85.09	11.310	28.230

The dose of energy entering the LC when the voltage changes within $(0.25 \text{ to } 0.5)U_n$ is 1.39 times greater than the dose of energy entering the SC, while the energy loss is 1.43 times higher in the charge circuit of the LC. With increasing voltage on the LC and SC in the range $(0.5 \text{ to } 0.75)U_n$, they accumulate almost the same energy, but the loss in the charge circuit of the LC is 5.3% higher. In the ranges of voltage change $(0.75 \text{ to } 0.9)U_n$ and $(0.9 \text{ to } 0.99)U_n$ dose of energy entering the SC in 1.17 and 1.3 times higher than the dose of energy entering the LC, but losses are 1.15 and 1.27 times more.

The dose of energy taken from the AB when charging the LC in the range $(0.25 \text{ to } 0.5)U_n$ is 1.4 times greater than the energy dose for the SC. In the voltage range $(0.75 \text{ to } 0.9)U_n$ and $(0.9 \text{ to } 0.99)U_n$ the dose of energy taken from the AB when the charge of the SC in 1.17 and 1.30 times higher the energy dose for the charge of the LC.

Thus, the most energy-efficient charge occurs in the voltage range $(0.9,\ 0.99)U_n$. At equally high energy transfer coefficients ($\eta_{SC} = \eta_{LC} = 0.95$), the advantage of SC is that it accumulates 30% more energy than LC.

When the LC and SC are charged to a voltage of $0.99U_n$, the doses of accumulated energies in them are the same (according to the selected initial conditions) $W_{LC} = W_{SC} = 2196$ (J), and the energy of losses in the LC charge circuit are 51% higher than the SC charge circuit losses. In such modes of operation, the supercapacitor is more energy efficient.

The advantage of supercapacitors is that they can have a thousand times higher energy than linear capacitors [12,24]. In terms of specific energy, SCs are equivalent to modern lead-acid batteries [8], and the specific power of SCs is 23 times higher than the specific power of serial lithium-ion batteries [11].

The development and improvement of technology for the production of supercapacitors has led to the emergence of new materials for electrodes, such as three-dimensional porous carbon. This type of carbon has the properties of a supercapacitor, with a thickness of only one atom. The surface area of one gram of the material is 3100 square meters. Treatment of the above material with potassium hydroxide leads to the creation of a large number of tiny pores in the carbon, which, in combination with the electrolyte are able to retain a colossal electric charge. This material has ideal properties for a supercapacitor and industry is fully prepared for its production.

With the advent of new serial samples of supercapacitors, it will be necessary to improve equivalent substitution schemes, taking into account the physical properties of the material of the electrodes of the SC. It will be important to take into account the dependence of the electrical parameters of the SC on frequency, voltage and ambient temperature. This substitution scheme can be refined to study longer processes that occur in the deep layers for more than 30 min.

4. Conclusions

1. Analysis was made of the energy characteristics of the linear capacitor and supercapacitor when charged from a lithium-ion battery, taking into account the condition that the capacitance of the linear capacitor C_W = 846.84 F is equivalent to the capacitance of the supercapacitor, provided they have accumulated energy $W_{LC} = W_{SC} = 2196$ (J) at $U_{LC} = U_{SC} = 0.99U_n$). The study was performed in the Matlab application package.
2. It was established that at charge of completely discharged SC and LC (provided that their internal resistances R_{LC} = R_1 = 0.0025 Ohm) the energy transfer coefficient increases nonlinearly. In order to reduce energy losses, the charge of SC and LC to voltages higher than $0.9U_n$ with the energy transfer coefficient $\eta_{SC} \geq 53.9$ and $\eta_{LC} \geq 45\%$, is optimal.
3. When charging a fully discharged supercapacitor to a voltage of $0.99U_n$, its energy transfer coefficient is η_{SC} = 59.7, which is 1.2 times higher than the energy transfer coefficient of a linear capacitor. When the LC and SC are charged to a voltage of $0.99U_n$, the doses of accumulated energies in them is the same (according to the selected initial conditions) $W_{LC} = W_{SC} = 2196$ (J), and the energy of losses in the LC charge circuit are 51% higher than the SC charge circuit losses. In such modes of operation, the supercapacitor is more energy efficient.
4. Comparison of research results for different ranges of voltage change during LC and SC charge was performed. It was established that in all studied ranges of voltage change, the energy transfer coefficient at the SC charge exceeds the energy transfer coefficient at the LC charge by 0.5 to 0.8%. Their values, depending on the range of voltage changes are 37.5 to 95% (with increasing voltage, the energy efficiency of the charge increases). When the SC and LC charge in the range $(0.9 \text{ to } 0.99)U_n$, the energy transfer coefficients in the SC and LC increase to 95%. The advantage of an SC is that it accumulates 30% more energy than an LC.
5. With the advent of new serial supercapacitors, it will be necessary to improve substitution schemes, taking into account the material properties of the electrodes of the SC. When using an equivalent substitution scheme for the analysis of longer energy processes in the SC (lasting more than 30 min), additional parallel branches in the scheme with large time constants will need to be entered.

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References

1. Thomas, S.; Amadou; Gueye, B.; Ram; Gupta, K. *Nanostructured Materials for Supercapacitors*; Springer Nature Switzerland AG.: Cham, Switzerland, 2022; 642p.
2. Izotov, V.Y.; Gromadsky, D.G.; Maletin, Y.A. Simulation and calculation of the operating parameters of the supercapacitor. *Naukovi Visti NTUU "KPI"* **2008**, *6*, 114–118.
3. Shcherba, A.A.; Suprunovskaya, N.I.; Beletsky, O.A. Power characteristics of supercapacitors during their charge from a source of voltage and discharge on resistive load. In *Pratsi Instytutu Elektrodynamiky Natsionalnoi Akademii Nauk Ukrainy*; Academy of Sciences of Ukraine: Kyiv, Ukraine, 2014; pp. 65–74.
4. Biletskyi, O.O.; Kotovsyi, V.Y. Energy processes in combined power supplies with linear capacitors and supercapacitors. *Semicond. Phys. Quantum Electron. Optoelectron.* **2019**, *22*, 326–332. [CrossRef]
5. Hassan, H.; El-Tawil, M.A. A new technique of using homotopy analysis method for second order nonlinear differential equations. *Appl. Math. Comput.* **2012**, *219*, 708–728. [CrossRef]

6. Zhao, J.; Burke, A.F. Electrochemical capacitors: Materials, technologies and performance. *Energy Storage Mater.* **2020**, *36*, 31–55. [CrossRef]
7. Jiya, I.N.; Gurusinghe, N.; Gouws, R. Electrical Circuit Modelling of Double Layer Capacitors for Power Electronics and Energy Storage Applications: A Review. *Electronics* **2018**, *7*, 268. [CrossRef]
8. Burke, A.F. Batteries and Ultracapacitors for Electric, Hybrid, and Fuel Cell Vehicles. *Proc. IEEE* **2007**, *95*, 806–820. [CrossRef]
9. Zhang, L.; Hu, X.; Wang, Z.; Sun, F.; Dorrell, D.G. A review of supercapacitor modeling, estimation, and applications: A control/management perspective. *Renew. Sustain. Energy Rev.* **2018**, *81*, 1868–1878. [CrossRef]
10. Kim, T.; Song, W.; Son, D.-Y.; Ono, L.K.; Qi, Y. Lithium-ion batteries: Outlook on present, future, and hybridized technologies. *J. Mater. Chem. A* **2019**, *7*, 2942–2964. [CrossRef]
11. Burke, A.; Miller, M. The power capability of ultracapacitors and lithium batteries for electric and hybrid vehicle applications. *J. Power Sources* **2011**, *196*, 514–522. [CrossRef]
12. Burke, A.; Miller, M.; Zhao, H. *Ultracapacitors in Hybrid Vehicle Applications: Testing of New High Power Devices and Prospects for Increased Energy Density*; Research Report—UCD-ITS-RR-12-06; Institute of Transportation Studies: Berkeley, CA, USA; University of California: Oakland, CA, USA, 2012.
13. Kurzwell, P.; Frenzel, B. Capacitance characterization methods and ageing behavior of supercapacitors. In Proceedings of the International Seminar on Double Layer Capacitors, Deerfield Beach, FL, USA, 5–7 December 2005.
14. Eleri, O.E.; Lou, F.; Yu, Z. Characterization Methods for Supercapacitors. In *Nanostructured Materials for Supercapacitors, Advances in Material Research and Technology*; Springer: Cham, Switzerland, 2022; pp. 101–128. [CrossRef]
15. Maletin, Y.; Novak, P.; Shembel, E.; Izotov, V.; Strizhakova, N.; Mironova, A.; Danilin, V.; Podmogilny, S. Matching the Nanoporous Carbon Electrodes and Organic Electrolytes in Double Layer Capacitors. *Appl. Phys. A Mater. Sci. Process.* **2006**, *82*, 653–657. [CrossRef]
16. Mihailescu, B.; Svasta, P.; Varzaru, G. Hybrid Supercapacitor-Battery electric system with low electromagnetic emissions for automotive applications. *UPB Sci. Bull. Ser. C* **2013**, *75*, 277–290.
17. Zuo, W.; Li, R.; Zhou, C.; Li, Y.; Xia, J.; Liu, J. Battery-Supercapacitor Hybrid Devices: Recent Progress and Future Prospects. *Adv. Sci.* **2017**, *4*, 1600539. [CrossRef] [PubMed]
18. Rafik, F.; Gualous, H.; Gallay, R.; Crausaz, A.; Berthon, A. Frequency, thermal and voltage supercapacitor characterization and modeling. *J. Power Sources* **2007**, *165*, 928–934. [CrossRef]
19. Kotovskyi, V.Y.; Biletskyi, O.O. Investigation of energy processes in circuits of oscillatory charge of supercapacitors. *Visnyk NTUU KPI Radiotekhnika Radioapar.* **2019**, *76*, 5–14.
20. Zubieta, L.; Bonert, R. Characterization of double-layer capacitors for power electronics applications. *IEEE Trans. Ind. Appl.* **2000**, *36*, 199–205. [CrossRef]
21. Tshiani, C.T.; Umenne, P. The Characterization of the Electric Double-Layer Capacitor (EDLC) Using Python/MATLAB/Simulink (PMS)-Hybrid Model. *Energies* **2022**, *15*, 5193. [CrossRef]
22. Shcherba, A.A.; Suprunovskaya, N.I.; Maslak, L.P.; Beletsky, O.A. Synthesis of charge-discharge circuits of thyristor discharge pulse generators with positive voltage feedback. *Energy Sav. Energy Energy Audit.* **2013**, *2*, 90–95.
23. Suprunovskaya, N.I.; Shcherba, A.A.; Beletsky, O.A. Processes of energy exchange between non-linear and linear links of the electric circuit of supercapacitor replacement. *Tech. Electrodyn.* **2015**, *201*, 3–11.
24. Biletskyi, O.O.; Suprunovs′ka, N.I.; Shcherba, A.A. Zalezhnist′ enerhetychnykh kharakterystyk kil zaryadu superkondensatoriv vid yikh pochatkovykh i kintsevykh napruh [Dependences of power characteristics of circuit at charge of supercapacitors on their initial and final voltages]. *Tekhnichna Elektrodynamika* **2016**, *1*, 3–10.