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Minimizing Intersection Waiting Time: Proposal of a Queue Network Model Using Kendall's Notation in Panama City

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Abstract: The paper presents a proposed queuing model based on Kendall's notation for the intersection of two streets in Panama City (53 East and 56 East). The proposed model is based on a set of traffic lights that controls the flow of vehicles at the intersection according to a predetermined schedule. The model analyzes the stability of the system and simulations are performed to evaluate its performance. The main objective of the paper is to optimize the vehicle flow by minimizing the waiting time for passage. In the study, it was observed that the current traffic light system on Calle 50 (50th Street) is unstable and oversaturated during weekdays, which generates large vehicle queues with no estimated exit times. It was proposed to increase the system capacity to 1300 vehicles per hour to achieve reasonable stability and provide a solution to improve traffic signal timing on 50th Street. The need to increase the system capacity has been demonstrated and an optimal value has been suggested. The evaluation of other models and the use of AI can further strengthen the system and improve the prediction accuracy in different traffic scenarios.

Keywords: vehicular traffic flow; urban intersection; optimal traffic light control; congestion; queuing theory; queuing network model; Kendall notation; optimization; simulation

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1. Introduction

Steady population growth and urbanization has generated a significant increase in the number of vehicles in cities, leading to traffic and congestion problems on roads and intersections [1,2]. These difficulties are the result of increasing traffic demand compared to the insufficient capacity of existing roads [3].

Intersections in urban centers are critical points where there is an accumulation of vehicles competing for right-of-way. Currently, traffic management at these intersections is performed in a static manner. The times assigned to traffic signals are constant and do not adjust to changing traffic conditions in real time. This results in suboptimal performance, as traffic signals may be assigning unnecessarily long or short green times, which interferes with efficient traffic flow [4–6].

Inefficient traffic management has a significant impact on the quality of life of citizens; long waits, delays and traffic jams generate considerable time loss and increase driver stress. In addition, traffic congestion also has a negative impact on the environment due to the emission of polluting gases and additional fuel consumption [7,8].

To address this problem, it is necessary to look for innovative solutions that optimize traffic management at intersections. In this sense, this article proposes a mathematical model based on queuing theory for the analysis of vehicular flow in one of the most congested arteries of Panama City. These mathematical models allow performing stability analysis and estimating the optimal capacity of the queuing system to achieve a more efficient traffic management. In addition, the use of artificial intelligence (AI) offers new opportunities to improve the accuracy and predictive capacity of traffic management models. AI can analyze large volumes of real-time data and adjust traffic signal timings dynamically according to traffic conditions, which can significantly improve the performance and efficiency of traffic management at intersections [9–11].

After the application of the method (for a large number of cases and under real traffic restriction systems), efforts are made to obtain temporal sequences for traffic lights that allow them to operate in a coordinated manner and allow the passage of a large number of vehicles through the intersection, thus reducing waiting times and queue lengths, allowing the proposed model to serve as a basis for the approach of similar problems and for the creation of better methods. The results may be of interest to researchers and practitioners in the field of transportation engineering, and the methodology and approach of the paper may serve as a valuable reference for similar studies in other cities facing similar problems.

The rest of the paper is as follows: Section 2, Related Research; Section 3, Theoretical Context; Section 4, Methodology; Section 5, Problem Definition and Motivation; Section 6, Mathematical Model; Section 7, Results and Discussion; Section 8, Conclusions; and finally in Section 9, Future Work.

2. Related Research

This section discusses related research in the field of traffic flow optimization.

A study by Manh and Thi [12] focuses on determining the queue model and performance measures of a motorcycle parking area during rush hours at Hanoi University of Science and Technology (HUST), specifically the D3–D5 parking lot. Data collection is done using the observation approach, which involves recording the number of arrivals and service time. The arrival distribution and service time distribution are tested using the one-sample Kolmogorov–Smirnov test with SPSS Statistics. The analysis of the collected data reveals that the arrival distribution follows a Poisson distribution, while the service time distribution follows an Exponential distribution. By applying Kendall Notation and considering the results of the data analysis, the queue model for the parking lot is determined to be (M/M/4): (FIFO/∞/∞). This notation indicates that the arrivals and service times both follow the Markovian process; the system has four servers, and the queue operates on a first-in, first-out (FIFO) basis with infinite queue capacity and infinite population capacity. The proposed queue model aligns well with the actual observed queue model at the D3–D5 parking lot. This finding provides valuable insights into understanding the queuing behavior and performance of the motorcycle parking area during rush hours at HUST.

The research of Liu et al. [13] proposes the development of a system that allows the visualization of information posted on social networks about traffic incidents. Feature engineering methods, such as vector counting and TF-IDF, were applied to process tweets into structured data. Machine Learning models were created for traffic-related tweet classification using SVM, Naïve Bayes, Random Forest, and XGBoost. The prediction models resulted in a classification model that detects incident or non-incident tweets and a categorization model that determines the type of incident (accident, hazard, or obstacle). This system has advantages, such as speeding up the detection and visualization of traffic incidents, which can significantly help the country's traffic authorities and the public.

Antoine et al. [14] present a new queuing model with multiple servers aimed at optimizing traffic signals to enhance the sustainability of urban mobility. The model focuses on analyzing the queue information for different movement patterns based on the arrival rate of cars on each road at the intersection. By utilizing queuing theory concepts, the collected data is processed and evaluated. The performance metrics considered in this proposed model include arrival rate, waiting time, average number of cars in the queue, and intersection utilization. These metrics are analyzed and assessed using ground truth data to determine the effectiveness of the queuing model. The numerical results obtained from the analysis are visually presented and indicate that the proposed approach of the queuing model reduces delays by improving the throughput of the intersection. Consequently, this leads to smoother traffic flow and decreased congestion for users of the Giporoso intersection in Kigali. The findings highlight the potential benefits of implementing the proposed queuing model in optimizing traffic signal operations and enhancing urban mobility sustainability.

Li et al. [15] introduce the concept of a birth and death process to analyze vehicle behavior and propose a traffic model based on queuing theory to simulate road traffic. They specifically focus on two scenarios: the steady state and the congested state of the traffic system. In the steady state, they obtain statistical variables such as the number of vehicles and the waiting time for vehicles to pass through a specific section of the road. These variables provide valuable information to drivers and contribute to their decision-making process. In addition, the authors use the proposed model to simulate traffic conditions and analyze the steady-state distribution of the number of vehicles. Comparison of the theoretical results with the simulation results allows validating the effectiveness of the model. Overall, the study demonstrates the applicability of the birth–death process and the traffic model based on queuing theory to describe and simulate traffic behavior. The results obtained confirm the validity and reliability of the proposed model, providing valuable insights for understanding and predicting traffic dynamics.

The research of Gunes et al. [16] presents the analysis results obtained from real data collected in the field. The study applies queuing models to analyze the collected data and explores the impact of improving signal durations on the obtained results. By optimizing the signal timings based on the data, the study examines the effects on parameters such as queue lengths and overall time spent in the system. The findings indicate that improving the signal durations leads to a reduction in queue lengths and the time consumed within the system. By utilizing queuing models, the study provides valuable insights into the relationship between signal timings and system performance, ultimately offering potential solutions for mitigating traffic-related challenges in urban areas.

3. Theoretical Context

Over the last fifty years, a wide range of traffic flow theories and models have been developed as tools to solve the economic and social problems arising from high vehicular demand. Research aims to optimize the efficiency of existing traffic systems, thereby increasing vehicle capacity [17,18].

3.1. Traffic and Vehicular Flow

Vehicular flow is the phenomenon caused by the flow of vehicles on a road, street, or highway. It also has many similarities in other phenomena, such as the flow of particles (liquids, gases, or solids) and pedestrians. In large cities, vehicular flow is present in almost all spheres of people's daily activities and causes numerous phenomena, among which congestion stands out [19,20].

3.2. Queuing Theory, Kendall's Notation

Queuing theory is the study of a technique based on operations research to solve problems that arise in situations where waiting for shifts or queues are formed for the provision of a service or execution of a job [21–23].

Among the most important terms that comprise the queuing theory [24–26] are "Customers," which refer to the entity that arrives at the system, such as: Cars waiting at a traffic light, machines waiting to be repaired, airplanes waiting to land, among others. "Arrivals" refers to the number of customers arriving at the service facility. "Service Rate" is used to designate the service capacity, which can be provided by one server or by multiple servers. "The Arrival Rate" describes in units of time the feeding of the system. "The

Server" oversees providing the respective service to the client. "The Queue Capacity" can be infinite or finite. Given the above, different queue models can be presented, with corresponding efficiency measures that characterize the system [27–29].

D.G. Kendall suggested a valuable notation for classifying the vast diversity of different wait-line models that have been developed [30]. Kendall's notation of three symbols is as follows: A/B/K, where A indicates the probability distribution of arrivals, B indicates the probability distribution of service times, and K indicates the number of channels. Various waiting line systems can be described depending on which letter appears in the A or B position.

Commonly used letters are M, which designates a Poisson probability distribution for arrivals or exponential probability distribution for service time; D, which designates the fact that arrivals or service time is deterministic or constant, and G, which indicates that arrivals or service time have a general probability distribution, with known mean and variance [31,32].

3.3. Artificial Intelligence (AI) Application

Artificial intelligence (AI) can be key in optimizing vehicular flow. For example, intelligent traffic control systems (ITS) use AI techniques to monitor traffic and adjust traffic lights and signals in real time [33]. These systems can help reduce waiting times and improve traffic flow at intersections. Intelligent transportation systems (ITS) can use technologies such as traffic sensors, surveillance cameras, and navigation systems to collect real-time traffic data and analyze it using AI techniques. ITS systems can automatically adjust traffic lights, direct traffic to alternate routes, and provide real-time information to drivers and pedestrians [34].

In recent years, intelligent transportation systems (ITS) has received considerable attention due to increased road safety and efficiency demands in highly interconnected road networks. As an essential part of ITS, traffic forecasting can provide support in many aspects, such as road routing, traffic congestion control, applications, etc., and analyze how traffic forecasting can improve the performance of these applications [35].

AI can predict traffic behavior and vehicular flow under different conditions. For example, machine learning models can analyze extensive traffic data sets to identify patterns and trends. These models can be used to predict traffic demand at different times of the day and in different weather conditions [36].

Some practical cases of the use of artificial intelligence in the optimization of vehicular traffic can be observed in the work of Guo and Yan [37]. They propose an intelligent network control architecture based on SDN and artificial intelligence. The proposed architecture consists of three modules: a network state collection/perception module, an AI intelligent analysis module, and an SDN controller module. The experimental results demonstrate that using SDN and artificial intelligence in operator networks can do intelligent network control and traffic optimization more intelligently.

Nam Bui and Jung [4] propose a game–theoretic approach of cooperative games among agents to improve traffic flow within a large network. For this purpose, a distributed merge-and-split algorithm for coalition formation is presented. This algorithm is applied to discover how to incorporate cooperation among agents to dynamically control the traffic light at intersections. In addition, a traffic simulation framework is constructed to evaluate our approach. With various parameters for traffic density, the proposed system can effectively improve both uniform and non-uniform traffic flow. Through inter-controller coordination, the waiting time of vehicles at intersections can be reduced by 15% to 25% compared with previous methods (e.g., Green Wave coordination).

Yang et al. [1] propose multiagent reinforcement learning for traffic signals (MARL4TS) to support traffic signal control and deployment. First, information about traffic flows and multiple intersections is formalized as input environments for reinforcement learning. Second, the authors design a new reward function to continuously select the most appropriate strategy as control during multi-agent learning to track traffic signal actions. Finally, they use a supporting tool, Simulation of Urban Mobility (SUMO), to simulate the proposed traffic signal control process and compare it with other methods. Experimental results show that our proposed MARL4TS method is superior to the baselines.

Li et al. [6] propose a deep feature learning approach using supervised learning techniques to predict the short-term traffic flow in the next multiple steps. To achieve next-day traffic flow forecasting, an advanced multi-objective particle swarm optimization algorithm is applied to optimize some parameters in deep belief networks.

The work of Shengdong et al. [38] aims to discuss problems such as complex object types, large amounts of data collection, high transmission and computational demand, and weak real-time control and scheduling capability in constructing modern intelligent traffic information, physical fusion networks and cloud-based control. The underlying theory for modern intelligent traffic network control system is a current research topic. In the same way, the best design of the physical scheme is investigated to achieve an integrated control system that allows to link the transport information in an intelligent way. The scheme includes intelligent transportation edge control technology and intelligent transportation network virtualization technology. Based on the intelligent traffic flow data, various deep learning study methods are implemented in the cloud to predict the traffic flow and congestion of urban roads.

4. Methodology

Figure 1 describes the development process of this mathematical model to optimize the vehicular flow of Calle 50 in Panama City:

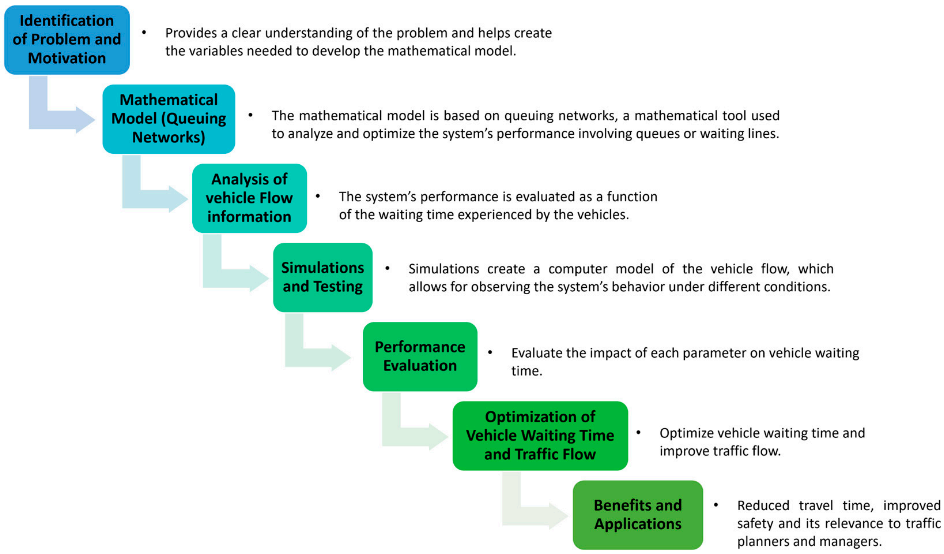

Figure 1. Block diagram of the model.

The project begins by identifying the problem and the motivation for the study. This step is essential, as it provides a clear understanding of the problem and helps create the variables needed to develop the mathematical model.

The mathematical model is based on queuing networks, a mathematical tool used to analyze and optimize the system's performance involving queues or waiting lines. In the context of the study, the queuing network is used to model the flow of vehicles. The system's performance is evaluated as a function of the waiting time experienced by the vehicles.

To check and verify the effectiveness of the mathematical model, simulations, and tests are performed. These simulations create a computer model of the vehicle flow, which allows for observing the system's behavior under different conditions. By varying different parameters, such as traffic volume, road capacity, and traffic control mechanisms, the impact of each parameter on the waiting time experienced by vehicles can be evaluated.

The research aims to optimize vehicle waiting time and improve traffic flow. This can bring several benefits, such as reduced travel time, increased safety, and improved fuel efficiency. The research can also help traffic planners and policymakers make more informed decisions and implement more effective traffic management strategies.

- Summary: The problem and motivation of the study are identified; this will allow us to create the necessary variables to develop the mathematical model based on queuing networks and thus be able to perform the information analysis of the vehicular flow. Finally, we perform tests and simulations to verify the optimization of the waiting time experienced by vehicles.

5. Problem Definition and Purpose

Traffic congestion is a common problem in many cities, and Panama City is no exception. The increase in population has led to an increase in vehicle traffic, especially during peak hours on weekdays. During these times, such as commute and lunch time, the city center becomes chaotic and traffic congestion becomes a frustrating experience for drivers. Despite the efforts made, such as the creation of exclusive lanes for the metro bus and the implementation of the Panama subway, this problem has not been completely solved [39–42].

In this context, this research focuses on addressing the problem of vehicular congestion in Panama, specifically on 50th Street, which is one of the most used roads for vehicular flow in the city (see Figures 2–4). The main objective is to develop mathematical models using queuing theory to understand and analyze the current congestion situation on 50th Street, particularly at the intersections of Street 53 East and Street 56 East [43].

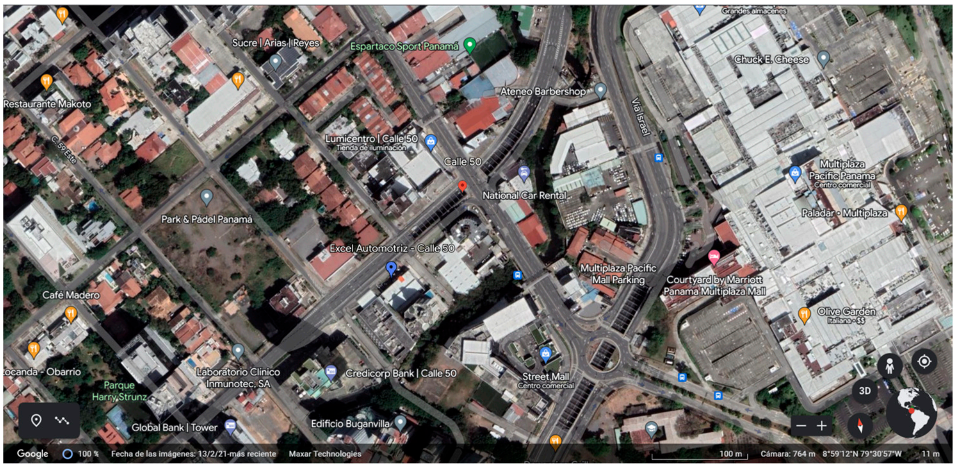

Figure 2. Calle 50, the most used route in Panama (Source: via Google Earth).

The proposed model seeks to reduce the time individuals spend daily in these vehicular congestions. To achieve this, queuing theory techniques will be used to model traffic flow at these intersections and establish optimal traffic signal timings. The goal is to improve the efficiency of vehicular flow and reduce congestion on the 50th Street section.

By using mathematical models and queuing theory techniques, it is hoped to obtain traffic signal timings that will help decongest the road and improve the driving experience for citizens. This, in turn, could have a positive impact on reducing travel time, reducing air pollution, and saving fuel. Ultimately, this research aims to provide recommendations and practical solutions to address the problem of vehicular congestion in Panama City and improve the quality of life of its inhabitants.

Figure 3. Route map of Calle 50 street in Panama City (Source: via Google Maps).

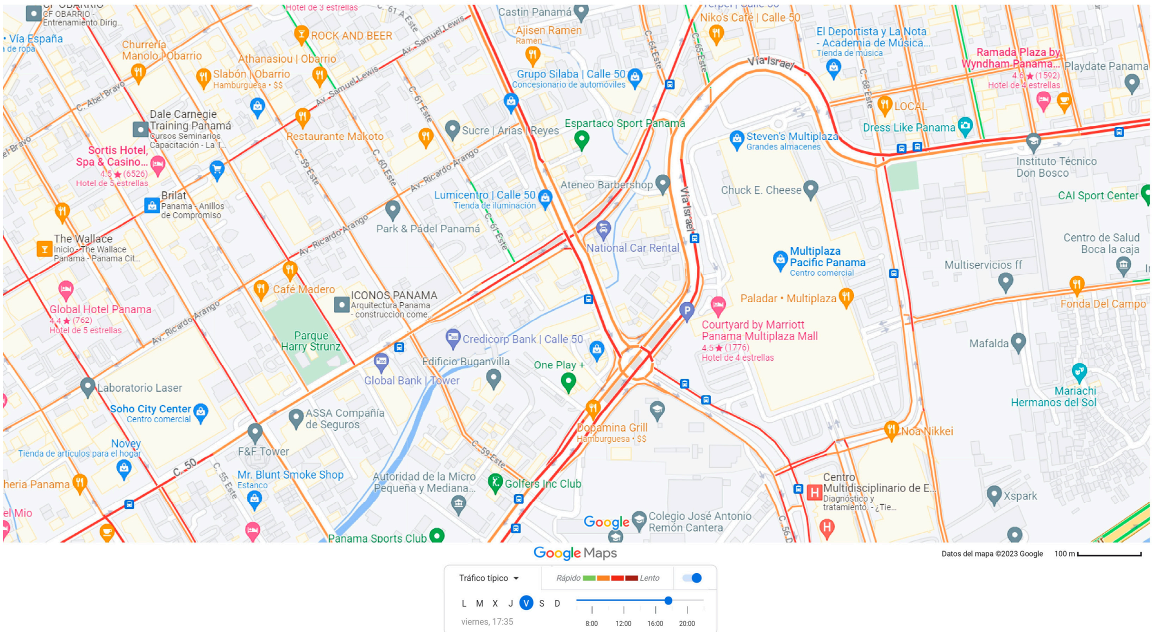

Figure 4. Traffic on 50th Street on a normal day (Source: via Google Maps).

6. Mathematical Model

The mathematical model proposed to address the problem of vehicular congestion on 50th Street is based on the implementation of an M/M/1 queuing system. This queuing system is characterized by having exponentially distributed arrival and service times, which means that the time intervals between the arrival of vehicles and the duration of their service follow an exponential distribution.

In this model, the system is considered to have a single server, which implies that only one vehicle can be served at a time. Queue discipline is governed by the FIFO (First In, First Out) principle, meaning that vehicles are serviced in the order in which they arrive at the intersection.

In addition, the size of the entry population is assumed to be infinite, implying that the number of vehicles entering the system does not affect the arrival rate of new vehicles.

This assumption is because vehicular congestion on 50th Street is primarily related to intersection capacity and waiting times, rather than the total number of vehicles in the city.

By implementing this M/M/1 queuing model, we seek to understand and analyze how traffic flow behaves on 50th Street and how it affects the waiting times experienced by drivers. Through mathematical techniques and statistical analysis, important performance measures such as arrival rate, average waiting time, and average number of vehicles in the queue can be obtained (see Figure 5) [44,45].

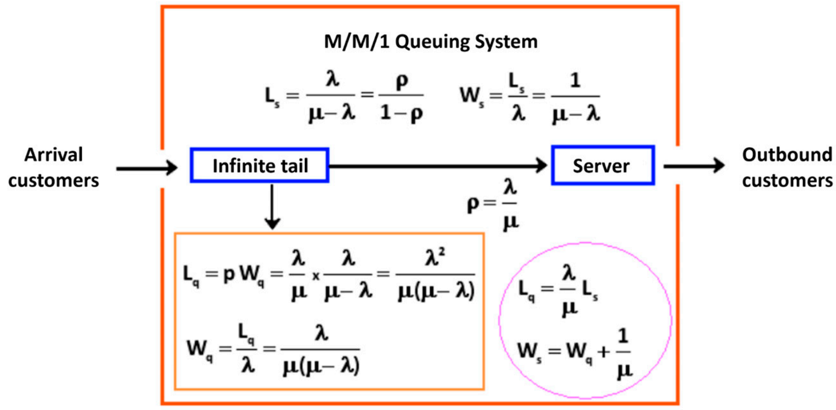

Figure 5. M/M/1 Queuing System.

For the development of this model, we will use:

- Population: The population to be used in the model to be generated is all vehicles that travel on the main avenues of Panama City.
- Sample: It will correspond to vehicles that circulate on the main road called Calle 50, Aquilino de la Guardia and also at its bifurcation with Calle 56.—. January 2020 will be considered, with a cycle of 180 s, from 3:00 p.m. to 6:00 p.m. (see Figure 6).

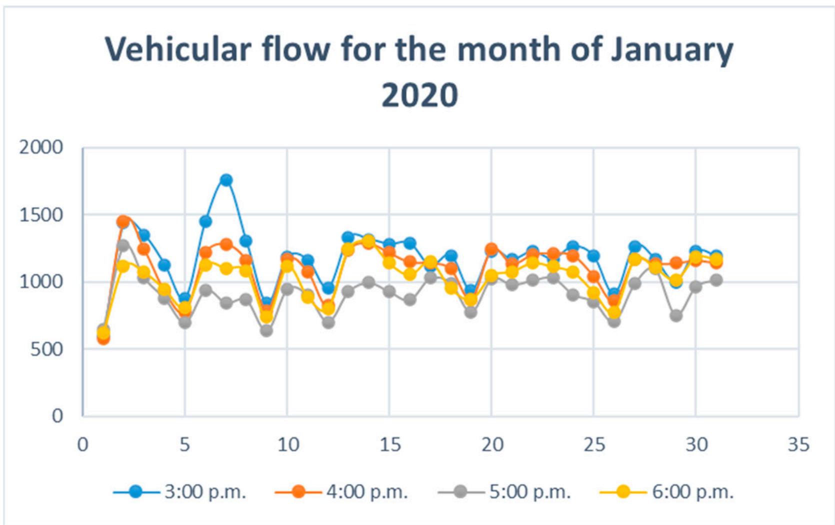

Figure 6. Vehicular flow for January 2020. Schedule from 3:00 p.m. to 7:00 p.m.

- Variables: The independent variables we will use are: λ, the number of arrivals, and μ, the number of departures or service rate. The dependent variables are:

ρ: average system utilization.
L: average number of vehicles in the service system.
Lq: average number of vehicles in the waiting queue.
W: average time elapsed in the system, including service.

Wq: average waiting time in the queue.

The information will be provided by the Panama Transit and Land Transportation Authority (ATTT). Here, we analyze the vehicular movement through two traffic lights, 044 and 045, located on Nicanor De Obarrio Avenue (50th Street), as shown in Figure 7.

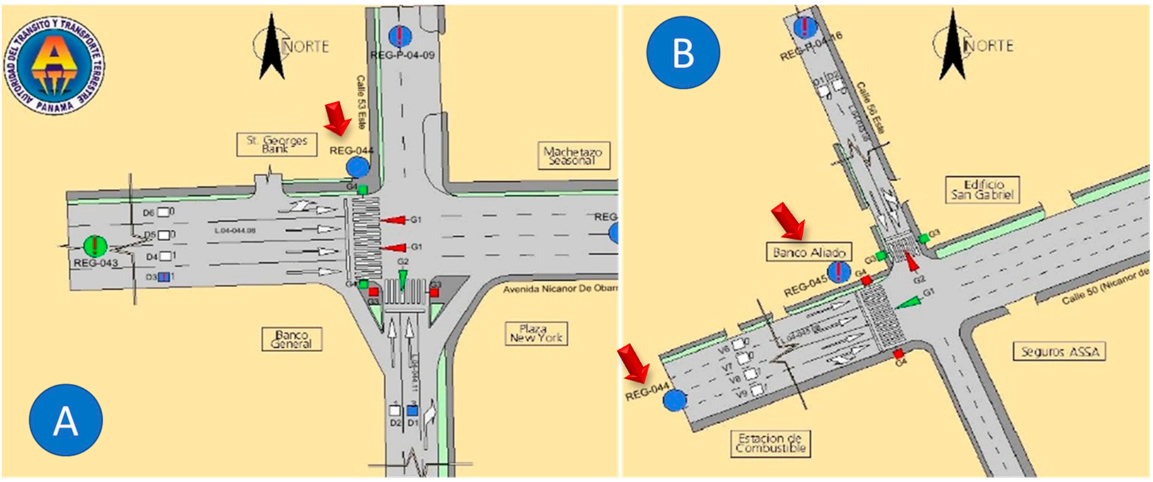

Figure 7. 50 Nicanor de Obarrio Street, A-53rd Street East; B-56th Street East.

From Figure 5 we can see, in the blue circle, the REG-044 and REG-045 that correspond the L04-044-08 and L04-045-08 counter, which provides the initial data for the queuing theory model, the number of customers or vehicles entering the system per hour. The queuing system study required a switch to a traffic network to understand the queuing processes at both server input and output, as shown in Figure 8.

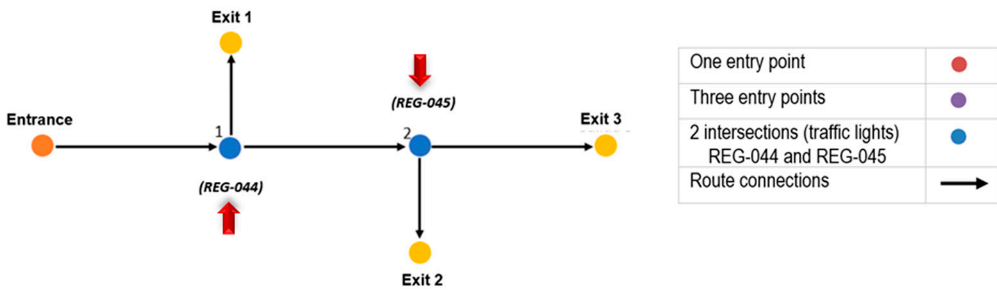

Figure 8. Vehicle traffic network.

7. Results and Discussions

For the stability analysis of the queuing system, we first adapted the traffic lights and the streets where the queues are formed to a serial M/M/1 queuing system; the traffic network was taken to a system where we have the queue vs. the servers (see Figure 9).

Applying Kendall's notation to these data, this waiting system is characterized by the fact that both the inter-arrival times and the service times are exponentially distributed, and the number of servers is one after another (in series). We analyze the stability of the vehicles entering the first traffic light at 50th Street to determine the % of the days of the month where the system is stable.

Through MATLAB software, a function called tcola50 was made to run through the vectors and generate the value of ρ, to see the days the system was stable. Moreover, thus calculate a percentage in January where the system was stable for the first traffic light (see Figure 10).

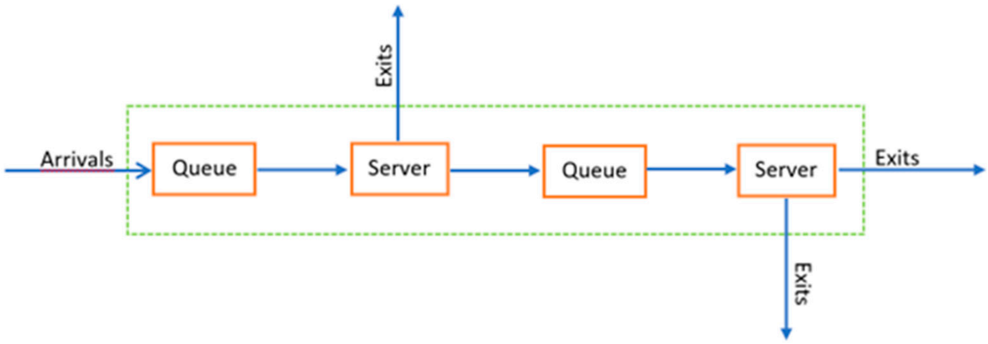

Figure 9. Queuing system at the 50th Street intersection (Nicanor de Obarrio Ave).

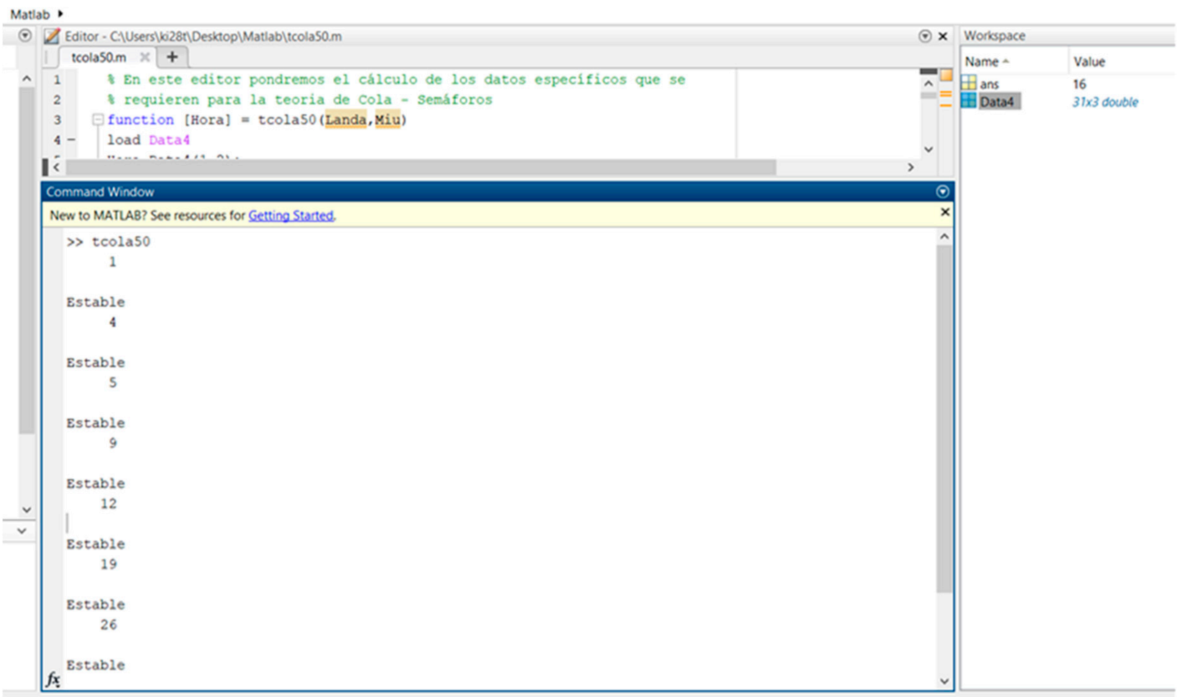

Figure 10. Results of the tcola50 function.

7.1. Optimization of the Queuing System

1. Analysis for the month of January for a traffic light capacity of 1000 vehicles/h.

The analysis of the first server (traffic light 1) shows the results: stability of 22.58% and instability of 77.42%. The results of the other variables analyzed on the days selected for this server are shown in Table 1.

Table 1. Results of L, Lq W, Wq. For a capacity of one thousand vehicles. First Server. 3:00 p.m.

January Day 2020	L	Lq	W	Wq
1	1.4510	0.8590	0.0025	0.0015
5	7.1301	6.2531	0.0081	0.0071
9	5.2112	4.3722	0.0062	0.0052
12	20.7391	19.7851	0.0217	0.0207
19	13.9254	12.9924	0.0149	0.0139
26	9.7527	8.8457	0.0108	0.0098
29	499	498.002	0.5	0.499

In addition, the following information is obtained for this queuing system:

- The average number of vehicles in the queue is 18 vehicles per light change.
- The average time to be served is: 65 s, under the condition that it only occurs on holidays and weekends.
- For the capacity of 1000 vehicles per hour, the system was 77% of the time over saturated, with an average occupancy rate of 118%.
- The results of the Kendall notation were obtained on holidays and weekends with a value of $\rho < 1$, and an average occupancy of 87% of the system.

Using the program, it was found that the average number of cars leaving the system after being served by the server (traffic light 1) corresponds to 20% and 80%. The system has four parallel lanes that continue, and one leaves the main road because the left lane forks. Taking this information into account, the percentage of distribution that will be the restriction to advance to the next server (traffic light 2) is calculated, and 100% of the cars entering the system is divided by five to obtain this percentage.

From the results obtained in the first server, 80% of cars waiting to be served by the second server (Traffic Light 2) are considered. In this second server, stability improves, and 64.5% stability and 35.5% instability are obtained. Additionally, the following results are obtained:

- The average number of vehicles in the queue is 11 vehicles per light change.
- The average time to be served is 40 s, under the condition of stability on holidays and weekends.
- For the capacity of 1000 vehicles per hour, the system was 65% stable and 35% unstable.
- For the full month, the system had an occupancy level of 94%, which confirms the improvement in stability.
- The system was at an average occupancy of 85% on days where the value of ρ is less than 1, i.e., $\rho < 1$.

Additionally, stability and instability values were obtained for the capacity of 1000 vehicles at the 4:00 p.m., 5:00 p.m. and 6:00 p.m. schedules. The results obtained are shown in Table 2:

Table 2. Stability and instability values for the capacity of 1000 vehicles during the hours of 4:00 p.m., 5:00 p.m. and 6:00 p.m.

Server	Hour	Stability	Instability
First traffic light	4:00 p.m.	23%	77%
Second traffic light		90%	10%
First traffic light	5:00 p.m.	74%	26%
Second traffic light		97%	3%
First traffic light	6:00 p.m	32%	68%
Second traffic light		97%	3%

From this second traffic light, we have two outputs. The second output corresponds to cars leaving the second server and leaving the system (see Figure 11), and the third output is not considered in this study.

2. Analysis for the month of January for a traffic light capacity of 1300 vehicles/h.

After several tests for the traffic light capacity of vehicles per hour, it was determined that the value of 1300 vehicles per hour for μ is where a stability of more than 70% is achieved. An improvement in stability of 77.4% was obtained and in instability of 22.6%. By having a stable queuing system, it was possible to calculate the average values by means of Kendall notation with a more accurate study of the system. For the first server (traffic light 1) the following results were obtained:

- The average number of vehicles in the queue is 13 vehicles per light change.
- The average time to be served is 37 s, if and only if the vehicle arrives when the light is green and there is no vehicle waiting.
- The system had an occupancy level of 91% for the entire month. This confirms the improvement in stability.

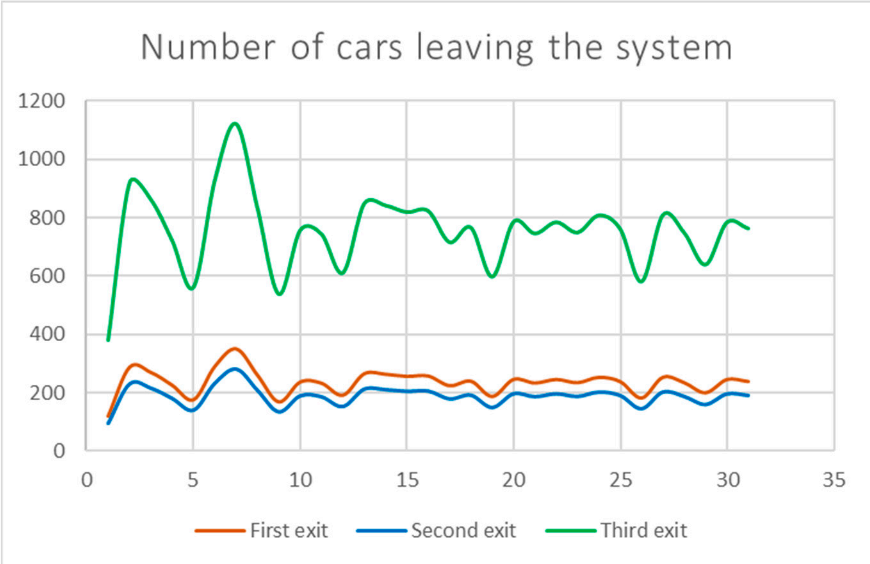

Figure 11. Number of cars that leave the system according to the outputs.

It is observed that 20% of the vehicles leave the system after being served by the first server, while 80% continue to be served by the second server (traffic light 2). This improves the stability in this second server by 96.8%, with an instability of 3.2% and the following results:

- The average number of vehicles in the queue is 3 vehicles per light change.
- The average time to be served is 11 s, if and only if the vehicle arrives when the light is green and there is no vehicle waiting.
- The system had an occupancy level of 72% for the entire month. This confirms the improvement in stability.

The values obtained for L, Lq, W, Wq at 4:00 p.m., 5:00 p.m. and 6:00 p.m. are shown in Table 3.

Table 3. Stability and instability values for the capacity of 1300 vehicles during the hours of 4:00 p.m., 5:00 p.m. and 6:00 p.m.

Server	Hour	Stability	Instability
First traffic light	4:00 p.m.	97%	3%
Second traffic light		100%	0%
First traffic light	5:00 p.m.	100%	0%
Second traffic light		100%	0%
First traffic light	6:00 p.m	97%	3%
Second traffic light		100%	0%

7.2. Simulation

For the simulation, a specific day in January was taken, 15 January 2020, for a capacity of 1300 vehicles. For this day there are 1280 vehicles per hour, which is equivalent to 98% of the system capacity. The Kendall notation has been used in theoretical analysis. For the first traffic light on this day, there are about 64 vehicles in the waiting queue at the first server of the system (traffic light 1); each vehicle takes about 180 s (3 min) to be served.

With this data, in Excel, we simulate the movement of these vehicles (see Figure 12), remembering that we are on the schedule from 3:00 p.m. to 4:00 p.m. and that each cycle takes 180 s. Two green cycles of 60 s (1 min) and one red cycle of 60 s are included. Each vehicle has their random arrival, and the time between arrivals is calculated, observing that their arrivals are very close to three vehicles every 3 s, which causes the queue to fill up with vehicles when in the red-light cycle with an average of 180 vehicles.

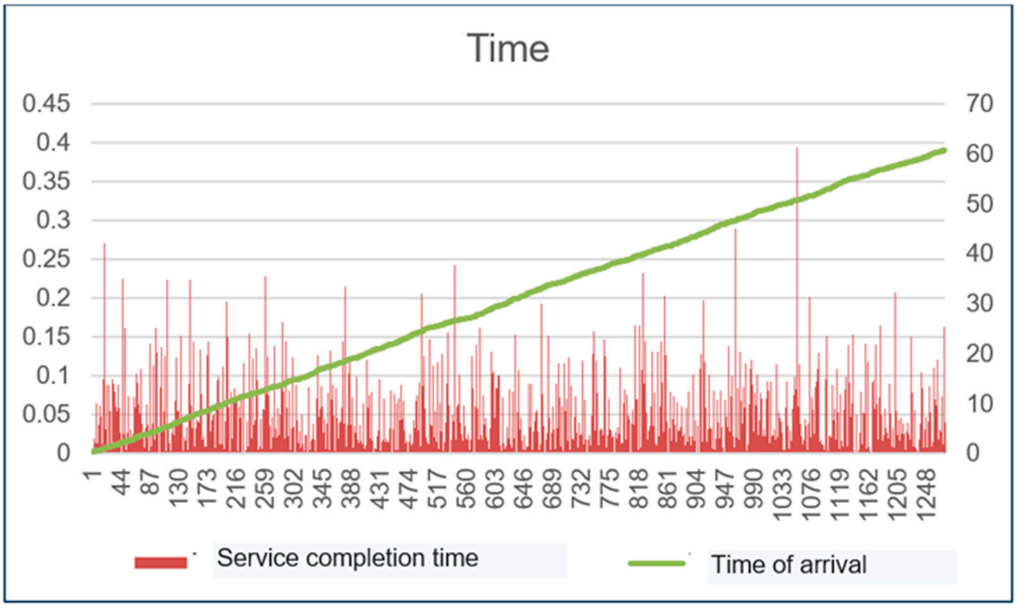

Figure 12. Simulated arrival times compared to vehicle completion times at 3:00 p.m. on 15 January 2021.

The time in the system is analyzed. At the beginning of the system, each vehicle takes about 180 s to be attended; however, as the seconds advance, the queue increases, and the last vehicles must wait approximately 60 min to be attended to, either to leave the server or to continue to the second traffic light.

7.3. AI Predictive Model

Currently, statistical, and artificial intelligence (AI) techniques are increasingly being used in combination with numerical models to generate more accurate predictions in various areas. In the context of queuing theory, one of the most widely used AI techniques is linear regression (LR) prediction [46–48].

LR is a data analysis technique that predicts the value of unknown data by using another related and known data value. It mathematically models the unknown or dependent variable and the known or independent variable as a linear equation [43,49–51]. The regression model consists of an approach to model the relationship between a dependent scalar variable "Y" and one or more explanatory variables named "X" and then to plot a line that will indicate the trend of a set of continuous data, whose formula is:

$$Y = mX + b$$

where Y is the result, X is the variable, m is the slope (or coefficient) of the line, and b is the constant or also known as the "point of intersection with the Y-axis" on the graph (when X = 0).

In the context of queuing theory, linear regression can be used to predict variables related to the performance of a queuing system, such as average waiting time as a function of independent variables such as customer arrival rate and server capacity, among others.

By applying linear regression to data collected from a queuing system, more accurate estimates and forecasts of system behavior and performance can be obtained [34,52,53].

Linear regression is just one of many statistical and AI tools that can be used in combination with queuing theory to analyze and optimize the performance of queuing systems. These techniques provide a solid foundation for understanding and making informed decisions about traffic management, urban planning, and other areas related to congestion and vehicle flow in cities [31,38,54,55].

The predictive models are obtained using simple linear regression using the demand data of each traffic flow intensity from 3:00 p.m. to 6:00 p.m. provided by Autoridad de Tránsito y Transporte Terrestre of Panamá (ATTT) [56]. Table 4 shows the data used to train the model.

Table 4. Maximum demand from traffic flow intensity by hours.

Date	3:00 p.m.	4:00 p.m.	5:00 p.m.	6:00 p.m.
1 January 2020	592	577	650	620
2 January2020	1437	1453	1270	1117
3 January2020	1348	1240	1029	1071
4 January2020	1127	929	877	945
5 January2020	877	778	699	809
6 January2020	1452	1219	938	1127
7 January2020	1754	1281	844	1103
8 January2020	1300	1156	865	1083
9 January2020	839	787	634	739
10 January2020	1183	1167	947	1116
11 January2020	1162	1076	902	888
12 January2020	954	824	701	802
13 January2020	1326	1236	928	1247
14 January2020	1316	1286	993	1301
15 January2020	1280	1216	926	1138
16 January2020	1287	1147	866	1053
17 January2020	1118	1142	1027	1150
18 January2020	1197	1096	991	952
19 January2020	933	871	774	870
20 January2020	1228	1245	1024	1047
21 January2020	1165	1135	979	1077
22 January2020	1226	1203	1010	1143
23 January2020	1171	1214	1031	1113
24 January2020	1263	1196	905	1072
25 January2020	1189	1042	853	923
26 January2020	907	856	704	777
27 January2020	1265	1164	985	1170
28 January2020	1169	1134	1097	1108
29 January2020	998	1138	753	1015
30 January2020	1225	1159	965	1185
31 January2020	1192	1140	1015	1169

The demand of the model variables w, x, y and z is obtained by applying simple linear regression on the data set that constitutes the traffic flow intensity demand for each hour. The variables used for the intensity are:

I_{15H}—traffic flow intensity demand at 3:00 p.m. (15 h)
I_{16H}—traffic flow intensity demand at 4:00 p.m. (16 h)
I_{17H}—traffic flow intensity demand at 5:00 p.m. (17 h)
I_{18H}—traffic flow intensity demand at 6:00 p.m. (18 h)

The models for each intensity are as follows:

$$w = i_1 + m_1 I_{15H}$$

$$x = i_2 + m_2 I_{16H}$$

$$y = i_3 + m_3 I_{17H}$$

$$z = i_4 + m_4 I_{18H}$$

where i_1, m_1, i_2, m_2, i_3, m_3, i_4, and m_4 are the coefficients of the linear regressions of the intensities.

Predictive models are obtained using simple linear regression and the demand values of each intensity.

These data were stored in Pandas DataFrame and were programmed using Python [27,39,57–61]. During the training of the models, 80% of the data for training and 20% of the data for testing were used. Tables 5–8 and Figures 13–16 show the values resulting from the model training for each intensity.

Table 5. Results of the prediction model for traffic flow intensity demand at 3:00 p.m.

Date	Traffic Flow Intensity Demand	Prediction
1 January 2020	592	699.61208
7 January 2020	1754	1323.617801
8 January 2020	1300	1212.821331
12 January 2020	954	918.545905
18 January 2020	1197	1159.639025

Table 6. Results of the prediction model for traffic flow intensity demand at 4:00 p.m.

Date	Traffic Flow Intensity Demand	Prediction
2 January 2020	1453	1501.728419
9 January 2020	787	804.933475
10 January 2020	1167	1147.852999
12 January 2020	824	878.337974
21 January 2020	1135	1182.911864

Table 7. Results of the prediction model for traffic flow intensity demand at 5:00 p.m.

Date	Traffic Flow Intensity Demand	Prediction
4 January 2020	877	845.417087
5 January 2020	699	758.486939
12 January 2020	701	754.012594
18 January 2020	991	849.891432
23 January 2020	1031	952.801386

Table 8. Results of the prediction model for traffic flow intensity demand at 6:00 p.m.

Date	Traffic Flow Intensity Demand	Prediction
1 January 2020	620	575.04631
9 January 2020	739	717.509395
22 January 2020	1143	1096.252718
25 January 2020	923	1110.151556
28 January 2020	1108	1078.879171

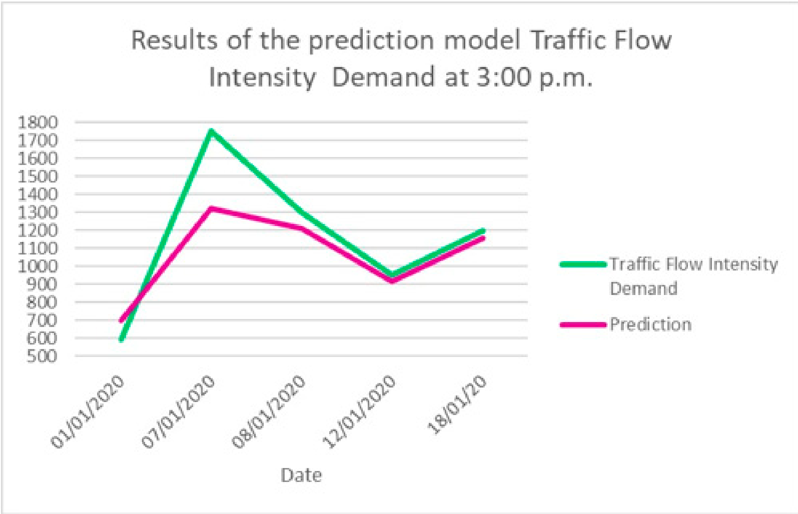

Figure 13. Prediction model for traffic flow intensity demand at 3:00 p.m.

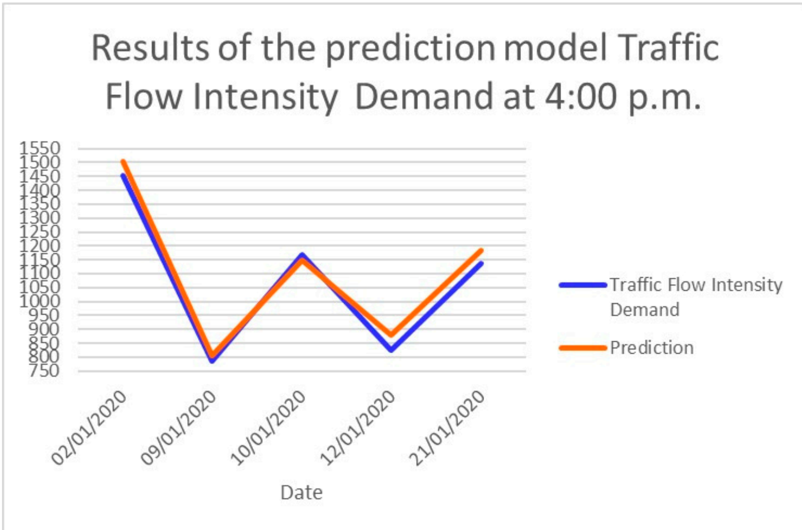

Figure 14. Prediction model for traffic flow intensity demand at 4:00 p.m.

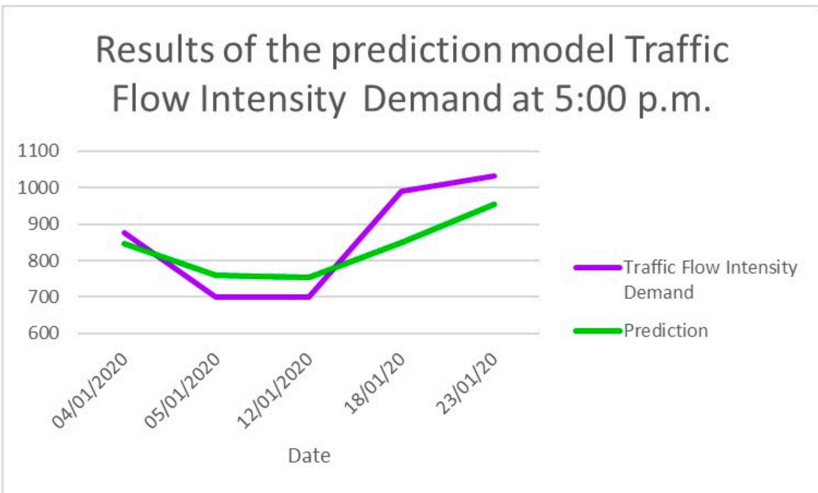

Figure 15. Prediction model for traffic flow intensity demand at 5:00 p.m.

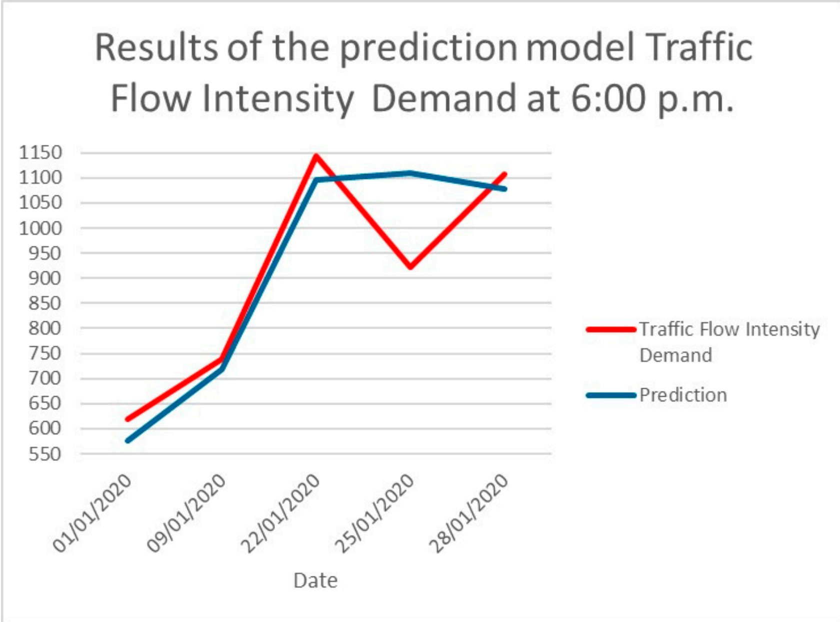

Figure 16. Prediction model for traffic flow intensity demand at 6:00 p.m.

The coefficient of determination (r^2) is used to evaluate how well the data of each model fit. In this, a value of 1 is equivalent to an optimal fit. The coefficients of determination (r^2) for each traffic flow intensity demand for hours are shown in Table 9.

Table 9. Coefficients of determination (r^2) by traffic flow intensity demand for hours.

Traffic Flow Intensity Demand at:	Coefficient r^2
3:00 p.m.	0.7148
4:00 p.m.	0.9713
5:00 p.m.	0.6423
6:00 p.m.	0.7262

8. Conclusions

An M/M/1 queuing model based on Kendall notation was proposed to solve the problem that currently exists in the synchronization of traffic lights on 50th Street in Panama City. The mathematical model included a stability analysis of the system, performing the analysis with two capacities of the system until achieving the stability of the queuing system at 1300 vehicles per hour.

For the AI component, we measured the accuracy, as shown in Table 9. We are considering evaluating other models to make the AI component even more robust, which will be studied in future research. Evaluating other models can help identify the strengths and weaknesses of the current model and compare its performance with other models in different scenarios. In addition, it can also improve the ability of the current model to make more accurate and valuable predictions in more diverse situations.

The algorithm developed in MATLAB was based on a stability analysis, which shows the stability and instability for January 2020. The stability analysis found that the system is not saturated on holidays and weekends. On weekdays, we observed that the system is oversaturated with the capacity currently having the traffic light cycle of 50th Street. With this, we can analyze that the current scheduling system of 50th Street is an unstable, oversaturated queuing system, which would generate large queues with non-estimated departure times.

A simulation was carried out in Excel over a single day. The chosen day was at 98% of its capacity. The results obtained are that the approximate duration in the queuing system is 0.002 s in the best scenario with 0 elements in the queue and with waiting times of up to 60 min. Regarding the service capacity of the system, several system stability analyses were carried out, where the current traffic light capacity is insufficient for the number of vehicles passing through the road at the 3:00 p.m. peak hour. The first analysis is of a capacity of one thousand vehicles per hour (current capacity), giving us a stability of 22.58% and an instability of 77.42%. With this capacity, the system was so saturated that it was not feasible to apply the model. Starting with a capacity of one thousand vehicles per hour, we continued analyzing one hundred at a time until we reached a capacity of one thousand three hundred vehicles per hour (suggested capacity). With this capacity, we obtained a stability of over 70%. Compared with the previous capacities, this result is feasible, and the system analysis could be carried out using Kendall's notation.

For the waiting time in the system, two scenarios were analyzed:

- Scenario 1. Vehicle capacity of 1000 vehicles per hour; in this scenario, employing only Kendall's notation model, it would be possible to estimate the average waiting values on holidays and weekends in January at 3:00 p.m., where a short queue with an average of 18 vehicles waiting for each cycle and a waiting time of 65 s on average was determined. In the rest of the days, the queuing system becomes unstable, and it would not be possible to estimate the waiting times in the system employing this model.
- Scenario 2. For a capacity of 1300 vehicles per hour, the service time for each vehicle is 37 s, having at least 13 vehicles in the queue in the best-case scenario where the light is green, and no vehicles are stacked in the queue. The capacity was analyzed from 3:00 p.m. to 4:00 p.m. every day of January 2020. It was observed that, at that time, there was more congestion. A vehicle could take between 2 to 60 min to be served.

9. Future Work

Considering the limitations and potential weaknesses of the current model, future research focuses on the evaluation of alternative queuing models. This analysis can help identify the strengths and weaknesses of the different models and compare their performance under various scenarios. By selecting the most suitable model, the overall robustness and accuracy of the AI component can be improved by leveraging advanced techniques and algorithms, contributing to better synchronization and optimization of traffic signals at 50th Street.

The stability analysis conducted for January 2020 provided information on system performance during vacations, weekends, and weekdays. As future research, it is intended to include analysis of system stability for a wider range of time periods and to consider additional factors that may affect traffic flow, such as weather conditions and special events. This expanded analysis will provide a more complete understanding of system behavior and help develop effective strategies for traffic signal scheduling.

Once the model has been validated and refined, a next step would be to focus on applying and testing it under real-world conditions on 50th Street. This would involve collaborating with relevant authorities and stakeholders to collect real-time traffic data, monitor system performance, and evaluate the effectiveness of the proposed model in improving traffic flow and reducing congestion.

As traffic patterns and road conditions evolve over time, it is essential to continuously monitor system performance and update the model accordingly. This would ensure that traffic signal timing remains optimized and efficient to cope with dynamic traffic demands on 50th Street.

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