

Article

Design Optimization of a Hydrodynamic Brake with an Electrorheological Fluid

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Abstract: This article describes the design optimization of a hydrodynamic brake with an electrorheological fluid. The design optimization is performed on the basis of mathematical model of the brake geometry and the brake's electrical circuit. The parameters of the mathematical models are selected based on experimental tests of the prototype brake. Six different objective functions are minimized during the design optimization. The functions are created taking into consideration the following factors: the braking torque, brake weight, electric power absorbed by the brake, and the torque rise time. The assumed design variables are: the number of blades and the radii (inner and outer) of the brake's working space. The optimization calculations are performed for two design variables intervals. The first interval is defined taking into consideration the accuracy of the mathematical model. The second, narrower interval is assumed for the tested prototypical brake. On the basis of the optimization calculation results, general guidelines are presented for the optimization of the hydrodynamic brakes with an ER fluid. In addition, the possibilities of optimizing the prototype brake are determined.

Keywords: electrorheological fluid; hydrodynamic ER brake; optimization method



Citation: Kęsy, Z.; Musiałek, I.; Choi, S.-B. Design Optimization of a Hydrodynamic Brake with an Electrorheological Fluid. *Appl. Sci.* **2023**, *13*, 1089. <https://doi.org/10.3390/app13021089>

Academic Editors: Antonio Concilio, Rosario Pecora, Ignazio Dimino, Salvatore Ameduri and Vikram G Kamble

Received: 5 December 2022

Revised: 5 January 2023

Accepted: 10 January 2023

Published: 13 January 2023



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1. Introduction

Hydrodynamic brakes (HB) belong to the group of hydraulic components of machines, as do clutches and hydrodynamic torque converters. They are used in drive systems of machines, mainly due to their favorable weight-to-torque ratio on the input shaft and the long durability [1]. The basic way of controlling the HBs is by changing their working fluid filling degree. The lower the filling degree, the smaller the torque on the input shaft of the brake. However, the dependence of the torque on the filling degree is not linear. Moreover, this manner of control has a significant disadvantage, which is the unstable operation of HB at low filling degrees. The analysis of the operation of hydrodynamic components leads to the conclusion that controlling them is also possible by changing properties of the working fluid [2]. Thus, in this case, it is possible to use smart fluids, which react with a change in shear stress when exposed to changes in the electrical field (electrorheological fluids) or in the magnetic field (magnetorheological fluids) [3,4]. Currently, smart fluids are used in numerous devices and components of machines, e.g., in viscous clutches and brakes [5–7], shock absorbers [8–10], valves [11–13], sealings [14], cantilever sandwich beams [15,16], landing gear systems in planes [17,18], and industrial robots [19–21]. In addition, prototypes of hydrodynamic clutches with electrorheological fluids (ER) and magnetorheological fluids (MR) are constructed [22–25].

A frequent method of perfecting the design of machine components is the use of construction optimization. The construction optimization is performed on the basis of the component's mathematical model. The model parameters whose values are changed during the optimization process are called the design variables. The objective function is determined, and then its global extreme is sought on the basis of the requirements analysis assigned to the machine component (i.e., criteria) and to the mathematical model. To search for the global extreme of the objective function, multiple methods have been developed. Among the most frequently used methods are randomization methods, including the Monte Carlo Method and the Genetic Algorithm method [26–28]. It is advantageous to use structure optimization methods in the design process due to the complex design of hydraulic clutches with ER or MR fluids, which results from the necessity to generate an electric field inside. For mathematical modeling of clutches, brakes and torque converters, and one-, two-, and three-dimensional flows within the rotors are taken into consideration [29–31]. In the model based on the one-dimensional flow, it is assumed that the working fluid moves with the mean flow path with a uniform through-flow velocity [32]. The high accuracy of this model, comparable to the accuracy of two- and three-dimensional models, is obtained as a result of parameter selection based on experimental research. Due to their simplicity, these models are particularly suitable for use in design optimization [33].

Publications concerning the optimization of brakes and hydraulic clutches with smart fluids most frequently concern viscous clutches and brakes with MR fluids [34,35]. The article [36] describes the design optimization of a disc viscous clutch with ER fluids. The optimization is performed with the use of two optimization methods: the Monte Carlo method and the Genetic Algorithm method. The purpose of the design optimization is to seek such values of design variables, for which the viscous clutch with the ER fluid transmits a large torque, while the clutch's dimensions are small and its determined operating temperature is low. The optimization calculations are performed for two models including three and eight design variables. As a result of the design optimization, a disc viscous clutch with the ER fluid is constructed. In the publication [37], the object of the optimization process is a disc brake with a MR fluid. The brake was intended for use in a motorcycle. The objective function is the brake's weight, and the assumed constraints of the design variables are the braking torque and the temperature of the MR fluid during braking. The optimization is performed with the use of optimization procedures from the ANSYS program. In the article [38] the optimized object is a disc brake with one disc and different locations of the electromagnet coils and with three types of MR fluid. The constraints of the design variables concern the radius of the brake shaft and the maximal current flowing through the electromagnet coil. The optimization is conducted on the basis of the optimization method included in the ANSYS software. In the objective functions, factors taken into consideration are the brake's weight, the power supply, and the braking torque. The work [39] shows the use of an original Taguchi method in the optimization of a clutch with a MR fluid. The method allows identification of decision variables whose influence on the objective functions is the greatest.

Apart from the use for hydraulic clutches and brakes with ER fluids, design optimization has also been successfully used to optimize devices with smart fluids [11,40]. Despite several works on design optimization for ER brakes or MR brakes, a study on the optimal design of ER brakes considering practical feasibility, in which many design parameters should be considered, is considerably rare. Thus, in this work, to achieve this target, many principal design parameters, such as the radius and electrode gap of ER brake, and several braking performance and product criteria also, such as torque level, blade number, brake weight, electric power and braking time, are included for the optimization. To accomplish such a target, the HB optimization method is used, and subsequently, the possibilities of optimal selection of considered parameters of the optimized prototypical hydrodynamic brake with the ER fluid (PHB) are discussed in detail, focusing on the performance efficiency with several ratios: the blade number versus torque level versus brake weight. Consequently, the main technical contribution of this work is to calculate the

optimized design and performance variables of the PHB that can be feasible in a practical environment. As far as the authors know, so far there is no report on this optimal design issue in the technology associated with ER fluid systems, such as hydrodynamic brakes.

2. Construction of the Hydrodynamic Brake

The HB has rotors with radial blades lying in planes passing through the rotor axis. These rotors have no inner rings. Such a design solution is frequently used in practice. The construction scheme of the HB is shown in Figure 1.

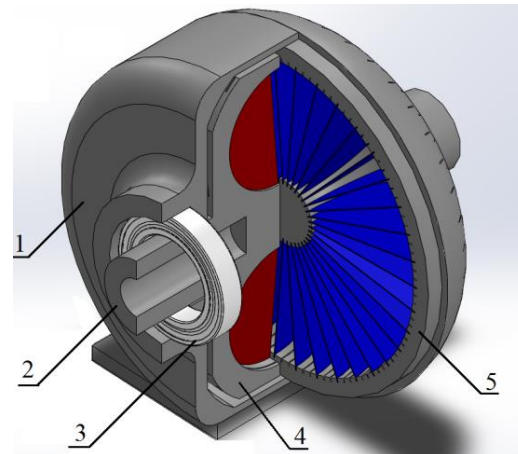


Figure 1. HB construction scheme: 1—casing, 2—input shaft, 3—bearing with sealing, 4—pump rotor, 5—turbine rotor connected to casing.

The HB is filled with the ER fluid. The pump rotor rotates with the angular velocity ω_1 , and the turbine rotor is immobile ($\omega_2 = 0$). The meridional cross-sections of both HB rotors are identical. The casing and rotors are made of an electrical insulator material, while the blades are made of an electrical conductor material. The pump and turbine blades are alternately connected to the (+) and (−) poles of the high voltage power supply with the use of slip rings, brushes, and electric wires. The HB operates as follows: the rotating pump rotor sets the fluid in motion; the fluid then flows into the immobile turbine rotor and presses on the blades, which causes the reaction torque to occur. After supplying the high voltage U to the rotor blades, the electric field is created, and it causes an increase in the shear stresses τ within the ER fluid. This causes an increase in the fluid flow resistance and, simultaneously, a decrease in the torque M affecting the pump rotor. Thus, the HB operates differently than a viscous brake with an ER fluid, in which an increase of the high voltage U causes an increase in the torque. The designations of the basic dimensions of the HB are shown in Figure 2.

Table 1. PHB data.

R mm	r_w mm	d_1 mm	d_2 mm	g_b mm	n -	r_s mm	r_1 mm	r_2 mm	ER Fluid LID 3354S
50.5	15.5	1.5	7.3	1.0	38	37.4	28.6	44.4	

The following notations are used in Figure 2: R —the outer radius of the rotors, r_w —the inner radius of the rotors, n —the number of blades, g_b —the blade thickness, d_1 —the smallest distance between blades, d_2 —the largest distance between blades, r_s —radius, determining the origin of the mean streamline, r_1 —the inner radius of the mean streamline, r_2 —the outer radius of the mean streamline, ω_1 —angular velocity of the pump rotor, ω_2 —angular velocity of the turbine rotor. The design solution for PHB is shown in Figure 3.

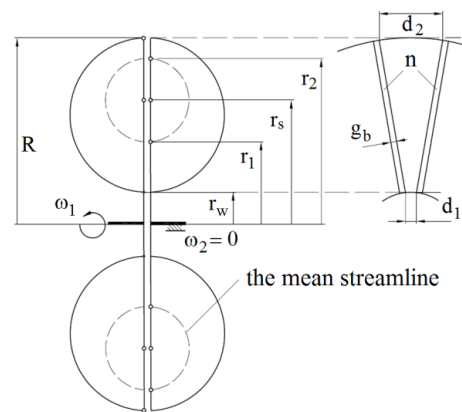


Figure 2. HB geometry for calculations (refer to Table 1).

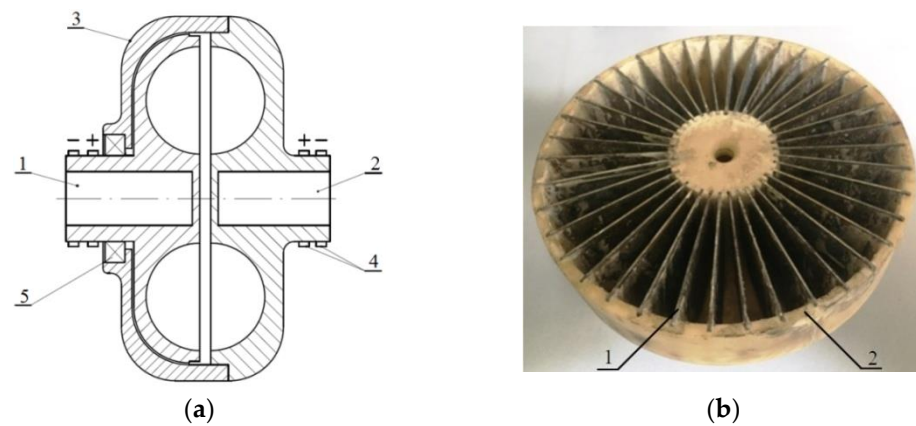


Figure 3. PHB with the ER fluid: (a)—design solution: 1—pump, 2—turbine, 3—casing, 4—slip rings supplying the high voltage, 5—sealing; (b)—view of the pump rotor after tests: 1—blade, 2—rotor casing.

The PHB rotors are made of 3D-printed nylon. They are equipped with flat steel sheet blades. The basic data concerning the PHB are shown in Table 1.

3. The Mathematical Model of the Hydrodynamic Brake

3.1. The Mathematical Model of the Hydrodynamic Brake Geometry

The parameters describing the geometry of the brake rotors are as follows: the outer radius of the rotors R , the inner radius of the rotors r_w , and the number of blades n . The radius R is larger than the radius r_w . The blade number n is even, so that in each of the channels formed by the blades, an electric field influences the ER fluid. Moreover, the blades cannot be set too close to each other due to the possibility of electric breakdown occurrence. Thus, the number of the blades should meet the following condition:

$$n < \frac{2 \pi r_w}{g_b + h_{\min}} \quad (1)$$

where g_b —blade thickness, $h_{\min}$ —the smallest gap width for which an electric breakdown occurs. The average blade distance d is calculated from the formula:

$$d = 0.5(d_1 + d_2) \quad (2)$$

wherein:

$$\begin{aligned} d_1 &= (2\pi r_w / n) - g_b \\ d_2 &= (2\pi R / n) - g_b \end{aligned} \quad (3)$$

where d_1 —the smallest distance between blades, r_w —inner radius of the rotors, R —outer radius of the rotors, d_2 —the largest distance between blades, n —number of blades.

In order to calculate the rotor volume V_t , the following formula is used:

$$V_t = 2\pi^2 \left(\frac{R - r_w}{2} \right)^2 \cdot \frac{R + r_w}{2} \quad (4)$$

where R —outer radius of the rotors, r_w —inner radius of the rotors.

The blades volume V_b is calculated on the basis of the following formula:

$$V_b = n \cdot \pi \left(\frac{R - r_w}{2} \right)^2 \cdot g_b \quad (5)$$

where n —number of blades, r_w —inner radius of the rotors, R —outer radius of the rotors, g_b —blade thickness.

The volume of the fluid V_{ER} contained in the HB is then:

$$V_{ER} = V_t - V_b \quad (6)$$

where V_t —the rotor volume, V_b —the blades volume. With the use of the Formulas (4)–(6), it is possible to formulate the weight G of the brake as follows:

$$G = V_{ER}\rho + V_b\rho_1 + \alpha V_t\rho_2 \quad (7)$$

where ρ —density of the working fluid, ρ_1 —density of the blade material, ρ_2 —density of the casing material, α —coefficient describing the casing density in relation to the rotor volume.

3.2. Mathematical Model of Hydraulic Interactions

In order to calculate the transferred torque M , the one-dimensional flow model of a hydrodynamic clutch is used [41,42]. The torque M for the HB is obtained from the equations of the one-dimensional flow model of a hydrodynamic clutch after assuming that $\omega_2 = 0$. It is assumed that the flow of the working fluid inside the hydrodynamic clutch is steady, continuous, and takes place along a single line of the mean stream. The radii r_1 , r_2 , r_s (Figure 3) determining the position of the mean streamline in the meridional cross-section are calculated on the basis of the following formulas:

$$\begin{aligned} r_s &= \sqrt{(R^2 + r_w^2)/2} \\ r_2 &= \sqrt{(r_s^2 + R^2)/2} \end{aligned} \quad (8)$$

where r_w —inner radius of the rotors, R —outer radius of the rotors. The unit torque dM in relations to the rotation axis of the rotor is only caused by the force dF caused by a change in speed c_u of the working fluid particles. The force dF and the unit torque dM are written as:

$$dF = \frac{d(mc_u)}{dt} \quad (9)$$

$$dM = dF \cdot r = \frac{d(m c_u r)}{dt} \quad (10)$$

where d/dt —differential operator, t —time, m —mass of the fluid particle, c_u —lifting speed, r —radius. The mass m of the fluid flowing over time dt can be calculated as:

$$m = \rho V = \rho Q dt \quad (11)$$

where V —fluid volume, t —time, Q —flow rate, c_m —meridional speed, A —rotor meridional cross-section. After taking into consideration the dependence (11) in relation to (10), the following is obtained:

$$dM = \rho Q dt \frac{d(c_u r)}{dt} = \rho Q d(c_u r) \quad (12)$$

where the product $c_u r$ is treated as a single variable. Integrating the dependence (12) from point 1 to point 2 positioned on the mean streamline leads to the following equation:

$$M = \rho Q \int_1^2 d(c_u r) = \rho Q [c_u r]_1^2 = \rho Q (c_{u2} r_2 - c_{u1} r_1) \quad (13)$$

where ρ —density of the working fluid, Q —flow rate, c_u —lifting speed, r —radius.

When the blades of the hydrodynamic clutch are positioned in the planes passing through the rotor axes, then the rotor lifting speed is, respectively:

$$\begin{aligned} c_{u1} &= \omega_1 r_1 \\ c_{u2} &= \omega_2 r_2 \end{aligned} \quad (14)$$

where ω_1 —angular velocity of the pump rotor, ω_2 —angular velocity of the turbine rotor, r_1 —the inner radius of the mean streamline, r_2 —the outer radius of the mean streamline. For the determined operation conditions of the hydrodynamic clutch:

$$Q = c_m A \quad (15)$$

where c_m —meridional speed, A —rotor flow field. Considering the fact that for the HB, the angular velocity of the clutch turbine rotor ω_2 equals zero, on the basis of dependences (13)–(15), the following is obtained:

$$M = \rho c_m A \omega_1 r_2^2 \quad (16)$$

where ρ —working fluid density, c_m —meridional speed, A —rotor flow field, ω_1 —angular velocity of the clutch pump rotor. After taking into consideration the blade thickness, the flow field of the HB is:

$$A = \pi (R^2 - r_s^2) - n g_b (R - r_s) \quad (17)$$

where n —number of blades, g_b —blade thickness. In order to calculate the meridional speed c_m , the head rise balance is considered, described as:

$$h = \Delta p / \rho g \quad (18)$$

where Δp —pressure change, ρ —working fluid density, g —gravitational acceleration. The balance of the head rise has the following form:

$$h_1 - h_u - h_t = 0 \quad (19)$$

where h_1 —head rise of the pump, h_u —impact loss, h_t —friction loss. For HBs with typical working fluids, the components of Equation (19) are written as follows:

$$h_1 = \omega_1^2 r_2^2 / g \quad (20)$$

$$h_u = \omega_1^2 (r_2^2 + r_1^2) / 2g \quad (21)$$

$$h_t = \xi c_m^2 / 2g \quad (22)$$

where ω_1 —angular velocity of the pump rotor, g —gravitational acceleration c_m —meridional speed, r_1 —the inner radius of the mean streamline, r_2 —the outer radius of the mean streamline, ξ —friction losses factor.

During the HB modeling, in the presence of an electrical field, when considering the friction losses value h_t (described with Formula (22)), it is necessary to take into account the decrease in pressure Δp_{ER} caused by the increase in the shear stresses τ . On the basis of the formulas used for calculation of the valves with ER fluids [11–13,43,44], it is assumed that:

$$\Delta p_{ER} = \frac{cl}{d} \tau_0 \quad (23)$$

where c —numerical coefficient, l —length of the channel formed by the blades, d —average distance between blades, τ_0 —yield stress. After taking into consideration Formula (23) in Formula (18), and subsequently adding the result to Formula (22), the following is obtained:

$$h_t = \xi \frac{c_m^2}{2g} + \frac{cl}{d} \frac{\tau_0}{\rho g} \quad (24)$$

where ξ —friction losses factor, c_m —meridional speed, g —gravitational acceleration, r_1 —the inner radius of the mean streamline, r_2 —the outer radius of the mean streamline, c —numerical coefficient, l —length of the channel formed by the blades, τ_0 —yield stress, d —average distance between blades. The meridional speed c_m , which enables the calculation of the torque M on the basis of Formula (16), is obtained by substituting Formulas (20), (21), and (24) to the equation for the head rise balance (19) and subsequently solving the equation:

$$c_m = (\sqrt{\omega_1^2(r_2^2 - r_1^2) - \zeta \tau_0}) / \xi \quad (25)$$

wherein:

$$\zeta = 2cl / (\rho d) \quad (26)$$

where ω_1 —angular velocity of the pump rotor, r_1 —the inner radius of the mean streamline, r_2 —the outer radius of the mean streamline, τ_0 —yield stress, ξ —friction loss coefficient, ζ —channel coefficient, c —numerical coefficient, l —length of the channel formed by the blades, ρ —density of the working fluid, d —average distance between blades.

3.3. Mathematical Model of the Electric Circuit of the Hydrodynamic Brake

Electrorheological properties of the ER fluids are presented with the use of the following formulas:

$$\tau_0 = aE^2 \quad (27)$$

$$i_g = bE^2 \quad (28)$$

wherein:

$$E = U/d \quad (29)$$

where τ_0 —yield stress, i_g —leakage current density, a , b —coefficients, E —the electric field intensity, U —voltage, d —average distance between blades. The dependence of τ_0 and i_g on the temperature Θ is not taken into account, because it is assumed that the HB operates at a constant temperature due to the use of a cooling system. The HB needs to be cooled, because all the energy supplied to the HB is converted to heat. The leakage current flowing between the electrodes is calculated on the basis of the formula:

$$i_c = S_e i_g \quad (30)$$

wherein:

$$S_e = n\pi(R - r_w)^2 / 4 \quad (31)$$

where S_e —surface of the electrodes, n —number of blades, r_w —inner radius of the rotors, R —outer radius of the rotors.

In the considered model, the increase of the high voltage U applied to the blades causes a decrease in the meridional speed c_m until it reaches zero. The meridional speed c_m

is described by Formula (25). The value of the voltage U_h for which $c_m = 0$ can be calculated on the basis of the equation:

$$\omega_1^2(r_2^2 - r_1^2) = \zeta\tau_0 \quad (32)$$

where ω_1 —angular velocity of the pump rotor, r_1 —the inner radius of the mean streamline, r_2 —the outer radius of the mean streamline, ζ —channel coefficient, τ_0 —yield stress. Having taken into consideration Formulas (27) and (29) in Equation (32), the following is obtained:

$$U_h = \sqrt{\omega_1^2(r_2^2 - r_1^2)\rho d^3 / (2acl)} \quad (33)$$

where ω_1 —angular velocity of the pump rotor, r_1 —the inner radius of the mean streamline, r_2 —the outer radius of the mean streamline, a , c —coefficients, l —length of the channel formed by the blades, ρ —working fluid density, d —average distance between blades. The maximal power P_e obtained from the power supply, while stopping the flow of the ER fluid in the brake, is:

$$P_{e\max} = U_h i_{c\max} \quad (34)$$

where U_h —voltage value for which $c_m = 0$, $i_{c\max}$ —leakage current calculated for $U = U_h$.

In order to assess the dynamics of the HB, the torque rise time M (after applying the high voltage to the electrodes U_h) is taken into consideration. The dynamics of the machine drive system containing the HB is not taken into account due to the fact that the total mass moment of inertia on the input shaft depends not only on the HB structure, but also on the structure of the drive system. The drive system structure, due to the general nature of the considerations, is not specified. The torque rise time for the HB are considered on the basis of the voltage courses in a simplified electrical circuit of the HB, as shown in Figure 4 [45].

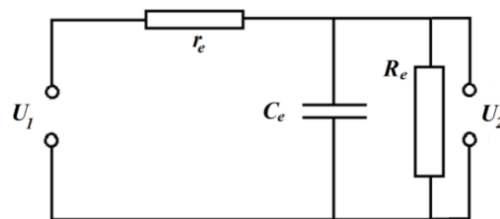


Figure 4. Electrical circuit of the HB: r_e —electrical resistance of the wires, R_e —electrical resistance of ER fluid, C_e —electric capacity of the HB, U_1 —power supply voltage, U_2 —voltage on the blades.

The blades of the HB, separated from each other by the ER fluid, form a capacitor whose capacity is C_e . The ER fluid provides the electrical resistance R_e during the leakage current flow i_c . After applying the supply voltage U_1 , the electric current flows through the wires (the resistance of the wires is r_e) to the condenser with the capacity of C_e and charges the condenser. After the power supply U_1 is switched off, the electric current flows through the ER fluid whose resistance is R_e , and the condenser discharges. Changes of the voltage U_2 over time, with a step voltage increase U_1 , is described by a first order system. It is assumed that the torque of the HB does not depend on time, but it is proportional to the voltage U_2 . While the condenser is charging, the time constant of the first order system is $T_c = r_e \cdot C_e$, whereas while the condenser discharges, it is $T_d = R_e \cdot C_e$. Due to the fact that $r_e \ll R_e$, $T_d \gg T_c$, the operation speed of the HB is assessed with the use of the time T described with the formula:

$$T = \beta R_e C_e \quad (35)$$

wherein:

$$R_e = U_h / i_{c\max} \quad (36)$$

$$C_e = \frac{\epsilon_0 \epsilon_r S_e}{d} \quad (37)$$

where β —number of time constants after which $U_1 \cong U_2$, U_h —value of the voltage for which $c_m = 0$, $i_{c\max}$ —leakage current calculated for $U = U_h$, ϵ_0 —electric permittivity of the

vacuum, ϵ_r —relative permittivity of the ER fluid, S_e —surface of the electrodes, d —average distance between blades.

3.4. Coefficient Selection for the Mathematical Model of the Hydrodynamic Brake

Before using the developed HB model in optimization calculations, it is necessary to determine the values of the model's coefficients: α , ξ , c , a , b , β , ϵ_r . The values of the coefficients are determined on the basis of the design analysis and PHB experimental research. The coefficient α is calculated on the basis of the PHB weight, assuming that the modelled HB has an identical structure and proportional dimensions. Calculating the coefficient ξ demanded PHB bench tests without connecting the blades to the high voltage power supply. The test bench scheme is shown in Figure 5.

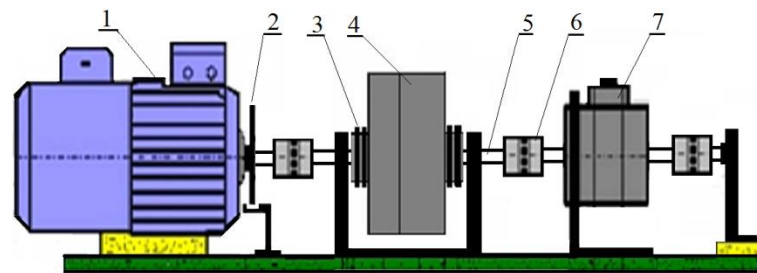


Figure 5. Test bench scheme: 1—electric motor, 2—speed sensor, 3—slip rings used to supply high voltage, 4—the tested brake with ER fluid, 5—shaft, 6—connecting clutch, 7—torque gauge.

The main elements of the test bench are: an electric drive motor and a torque gauge. The computer control system of the test bench rendered it possible to obtain the measurement values specified in the test program, as well as to collect and process measurement data. The values recorded during the test are: the angular velocity of the pump shaft ω_1 , the braking torque M , and the temperature of the working fluid Θ . The temperature Θ is maintained at 35 °C by cooling the PHB with compressed air. Subsequently, the braking torque M is measured at a predetermined angular velocity of the PHB input shaft. The coefficient ξ is determined on the basis of Formulas (14) and (23). During the measurement, apart from the braking torque M , the rotational torques are of resistance to motion of the bearings and sealing. In further considerations, the torques of resistance are omitted due to their low values [42]. The value of the a coefficient is determined on the basis of the data from the manufacturer of the fluid LID3354S [46].

In order to determine the coefficient c for the immobile turbine shaft and for a specified angular velocity of the pump shaft ω_1 , the braking torque M of the PHB is measured for a few selected voltages U . To generate the high voltage U , an electric power supply with a rated power of 18W is used. The maximal voltage U obtained during these measurements is $U_{\max} = 3$ kV. The coefficient c is selected, so that for the voltage $U_{\max} = 3$ kV, the torque calculated with the use of the dependence (22) is equal to the torque measured with the use of the test bench, i.e., $M = 1.733$ Nm, Figure 6.

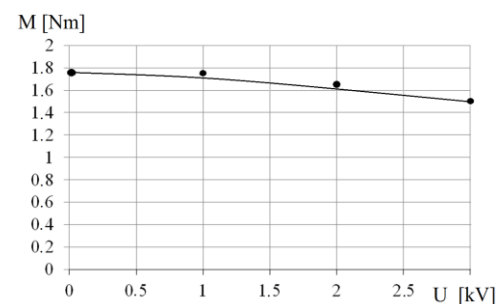


Figure 6. Method of determining the coefficient c : solid line—model, points—test.

The coefficient b is selected in order to obtain the measured total leakage current $i_c = 6$ mA for the voltage $U = 3$ kV. The value of the relative electric permittivity ϵ_r of the ER fluid is determined on the basis of [47,48]. The coefficient values of the HB mathematical model determined for $\omega_1 = 200$ rad/s and for the temperature $\Theta = 35$ °C are presented in Table 2.

Table 2. Mathematical model coefficients for the researched PHB.

Coefficient	α -	ξ -	c -	a kPa mm ² /kV ²	b μ A mm ² /(cm ² kV ²)	β -	ϵ_r -
Value	2.30	6.64	3.93	0.41	35.31	3.00	5.00
Formula	(7)	(22)	(23)	(27)	(28)	(35)	(37)

Due to the chosen method of selecting the coefficients shown in Table 2, only the modelling error of the dependence of the torque M on the high voltage U can be assessed. The solid line shown in Figure 6, obtained on the basis of the mathematical model, according to the assumptions, passes through the points for $U = 0$ kV and $U = 3$ kV. The average relative error calculated for $U = 1$ kV and $U = 2$ kV is under 2%.

4. Optimization Method for the Hydrodynamic Brake Design

In order to obtain guidelines for optimization of the HB design (concerning the selection of design variables and objective functions), calculations are performed on the basis of the developed mathematical model. Two calculation groups are performed. One group includes the detailed PHB calculations, on the basis of which the design variables are selected. The other group contains the general HB calculations, on the basis of which the optimization criteria are selected. For the HB, the calculation data are randomly selected parameters from determined ranges.

4.1. Selection of Design Variables

The calculations for the PHB are performed for the data shown in Table 1. The torque M and the voltage U_h are calculated, while one of the parameters r_w , R , n is changed, and the remaining parameters are fixed. Examples of the calculation results for $\omega_1 = 200$ rad/s, $\Theta = 35$ °C are shown in Figures 7–9.

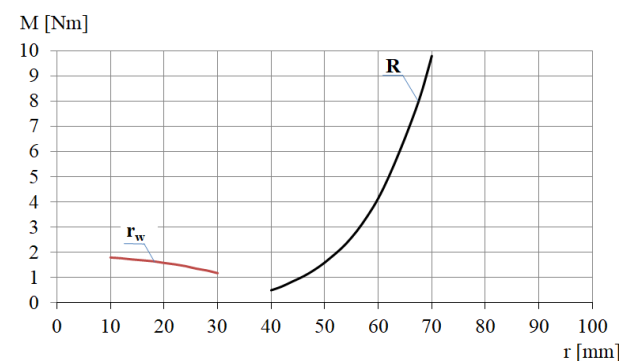


Figure 7. Dependence of the torque on the radius r_w and on the radius R .

Figure 7 presents the influence of the radius r_w and the radius R on the braking torque M . Figure 8 shows the dependence of the braking torque M on the blade number n . Figure 9 shows the influence that the number of blades n in the PHB has on the value of the voltage U_h . As shown in Figure 7, increasing the inner radius r_w causes a decrease in the braking torque M , while increasing the outer radius R causes an increase in the torque M . This is related to the opposing influence of these radii on the flow velocity of the ER fluid within the rotors. Increasing the blade number n , Figure 8, causes a decrease in the torque M , which is connected to the decrease of the flow cross-sectional area A of the rotors. Figure 9

shows a decrease in voltage U_h as the number of blades increase, with a constant value of the torque M . The decrease is a result of decreasing average distance between blades d when the number of the blades increases. Decreasing the value of d causes an increase in the electric field strength E , which has a direct influence on the value of the yield stresses τ_0 . A significant (and different) influence of the radii r_w and R , as well as the number of blades n , on the PHB characteristics justifies the purposefulness of selecting these parameters as design variables for optimization of the PHB design.

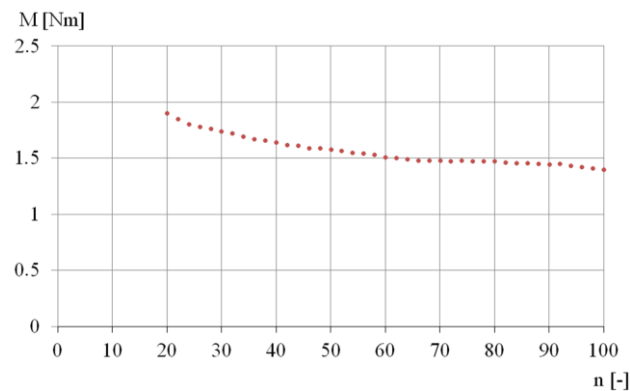


Figure 8. Dependence of the torque M on the blade number n .

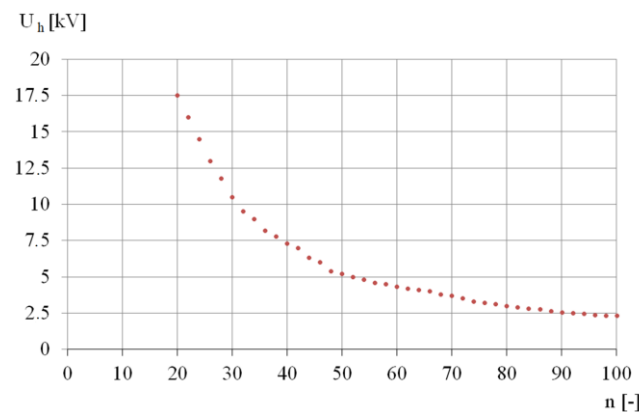


Figure 9. Dependence of the voltage U_h on the blade number n .

4.2. Selection of Optimization Criteria

The calculations included the dependence of weight G , electric power P_e , and time T on the torque M , as well as the dependence of the torque M , weight G , electric power P_e , and time T on the radii r_w , R , and the blade number n . The intervals of the HB parameters and additional conditions ensuring the absence of electric breakdowns are shown in Table 3.

Table 3. Intervals and conditions concerning the parameters.

Parameter Interval			Conditions	
r_w mm	R mm	n -	d_1 mm	U_h kV
$10 < r_w < 30$	$40 < R < 70$	$10 < n < 120$	$d_1 > h_{\min} = 1$	$U_h < 20$ kV

The HB parameter intervals are determined arbitrarily, aiming at possibly small differences between the parameters of the calculated HB and the parameters of the PHB used to determine the coefficients of the mathematical model. This allows obtaining the necessary accuracy of the used mathematical model. The HB calculations are performed as follows:

- it is assumed that $\omega_1 = 200$ rad/s;
- the radii r_w and R are randomly selected from the intervals given in Table 3;
- it is assumed that $g_b = 0.02 R$;
- calculations are performed according to the formulas of the mathematical model described in Section 2;
- the calculation results not meeting the requirements given in Table 3 are rejected.

The calculations are performed on the basis of a computer program written in Turbo Pascal 7. Examples of calculation results for 5000 draws and for $\omega_1 = 200$ rad/s and $\Theta = 35$ °C are shown in Figures 10–18.

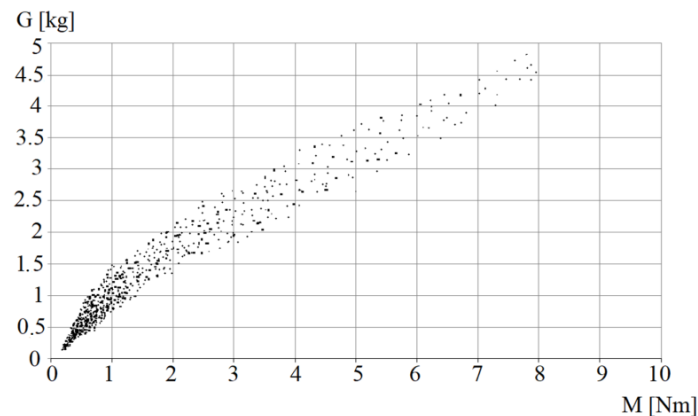


Figure 10. Dependence of the weight G on the torque M .

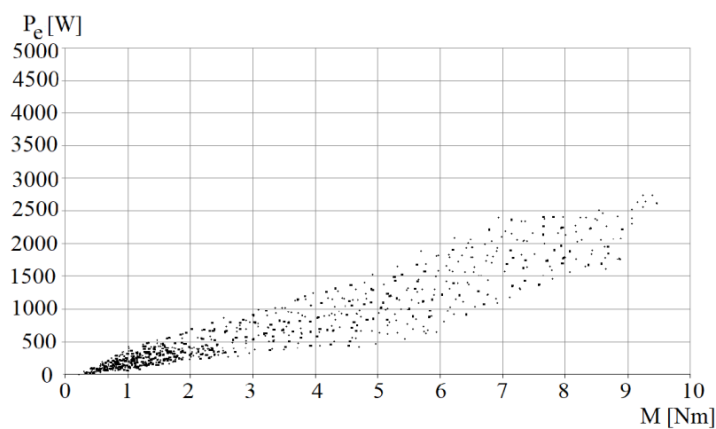


Figure 11. Dependence of the electric power P_e on the torque M .

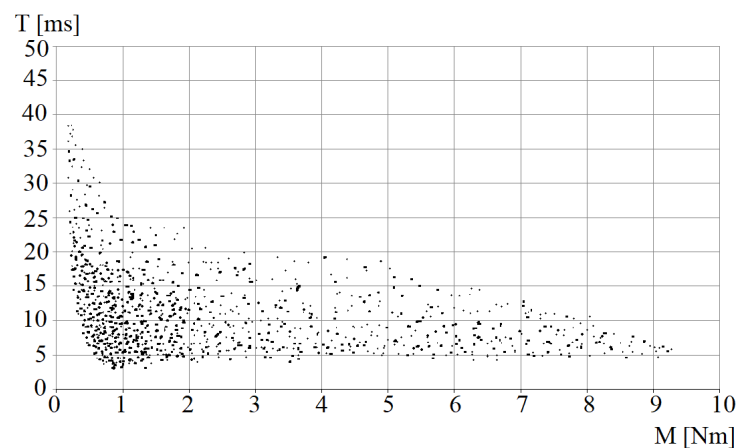


Figure 12. Dependence of the time T on the torque M .

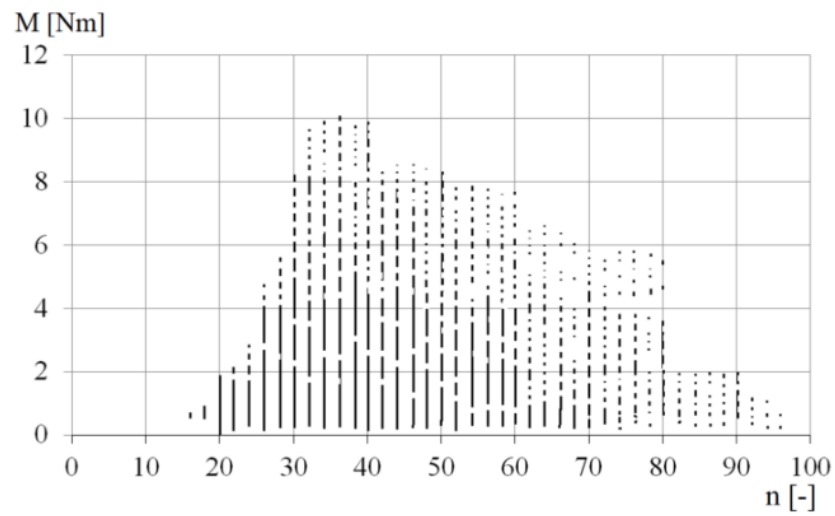


Figure 13. Dependence of the torque M on the number of blades n .

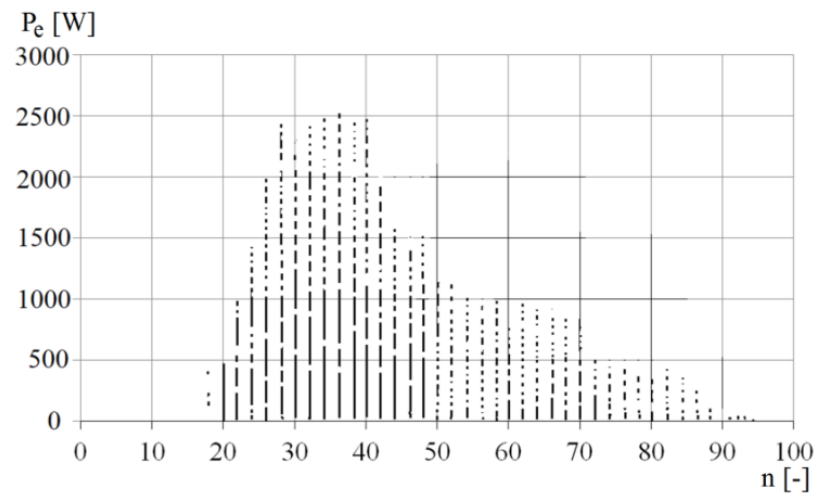


Figure 14. Dependence of the electric power P_e on the number of blades n .

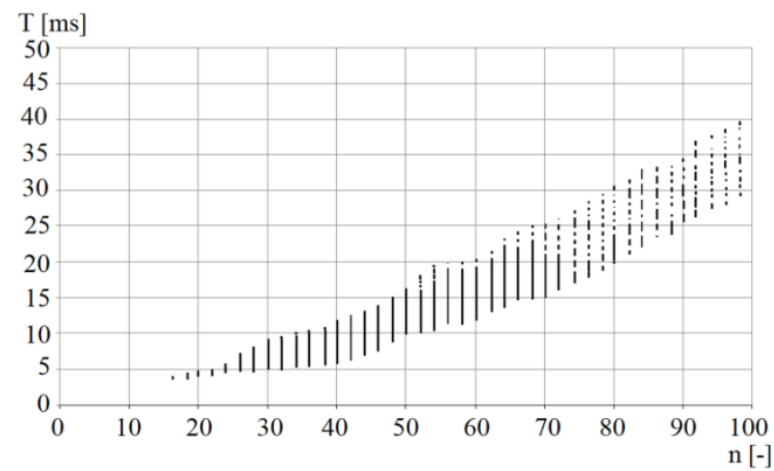


Figure 15. Dependence of the time T on the number of blades n .

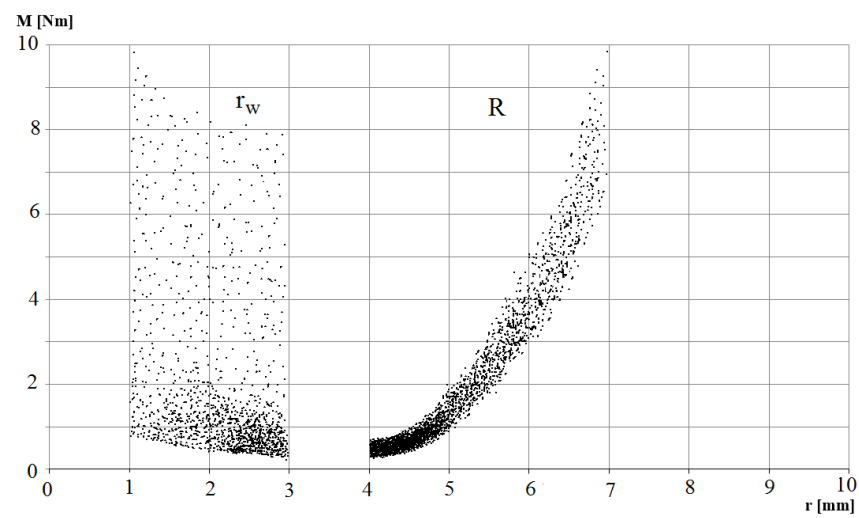


Figure 16. Dependence of the torque M on the radius r_w and on the radius R .

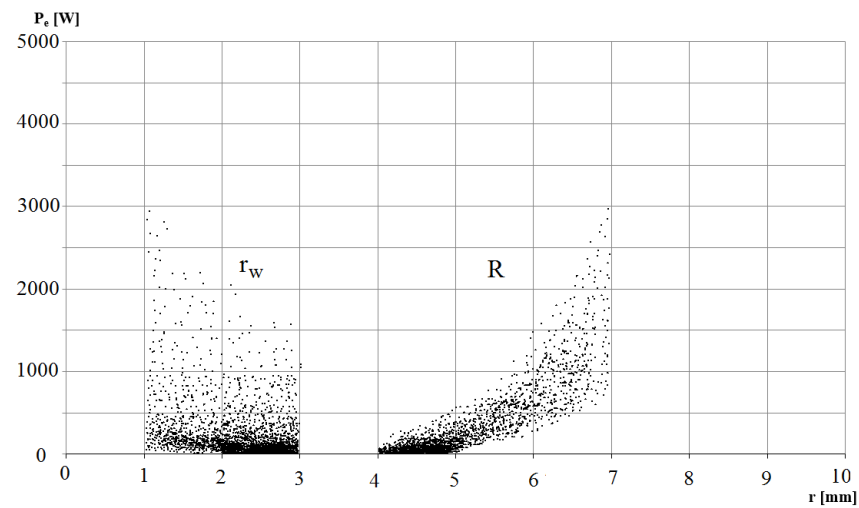


Figure 17. Dependence of the electric power P_e on the radius r_w and on the radius R .

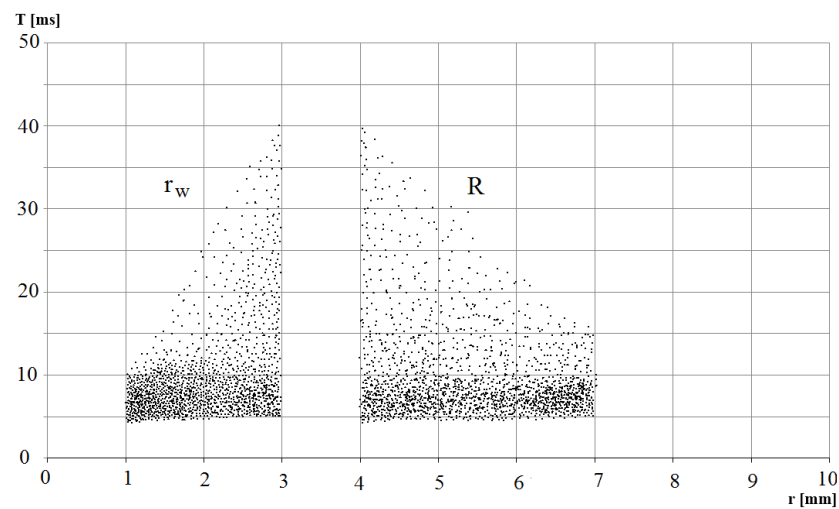


Figure 18. Dependence of the time T on the radius r_w and on the radius R .

Figures 10 and 11 show the proportional dependence of the weight G and electric power P_e on the torque M . Different values of the weight G and the power P_e occurring

for the same values of the torque M are mostly a result of the difference in the number of blades n . For fixed values of the radius r_w and the radius R , the smaller the number of blades n , the smaller the weight G . In this case, the dependence of the power P_e on the number of blades is the opposite; the smaller the number of blades n , the higher the voltage U_h necessary to reach the yield stress τ_0 needed to stop the flow of the ER fluid (and, therefore, the power P_e is greater). Figure 12 shows that small values of time T occur for all values of the torque M . A wider range of the time values T for smaller torques M is a result of smaller dimensions of the HB and, therefore, smaller distances between blades and a larger capacity C_e . As shown in Figures 13 and 14, the dependence of the torque M and the electric power P_e on the number of blades are similar. The largest values of M and P_e occur within the range from $n = 30$ to $n = 40$. The increase of the average time values T with the increase in the number of blades n can be observed in Figure 15. It is a result of the smaller distances between blades and, therefore, larger capacities C_e on which the time T depends. As shown in Figures 16 and 17, the dependence of the torque M and the electric power P_e on the radius r_w and on the radius R are similar. As can be seen in Figure 18, by changing the value of the radius r_w and the radius R in the intervals presented in Table 3, one can obtain similar values of time T .

The dependences of weight G , electric power P_e , and time T on the torque M are not functions; thus, HBs with the same value of the torque M can have different weights G , electric powers P_e , and times T . Moreover, M , G , P_e , and T significantly (and in different ways) depend on the assumed design variables r_w , R , and n . Due to that, the torque M , weight G , electric power P_e , and time T_s are selected as optimization criteria for the design and are used to formulate the objective functions F . The selection of the criteria is consistent with the pursuit of mechanical engineering to increase efficiency, reduce weight, and save energy. The requirements concerning the assumed design optimization criteria are presented in Table 4.

Table 4. Requirements concerning the HB optimization criteria.

Name	Designation	Requirements	Main Calculation Formula Numbers
Torque	M	Maximal	(16), (17), (25), (26)
Weight	G	Minimal	(4)–(7)
Electric power	P_e	Minimal	(30), (31), (33), (34)
Time	T	Minimal	(35)–(37)

4.3. The Algorithm for Design Optimization of the Hydrodynamic Brake

On the basis of the criteria given in Table 4, the following objective functions are assumed:

$$\begin{aligned} F_G &= G - 0.5 \cdot M \\ F_{G1} &= G/M \end{aligned} \quad (38)$$

$$\begin{aligned} F_P &= 0.1 \cdot P_e - M \\ F_{P1} &= P_e/M \end{aligned} \quad (39)$$

$$\begin{aligned} F_T &= T - 0.3 \cdot M \\ F_{T1} &= T/M \end{aligned} \quad (40)$$

The objective functions F_G , F_P , F_T , F_{G1} , F_{P1} , F_{T1} described by Formulas (38)–(40) cannot be written using simple formulas, in which there are the design variables, due to the complicated form of the applied mathematical model. The values of the criteria G , M , P_e , T were calculated on the basis of the formulas given in Table 4. The numerical coefficients appearing in the Formulas (38)–(40) are arbitrarily selected during initial calculations, so that minimal values of the objective functions F_{Gmin} , F_{Pmin} , F_{Tmin} are as close to zero as possible. The objective function, which is the difference between G and M , is also assumed in the articles [49,50]. The minimalization of the objective functions (38)–(40) leads to finding the design variables, for which the unit of the torque M will be assigned with,

respectively, the lowest weight G , the lowest electrical power P_e , and the shortest time T . The design variables intervals r_w , R , n and constraints concerning d_1 and U_h are the ones given in Table 3. The optimization calculations for the HB are aimed at minimizing the selected objective function F described with one of the Formulas (38)–(40). The calculations are performed as follows:

- the decision variables r_w , R , and n are randomly selected from the assumed intervals with the use of a random number generator;
- the value of ω_1 is determined and $g_b = 0.02R$ is calculated;
- calculations are performed according to the formulas of the mathematical model;
- the constraints of d_1 and U_h are checked;
- when the randomly selected values of the design variables meet the conditions of the constraints, the objective function F is calculated, its result is saved, and the calculations are repeated;
- the smaller value of the objective function from the previous F_p and present step of the calculations F_a is selected;
- the values of the design variables r_w , R , and n are saved, and on the basis of these values, an objective function of a smaller value is obtained.

The scheme of calculations carried out in one calculation step of a computer program is shown in Figure 19.

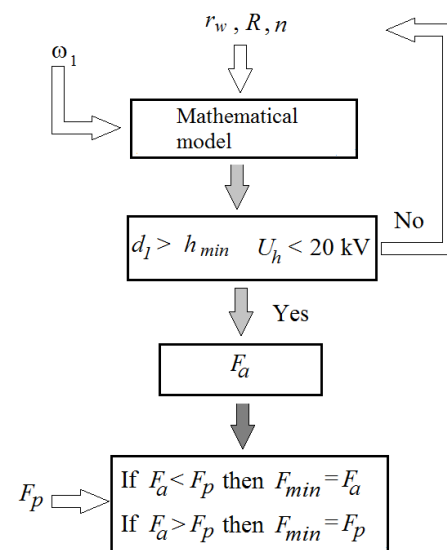


Figure 19. Algorithm of calculations carried out in one calculation step.

5. Design Optimization Results and Discussion

The results of the calculations shown in Figures 10–12 demonstrate that it is not possible to determine intervals of the torque M , for which there are minima of the considered objective functions. Due to that, initially, the design optimization of the HB is carried out for the entire range of decision variables. The results of the HB design optimization for 150,000 draws are shown in Table 5. The placement of the minima F_{min} of the objective function F on the background of the “Pareto chart” showing the dependence of the “lightness of construction” (described as $1/G$) on the torque M is depicted in Figure 20. Table 6 contains the values of the ratios: G/M , P_e/M , T/M and G , P_e , T calculated on the basis of the data from Table 1. In order to assess the possibility of PHB optimization, calculations are made with the assumption that the braking torque M meets the requirement $1.700 \text{ Nm} < M < 1.800 \text{ Nm}$, i.e., it is close to the torque of the experimentally tested PHB: $M = 1.733 \text{ Nm}$. Assuming such a small range of torque M variations increases the possibility of finding the minima of the objective functions. A few percent change of the torque M during the HB selection for a machine’s drive system is acceptable. Table 7 shows the results of these calculations for 150,000 draws. It is justified to assume the quotients of the criteria

listed in Table 4 as the functions of the objectives, because the quotients G/M , P_e/M , T/M can be interpreted as efficiency related to 1 Nm. For instance, $G/M = 0.497 \text{ kg/Nm}$ means that 1 Nm of the braking torque M corresponds to 0.497 kg of the HB mass. Table 5 shows that the minimum of the objective function is achieved for torques M of approximately 7.5 Nm for the objective functions F_G , F_{G1} ; 0.2 Nm for the objective functions F_{P_e} , $F_{P_{e1}}$; and 9.9 Nm for the objective functions F_T , F_{T1} . Therefore, the HBs having a large braking torque M have a low G/M ratio and a low T/M ratio, which is advantageous. On the contrary, the HBs having a low braking torque M have unfavorable values of G/M ratio and especially T/M ratio, but the value of the P_e/M ratio is advantageous. The minimal value of the G/M ratio does not correspond to the minimal value of the HB, due to the large differences between the values of the torques. However, the minimal values of the P_e/M and T/M ratios occur for the minimal values, respectively, P_e and T . Proximate values of the G/M , P_e/M , T/M ratios, defined for the objective functions as differences and quotients of criteria, prove the correctness of the performed calculations. Table 6 and Figure 20 show that the PHB is not an optimal brake in terms of the considered criteria. This is proven by the high values of the G/M , P_e/M , T/M ratios, as well as the placement of point 7 beyond the “Pareto front” (which is a set of points with the lowest values of $1/G$). Table 7 shows that the minimal values of the G/M , P_e/M , and T/M ratios occur for the minimal values of, respectively, G , P_e , and T . The optimized PHBs with a lower weight also have lower mass moments of inertia. This results in the fact that these brakes have a short head rise time of the torque M . Table 7 also shows that the values of the braking torque M , differing from each other by less than 4%, are obtained for significantly different values of design variables. The relative differences between the highest and lowest values of individual design variables, related to the average values, are 80% for r_w , 12% for R , and 83% for n . Whereas, the relative differences between the highest and lowest values of the ratios G/M , P_e/M , and T/M are respectively 54%, 140%, and 146%.

Table 5. Optimization results for the whole intervals of decision variables.

$F_{\min}$	M Nm	r_w mm	R mm	n -	G/M kg/Nm	P_e/M W/Nm	T/M ms/Nm	G kg	P_e W	T ms
$F_{G\min}$	7.478	29.1	69.9	42	0.497	181.4	0.790	3.717	1356	5.9
$F_{G1\min}$	7.473	29.9	69.4	38	0.446	231.6	0.681	3.333	1731	5.1
$F_{P\min}$	0.218	28.8	40.9	98	1.136	38.5	201.821	0.248	8.42	44.4
$F_{P1\min}$	0.222	28.9	41.1	100	1.150	38.2	206.113	0.255	8.45	45.8
$F_{T\min}$	9.850	11.6	69.8	30	0.601	254.6	0.358	5.920	2508	3.5
$F_{T1\min}$	9.886	10.6	69.9	30	0.611	313.3	0.489	6.040	3096	4.8

The initial values of PHB presented in Table 6 can be compared with the optimized values presented in Table 7. The comparison of the values of the G/M , P_e/M and T/M ratios (presented in Tables 6 and 7) show that, on the basis of the design optimization, it is possible to reduce the following factors: the weight of PHB 1.7 times, the maximum electric power 1.5 times, and the time T 2.5 times. However, these benefits cannot be gained simultaneously. The results shown in Table 7 confirm that it is advisable to improve the PHB design on the basis of design optimization methods. Due to the fact that the selection of decision variables is random, repeating the optimization calculation always provides different results. Moreover, it is not possible to unequivocally determine the optimal number of draws because there is always a possibility that the following draws will provide better results. However, it is necessary to take into consideration the fact that, during the design optimization, only the changes of radii r_w and R by tenths of a millimeter are practically significant.

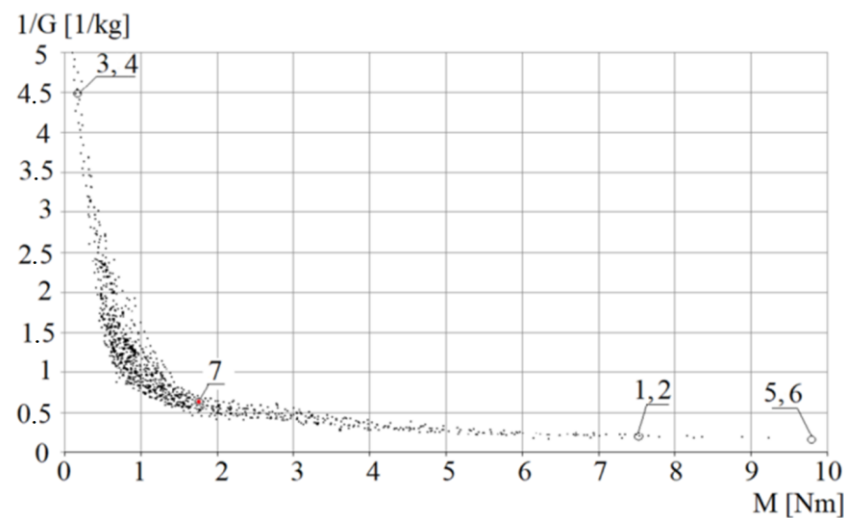


Figure 20. Optimization results on the background of the Pareto chart: 1— F_{Gmin} , 2— F_{G1min} , 3— F_{Pmin} , 4— F_{P1min} , 5— F_{Tmin} , 6— F_{T1min} , 7—PHB.

Table 6. Values of the ratios G/M , P_e/M , T/M and G , P_e , T for the optimized PHB.

M Nm	r_w mm	R mm	n -	G/M kg/Nm	P_e/M W/Nm	T/M ms/Nm	G kg	P_e W	T ms
1.733	15.5	50.5	38	1.013	97.9	5.72	1.756	170	9.9

Table 7. PHB optimization results for $1.700 \text{ Nm} < M < 1.800 \text{ Nm}$.

F_{min}	M Nm	r_w mm	R mm	n -	G/M kg/Nm	P_e/M W/Nm	T/M ms/Nm	G kg	P_e W	T ms
F_{Gmin}	1.728	29.7	53.6	30	0.570	239.0	3.027	0.985	413	5.2
F_{G1min}	1.782	28.9	53.6	30	0.586	240.9	3.070	1.044	429	5.4
F_{Pmin}	1.749	29.6	55.8	88	0.837	64.8	14.603	1.464	113	25.5
F_{P1min}	1.753	29.6	55.9	86	0.823	66.5	13.986	1.443	116	24.5
F_{Tmin}	1.746	16.2	50.0	22	0.894	356.1	2.245	1.561	621	3.9
F_{T1min}	1.795	12.6	49.8	22	0.992	351.0	2.269	1.781	630	4.0

6. Conclusions

On the basis of the performed optimization calculations concerning the design of the HB with the ER fluid, the following conclusions have been achieved:

1. Performing the optimization of the HB with the ER fluid requires formulating a mathematical model, establishing constraints, and selecting design variables and the objective function. While planning the design optimization, it is advisable to aim at a simple mathematical model and a low number of design variables, especially when the optimization calculations are performed with the use of the Monte Carlo method (so that it is possible to perform a large number of calculations within an acceptable time). On the basis of the obtained results of the optimization of the HB with the ER fluid, it can be presumed that the assumptions are correct.
2. The form of the objective function has a significant influence on the optimization results. However, there are no strict rules as to how the objective function should be formulated. This is why the selection of the objective function is always arbitrary. It is advisable to formulate the objective function in a simple way, so that the results of optimization calculations are easy to interpret. Defining two different objective

functions on the basis of the same criteria increases the probability of finding the optimal solution. Due to that, it is justifiable to use the difference and quotient of only two criteria when optimizing the design of the HB with the ER fluid in order to determine the objective function.

3. The Monte Carlo method can be successfully used for design optimization of brakes with ER fluid for three design variables. Increasing the number of the design variables causes an exponential increase in the number of draws necessary to obtain the required accuracy of calculations. Not all optimization methods can be used for this purpose because the number of blades, which is a natural number, causes the calculations to be discrete.
4. In order to obtain credible and reliable results of the optimization calculations, it is advisable to select coefficients for the assumed mathematical model, as well as for the range of the mathematical model. It is especially important when the HB optimization calculations are performed with the use of a mean streamline model whose accuracy depends on the correctness of the selected coefficients.

It is finally remarked that the optimized method proposed based on the mathematical model can be used when the design of the transmission system is specified only. Therefore, it is a general optimization method that can be applied to optimally design several application systems utilizing ER fluid and MR fluid also.

Author Contributions: I.M. and Z.K. contributed conception and design of the study. Z.K. conducted optimization analysis and calculations. Z.K. and I.M. carried out tests. Z.K. and I.M. wrote the first draft of the manuscript. S.-B.C. contributed to idea analysis, manuscript revision, reading, and approving the submitted version. All authors have read and agreed to the published version of the manuscript.

Funding: The authors received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no potential conflict of interest with respect to the research, authorship, and publication of this article.

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