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A New Effective Narrowband Active Noise Control System for Accommodating Frequency Mismatch

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Abstract: Narrowband active noise control (NANC) has shown excellent performance in dealing with the low frequency periodic noise generated by rotating machines, such as fans, engines and power transformers. Accommodating large frequency mismatch (FM) and improving its tracking capability is required for the NANC system. The existence of FM influences the noise cancellation performance. In this paper, a frequency correction algorithm based on least mean p-power (LMP) combined with the autoregressive (AR) model is designed for the NANC system, which is simple and feasible, and has a good performance under a large step size. In the NANC system, the reference signal is handled by a delay unit and AR model, and the coefficients of the AR model are adjusted by the LMP algorithm, which fine-tunes the coefficients and offers the reference signals to the NANC system. The stability bounds for the step size parameter have also been derived in the mean sense. The designed mechanism converges fast and enhances the noise decrement. Extensive simulations are performed to demonstrate the superior performance of the proposed NANC in dealing with periodic noises.

Keywords: narrowband active noise control; convergence rate; frequency mismatch



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1. Introduction

In our daily work and life, many disturbing noises can not only cause psychological discomfort for humans, but could also strengthen the structural load and, hence, give rise to fatigue damage [1–5]. These noises are generated by rotating machines such as cutting machines, electric fans, and motors. The other characteristic low frequency humming noise is due to the periodic vibration of the iron core with the change of the excitation frequency in the power transformer box [6]. A similar noise to this may be modeled as a sinusoidal signal in additive noise. Usually, the frequencies of the noise signals are arbitrary, and their magnitudes are time-varying. The noise generated by rotating or reciprocating equipment is mainly low frequency harmful noise [7]. Narrowband active noise control (NANC) is commonly used in noise-cancelling headsets [8,9], modern automobiles [10–13], and vehicle noise [14–16]. The conventional NANC system has shown great superiority in low frequency signal processing [17], while the use of sound-absorbing and sound-insulating materials shows a poor performance in dealing with the low frequency part of this noise [18].

In general, there are two ways to obtain reference signals. One is to obtain the noise directly by placing an acoustic signal sensor in the noise region, and the other is to use a piece of electronics hardware to convert the detected velocity or acceleration into the same

frequency signal via a linear conversion relation. However, to avoid the acoustic feedback problem caused by acoustic sensors, the aforementioned non-acoustic sensors can be used instead in most conventional feedforward NANC systems. Compared with the former method, it is a good choice to detect the fundamental frequency information of the primary noise to generate a reference signal in [19]. Typically, the reference signal is generated by a waveform synthesis method to control the narrowband noise, whose amplitude is the same as and the phase is opposite to the primary noise. The secondary signal produced by the filtered reference signal is then fed to the loudspeaker to attenuate the noise.

In real life, non-acoustic sensors or cosine wave generators may not be very precise, fatigue and aging, which could cause indeterminacy. If the non-acoustic sensor error is nonnegligible or sufficiently large, the frequency of the reference signal fed to each channel will be at odds with the frequency possessed by the primary noise signal, and the NANC system will present worse performance. Therefore, the difference in frequencies between the reference signal and the primary noise is defined as the frequency mismatch (FM). The simulation results have shown that the noise reduction performance of active noise control system will be severely degraded even if it is just 1% FM [20].

Various different adaptive filtering algorithms have been proposed to deal with FM. Based on the theory of acoustic superposition, a typical parallel adaptive NANC system designed by Widrow et al. could eliminate the target single noise by means of a second-order filter in [21], but a problem that existed in the system is that the frequency of the reference signal was inconsistent with the target noise signal. Considering the flaws in the system, a new auto-regressive (AR) ANC system was proposed by Xiao and other scholars in [22], whose reference signal is preprocessed by the AR model in advance, thus correcting the reference signal. In this way, the system was found to compensate the FM by 10%. The statistical analysis of the NANC system is presented in [23,24], where Xiao and Liu Jian elaborated on the condition in which the system reaches a steady state with the FM, such as the optimum step size and excess mean square error for the NANC system. However, this construction makes it difficult to handle large misalignments. In [25], from the view of the complex domain, the ANC system dealing with the FM is experimentally and theoretically analyzed, and a new adaptive algorithm, named the variable step-size FXLMS algorithm, was used to improve the system performance. However, this is constrained by the fact that, in real life, there is no manipulation of complex signals, and the components of the primary noise are very complex. Another frequency correction method based on the iterative minimum variance distortionless responses (MVDR) spectrum estimator was proposed by Hyeo, in [26], for nullifying the ill effect of the FM. A momentum least mean square (MLMS) algorithm based on the ANC algorithm, in which a momentum correction term is artificially added between the weight vector correction and the gradient estimation, has been proposed by Huang to develop the NANC algorithm in [27]; this momentum term will quickly increase at the beginning of the iteration, thus accelerating the gradient decline, making the weight of the coefficient more stable. There was also an adaptive algorithm with similar functions proposed by Xia et al.; the phase of the reference signal is changed by introducing a delay module in the i -th frequency channel, and then adjusted by the adaptive linear combination, and the correctness of the results was simulated by Simulink [28,29]. In [30], the authors present the steady state analysis of the p-power algorithm for the sinusoidal signal.

The presence of FM at the reference signal needs a large step size to maintain the noise-canceling performance in the NANC system. However, the large step size poses a bad steady state. Therefore, another LMP algorithm concept is applied to the NANC algorithm with a frequency correction feature. In light of the issues mentioned above, a new NANC system is developed in this paper, which attempts to address the convergence rate and the robustness of conventional NANC systems.

The main contributions of this paper are summarized as follows: Firstly, we have dramatically reduced the time it takes for the system to converge with the FM. Secondly, we consider that a small step size can obtain a larger steady-state error with the FM,

as the conventional variable step size algorithms are not useful. With the LMP, a large step size could achieve a better performance. Therefore, we use the LMP to optimize the dual frequency compensation so that the system could accommodate for a large FM and improve the tracking ability. Thirdly, the closed-form convergence analysis of the mean of the proposed system for discrete Fourier coefficients (DFC) are derived, along with the stability condition on the step size.

The remainder of this paper is organized as follows. Section 2 describes the structure of the NANC system and the FM. The proposed method and stability analysis with a delay unit in the mean sense are described in detail in Section 3. Section 4 presents the representative simulations to clarify the validity of the proposed method. The conclusions are presented in Section 5.

2. Materials and Methods

2.1. The Conventional NANC System

Figure 1 depicts the structural block diagram of the conventional parallel narrowband ANC system [21]:

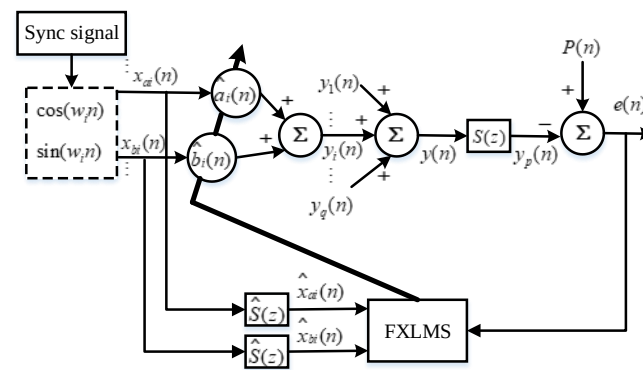


Figure 1. Block diagram of conventional FXLMS algorithm.

It is assumed that the primary noise that is removed has multiple fundamental frequency noises, and such noise signals can be expressed as:

$$P(n) = \sum_{i=1}^{i=q} [a_{p,i} \cos(w_{p,i}n) + b_{p,i} \sin(w_{p,i}n)] + v_p(n) \quad (1)$$

In this formula, q is the number of frequency components contained in the primary noise, $\omega_{p,i}$ ($i = 1, 2, 3$) is the frequency of the i -th component, $\{a_{p,i}, b_{p,i}\}_{i=1}^{i=q}$ correspond to the DFCs of the different components of the primary noise. $V_p(n)$ represents the additive background noise, which is a zero-mean additive white Gaussian noise with variance δ_p^2 .

The reference signals are the software synthesized sinusoidal signal according to the information of the estimated frequencies. The i -th input channel can be, respectively, modeled as:

$$x_{ai}(n) = a_i \cos(w_{r,i}n) \quad x_{bi}(n) = b_i \sin(w_{r,i}n) \quad (2)$$

where $\{a_i, b_i\}_{i=1}^{i=q}$ represent the weights of the controller filter, $\omega_{r,i}$ represents the frequency captured by a non-acoustic sensor.

The sum of the synthesized reference signals in all channels are:

$$y_i(n) = \sum_{i=1}^{i=q} [\hat{a}_i(n)x_{ai}(n) + \hat{b}_i(n)x_{bi}(n)] \quad (3)$$

where $\{\hat{a}_i, \hat{b}_i\}_{i=1}^{i=q}$ denote the estimations of the controller filter weights, $\sum_{m=1}^{m=M} \hat{s}_m$ is the coefficients of the estimated secondary path. Generally, the block $S_{(z)}$ is assumed to be known, which corresponds to the secondary-path and is considered as a FIR low pass filter with length M , $\sum_{m=1}^{m=M} S_m$ represents the weights of the filter.

The error signal that consists of q residual noise components in the error microphone is expressed as:

$$e(n) = P(n) - \sum_{m=1}^{m=M} \hat{s}_m y(n-m) \quad (4)$$

Then, the error signal is fed to the DFC estimation by FXLMS:

$$\hat{a}_i(n+1) = \hat{a}_i(n) + \mu_{a,i} e(n) \hat{x}_{ai}(n) \quad (5)$$

$$\hat{b}_i(n+1) = \hat{b}_i(n) + \mu_{b,i} e(n) \hat{x}_{bi}(n) \quad (6)$$

where $\{u_{a,i}, u_{b,i}\}_{i=1}^{i=q}$ is the step size of the DFCs coefficients, $\{\hat{x}_{ai}(n), \hat{x}_{bi}(n)\}_{i=1}^{i=q}$ are the input signals filtered by $\hat{S}_{(z)}$, which are as follows:

$$\hat{x}_{ai}(n) = \sum_{m=1}^{m=M} \hat{s}_m x_{ai}(n-m) = \hat{\alpha}_i x_{ai}(n) + \hat{\beta}_i x_{bi}(n) \quad (7)$$

$$\hat{x}_{bi}(n) = \sum_{m=1}^{m=M} \hat{s}_m x_{bi}(n-m) = -\hat{\beta}_i x_{ai}(n) + \hat{\alpha}_i x_{bi}(n) \quad (8)$$

where

$$\hat{\alpha}_i = \sum_{m=1}^{m=M} \hat{s}_m \cos(m\omega_{r,i}) \quad \hat{\beta}_i = \sum_{m=1}^{m=M} \hat{s}_m \sin(m\omega_{r,i}) \quad (9)$$

2.2. The Limitations of Conventional NANC System

If the frequencies of the reference signal have no discrepancy with the original signal, as shown in Figure 2a, the mean square error (MSE) value gradually decreases and tends to be stable in the end. However, as shown in Figure 2b, even if a 1% FM appears in the NANC system, the system performance will sharply deteriorate, the MSE curve will oscillate periodically, and the noise suppression performance of the traditional NANC system is worse.

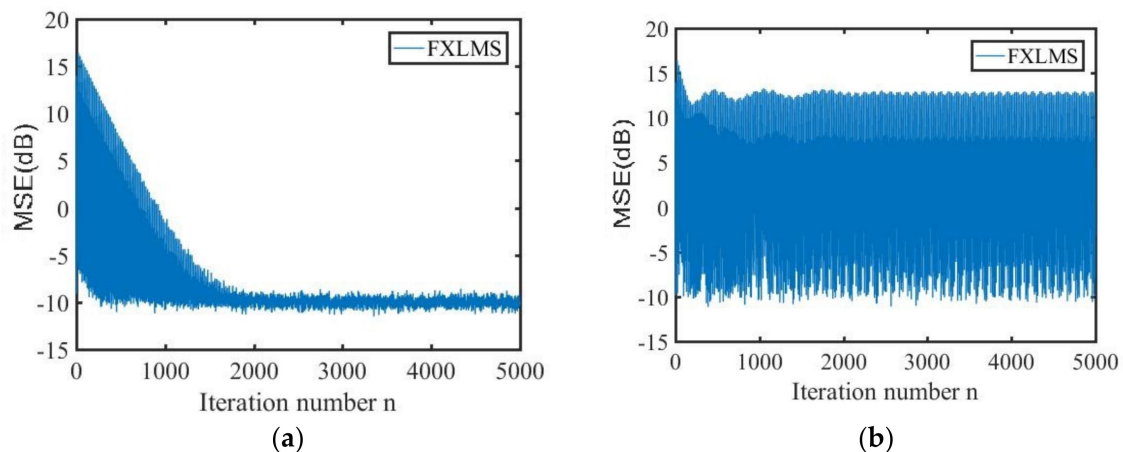


Figure 2. Levels of residual noise signals for the conventional NANC system (true primary noise frequency: $\omega_p = [0.1\pi \ 0.2\pi \ 0.3\pi]$, reference signal frequency: $\omega_r = [0.101\pi \ 0.202\pi \ 0.303\pi]$, original amplitude: $a_p = [2 \ 1 \ 1]$, $b_p = [1 \ 1 \ 1]$, reference signal: $a_r = b_r = [1 \ 1 \ 1]$, the step size: $\mu = 0.005$, the secondary path: $M = 11$, $S_{(z)} = \hat{S}_{(z)}$, $\delta_p^2 = 0.1$, 40 runs, (a) Conventional NANC system (without FM). (b) Conventional NANC system (with 1% FM).

2.3. NANC System with AR Model

Xiao et al. proposed a second order AR with reference signals to mitigate the effect of the FM; the i -th channel of reference signal can be expressed as:

$$x_{ai}(n) = -c_i(n)x_{ai}(n-1) - x_{ai}(n-2), \quad n \geq 2 \quad (10)$$

$$x_{bi}(n) = -c_i(n)x_{bi}(n-1) - x_{bi}(n-2), \quad n \geq 2 \quad (11)$$

where $c_i(n)$ is the coefficient the of frequency compensation in the i -th channel, respectively, its initial value is $-2 \cos(\omega_i)$. The related coefficients could be updated to obey a LMS recursion:

$$c_i(n+1) = c_i(n) - \mu_{c_i} e(n) [\hat{a}_i(n) \hat{x}_{a_i}(n-1) + \hat{b}_i(n) \hat{x}_{b_i}(n-1)] \quad (12)$$

where u_{c_i} is the updating step size.

2.4. NANC System with MLMS

In order to improve the lag of the tracking curve, a method based on the momentum minimum least mean square (MLMS) error is proposed; that is, a simple predictor is added when updating the coefficients of the AR model [31]:

$$c_i(n+1) = c_i(n) - \mu_{c_i} e(n) [\hat{a}_i(n) \hat{x}_{a_i}(n-1) + \hat{b}_i(n) \hat{x}_{b_i}(n-1)] + \lambda_{c_i} [c_i(n) - c_i(n-1)] \quad (13)$$

The formula for updating the AR coefficients based on the MLMS algorithm is shown above. The principle of the MLMS algorithm is to make use of the correlation of coefficients in the adaptive process, and then bring in a momentum term $\lambda_{c_i} (c_i(n) - c_i(n-1))$ in the iteration process. However, its convergence is relatively slow with 10% FM and may not meet the requirements of some applications.

3. Proposed Algorithm

In this section, inspired by the convex combination structures and FE-NANC system [32,33], a reference signal is generated by adding the weighted sum of both the cosine and sine components of each frequency present in the noise, and then the frequency compensation and the AR model work simultaneously. Excellent steady-state errors can be obtained after considering larger step sizes. We thoroughly demonstrate the proposed algorithm with the block diagram, shown in Figure 3 below.

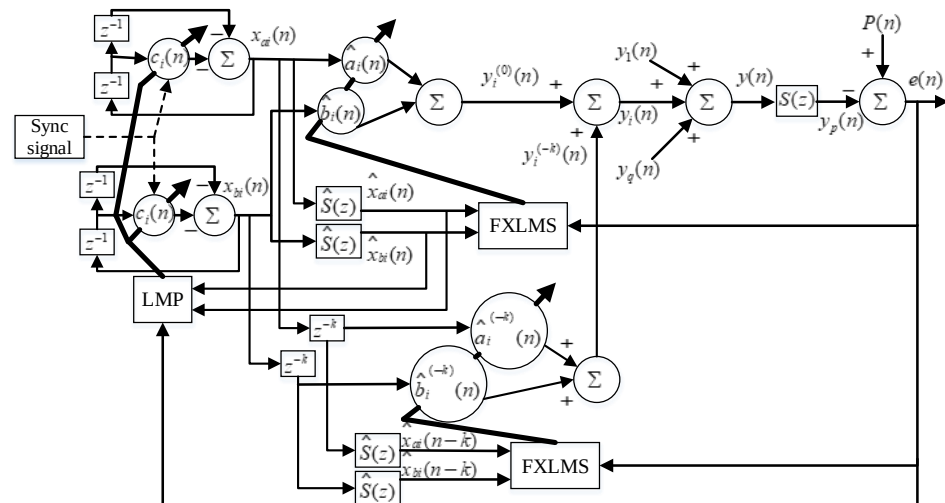


Figure 3. The block of proposed NANC system.

The AR model is optimized by the LMP criterion; with the coefficients of the AR model adjusted by the LMP and good performance of the AR model in the tracking frequencies, the NANC system can achieve a high standard in canceling the narrowband noises and fast convergence.

In the above graph, the reference signal synthesized in each frequency channel is composed of two parts: one part is that the cosine and sine signal are adjusted by the

AR model, and then filtered by FXLMS; the other part is the frequency compensation combination with the delay module, which is expressed as:

$$y_i(n) = y_i^{(0)}(n) + y_i^{(-k)}(n) \quad (14)$$

where

$$y_i^{(0)}(n) = \hat{a}_i(n)x_{ai}(n) + \hat{b}_i(n)x_{bi}(n) \quad (15)$$

$$y_i^{(-k)}(n) = \hat{a}_{k_i}(n)x_{ai}(n-k) + \hat{b}_{k_i}(n)x_{bi}(n-k) \quad (16)$$

According to the FXLMS algorithm, we have:

$$\hat{a}_{k_i}(n+1) = \hat{a}_{k_i}(n) + \mu_{k_{a_i}} e(n) \hat{x}_{ai}(n-k) \quad (17)$$

$$\hat{b}_{k_i}(n+1) = \hat{b}_{k_i}(n) + \mu_{k_{b_i}} e(n) \hat{x}_{bi}(n-k) \quad (18)$$

where k is the delay unit, $\hat{x}_{ai}(n-k)$, $\hat{x}_{bi}(n-k)$ are the input cosine and sine waves filtered by $\hat{S}_{(z)}$; $u_{k_{a_i}}$, $u_{k_{b_i}}$ is the step size parameter of the delay model.

Where

$$\hat{x}_{ai}(n-k) = \sum_{m=1}^{m=M} \hat{s}_m x_{ai}(n-k-m) = \hat{\alpha}_{k_i} x_{ai}(n) + \hat{\beta}_{k_i} x_{bi}(n) \quad (19)$$

$$\hat{x}_{bi}(n-k) = \sum_{m=1}^{m=M} \hat{s}_m x_{bi}(n-k-m) = -\hat{\beta}_{k_i} x_{ai}(n) + \hat{\alpha}_{k_i} x_{bi}(n) \quad (20)$$

where

$$\hat{\alpha}_{k_i} = \sum_{m=1}^{m=M} \hat{s}_m \cos[(m+k)\omega_{r,i}] \quad \hat{\beta}_{k_i} = \sum_{m=1}^{m=M} \hat{s}_m \sin[(m+k)\omega_{r,i}] \quad (21)$$

Based on the LMP cost function $J(n) = 1/3|e(n)|^3$, the frequency-related coefficients may be updated using the steepest descent gradient expression:

$$c_i(n+1) = c_i(n) - \mu_{c_i} \nabla_{c_i(n)} J(n) \quad (22)$$

This algorithm is derived as follows:

$$\begin{aligned} \nabla_{c_i(n)} J(n) &= -|e(n)|(e(n))^2 \frac{\partial y_p(n)}{\partial c_i(n)} \\ &= -|e(n)|(e(n))^2 \frac{\partial (\sum_{m=1}^{m=M} s_m y_i(n-m))}{\partial c_i(n)} \\ &= -|e(n)|(e(n))^2 \frac{\partial [\sum_{m=1}^{m=M} s_m (y_i^{(0)}(n-m) + y_i^{(-k)}(n-m))]}{\partial c_i(n)} \\ &= -|e(n)|(e(n))^2 s_0 \left[\frac{\partial y_i^{(0)}(n)}{\partial \hat{x}_{ai}(n)} \frac{\partial \hat{x}_{ai}(n)}{\partial c_i(n)} + \frac{\partial y_i^{(0)}(n)}{\partial \hat{x}_{bi}(n)} \frac{\partial \hat{x}_{bi}(n)}{\partial c_i(n)} \right. \\ &\quad \left. + \frac{\partial y_i^{(-k)}(n-k)}{\partial \hat{x}_{ai}(n-k)} \frac{\partial \hat{x}_{ai}(n-k)}{\partial c_i(n)} + \frac{\partial y_i^{(-k)}(n-k)}{\partial \hat{x}_{bi}(n-k)} \frac{\partial \hat{x}_{bi}(n-k)}{\partial c_i(n)} \right] \end{aligned} \quad (23)$$

As the system stabilizes, we have $c_i(n) \approx c_i(n-k)$, so (23) can be written as:

$$\nabla_{c_i(n)} J(n) = |e(n)|(e(n))^2 s_0 [\hat{a}_i(n) \hat{x}_{ai}(n-1) + \hat{b}_i(n) \hat{x}_{bi}(n-1) + \hat{a}_{k_i}(n) \hat{x}_{ai}(n-k-1) + \hat{b}_{k_i}(n) \hat{x}_{bi}(n-k-1)] \quad (24)$$

By substituting (24) into (22), we have the updated equation:

$$c_i(n+1) = c_i(n) - \mu_{c_i} |e(n)|(e(n))^2 [\hat{a}_i(n) \hat{x}_{ai}(n-1) + \hat{b}_i(n) \hat{x}_{bi}(n-1)] \quad (25)$$

3.1. The Relationship between MSE and Delay Unit

The FM converts the amplitude compensation system of the reference signal into an amplitude/phase compensation system. Therefore, we need to understand the relationship between the steady-state MSE and the value of k in order to determine the optimal k value.

As an index of accuracy, we use the MSE $E(e(n)^2)$. The lower the MSE value, the better the k value.

In Figure 4, it can be seen that the delay unit k can affect the stability under different FMs. When the delay unit k is changed, the convergence rate and steady-state performance of the proposed system are also affected. The symbol of the red triangle represents that the k value will lead to an optimal MSE of the system, but the black circle is the opposite. When k is set to 3, 11, 17, 24 or 31, it is easy to cause the decline of the system performance in dealing with 15% FM, as shown in Figure 4d; $k = 9, 10$ or 27 will result in system divergence, and $k = 8, 14, 20$ or 32 will obtain a better performance. Hence, it is essential to place the above analysis in the first consideration to choose an appropriate value. Based on the existing results, it is clear that $k = 14$ is able to achieve the best MSE over the others under different FMs, as shown in Table 1. We therefore set the value of k to 14 for obtaining a good system performance in different cases.

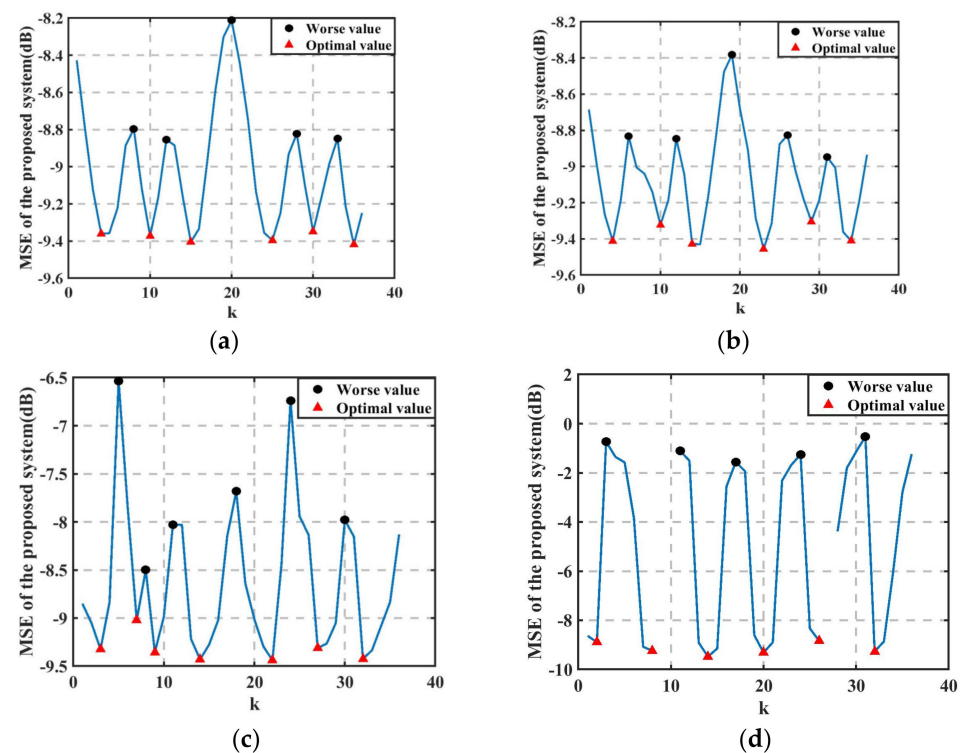


Figure 4. MSE of different k under different FM. (a) Without FM (b) Under 5%FM. (c) Under 10%FM (d) Under 15%FM.

Table 1. The MSE with Different k under Different FMs(dB).

Parameters	K = 8	K = 14	K = 20	K = 32
FM = 10%	−8.4999	−9.43	−9.012	−9.424
FM = 5%	−9.041	−9.428	−8.677	−9.007
Without FM	−8.796	−9.159	−8.212	−8.972

3.2. Stability Analysis in the Mean Sense

The stability analysis of the proposed algorithm is undertaken by the following. Using (1)–(4), (5), (7)–(9) and (14)–(21), the error signal (4) can be represented as:

$$e(n) = \sum_{i=1}^q \left\{ \left[a_{p,i} - \left(\alpha_i \hat{a}_i(n) - \beta_i \hat{b}_i(n) + \alpha_{k_i} \hat{a}_{k_i}(n) - \beta_{k_i} \hat{b}_{k_i}(n) \right) \right] x_{ai}(n) + \left[b_{p,i} - \left(\beta_i \hat{a}_i(n) + \alpha_i \hat{b}_i(n) + \beta_{k_i} \hat{a}_{k_i}(n) + \alpha_{k_i} \hat{b}_{k_i}(n) \right) \right] x_{bi}(n) \right\} + v_p(n) \quad (26)$$

Obviously, from the above error signal expression,

$$\begin{pmatrix} a_{p,i} \\ b_{p,i} \end{pmatrix} = \begin{pmatrix} \alpha_i \hat{a}_i(n) - \beta_i \hat{b}_i(n) + \alpha_{k_i} \hat{a}_{k_i}(n) - \beta_{k_i} \hat{b}_{k_i}(n) \\ \beta_i \hat{a}_i(n) + \alpha_i \hat{b}_i(n) + \beta_{k_i} \hat{a}_{k_i}(n) + \alpha_{k_i} \hat{b}_{k_i}(n) \end{pmatrix} = \begin{pmatrix} \alpha_i & -\beta_i \\ \beta_i & \alpha_i \end{pmatrix} \begin{pmatrix} \hat{a}_i(n) \\ \hat{b}_i(n) \end{pmatrix} + \begin{pmatrix} \alpha_{k_i} & -\beta_{k_i} \\ \beta_{k_i} & \alpha_{k_i} \end{pmatrix} \begin{pmatrix} \hat{a}_{k_i}(n) \\ \hat{b}_{k_i}(n) \end{pmatrix} \quad (27)$$

We assume that, it is worth emphasizing that m is an arbitrary constant

$$\begin{pmatrix} \alpha_i & -\beta_i \\ \beta_i & \alpha_i \end{pmatrix} \begin{pmatrix} \hat{a}_{i,opt} \\ \hat{b}_{i,opt} \end{pmatrix} = m \begin{pmatrix} a_{p,i} \\ b_{p,i} \end{pmatrix} \quad (0 < m < 1) \quad (28)$$

$$\begin{pmatrix} \alpha_{k_i} & -\beta_{k_i} \\ \beta_{k_i} & \alpha_{k_i} \end{pmatrix} \begin{pmatrix} \hat{a}_{k_i}(n) \\ \hat{b}_{k_i}(n) \end{pmatrix} = (1-m) \begin{pmatrix} a_{p,i} \\ b_{p,i} \end{pmatrix} \quad (29)$$

Therefore, the optimal DFCs for a perfect performance of targeted noises are given by:

$$\begin{pmatrix} \hat{a}_{i,opt} \\ \hat{b}_{i,opt} \end{pmatrix} = m \begin{pmatrix} \alpha_i & -\beta_i \\ \beta_i & \alpha_i \end{pmatrix}^{-1} \begin{pmatrix} a_{p,i} \\ b_{p,i} \end{pmatrix} \quad (30)$$

$$\begin{pmatrix} \hat{a}_{k_i,opt} \\ \hat{b}_{k_i,opt} \end{pmatrix} = (1-m) \begin{pmatrix} \alpha_{k_i} & \alpha_{k_i} \\ \beta_{k_i} & \alpha_{k_i} \end{pmatrix}^{-1} \begin{pmatrix} a_{p,i} \\ b_{p,i} \end{pmatrix} \quad (31)$$

Define the estimation errors of DFCs as:

$$\begin{pmatrix} \varepsilon_{a_i}(n) & \varepsilon_{a_{k_i}}(n) \\ \varepsilon_{b_i}(n) & \varepsilon_{b_{k_i}}(n) \end{pmatrix} = \begin{pmatrix} \hat{a}_{i,opt} - \hat{a}_i(n) & \hat{a}_{k_i,opt} - \hat{a}_{k_i}(n) \\ \hat{b}_{i,opt} - \hat{b}_i(n) & \hat{b}_{k_i,opt} - \hat{b}_{k_i}(n) \end{pmatrix} \quad (32)$$

The error signal eventually reduces to:

$$\begin{aligned} e(n) &= \sum_{i=1}^q \left\{ \left[\alpha_i \varepsilon_{a_i}(n) - \beta_i \varepsilon_{b_i}(n) + \alpha_{k_i} \varepsilon_{a_{k_i}}(n) - \beta_{k_i} \varepsilon_{b_{k_i}}(n) \right] x_{ai}(n) \right. \\ &\quad \left. + \left[\left(\beta_i \varepsilon_{a_i}(n) + \alpha_i \varepsilon_{b_i}(n) + \beta_{k_i} \varepsilon_{a_{k_i}}(n) + \alpha_{k_i} \varepsilon_{b_{k_i}}(n) \right) \right] x_{bi}(n) \right\} + v_p(n) \end{aligned} \quad (33)$$

Putting the above error signal, (7) and (8) in the FXLMS recursions (5) and (6), (19) and (20) in the delayed FXLMS recursions (17) and (18), and taking the ensemble average, one yields, after some manipulations:

$$E[\varepsilon_{a_i}(n+1)] = \underbrace{\left[1 - \frac{u_i}{2} (\alpha_i \hat{\alpha}_i + \beta_i \hat{\beta}_i) \right]}_{=p_1} E[\varepsilon_{a_i}(n)] + \underbrace{\frac{u_i}{2} (\hat{\alpha}_i \beta_i - \alpha_i \hat{\beta}_i)}_{=p_2} E[\varepsilon_{b_i}(n)] \quad (34)$$

$$E[\varepsilon_{b_i}(n+1)] = \underbrace{\frac{u_i}{2} (\alpha_i \hat{\beta}_i - \hat{\alpha}_i \beta_i)}_{=p_2} E[\varepsilon_{a_i}(n)] + \underbrace{\left[1 - \frac{u_i}{2} (\alpha_i \hat{\alpha}_i + \beta_i \hat{\beta}_i) \right]}_{=p_1} E[\varepsilon_{b_i}(n)] \quad (35)$$

$$E[\varepsilon_{a_{k_i}}(n+1)] = \underbrace{\left[1 - \frac{u_{k_i}}{2} (\alpha_{k_i} \hat{\alpha}_{k_i} + \beta_{k_i} \hat{\beta}_{k_i}) \right]}_{=p_{k1}} E[\varepsilon_{a_{k_i}}(n)] + \underbrace{\frac{u_{k_i}}{2} (\hat{\alpha}_{k_i} \beta_{k_i} - \alpha_{k_i} \hat{\beta}_{k_i})}_{=p_{k2}} E[\varepsilon_{b_{k_i}}(n)] \quad (36)$$

$$E[\varepsilon_{b_{k_i}}(n+1)] = \underbrace{\frac{u_{k_i}}{2} (\alpha_{k_i} \hat{\beta}_{k_i} - \hat{\alpha}_{k_i} \beta_{k_i})}_{=p_{k2}} E[\varepsilon_{a_{k_i}}(n)] + \underbrace{\left[1 - \frac{u_{k_i}}{2} (\alpha_{k_i} \hat{\alpha}_{k_i} + \beta_{k_i} \hat{\beta}_{k_i}) \right]}_{=p_{k1}} E[\varepsilon_{b_{k_i}}(n)] \quad (37)$$

Combining (34)–(37), we obtain the transformation matrix:

$$A_i = \begin{pmatrix} p_1 & p_2 & 0 & 0 \\ p_2 & p_1 & 0 & 0 \\ 0 & 0 & p_{k1} & p_{k2} \\ 0 & 0 & p_{k2} & p_{k1} \end{pmatrix} \quad (38)$$

The condition for convergence of the algorithm from the spectral radius is:

$$\rho(A_i) < 1 \quad (39)$$

By analyzing (39), the stability bound of step size can be derived as below:

$$0 < [u_i, u_{k_i}] < \left[\left(\frac{4}{\alpha_i^2 + \beta_i^2} \right), \left(\frac{4}{\alpha_{k_i}^2 + \beta_{k_i}^2} \right) \right] \quad (40)$$

4. Simulation Results and Discussion

In this section, some representative simulations are conducted to prove the fast convergence and better noise reduction in the proposed NANC algorithms under FMs. The mean square error (MSE) $E(e(n)^2)$ in dB is obtained to compare the performance of the above three NANC systems. We consider three tonal noise frequencies $f_i = [50 \ 100 \ 150]$: ($i = 1, 2, 3$) in the primary noise, where 100 Hz and 150 Hz are the harmonic of 50 Hz, and then convert all three frequencies to the digital ones by $2\pi f_i / f_s$. A hundred independent trials were performed to evaluate the mean square behavior of all of the systems. The optimal step-size is chosen to reach the minimum MSE for each algorithm, respectively. All the basic conditions are listed in Table 2. This section may be divided into subheadings. It should provide a concise and precise description of the experimental results, their interpretation, as well as the experimental conclusions that can be drawn.

Table 2. The basic conditions.

Parameters	Value
f_s	1 kHz
δ_p^2	0.1
$\omega_p = [\omega_{p,1} \ \omega_{p,2} \ \omega_{p,3}]^T$	$[0.1\pi \ 0.2\pi \ 0.3\pi]^T$
$a_p = [a_{p,1} \ a_{p,2} \ a_{p,3}]^T$	$[2 \ 1 \ 1]^T$
$b_p = [b_{p,1} \ b_{p,2} \ b_{p,3}]^T$	$[1 \ 1 \ 1]^T$
$a_i = b_i = a_{k_i} = b_{k_i}$	$[1 \ 1 \ 1]^T$
FM C% (C is a random constant)	$\omega_{p,i}(1 + C\%)$
$S(z)$	$M = 11, \text{cut_off frequency } 0.4\pi$

4.1. Simulation 1: The MSE of All NANC System

This simulation was carried out to demonstrate the MSE and convergence performance of the above NANC algorithms under different FMs. The step sizes were set as: $\mu_i = [0.1 \ 0.1 \ 0.1]$, ($i = 1, 2, 3$), $\mu_{c_i} = [0.00025 \ 0.00025 \ 0.00025]$, ($i = 1, 2, 3$) in the conventional NANC system and the NANC system with MLMS; $\mu_i = [0.1 \ 0.1 \ 0.1]$, ($i = 1, 2, 3$), $\mu_{c_i} = [0.00025 \ 0.00025 \ 0.00025]$, ($i = 1, 2, 3$) in the proposed NANC system. The momentum factor $\lambda = 10$ was set in the NANC system with MLMS. The step sizes of the delay model were set as $\mu_{k_{ai}} = \mu_{k_{bi}} = [0.05 \ 0.05 \ 0.05]$, ($i = 1, 2, 3$) and the delay unit k was set to 14 in the proposed NANC system. Figures 5–8 investigate the steady-state MSE in the different NANC systems, where the FM varied between 1% and 20%.

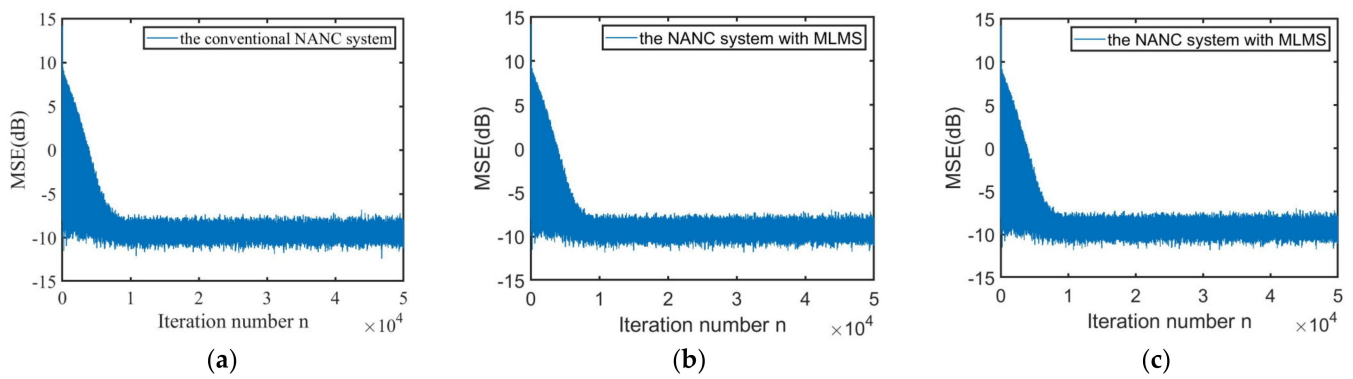


Figure 5. Under 5%FM (a) the conventional NANC system (b) the system with MLMS (c) the proposed system.

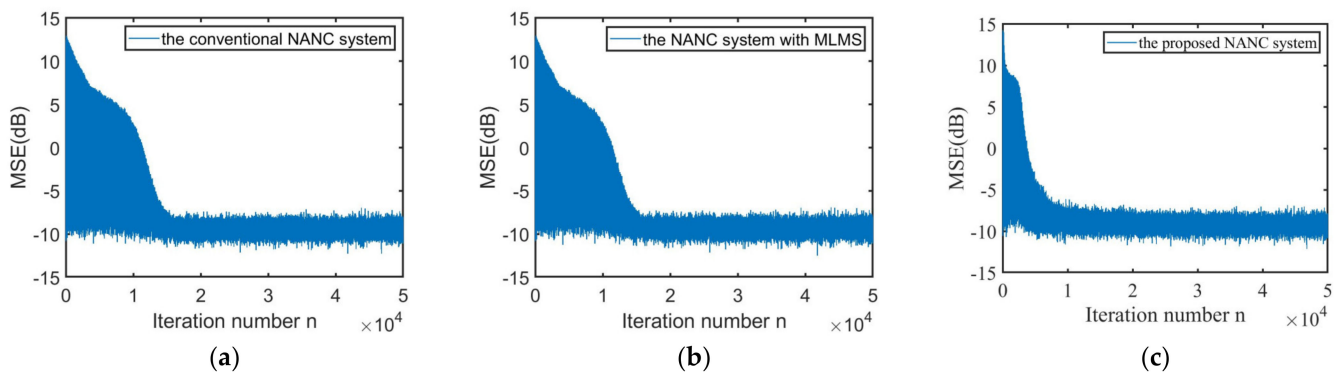


Figure 6. Under 10%FM (a) conventional NANC system (b) the system with MLMS (c) proposed system.

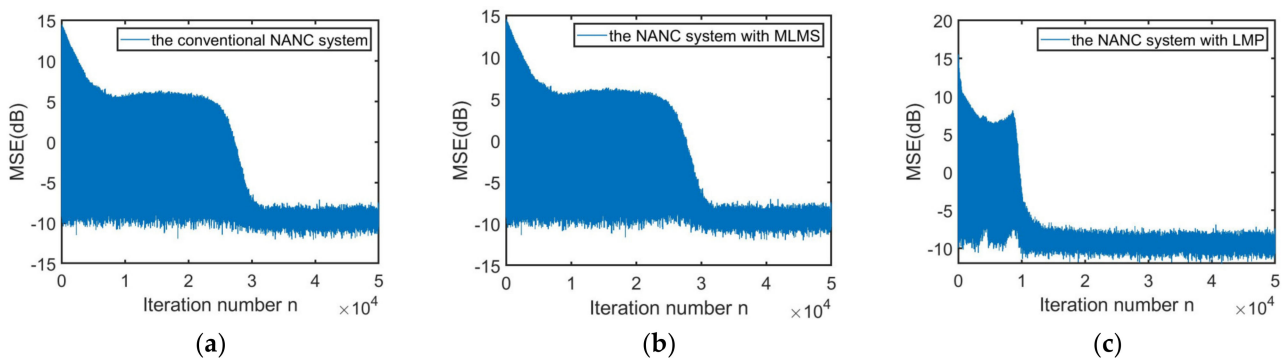


Figure 7. Under 15%FM (a) the conventional NANC system (b) the system with MLMS (c) proposed system.

It is clear that the system eventually converges to -10 dB when the FM varies between 5% and 15%. Moreover, the MSE and convergence rate of the conventional NANC system are similar to the system with MLMS. In Figure 5, given the same 5% FM, it appears that the proposed NANC system can achieve the same steady state and the same convergence rate as the others, but the curve of the new proposed NANC system has the fastest downward trend. Clearly, in Figure 6, the proposed system converges fastest and significantly outperforms its counterpart in the 10% FM condition. In Figure 7, a great superiority in the convergence was shown in the proposed system.

However, at 20% FM, it is clear that a serious performance deterioration occurs in the NANC system with MLMS, as shown in Figure 8a. Obviously, in Figure 8b, even if it is unstable at the beginning, the proposed system achieves good noise cancellation after

multiple iterations. Although the latter takes a long time to reach a stable state, it provides the possibility for accommodating a larger FM.

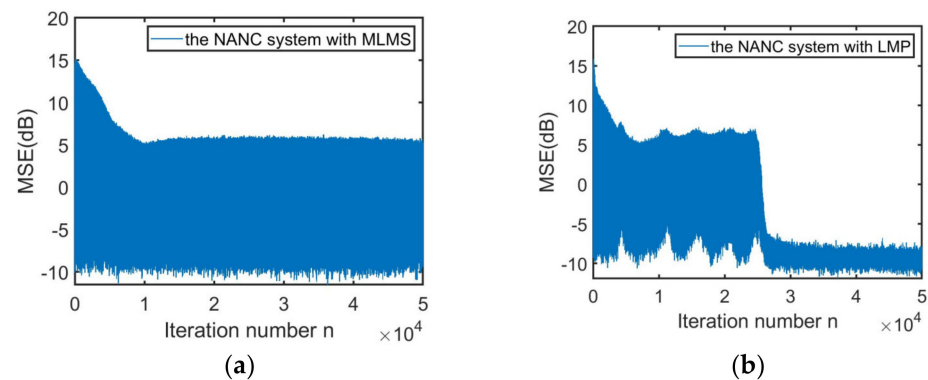


Figure 8. Under 20% FM (a) the system with MLMS (b) the proposed system.

In conclusion, all of the results further reveal that the proposed system converges faster than the others without damaging the noise reduction performance of the system, while both the NANC system and the NANC system with the MLMS algorithm fail to suppress the noise well with a large FM. Certainly, by comparing the two systems, it is confirmed that the method of the MLMS algorithm has little contribution to the noise reduction in the article [27].

Second, the improvement in the proposed NANC system benefits from the new adjustment strategy for the reference signal. When the reference signal is updated, the LMP is used to optimize the AR coefficient. The convergence of the AR coefficients in the NANC system is depicted in Figure 9. The red, pink, and blue lines draw the MSE of c_i in the conventional NANC system, the ones with MLMS and the proposed system, respectively. As it can be seen from (a), the proposed method converges faster than the others. Furthermore, the proposed system shows the best convergent performance in Figure 9b,c, while the system with MLMS cannot achieve similar results, but it is slightly better than conventional NANC system.

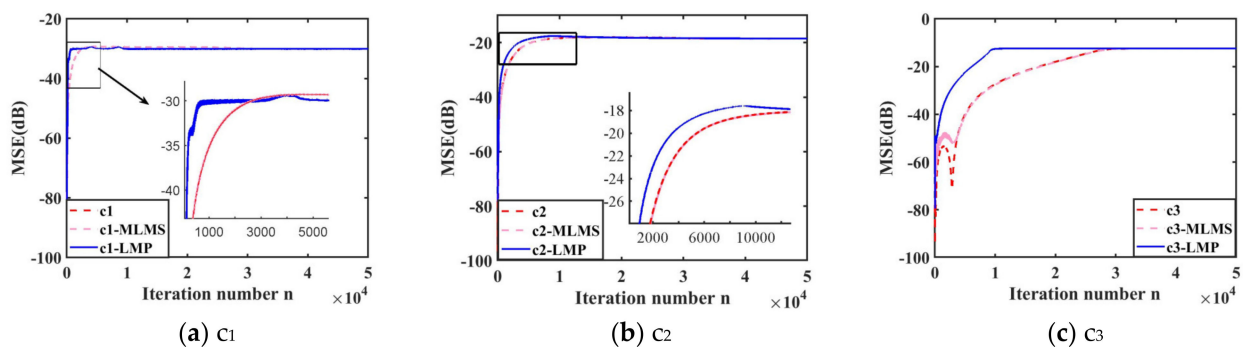


Figure 9. MSE of AR-related coefficients c_i performance for various methods under 15% FM.

4.2. Simulation 2: The Tracking Performance of NANC System

The amplitude and frequency mutation of primary noise are also FM phenomena that are often encountered in practical applications. The change in the FM usually influences the system stability. As the system with MLMS performs better than the regular system, we chose the former as the comparison experiment. Therefore, this case is used to simulate the tracking performance of the two compared NANC systems when the input signal is unstable. The FM jump is artificially introduced at the midpoint of the iteration. The same basic conditions of simulation 1 are applied to this simulation. The MSE and frequency tracking plots of the two NANC systems are demonstrated in Figure 10.

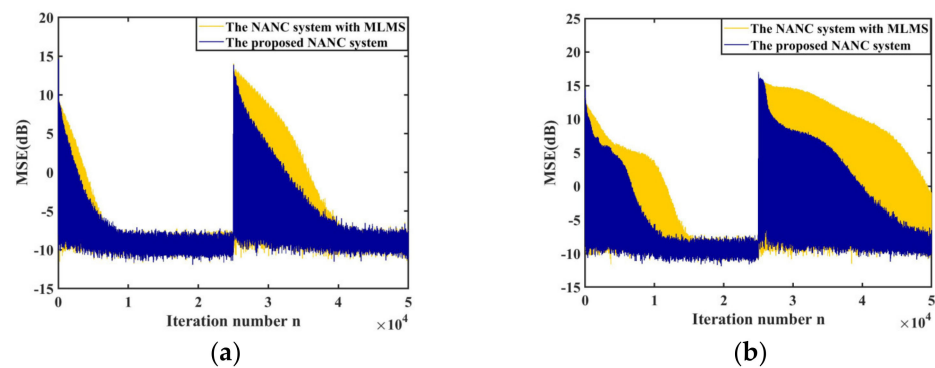


Figure 10. Tracking ability of the proposed algorithm versus its counterparts with different FM. (a) 5% FM. (b) 10% FM.

We assume that there is a 5% FM up to 25,000 samples and a FM of -5% is introduced at 25,000 samples; Figure 10a shows that the error microphone signal is reduced adaptively, even after the transition of FM. As shown in (a), the proposed system can provide a faster tracking ability without damaging the system performance on the whole. Secondly, it is shown in Figure 10b that, even if we introduce a jump from 10% FM to -10% FM at the middle of the iteration, the proposed algorithm is able to track the FM efficiently, and the proposed NANC system converges faster than the NANC system with MLMS. Obviously, the new proposed NANC system provides excellent robustness against non-acoustic sensor errors.

4.3. Simulation 3: Power Spectrum Density

The power spectrum density (PSD) of the primary noise and noises of three compared algorithms under different FM

Figure 11 shows the peaks of the PSDs at different frequencies in three algorithms. In the above pictures, the proposed NANC system has the lowest PSD among the three algorithms under different FMs. Clearly, it is capable for all three algorithms to eliminate the main tone in the primary noise. In Figure 11b, it can be seen the higher the frequency, the smaller the PSD of the error signal in the curve of the proposed NANC system, 5 dB, 10 dB, 13 dB, respectively, decreased compared with the other NANC systems. Similar results could be obtained in Figure 11c. Overall, the proposed NANC system has potential advantages over other frequency correction methods.

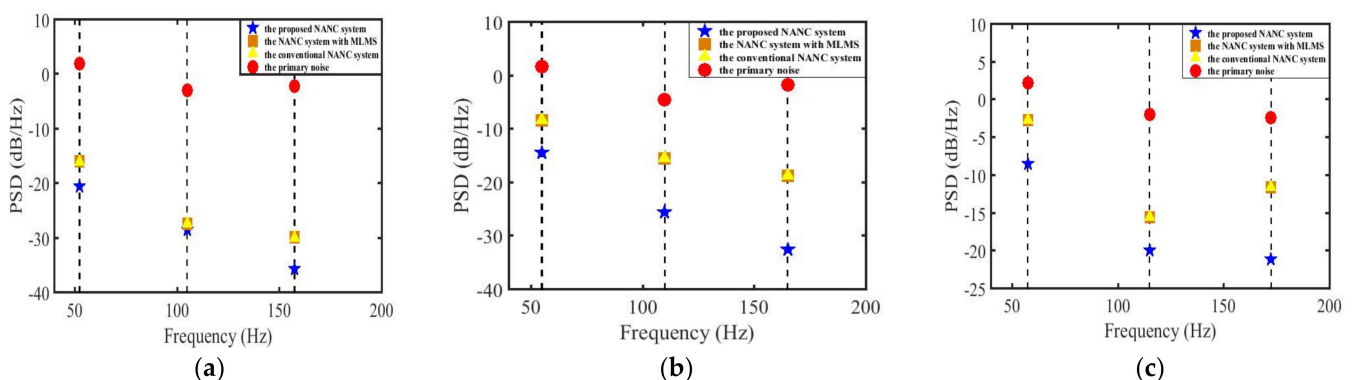


Figure 11. The peaks of PSDs of primary noise and residual noises of the three algorithms. (a) 5% FM. (b) 10% FM. (c) 15% FM.

5. Conclusions

In this paper, a structure of a NANC system with the combination of frequency compensation and the LMP algorithm was proposed to accommodate large FM and uplink robustness. This method can not only accommodate a 15% FM, but also significantly

improves the convergence rate, especially for a large FM. In addition, the tracking ability has been greatly improved for the sudden acceleration and deceleration. When a 10% FM occurs, the proposed system is drastically reduced from 7.5 s to 1.5 s when the system reaches the steady state. Even for a larger FM, with 15%, the proposed system converges in almost five seconds. It demonstrates the superior convergence rate of the proposed system when facing the FM caused by the sensors. Furthermore, the simulation results for the tracking capability demonstrate that the new frequency compensation performs better in handling the instability problem in the primary noise. Furthermore, the application of the proposed system will be conducted in our future work.

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