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Dynamic Characteristics and Fault Mechanism of the Gear Tooth Spalling in Railway Vehicles under Traction Conditions

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Abstract: Gear tooth spalling is one of the inevitable fault modes in the long-term service of the traction transmission system of railway vehicles, which can worsen the dynamic load of the rotating mechanical system and reduce the operating quality. Therefore, it is necessary to study its fault mechanism to guide fault diagnosis scientifically. This paper established a planar railway vehicle model with a traction transmission system and an analytical time-varying meshing stiffness (TVMS) model of the spalling spur gear. Then, it analyzed the dynamic characteristics under traction conditions. The research found that the spalling length and depth affect the amplitude of the TVMS at the defect, while the width affects the range of the TVMS loss. The crest factor is the best evaluation indicator in ideal low-noise environments due to its sensitivity and linearity, but it is not good in strong-noise environments. Similarly, a time–frequency analysis tool cannot significantly detect the sideband characteristics that are excited by spalling. After high-pass filtering, the root mean square and variance exhibit excellent classification and vehicle speed independence in strong-noise environments. This research achievement can provide adequate theoretical support for feature selection and making strategies for fault diagnosis of railway vehicle gear systems.

Keywords: spalling; spur gear; railway vehicle; traction condition; dynamic characteristics; TVMS



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1. Introduction

The safety of railway vehicles in service is key to ensuring the efficient, reliable, and rapid operation of transportation systems. Due to frequent and high-load gear tooth meshing, the problem of spalling and pitting caused by contact fatigue has become one of the inevitable problems during the operation of railway vehicles. The propagation of gear tooth spalling can lead to increased abnormal wear and can seriously affect vehicle dynamics. Based on dynamic modeling technology, many pioneering researchers have studied the mapping relationship between fault evolution and health indicators, providing scientific and systematic theoretical guidance for fault diagnosis [1–5]. Thus, studying the fault mechanism of gear tooth spalling on railway vehicles is necessary, providing theoretical support for accurate fault diagnosis.

The time-varying meshing stiffness (TVMS) is the research core of the fault mechanism for gear systems, and it is the main factor in describing gear meshing excitation. TVMS modeling based on the potential energy method (PEM) has the advantages of good interpretation, fast calculation, and high accuracy, and it is currently a hot research topic in the field [6–12]. The TVMS model based on PEM plays a significant role in the study of the fault mechanism and dynamic analysis of gear cracks [13], wear [14], spalling, pitting [15], and other faults [16]. Yang et al. [17] established a TVMS model with a gear tooth surface crack and analyzed the effect of the surface crack on the dynamic system. Shi et al. [18] investigated the dynamic characteristics of the gear system with a double-teeth spalling fault. They utilized some advanced signal processing methods, including variational mode decomposition, local mean decomposition, and empirical mode decomposition, to successfully

extract the fault feature. Considering the effect of tooth surface roughness and geometric deviations, Luo et al. [19] presented a novel gear model with tooth pitting and spalling. The proposed model is validated by experiments on time and frequency domains under different rotation speeds and fault severity conditions and exhibits an excellent consistency. Chen et al. [20] proposed a new distribution model to describe the multiple teeth pits and established the TVMS model. This combination of probabilistic distribution models and deterministic analytical models provides a new approach to modeling distributed faults. Wu et al. [21] proposed an advanced TVMS model of the helical gears with tooth spalls with curved-bottom features and consequently demonstrated better accuracy. Based on the modeling of TVMS, some scholars have conducted phenomenological research on the dynamic response induced by gear excitation [22], and others have researched model-based prediction methods for the degradation process of gear system [23]. In summary, it is critical to establish an accurate TVMS model when studying the influence of fault modes on dynamics.

A gear is a key mechanical component of the traction transmission system of railway vehicles, and its research focuses mainly on operational status assessment and health maintenance [24–26]. The current research mainly uses a dynamic modeling approach to study its fault mechanism [27–30], nonlinear behavior [31–33], and impact on vehicle dynamics [34–39]. Zhang et al. [40] proposed electromechanical coupling modeling to describe the interaction between the electrical and mechanical parts of the traction transmission system in a high-speed train. Meanwhile, Wang et al. [41] verified the electromechanical coupling mechanism of the traction transmission system using a field test. Wang et al. [42] studied the coupled dynamic characteristics of a transmission system with gear eccentricities in a high-speed train. They found that pinion eccentricity has a significant effect on the vibration of the wheelset that is successfully neglected in previous studies. Yang et al. [43] established a local model of the gear system of high-speed trains and conducted in-depth research on the variation law of bifurcation behavior with rotational speed under wear faults. Liu et al. [44] established a dynamic locomotive model with gear transmission and analyzed the dynamic response to assist the feature selection in fault detection. Chen et al. [45] applied a 2D vehicle-track-coupled model to investigate the vibration feature evolution and proposed an angular synchronous average technique to suppress the noise caused by track irregularities. The research provides a strong theoretical guideline for selecting suitable indicators in dynamic monitoring, fault diagnosis, and degradation assessment. Therefore, it can be seen that fault mechanism research is an important basis for fault diagnosis.

To realize the accurate identification of gear spalling in railway vehicles, dynamic modeling and fault mechanism investigation will be carried out from the perspective of dynamics. Based on the simulation results, the vibration feature evolution and time–frequency characteristics are analyzed to provide theoretical support for fault diagnosis. The paper is organized as follows: Section 2 presents the railway vehicle model with a traction transmission system. Section 3 presents the TVMS model with tooth gear spalling. Section 4 analyzes and discusses the TVMS and its dynamic characteristics at different fault severities. Section 5 concludes the research work.

2. Dynamic Modeling of a Railway Vehicle

This paper establishes a planar railway vehicle model with a traction transmission system. Since the vibrations are mainly caused by vertical track excitation, the longitudinal, bounce, and pitch motions are analyzed. The track excitation is transmitted upward from the wheelset, axlebox, gearbox, motor, and bogie frame to the car body. Herein, the planar railway vehicle model with a traction transmission system is shown in Figure 1.

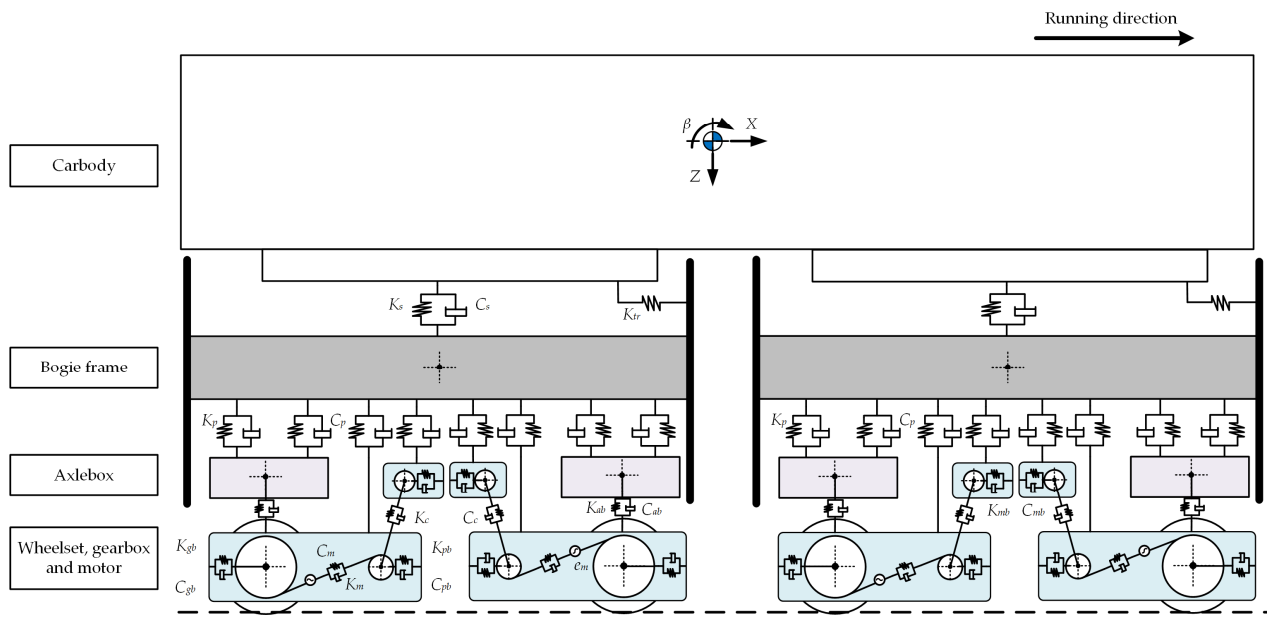


Figure 1. The planar dynamic model of a railway vehicle with the traction transmission system.

2.1. Vehicle Model

The primary suspension, secondary suspension, tractive rod, bearing, and suspender can be simplified as the spring-damping elements to connect different rigid bodies and reduce the vibration. The primary suspension force acting between the axlebox and wheelset can be calculated by

$$F_{zep,i} = K_{zp} \left[z_{b,j} - z_{a,i} + (-1)^{i+1} l_{epkb} \beta_{b,j} + (-1)^i l_{epka} \beta_{a,i} \right] + C_{zp} \left[\dot{z}_{b,j} - \dot{z}_{a,i} + (-1)^{i+1} l_{epkb} \dot{\beta}_{b,j} + (-1)^i l_{epka} \dot{\beta}_{a,i} \right], \quad (1a)$$

$$F_{xep,i} = K_{xp} \left(x_{b,j} - x_{a,i} - H_{epkb} \beta_{b,j} - H_{epka} \beta_{a,i} \right) + C_{xp} \left(\dot{x}_{b,j} - \dot{x}_{a,i} - H_{epkb} \dot{\beta}_{b,j} - H_{epka} \dot{\beta}_{a,i} \right), \quad (1b)$$

$$F_{zip,i} = K_{zp} \left[z_{b,j} - z_{a,i} + (-1)^{i+1} l_{ipkb} \beta_{b,j} + (-1)^{i+1} l_{ipka} \beta_{a,i} \right] + C_{zp} \left[\dot{z}_{b,j} - \dot{z}_{a,i} + (-1)^{i+1} l_{ipkb} \dot{\beta}_{b,j} + (-1)^{i+1} l_{ipka} \dot{\beta}_{a,i} \right], \quad (1c)$$

$$F_{xip,i} = K_{xp} \left(x_{b,j} - x_{a,i} - H_{ipkb} \beta_{b,j} - H_{ipka} \beta_{a,i} \right) + C_{xp} \left(\dot{x}_{b,j} - \dot{x}_{a,i} - H_{ipkb} \dot{\beta}_{b,j} - H_{ipka} \dot{\beta}_{a,i} \right), \quad (1d)$$

The secondary suspension force acting between the bogie frame and the car body can be calculated by

$$F_{zs,j} = K_{zs} \left[z_c - z_{b,j} + (-1)^{j+1} l_{skc} \beta_c \right] + C_{zs} \left[\dot{z}_c - \dot{z}_{b,j} + (-1)^{j+1} l_{skc} \dot{\beta}_c \right], \quad (2a)$$

$$F_{xs,j} = K_{xs} \left(x_c - x_{b,j} - H_{skc} \beta_c - H_{skb} \beta_{b,j} \right) + C_{xs} \left(\dot{x}_c - \dot{x}_{b,j} - H_{skc} \dot{\beta}_c - H_{skb} \dot{\beta}_{b,j} \right), \quad (2b)$$

The tractive rod can be described by

$$F_{tr,j} = K_{tr} \left(x_c - x_{b,j} - H_{trc} \beta_c + H_{trb} \beta_{b,j} \right), \quad (3)$$

The support force of the axlebox bearing can be written by

$$F_{xab,i} = K_b (x_{a,i} - x_{w,i}) + C_b (\dot{x}_{a,i} - \dot{x}_{w,i}), \quad (4a)$$

$$F_{zab,i} = K_b(z_{a,i} - z_{w,i}) + C_b(\dot{z}_{a,i} - \dot{z}_{w,i}), \quad (4b)$$

The support force of the pinion side bearing can be written by

$$F_{xpb,i} = K_b(x_{p,i} - x_{gh,i} + H_{pbgh}\beta_{gh,i}) + C_b(\dot{x}_{p,i} - \dot{x}_{gh,i} + H_{pbgh}\dot{\beta}_{gh,i}), \quad (5a)$$

$$F_{zpb,i} = K_b[z_{p,i} - z_{gh,i} + (-1)^{i+1}l_{pbgh}\beta_{gh,i}] + C_b[\dot{z}_{p,i} - \dot{z}_{gh,i} + (-1)^{i+1}l_{pbgh}\dot{\beta}_{gh,i}], \quad (5b)$$

The support force of the gear side bearing can be written by

$$F_{xgb,i} = K_b(x_{w,i} - x_{gh,i} + H_{gbgh}\dot{\beta}_{gh,i}) + C_b(\dot{x}_{w,i} - \dot{x}_{gh,i} + H_{gbgh}\dot{\beta}_{gh,i}), \quad (6a)$$

$$F_{zgb,i} = K_b[z_{w,i} - z_{gh,i} + (-1)^i l_{gbgh}\beta_{gh,i}] + C_b[\dot{z}_{w,i} - \dot{z}_{gh,i} + (-1)^i l_{gbgh}\dot{\beta}_{gh,i}], \quad (6b)$$

The support force of the motor bearing can be written by

$$F_{xmb,i} = K_b(x_{mr,i} - x_{mh,i}) + C_b(\dot{x}_{mr,i} - \dot{x}_{mh,i}), \quad (7a)$$

$$F_{zmb,i} = K_b(z_{mr,i} - z_{mh,i}) + C_b(\dot{z}_{mr,i} - \dot{z}_{mh,i}), \quad (7b)$$

The coupling force between the motor and the pinion can be given by

$$F_{xmp,i} = K_{mp}(x_{mr,i} - x_{p,i}) + C_{mp}(\dot{x}_{mr,i} - \dot{x}_{p,i}), \quad (8a)$$

$$F_{zmp,i} = K_{mp}(z_{mr,i} - z_{p,i}) + C_{mp}(\dot{z}_{mr,i} - \dot{z}_{p,i}), \quad (8b)$$

$$T_{mp,i} = K_c(\beta_{mr,i} - \beta_{p,i}) + C_c(\dot{\beta}_{mr,i} - \dot{\beta}_{p,i}), \quad (8c)$$

The suspender force of the gearbox housing can be written by

$$F_{xgh,i} = K_{xgh}(x_{gh,i} - x_{b,j} - H_{ghgh}\beta_{gh,i} - H_{ghhb}\beta_{b,j}) + C_{xgh}(\dot{x}_{gh,i} - \dot{x}_{b,j} - H_{ghgh}\dot{\beta}_{gh,i} - H_{ghhb}\dot{\beta}_{b,j}), \quad (9a)$$

$$F_{zgh,i} = K_{zgh}[z_{gh,i} - z_{b,j} + (-1)^i l_{ghgh}\beta_{gh,i} + (-1)^i l_{ghhb}\beta_{b,j}] + C_{zgh}[\dot{z}_{gh,i} - \dot{z}_{b,j} + (-1)^i l_{ghgh}\dot{\beta}_{gh,i} + (-1)^i l_{ghhb}\dot{\beta}_{b,j}], \quad (9b)$$

The suspender force of the motor can be given by

$$F_{xmh,i} = K_{xmh}(x_{mh,i} - x_{b,j} + H_{mhmh}\beta_{mh,i} - H_{mhbb}\beta_{b,j}) + C_{xmh}(\dot{x}_{mh,i} - \dot{x}_{b,j} + H_{mhmh}\dot{\beta}_{mh,i} - H_{mhbb}\dot{\beta}_{b,j}), \quad (10a)$$

$$F_{zmh,i} = K_{zmh}[z_{mh,i} - z_{b,j} + (-1)^i l_{mhmh}\beta_{mh,i} + (-1)^i l_{mhbb}\beta_{b,j}] + C_{zmh}[\dot{z}_{mh,i} - \dot{z}_{b,j} + (-1)^i l_{mhmh}\dot{\beta}_{mh,i} + (-1)^i l_{mhbb}\dot{\beta}_{b,j}], \quad (10b)$$

$$F_{xmh,i} = K_{xmh}(x_{mh,i} - x_{b,j} - H_{mhmh}\beta_{mh,i} + H_{mhbb}\beta_{b,j}) + C_{xmh}(\dot{x}_{mh,i} - \dot{x}_{b,j} - H_{mhmh}\dot{\beta}_{mh,i} + H_{mhbb}\dot{\beta}_{b,j}), \quad (10c)$$

$$F_{zmh,i} = K_{zmh}[z_{mh,i} - z_{b,j} + (-1)^i l_{mhmh}\beta_{mh,i} + (-1)^i l_{mhbb}\beta_{b,j}] + C_{zmh}[\dot{z}_{mh,i} - \dot{z}_{b,j} + (-1)^i l_{mhmh}\dot{\beta}_{mh,i} + (-1)^i l_{mhbb}\dot{\beta}_{b,j}], \quad (10d)$$

where l_{epkb} is the longitudinal distance from the external primary suspension to the center of the bogie frame; l_{epka} is the longitudinal distance from the external primary suspension to the center of the axlebox; l_{ipkb} is the longitudinal distance from the inner primary suspension to the center of the bogie frame; l_{ipka} is the longitudinal distance from the inner primary suspension to the center of the axlebox; l_{skc} is the longitudinal distance

between the secondary suspension and the center of the carbody; l_{pbg} is the longitudinal distance between the pinion side bearing and the center of the gearbox housing; l_{gbg} is the longitudinal distance between the gear side bearing and the center of the gearbox housing; l_{ghg} is the longitudinal distance between the gearbox suspender and the center of the gearbox housing; l_{ghb} is the longitudinal distance between the gearbox suspender and the center of the bogie frame; l_{mgh} is the longitudinal distance between the motor suspender and the center of the motor housing; l_{mhb} is the longitudinal distance between the motor housing and the center of the bogie frame; H_{epkb} is the vertical distance from the external primary suspension to the center of the bogie frame; H_{epka} is the vertical distance from the external primary suspension to the center of the axlebox; H_{ipkb} is the vertical distance from the inner primary suspension to the center of the bogie frame; H_{ipka} is the vertical distance from the inner primary suspension to the center of the axlebox; H_{skc} is the vertical distance between the secondary suspension and the center of the carbody; H_{skb} is the vertical distance from the secondary suspension to the center of the bogie frame; H_{trc} is the vertical distance between the tractive rod and the center of the carbody; H_{trb} is the vertical distance between the tractive rod and the center of the bogie frame; H_{ghg} is the vertical distance between the gearbox suspender and the center of the gearbox housing; H_{ghb} is the vertical distance between the gearbox suspender and the center of the bogie frame; H_{mgh} is the vertical distance between the motor suspender and the center of the motor housing; H_{mhb} is the vertical distance from the motor suspender to the center of the bogie frame; K_{xp} and K_{zp} are the longitudinal and vertical stiffness of the primary suspension; C_{xp} and C_{zp} are the longitudinal and vertical damping of the primary suspension; K_{xs} and K_{zs} are the longitudinal and vertical stiffness of the secondary suspension; C_{xs} and C_{zs} are the longitudinal and vertical damping of the secondary suspension; K_{tr} is the stiffness of the tractive rod; and K_{ab} and C_{ab} are the support stiffness and damping of the bearing.

According to the Newton–Euler theory, the dynamic governing equation of the railway vehicle, as shown in Figure 1, can be further established. The equations of the car body's longitudinal, vertical, and pitch motions are as follows:

$$M_c \ddot{x}_c = -2F_{xs,1} - 2F_{xs,2} - 2F_{tr,1} - 2F_{tr,2}, \quad (11a)$$

$$M_c \ddot{z}_c = -2F_{zs,1} - 2F_{zs,2} + M_c g, \quad (11b)$$

$$I_c \ddot{\beta}_c = (2F_{zs,2} - 2F_{zs,1})l_{skc} + (2F_{xs,1} + 2F_{xs,2})H_{skc} + (2F_{tr,1} + 2F_{tr,2})H_{trc}, \quad (11c)$$

The equations of the bogie frame's longitudinal, vertical, and pitch motions are as follows:

$$M_b \ddot{x}_{b,i} = 2F_{xs,i} + 2F_{tr,i} - 2F_{xep,2i-1} - 2F_{xep,2i} - 2F_{xip,2i-1} - 2F_{xip,2i} + F_{xgh,2i-1} + F_{xgh,2i} + 2F_{xmhu,2i-1} + 2F_{xmhu,2i} + 2F_{xmhl,2i-1} + 2F_{xmhl,2i}, \quad (12a)$$

$$M_b \ddot{z}_{b,i} = 2F_{zs,i} - 2F_{zep,2i-1} - 2F_{zep,2i} - 2F_{zip,2i-1} - 2F_{zip,2i} + F_{zgh,2i-1} + F_{zgh,2i} + 2F_{zmhu,2i-1} + 2F_{zmhu,2i} + 2F_{zmhl,2i-1} + 2F_{zmhl,2i} + M_b g', \quad (12b)$$

$$I_b \ddot{\beta}_{b,i} = 2F_{xs,i}H_{skb} + (2F_{zep,2i} - 2F_{zep,2i-1})l_{epkb} + (2F_{xep,2i-1} + 2F_{xep,2i})H_{epkb} + (2F_{zip,2i} - 2F_{zip,2i-1})l_{ipkb} + (2F_{xip,2i-1} + 2F_{xip,2i})H_{ipkb} + F_{xgh,2i-1}H_{ghb} + F_{xgh,2i}H_{ghb} + F_{zgh,2i-1}l_{ghb} - F_{zgh,2i}l_{ghb} + 2F_{xmhu,2i-1}H_{mhb} + F_{xmhu,2i-1}l_{mhb} - F_{xmhu,2i}l_{mhb} - 2F_{xmhl,2i-1}H_{mhb} - 2F_{xmhl,2i}H_{mhb} + F_{zmhl,2i-1}l_{mhb} - F_{zmhl,2i}l_{mhb} + 2F_{xmhu,2i}H_{mhb}, \quad (12c)$$

The equations of the longitudinal, vertical, and pitch motions of the axlebox are given as follows:

$$M_a \ddot{x}_{a,i} = F_{xep,i} + F_{xip,i} - F_{xab,i}, \quad (13a)$$

$$M_a \ddot{z}_{a,i} = F_{zep,i} + F_{zip,i} - F_{zab,i} + M_a g, \quad (13b)$$

$$I_a \ddot{\beta}_{a,i} = F_{xep,i}H_{epka} + F_{xip,i}H_{ipka} + (-1)^{i+1}F_{zep,i}l_{epka} + (-1)^iF_{zip,i}l_{ipka}, \quad (13c)$$

The equations of the wheelset's longitudinal, vertical, and rotational motions are as follows:

$$M_w \ddot{x}_{w,i} = F_{m,i} \cos(\alpha'_0) + 2F_{xw,i} + 2F_{xab,i} - 2F_{xgb,i}, \quad (14a)$$

$$M_w \ddot{z}_{w,i} = -F_{m,i} \sin(\alpha'_0) - 2F_{zw,i} + 2F_{zab,i} - 2F_{zgb,i} + M_w g, \quad (14b)$$

$$I_w \ddot{\beta}_{w,i} = (-1)^{i+1} F_{m,i} r_g - 2F_{xw,i} r_w, \quad (14c)$$

The equations of the pinion's longitudinal, vertical, and rotational motions are as follows:

$$M_p \ddot{x}_{p,i} = -F_{m,i} \cos(\alpha'_0) - 2F_{xpb,i}, \quad (15a)$$

$$M_p \ddot{z}_{p,i} = F_{m,i} \sin(\alpha'_0) - 2F_{zpb,i} + M_p g, \quad (15b)$$

$$I_p \ddot{\beta}_{p,i} = (-1)^{i+1} F_{m,i} r_p + T_{mp,i}, \quad (15c)$$

The equations of the gearbox housing's longitudinal, vertical, and pitch motions are as follows:

$$M_{gh} \ddot{x}_{gh,i} = 2F_{xpb,i} + 2F_{xgb,i} - F_{xgh,i}, \quad (16a)$$

$$M_{gh} \ddot{z}_{gh,i} = 2F_{zpb,i} + 2F_{zgb,i} - F_{zgh,i} + M_{gh} g, \quad (16b)$$

$$I_{gh} \ddot{\beta}_{gh,i} = (-1)^i 2F_{zpb,i} l_{pbgh} + (-1)^{i+1} 2F_{zgb,i} l_{gbgh} + F_{xgh,i} H_{ghgh} + (-1)^{i+1} F_{zgh,i} l_{ghgh} - 2F_{xpb,i} H_{pbgh} - 2F_{xgb,i} H_{gbgh}, \quad (16c)$$

The equations of the motor rotor's longitudinal, vertical, and rotational motions are as follows:

$$M_{mr} \ddot{x}_{mr,i} = -2F_{xmb,i}, \quad (17a)$$

$$M_{mr} \ddot{z}_{mr,i} = -2F_{zmb,i} + M_{mr} g, \quad (17b)$$

$$I_{mr} \ddot{\beta}_{mr,i} = -T_{input,i} - T_{mp,i}, \quad (17c)$$

The equations of the motor housing's longitudinal, vertical, and pitch motions are as follows:

$$M_{mh} \ddot{x}_{mh,i} = 2F_{xmb,i} - 2F_{xmhu,i} - 2F_{xmhl,i}, \quad (18a)$$

$$M_{mh} \ddot{z}_{mh,i} = 2F_{zmb,i} - 2F_{zmhu,i} - 2F_{zmhl,i} + M_{mh} g, \quad (18b)$$

$$I_{mh} \ddot{\beta}_{mh,i} = T_{input,i} - 2F_{xmhu,i} H_{mhmh} + 2F_{xmhl,i} H_{mhmh} + (-1)^{i+1} 2(F_{zmhu,i} + F_{zmhl,i}) l_{mhmh}, \quad (18c)$$

where M_c and I_c represent the mass and moment of inertia of the car body; M_b and I_b represent the mass and moment of inertia of the bogie frame; M_a and I_a represent the mass and moment of inertia of the axlebox; M_w and I_w represent the mass and moment of inertia of the wheelset; M_p and I_p represent the mass and moment of the inertia of the pinion; M_{gh} and I_{gh} represent the mass and moment of inertia of the gearbox housing; M_{mr} and I_{mr} represent the mass and moment of inertia of the motor rotor; and M_{mh} and I_{mh} represent the mass and moment of the inertia of the motor housing.

2.2. Wheel–Rail Interaction

According to the Hertzian contact theory, the wheel–rail vertical force can be described using the vertical displacements of the wheelset, the track irregularity as follows:

$$F_{zwi} = \begin{cases} (z_{wri}/G_{wr})^{3/2}, & z_{wri} > 0 \\ 0, & z_{wri} \leq 0 \end{cases}, \quad (19)$$

where G is the wheel–rail contact constant, z_r is the vertical displacement of the rail, and z_0 is the track irregularity.

For the longitudinal force, the slip between the wheel and the rail can be represented by the adhesive force, which is usually determined by material properties, relative slip speed, and surface conditions. This wheel–rail longitudinal force is proportional to the wheel–rail vertical force and can be written as

$$F_{xwi} = \mu_i F_{zwi}, \quad (20)$$

where μ_i is the adhesive coefficient. From Ref. [45], it can be given by

$$\mu_i = c_\mu \cdot \exp(-a_\mu \cdot v_{slip}) - d_\mu \cdot \exp(-b_\mu \cdot v_{slip}), \quad (21)$$

where a_μ , b_μ , c_μ , and d_μ are the calculation parameters of the adhesive coefficient. The relative velocity v_{slip} indicates the slip characteristics of the wheel–rail interaction and can be expressed as

$$v_{slip} = \dot{\beta}_{wi} r_w - \dot{x}_{wi}, \quad (22)$$

where $\dot{\beta}_{wi}$ and $\dot{x}_{wi}$ are the angular velocity and longitudinal velocity of the wheelset, and r_w is the nominal wheel radius.

2.3. Gear Meshing Force

The gear system of a railway vehicle is composed of a pair of involute external spur gears. We have involved a parallel spring-damping element to describe gear meshing for the complex interaction caused by tooth deformation and contact behavior. The meshing force can be written by

$$F_{m,i} = K_m(t) \delta_{m,i} + C_m \dot{\delta}_{m,i}, \quad (23)$$

where $K_m(t)$ is the time-varying meshing stiffness, C_m is the meshing damping. $\delta_{m,i}$ is the dynamic transmission error (DTE) and $\dot{\delta}_{m,i}$ is the differential form, which can be written as follows

$$\delta_{m,i} = (x_{p,i} - x_{w,i}) \cos \alpha'_0 - (z_{p,i} - z_{w,i}) \sin \alpha'_0 + (-1)^i (\beta_{p,i} r_p + \beta_{w,i} r_g) - e_i(t), \quad (24a)$$

$$\dot{\delta}_{m,i} = (\dot{x}_{p,i} - \dot{x}_{w,i}) \cos \alpha'_0 - (\dot{z}_{p,i} - \dot{z}_{w,i}) \sin \alpha'_0 + (-1)^i (\dot{\beta}_{p,i} r_p + \dot{\beta}_{w,i} r_g) - \dot{e}_i(t), \quad (24b)$$

where $e(t)$ is a static transmission error, which is a deterministic excitation composed of two components, one is from the long-period error caused by the geometric eccentricity of the gears, and the other is from the short-period error, which is caused by the individual differences in the gear manufacturing process. Therefore, it can be expressed by

$$e_i(t) = A_{l,i} \cos(\beta_{p,i} Z_p + \varphi_s) + A_{s,i} \cos(\beta_{p,i} Z_p + \varphi_m), \quad (25)$$

where A_l and A_s are the gear eccentricity and gear manufacturing error, ω_s and ω_m are the shaft frequency and gear meshing frequency, and ϕ_s and ϕ_m are the relative phases.

According to the length a_s , width b_s , and depth c_s of the spalling, the change in contact length, section area, and moment of inertia can be calculated as

$$\Delta L_x = \begin{cases} a_s, & u - r \leq x \leq u + r \\ 0, & \text{others} \end{cases}, \quad (27a)$$

$$\Delta A_x = \begin{cases} \Delta L_x c_s, & u - r \leq x \leq u + r \\ 0, & \text{others} \end{cases}, \quad (27b)$$

$$\Delta I_x = \begin{cases} \frac{1}{12} \Delta L_x c_s^3 + \frac{A_x \Delta A_x (h_x - c_s/2)^2}{A_x - \Delta A_x}, & u - r \leq x \leq u + r \\ 0, & \text{others} \end{cases}, \quad (27c)$$

According to the expression of gear tooth geometric parameters in Reference [12], we can further derive all gear stiffness components as follows: For scenario I, the bending stiffness, shear stiffness, and axial compression stiffness can be written by

$$\begin{aligned} \frac{1}{k_b} = & - \int_{\varphi_2}^{\varphi_3} \frac{3a_x (R_b - R_f \cos \varphi_3 \cos \varphi_1 - a_x \varphi \cos \varphi_1 - b_x \cos \varphi_1)^2}{2EL [R_b \sin \varphi_2 + r_f - \sqrt{r_f^2 - (x - d_1)^2}]^3} d\varphi \\ & + \int_{-\varphi_1}^{\varphi_2} \frac{3\{1 + \cos \varphi_1[(\varphi_2 - \varphi) \sin \varphi - \cos \varphi]\}^2 (\varphi_2 - \varphi) \cos \varphi}{E \left\{ 2L [\sin \varphi + (\varphi_2 - \varphi) \cos \varphi]^3 - 3 \frac{\Delta I_x}{R_b^3} \right\}} d\varphi, \end{aligned} \quad (28a)$$

$$\frac{1}{k_s} = - \int_{\varphi_2}^{\varphi_3} \frac{1.2a_x(1 + \nu) \cos^2 \varphi_1}{EL [R_b \sin \varphi_2 + r_f - \sqrt{r_f^2 - (x - d_1)^2}]} d\varphi + \int_{-\varphi_1}^{\varphi_2} \frac{1.2(1 + \nu)(\varphi_2 - \varphi) \cos \varphi \cos^2 \varphi_1}{E \left\{ L [\sin \varphi + (\varphi_2 - \varphi) \cos \varphi] - \frac{\Delta A_x}{2R_b} \right\}} d\varphi, \quad (28b)$$

$$\frac{1}{k_a} = - \int_{\varphi_2}^{\varphi_3} \frac{a_x \sin^2 \varphi_1}{2EL [R_b \sin \varphi_2 + r_f - \sqrt{r_f^2 - (x - d_1)^2}]} d\varphi + \int_{-\alpha_1}^{\alpha_2} \frac{(\varphi_2 - \varphi) \cos \varphi \sin^2 \varphi_1}{E \left\{ 2L [\sin \varphi + (\varphi_2 - \varphi) \cos \varphi] - \frac{\Delta A_x}{2R_b} \right\}} d\varphi, \quad (28c)$$

For scenario II, the bending stiffness, shear stiffness, and axial compression stiffness can be written by

$$\frac{1}{k_b} = \int_{-\varphi_1}^{\varphi_5} \frac{3\{1 + \cos \varphi_1[(\varphi_2 - \varphi) \sin \varphi - \cos \varphi]\}^2 (\varphi_2 - \varphi) \cos \varphi}{E \left\{ 2L [\sin \varphi + (\varphi_2 - \varphi) \cos \varphi]^3 - 3 \frac{\Delta I_x}{R_b^3} \right\}} d\varphi, \quad (29a)$$

$$\frac{1}{k_s} = \int_{-\varphi_1}^{\varphi_5} \frac{1.2(1 + \nu)(\varphi_2 - \varphi) \cos \varphi \cos^2 \varphi_1}{E \left\{ L [\sin \varphi + (\varphi_2 - \varphi) \cos \varphi] - \frac{\Delta A_x}{2R_b} \right\}} d\varphi, \quad (29b)$$

$$\frac{1}{k_a} = \int_{-\varphi_1}^{\varphi_5} \frac{(\varphi_2 - \varphi) \cos \varphi \sin^2 \varphi_1}{E \left\{ 2L [\sin \varphi + (\varphi_2 - \varphi) \cos \varphi] - \frac{\Delta A_x}{2R_b} \right\}} d\varphi, \quad (29c)$$

4. Dynamic Characteristics and Fault Mechanism Analysis

This section studies the fault mechanism of gear tooth spalling based on the proposed dynamic model and TVMS model. On the one hand, the fault severity of different dimensions was simulated by setting different fault sizes to understand the most sensitive geometrical dimension of defects to TVMS. On the other hand, the influence of defects on vehicle dynamics is studied to provide theoretical support for fault diagnosis. The vehicle parameters used in this study are listed in detail in Table 1. Meanwhile, the gear parameters are given in Table 2.

Table 1. Railway vehicle parameters.

Symbol	Value	Symbol	Value
M_c	35,400 kg	I_c	1,970,300 kg·m ²
M_b	3188 kg	I_b	2710 kg·m ²
M_a	85 kg	I_a	2.45 kg·m ²
M_w	1024 kg	I_w	78 kg·m ²
M_p	5.15 kg	I_p	0.007 kg·m ²
M_{gh}	149.75 kg	I_{gh}	10.45 kg·m ²
M_{mr}	178.36 kg	I_{mr}	1.9 kg·m ²
M_{mh}	422.82 kg	I_{mh}	24.7 kg·m ²
K_{xs}	4.41×10^5 N/m	K_{zs}	2.06×10^5 N/m
C_{xp}	1.25×10^3 N·s/m	C_{zp}	2×10^3 N·s/m
C_{xs}	2.5×10^3 N·s/m	C_{zs}	6×10^4 N·s/m
K_b	1×10^8 N/m	C_b	1×10^3 N·s/m
K_{zgh}	5×10^6 N/m	C_{zmh}	1×10^3 N·s/m
K_{xmh}	1×10^9 N/m	C_{xmh}	1×10^3 N·s/m
K_{zmh}	1×10^9 N/m	C_{zmh}	1×10^3 N·s/m
l_{skc}	6.3 m	l_{epkb}	1.375 m
l_{ipkb}	0.825 m	l_{epka}	0.275 m
l_{ipka}	0.275 m	l_{pbgh}	0.178 m
l_{gbgh}	0.183 m	l_{mhmh}	0.329 m

Table 2. Gear system parameters.

Parameters	Pinion	Gear
Tooth number	22	133
Module (mm)	5	5
Pressure angle (°)	20	20
Young's modulus (Gpa)	206	206
Poisson's ratio	0.3	0.3

4.1. TVMS Analysis

This subsection discusses the effects of TVMS on spalling length, width, and height, respectively. When studying the effects of a particular geometric dimension, the other two variables are fixed. Therefore, we will analyze the impact of the spalling defects' length, width, and depth sequentially.

Figure 4 compares meshing stiffness under different spalling lengths when the width and depth are set at 4 mm. As can be seen from the figure, the spalling length causes a decrease in the TVMS of the gear fault location, and the influence of the contact position is obvious, indicating that the decline in the Hertzian stiffness is relatively significant. As the fault severity increases, the loss range of TVMS does not expand, only reducing the stiffness. In addition, with the meshing progress, the loss of the TVMS near the pitch circle presents a maximum value.

Figure 5 shows a comparison of TVMS for different spalling widths with a spalling length of 10 mm and a spalling depth of 4 mm. Unlike the effect of the spalling length, the increase in the spalling width gradually expands the range of a loss of TVMS centered on the pitch circle, but does not lead to any further losses of the stiffness amplitude. Therefore, the impact of the spalling width on the amplitude loss of TVMS on localized defects is limited. It has a lesser effect on the dynamic response than in the case of a large reduction in the localized defect.

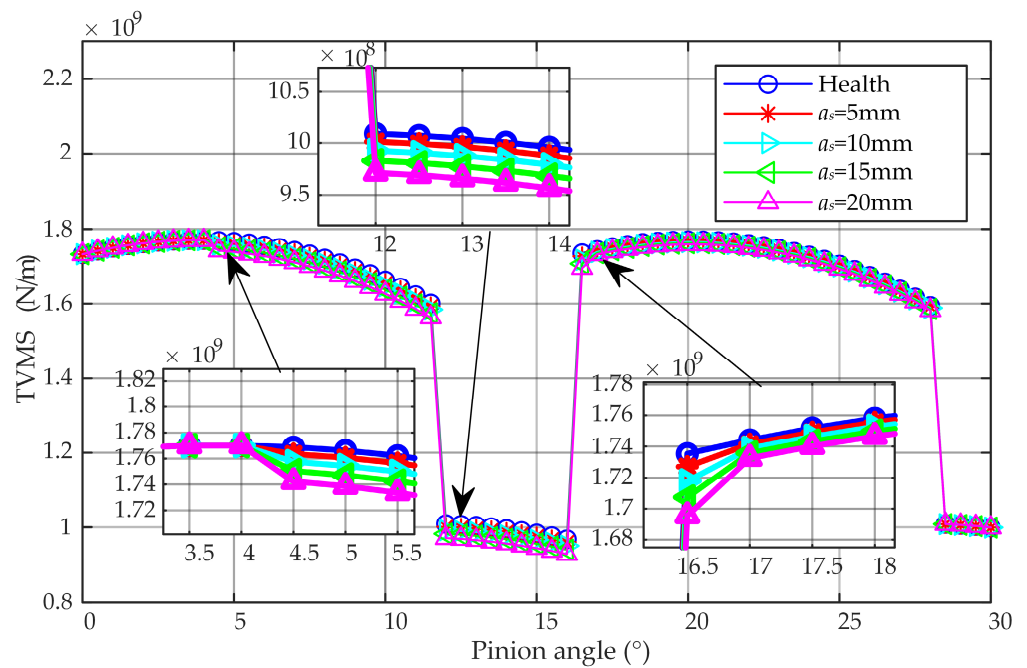


Figure 4. Comparison of TVMS of gears with different spalling lengths under fixed width and depths.

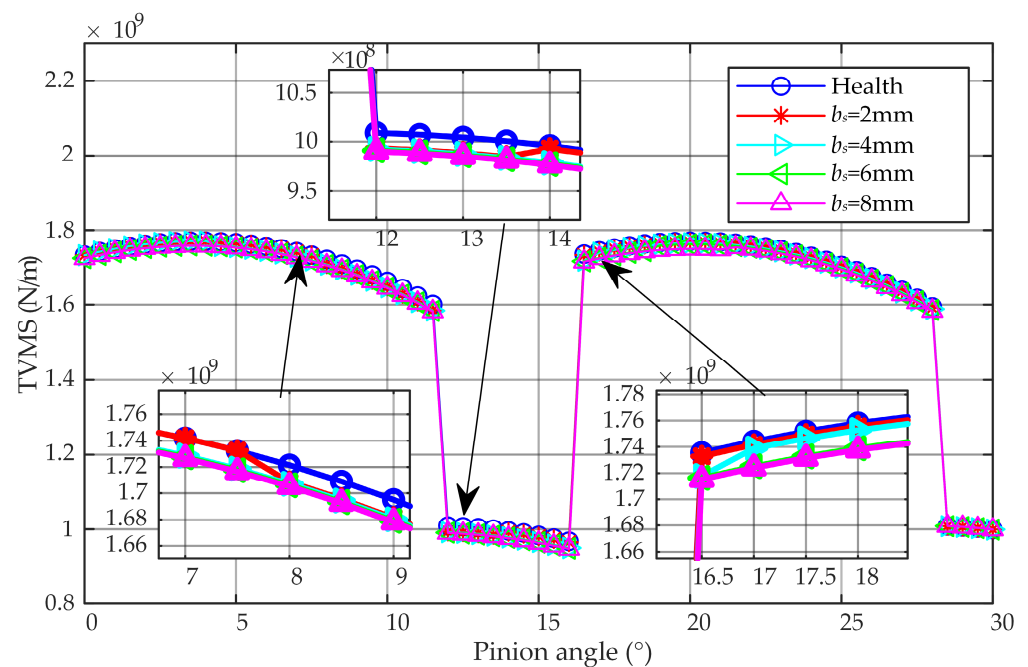


Figure 5. Comparison of TVMS of gears with different spalling widths under fixed lengths and depths.

Figure 6 shows the comparison of TVMS at different spalling depths with a spalling length of 10 mm and a spalling width of 4 mm. Similar to the effect of spalling length, the spalling depth decreases the overall meshing stiffness at the gear fault location. Compared to the spalling length, the spalling depth does not affect the change in the Hertzian stiffness, so its impact effect is smaller. Thus, it can be seen that the most significant impact on the peak value of the loss of TVMS is the spalling length, while the largest impact on the loss range of the TVMS is the spalling width.

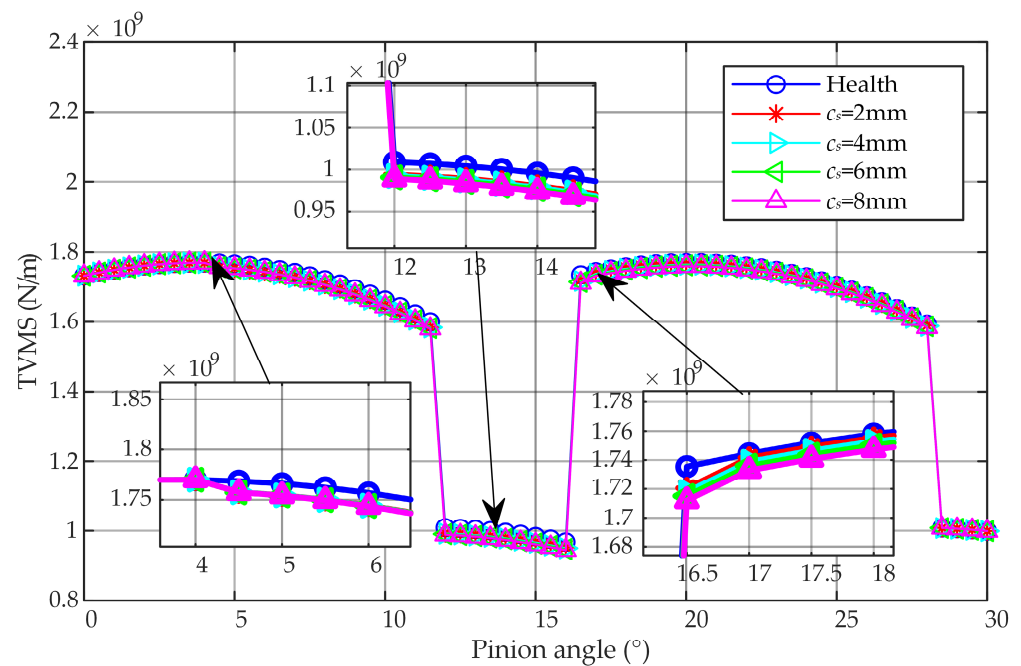


Figure 6. Comparison of TVMS of gears with different spalling depths under fixed length and width.

4.2. Dynamic Characteristics Analysis

This section mainly studies the law of the most significant impact of spalling length on vehicle dynamic characteristics. Due to the most significant impact of spalling length on TVMS, we fixed the spalling width and depth to 4 mm and set TVMS with spalling lengths of 4 mm, 8 mm, 12 mm, 16 mm, and 20 mm, respectively, as a research group,. Firstly, we chose the non-stationary process of the traction condition as the simulation environment, where the strong traction force better excites the characteristic frequency of gear meshing. We set the railway vehicle to an initial speed of 10 km/h and accelerated for 20 s after a stable dynamic period of 5 s. In this simulation, the Shanghai track spectrum is used as an input to the vehicle system because of its fidelity to urban rail excitation [27]. Specifically, the driving torque can be represented by the mechanical characteristics of the traction motor in Figure 7.

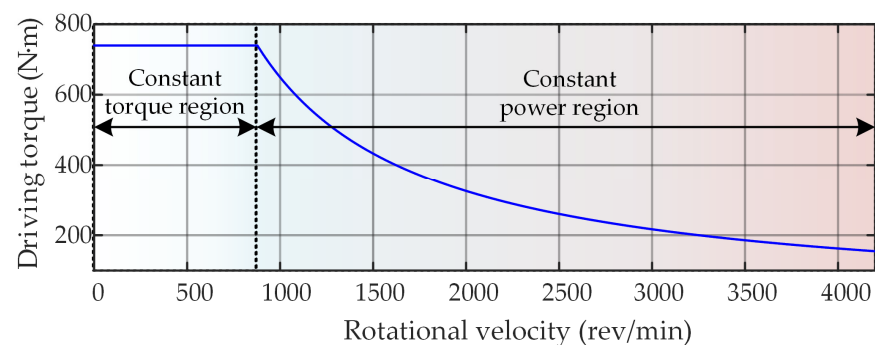


Figure 7. Mechanical characteristics of traction motors.

Then, we chose the root mean square (RMS), kurtosis, crest factor (CF), and variance (VAR) as candidate indicators to measure the health of the gear system. The mathematical expressions for these indicators can be obtained from Ref. [45]. To compare the sensitivity of these three indicators, we selected the numerical data from the last 2 s of the traction process to analyze the fault severity. Additionally, the vertical vibration on the first gearbox housing is carried out as research data of interest. To describe the differences more accurately in

these indicators, we chose the ratio of the differences with the health gear, which can be written as follows:

$$P_{Y_i} = \frac{Y_i - Y_1}{Y_1} \times 100\%, \quad (30)$$

where Y denotes the relative indicator, the subscript is fault severity, and one represents the health gear.

Figure 8 shows the comparison of the four time-domain statistical indicators for the last 2 s of vertical vibration without track irregularity. All indicators show excellent monotonicity so they can describe the severity of the fault. The CF amplitude at 20 mm spalling is approximately 8% higher than the health gear, while the RMS, kurtosis, and VAR variations are still less than 1% at the maximum spalling length of 20 mm. The CF curve exhibits a strong linear relationship with the spalling length after the fault occurs. Due to its sensitivity and linearity, CF is considered the most suitable indicator for analyzing spalling evolution.

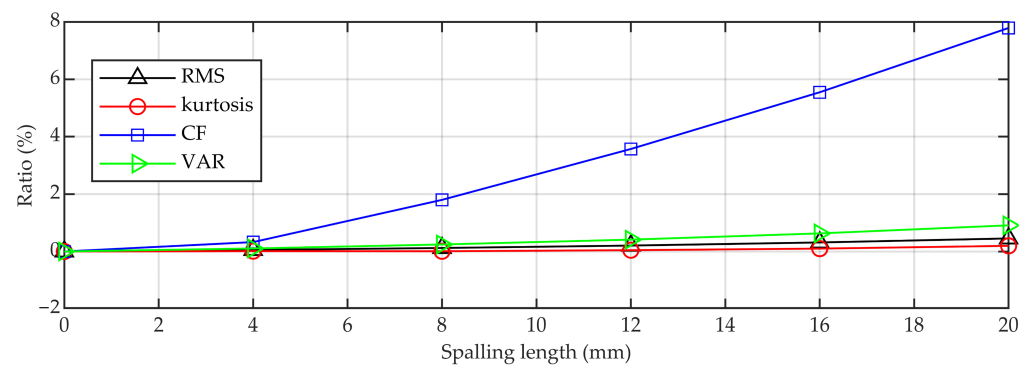


Figure 8. The variations of the RMS, kurtosis, CF, and VAR of the first gearbox housing vertical vibration at different spalling lengths.

Further, we select the most sensitive indicator CF to evaluate the feature evolution trends at different speed stages. Figure 9 shows the variation of CF at varying times under traction conditions, where these data points are calculated every 2 s. Figure 9a describes the indicator changes in the railway vehicles without track irregularities, which shows the ideal denoising results. All curves can orderly represent the indicator rise characteristics due to the spalling propagation. However, regardless of the fault severity, the indicators at low speeds are lower than the amplitude at high speeds. It demonstrates that the impact of vehicle speed on the indicator is more significant than the fault severity. Figure 9b describes the indicator changes in railway vehicles with track irregularities. In contrast, the changes in these indicators exhibit a strong irregularity. This indicates that the strong noise caused by track irregularities significantly impacts the indicator.

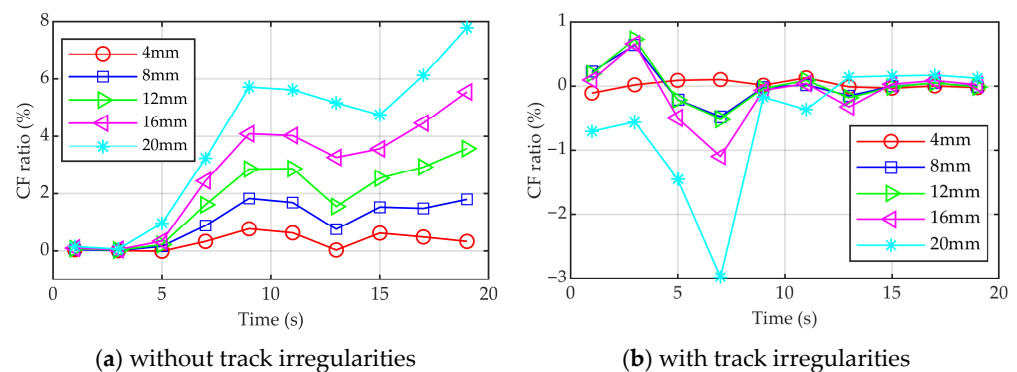


Figure 9. Feature evolution of vertical vibration of the first gearbox housing under traction condition.

At the same time, the four time-domain indicators of longitudinal acceleration for the last 2 s are also used for comparative research, as shown in Figure 10. Similarly to the vertical indicators results, the four indicators exhibit excellent monotonicity. Moreover, the CF amplitude is still the most sensitive but it decreases nearly 3% more than the vertical amplitude at the maximum spalling length of 20 mm.

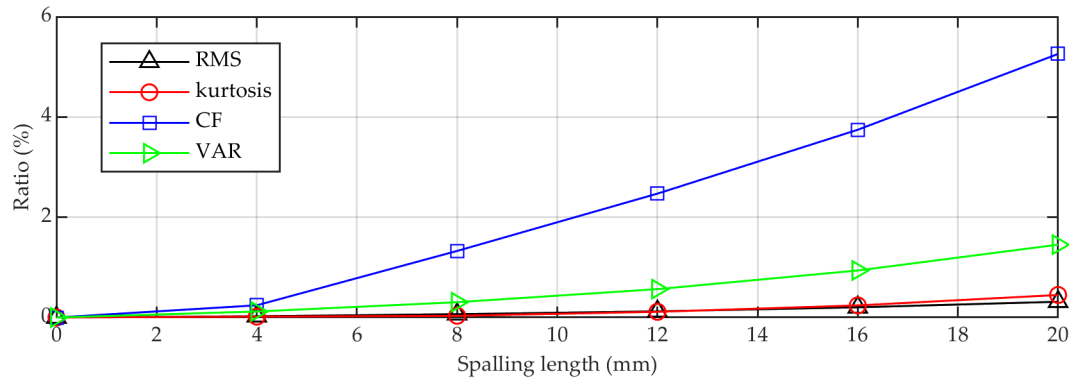


Figure 10. The variations of the RMS, kurtosis, CF, and VAR of the first gearbox housing longitudinal vibration at different spalling lengths.

Similarly, we select the most sensitive indicator CF to evaluate the feature evolution trends at different speed stages. Figure 11 shows the variation of CF at varying times under traction conditions, where these data points are calculated every 2 s. Figure 11a describes the longitudinal indicator changes without track irregularities, which show the ideal denoising results. All curves can orderly represent the indicator rise characteristics due to the spalling propagation. Unlike the case of vertical vibration, these curves exhibit more significant oscillations as the vehicle speed changes. This indicates that longitudinal vibration makes it is more challenging to measure the disturbance of the indicator by velocity changes than vertical vibration and determines the fault threshold with difficulty. Figure 11b describes the longitudinal indicator changes in railway vehicles with track irregularities. In contrast, the changes in these indicators exhibit a strong irregularity. It proves that even longitudinal vibration acceleration can significantly disturb the indicator due to the impact of track irregularity caused by the vibration coupling effect.

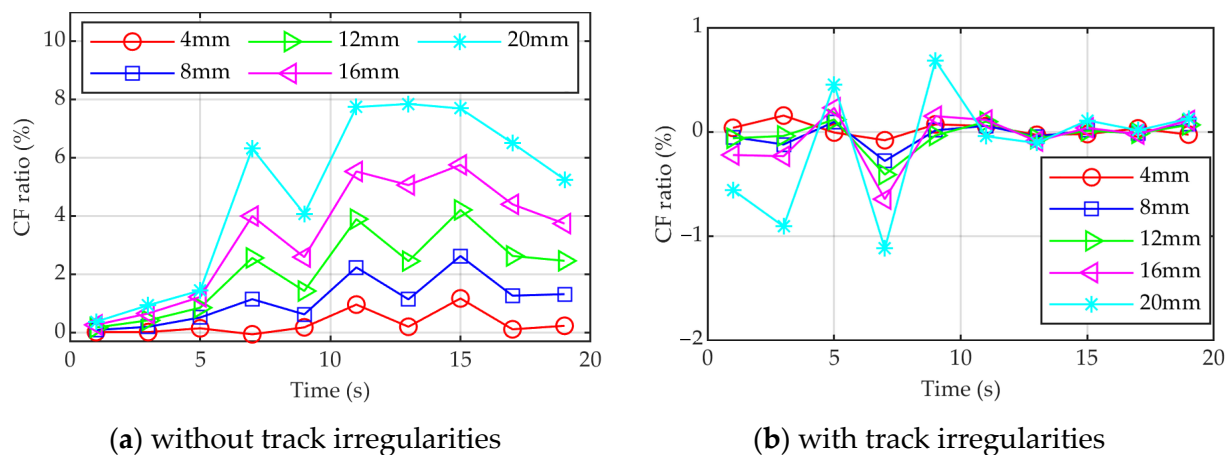


Figure 11. Feature evolution of longitudinal vibration of the first gearbox housing under traction condition.

In addition, we use a time–frequency analysis tool to analyze the characteristic frequencies caused by gear tooth spalling. Figure 12 shows the time–frequency spectrum of

the vertical vibration of the first gearbox housing with health conditions and a spalling length of 20 mm. The natural frequency, meshing, and harmonic components are evident in both the time–frequency spectra. Theoretically, local faults such as cracks and spalling can lead to significant sidebands in the frequency spectrum around the meshing frequency, which is the central frequency. However, there is no significant difference between the two time–frequency spectra. The reason is that the excitation generated by gear tooth spalling is weak for railway vehicles and insufficient to excite significant characteristic frequencies. In contrast, it excites a more substantial natural frequency of the system. The research result indicates that it is difficult to apply a time–frequency analysis tool for fault diagnosis of spalling gear of railway vehicles under strong noise and non-stationary environments.

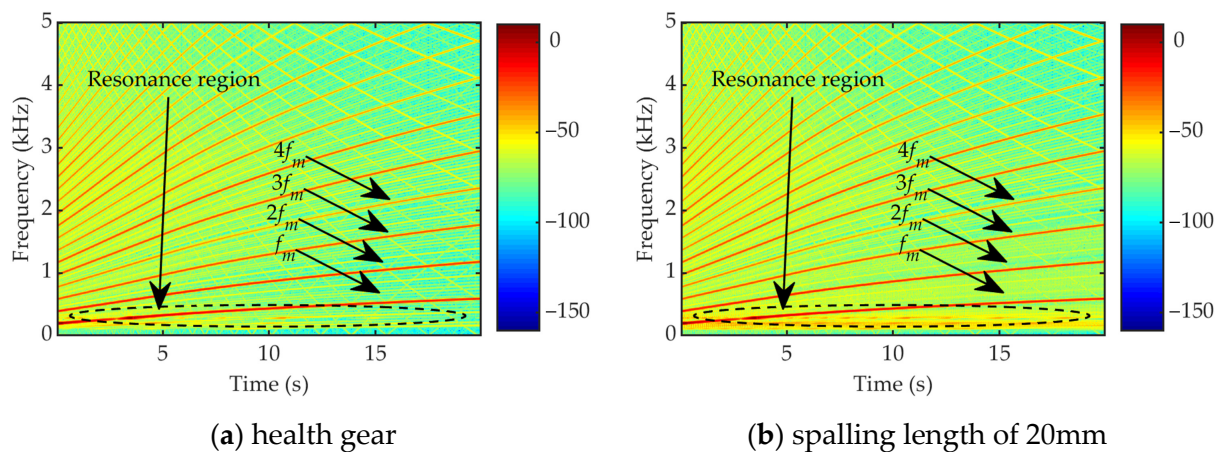


Figure 12. Comparison of time–frequency spectrum of healthy and maximum spalling length gears without track irregularities.

Theoretically, as a random excitation with a wavelength of more than 1 m, the track spectrum has a low-frequency effect on the vehicle vibration compared to gear engagement. Therefore, we designed a simple high-pass filter to suppress the impact of the track spectrum, thereby using indicators of interest for the health assessment. To evaluate the effect of the passband frequency selection, we show the comparison of time histories. Figure 13 shows the vertical acceleration comparison of the raw signal without a track spectrum, the raw signal with a track spectrum, and the filtered signal with a track spectrum at a passing frequency of 350 Hz. The filtered signal is consistent with the raw signal without a track spectrum in the later stage of the traction condition. However, the low-frequency characteristics caused by resonance in the early stage are ignored.

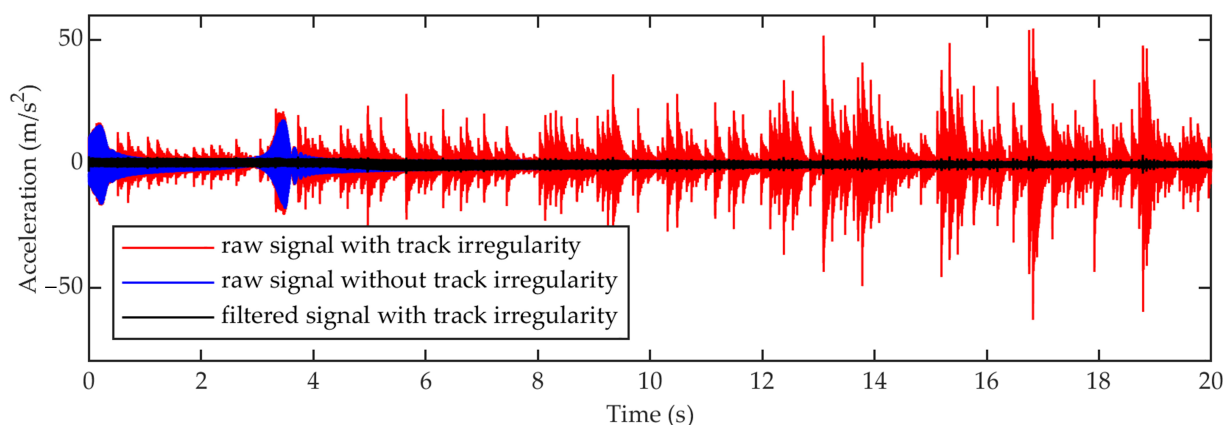


Figure 13. The comparison of the time histories of the first gearbox housing under traction condition.

Finally, we show the variation in the four time-domain indicators of the filtered signal, as shown in Figure 14. Even when applying the ideal filtering techniques, RMS and VAR present clear classifications for different severity levels. However, the indicators under different severities show little difference. Interestingly, these indicators are no longer sensitive to changes in vehicle speed. The reason is that the increased vehicle speed leads to low-frequency vibration caused by random excitation of track irregularities. Thus, high-pass filtering can easily avoid adverse effects.

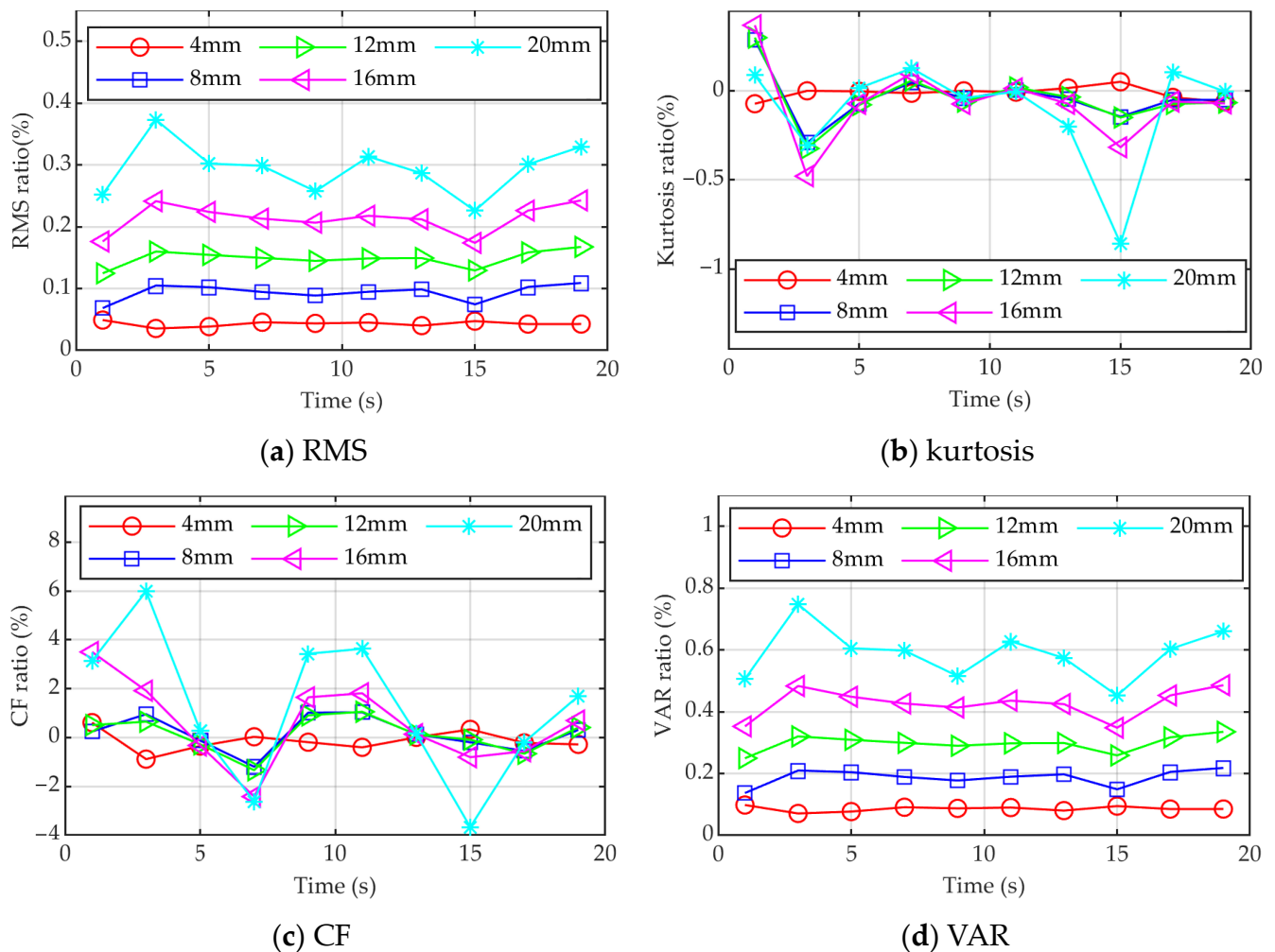


Figure 14. The variation of vertical vibration indicators of the first gearbox housing after filtering.

5. Conclusions

A planar railway vehicle with a traction transmission model and the TVMS model of spalling spur gears is established. Then, the influence of the spalling length, width, and depth on TVMS are analyzed, and the most sensitive geometrical dimension is found. Furthermore, the evolution law of dynamic characteristics under the change in tooth spalling length was further explored. Based on the above research, the following conclusions can be obtained:

1. The spalling length and depth both reduce the TVMS amplitude at the defect location, while the spalling width increases the loss range of TVMS. Overall, the spalling length has the most significant impact on TVMS.
2. From a comparative perspective, the CF is the best candidate indicator of sensitivity and linearity under the ideal noiseless environment. Unfortunately, the strong noise caused by track irregularities leads to disordered fault evaluation. After simple high-pass filtering,

the adverse effects of vehicle speed and track irregularities can be eliminated on the RMS and VAR. However, the sensitivity of these indicators is poor.

3. From the time–frequency spectrum, gear tooth spalling cannot exhibit a significant sideband in the operating environment of railway vehicles. This is very different from faults such as root cracks and missing teeth. Therefore, it can be considered a weak signal fault diagnosis and requires effective denoising and weak feature extraction algorithms in fault diagnosis to accurately extract its characteristic frequency.

Based on the above conclusions, this paper recommends setting a threshold value at a specific speed and applying advanced denoising methods to evaluate gear spalling. This strategy is simple and effective while it avoids the adverse effects that are caused by strong-noise and non-stationary processes.

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