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Generalized Electron Emission Theory for One-Dimensional Conducting Materials

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Featured Application: This research is useful to understanding generalized electron emission in low-dimensional conducting materials.

Abstract: This research focuses on exploring generalized electron emissions in one-dimensional conducting materials, specifically examining field emission and thermionic emission. Additionally, this study investigates one-dimensional thermal radiation. The current density is derived for both one-dimensional thermionic emission and one-dimensional field emission. A generalized current density in one dimension is devised, taking into account arbitrary accelerating fields and temperature conditions. The formula for the one-dimensional current density is depicted with accelerating field and temperature across various work function materials. The emission current is represented as a function of temperature for various electric fields. The emission current is also represented as a function of the electric field for various temperatures.

Keywords: field emission; thermionic emission; electron emission; one-dimensional conductors; low dimensional materials

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1. Introduction

Photons are emitted by a heated object. Max Planck's exploration of blackbody radiation led to the discovery of angular momentum quantization and the Planck constant (h) [1]. The Planck constant, along with its reduced form ($h/2\pi$), representing the minimum action, serves as the foundation for quantization, encompassing phenomena such as photons, the Josephson constant, the von Klitzing constant, the conductance quantum, the inverse conductance quantum, and the magnetic flux quantum. The study of n-dimensional blackbody radiation revealed generalized Planck energy and the Stefan–Boltzmann law [2]. The characteristics of thermal radiation were deduced across arbitrary fractional dimensions, ascertainable through the measurement of spectral radiance [3]. The extent of thermal radiation is influenced by the dimensions of the object. As the size of the object decreases significantly, quantum effects become apparent due to the minimum wavelength of photons being comparable to the size of the object. Researchers have examined the size-dependent impact on the energy density of thermal radiation [4]. Additionally, thermionic emission, wherein electrons with energy levels surpassing their work function are emitted from a heated conducting metal, has been studied. Initially, thermionic emission was investigated in metallic materials [5], while subsequent studies extended to semiconductor materials to explore scaling laws [6]. The derivation of the thermionic emission current density was conducted for a two-dimensional crystal material and generalized for arbitrary dimensions [7,8]. In conducting metals, electrons near the Fermi energy level undergo quantum tunneling when subjected to a strong electric field, a phenomenon referred to as field emission. The exploration of field emission began in metallic materials utilizing quantum mechanical tunneling principles [9]. Comprehensive investigations were carried out on both field emission and field ionization phenomena [10]. Superconducting niobium cavities are extensively utilized for accelerating electrons or heavy ions. However, in high-accelerating electric fields, field emissions originating from micron-sized particles or surface irregularities in these niobium superconducting cavities can lead to performance degradation. Thorough research efforts have been made to comprehend the impacts of field emission on these niobium superconducting cavities [11–14]. Furthermore, research has explored field emission phenomena in carbon nanotubes and carbon nanotube field emitter arrays [15–17], with efforts to calculate the generalized current density for field emission across arbitrary dimensions [18]. Carbon nanotubes can have the highest field enhancement, and the field enhancement factor for carbon nanotubes and carbon nanotube displays has been intensively studied [19–22]. Additionally, a comprehensive electron emission theory has been developed by integrating field emission and thermionic emission principles within two-dimensional conducting materials [23]. Thermoelectric materials were investigated, and the thermoelectric figure of merit was well studied in low-dimensional materials [24–26].

In this study, we compute the universal current density encompassing both thermally emitted electrons and field-emitted electrons within one-dimensional materials. We delve into the spectral radiance of one-dimensional entities. The current density is specifically determined for generalized thermionic emission and generalized field emission within the one-dimensional framework. A comprehensive, one-dimensional current density is then calculated to accommodate diverse electric fields and temperatures. Furthermore, the expression of this current density varies with the temperature and electric field for various work functions.

2. Thermal Radiation in One Dimension

One-dimensional blackbody radiation arises from an infinite array of modes within a one-dimensional cavity, where photon energy is quantized. This quantized photon energy can be represented as

$$E_i = n_i h\nu, \tag{1}$$

where h denotes the Planck constant,n_i denotes a positive integer, and ν denotes the frequency. According to Planck's distribution, the mean photon count is calculated as

$$\overline{n}_{ph} = \frac{1}{\exp(\frac{h\nu}{k_B T}) - 1}, \tag{2}$$

where k_B stands for the Boltzmann constant and T stands for the absolute temperature.

The mean energy within the mode is given by

$$\overline{E} = \frac{h\nu}{\exp(\frac{h\nu}{k_B T}) - 1}. \tag{3}$$

The spectral density for one-dimensional blackbody radiation, derived from Planck's radiation law, is expressed as

$$u(\nu, T) = (\frac{4}{c})\frac{\nu}{\exp(\frac{h\nu}{k_B T}) - 1}, \tag{4}$$

where c represents the speed of light.

The energy density for photon energy E at temperature T is formulated as

$$u(E, T) = (\frac{4}{hc})\frac{E}{\exp(\frac{E}{k_B T}) - 1}. \tag{5}$$

The energy density of the photon gas, assuming an infinite body length, and integrating Equation (5) from zero to infinity, yields

$$u(T) = (\frac{4}{hc})\int_0^\infty \frac{EdE}{\exp(\frac{E}{k_B T}) - 1}. \tag{6}$$

By introducing $x = E/k_B T$, Equation (6) is simplified to

$$u(T) = (\frac{4(k_B T)^2}{hc})\int_0^\infty (\frac{xdx}{\exp(x) - 1}). \tag{7}$$

Equation (7) can be evaluated to obtain [4]

$$u_B(T) = \left(\frac{4(k_B T)^2}{hc}\right)\left(\frac{\pi^2}{6}\right), \tag{8}$$

which represents one-dimensional blackbody radiation. This derivation follows the conventional approach for one-dimensional blackbody radiation, where the radiation's maximum wavelength is large enough to approximate the minimum energy to zero. In a one-dimensional blackbody, the energy density is proportional to T^2, contrasting with a three-dimensional blackbody, where the energy density is proportional to T^4.

The spectral radiance in one dimension is given by [3]

$$\Phi_v = \frac{4\pi h v}{\exp(hv/k_B T) - 1}. \tag{9}$$

This spectral radiance can be represented in terms of dimensionless photon energy as $(\frac{h\nu}{k_B T})$.

3. One-Dimensional Thermionic Emission

At zero temperature, electrons within a conductor occupy energy levels up to the Fermi energy. As the temperature increases, some electrons are excited to energy levels exceeding the work function. Electrons thermally emitted in this process, possessing energy surpassing the vacuum energy, participate in thermionic emissions.

The one-dimensional kinetic energy of an electron is given by

$$E_{1D} = \frac{1}{2}mv_x^2, \tag{10}$$

where m denotes the electron mass, and v_x denotes the electron velocity in the x-direction. This Equation (10) can also be represented as $E_{1D}(k_x) = \frac{\hbar^2 k_x^2}{2m}$ when the wave number is incorporated, given that the electron's velocity is significantly lower than the speed of light. The electron velocity can be expressed as $v_x = \frac{\hbar k}{m}$ by considering $v_x = \frac{1}{\hbar}\nabla_{k_x} E(k_x)$.

The one-dimensional current density associated with thermionic emission becomes

$$J_{1D} = e\int_0^\infty dv_x v_x n(v_x) D(E_x), \tag{11}$$

where n signifies the density of electrons, D represents the tunneling probability, and e denotes the electric charge. The electron density, represented as $n(E) = g(E)f(E)$, incorporates the density of states $g(E)$ and the Fermi factor $f(E)$, indicating the probability of a specific state having energy E. When the electron energy exceeds the vacuum energy, the tunneling probability is 1. Utilizing Equation (11), the thermionic current density in one dimension is derived as [8]

$$J_{1D} = e\int_0^\infty (\frac{dE_x}{m})(2\frac{m}{h})\frac{D(E_x)}{\exp\left(\frac{E_x - E_F}{k_B T}\right) + 1}, \tag{12}$$

where E_F represents the Fermi energy and k_B denotes the Boltzmann constant. Taking into account the vacuum energy, Equation (12) is rewritten as [8]

$$J_{1D} = \frac{2ek_BT}{h}\int_{\Phi_w+E_F}^{\infty} \frac{dE_x/k_BT}{\exp\left(\frac{E_x-E_F}{k_BT}\right)+1}, \tag{13}$$

where Φ_W denotes the work function of the one-dimensional material. By introducing $x = \frac{E_x-E_F}{k_BT}$, Equation (13) can be expressed as

$$J_{1D} = \frac{2ek_BT}{h}\int_{\frac{\Phi_w}{k_BT}}^{\infty} \frac{dx}{\exp(x)+1}. \tag{14}$$

Integration of Equation (14) yields [10]

$$J_{1D} = \frac{2ek_BT}{h}[\ln(1+e^{\frac{\Phi_w}{k_BT}}) - \frac{\Phi_w}{k_BT}]. \tag{15}$$

Equation (15) can alternatively be written as

$$J_{1D} = \frac{2ek_BT}{h}[\ln(1+e^{-\frac{\Phi_w}{k_BT}})]. \tag{16}$$

Equation (16) represents the thermionic current density in one-dimensional materials. The current density given by Equation (16) is applicable across varying work functions and temperatures.

When the work function Φ_W is much bigger than the thermal energy k_BT, Equation (14) simplifies to [8]

$$J_{1D} = \frac{2e}{h}k_BT\int_{\frac{\Phi_w}{k_BT}}^{\infty} \exp(-x)dx. \tag{17}$$

From Equation (17), the thermionic current density for one-dimensional materials is obtained as

$$J_{1D} = \frac{2ek_BT}{h}e^{-\frac{\Phi_w}{k_BT}}. \tag{18}$$

This Equation (18) represents the one-dimensional current density for thermionic emission.

When thermally excited electrons have energy exceeding the vacuum energy, they might undergo reflection at the boundary between the metal and vacuum. The generalized one-dimensional current density for thermally emitted electrons is then described as [8]

$$J_{1D} = (1-r)\frac{2ek_BT}{h}e^{-\frac{\Phi_w}{k_BT}}, \tag{19}$$

where r denotes the probability of reflection for the electrons.

4. One-Dimensional Field Emission

In a metal, free electrons adhere to a Fermi–Dirac distribution. Concerning field emission, the current density in a one-dimensional conductor is as follows:

$$J_{1D} = e\int_0^{E_F} dv_x v_x n_{1D}(E_x)D(E_x), \tag{20}$$

where v_x represents the x-directional velocity of electrons, E_F signifies the Fermi energy, n_{1D} denotes the one-dimensional density of electrons, and D represents the tunneling probability. $D(E_x)$ signifies the probability of tunneling, indicating the likelihood that an electron with energy E_x traverses into a vacuum through the potential barrier.

Figure 1 depicts a schematic representation of one-dimensional field emission, showcasing the application of the x-directional electric field to the conducting material. The electron tunneling probability D for the conductor exposed to a steady electric field is provided by [9]

$$D = \left(\frac{4\sqrt{E_x}\sqrt{E_F + \Phi_w - E_x}}{E_F + \Phi_w}\right)e^{-4k(E_F+\Phi_w-E_x)^{3/2}/3F}, \tag{21}$$

where $k = \sqrt{\frac{8\pi^2 m}{h^2}}$ signifies the electron wave number, E_x denotes the electron energy along the x-direction, Φ_w represents the work function of the metal, and F signifies the accelerating field to the metal. The electron tunneling probability described by Equation (21) is contingent on the Fermi energy, work function, and accelerating electric field.

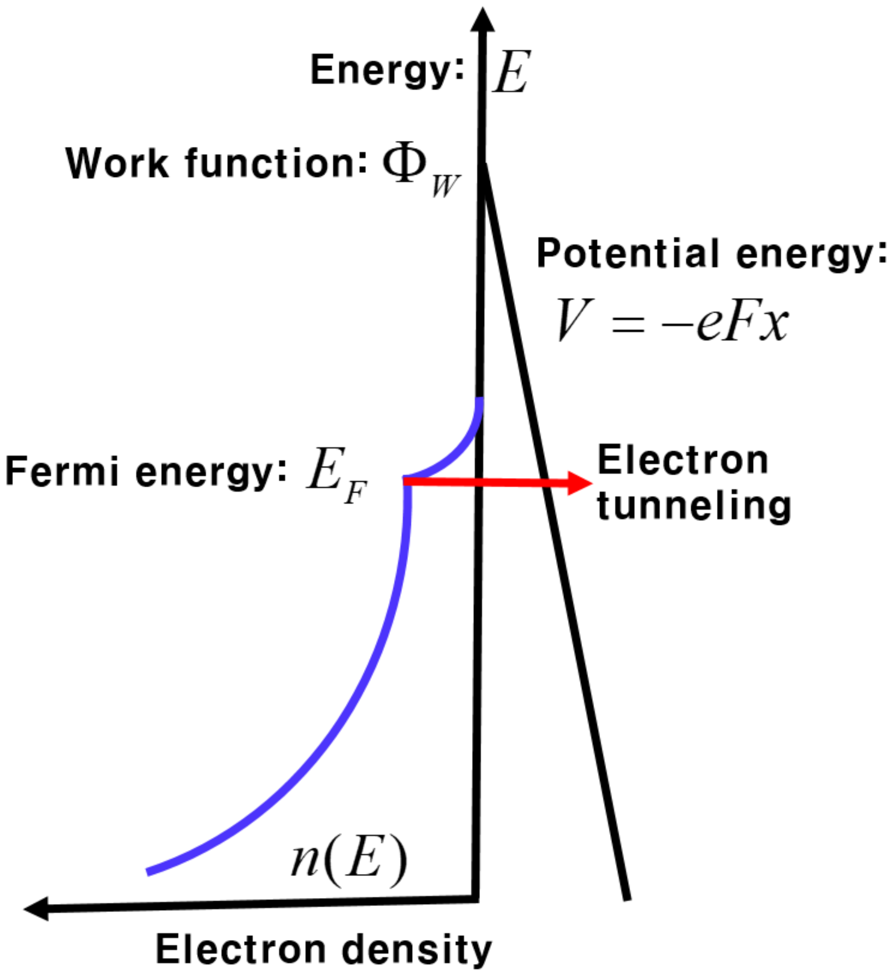

Figure 1. Diagram illustrating generalized electron emission in one-dimensional materials. An accelerating field is directed along the x-axis and applied to the metal. Thermally excited electrons with energy levels surpassing the vacuum threshold engage in thermionic emissions. Electrons experiencing a strong electric field, with energy levels below the vacuum threshold, partake in quantum tunneling, constituting the generalized field emission.

Particles cannot tunnel in classical mechanics when there is a potential barrier. Electrons can participate in quantum tunneling in quantum mechanics when the potential barrier thickness is comparable to the wavelength of the electron. The electron having the highest energy occupied by the Fermi energy can go through the potential barrier under a strong electric field in which the emitted electron energy is the same as the Fermi energy. This is called field emission because electrons under a strong electric field participate in quantum tunneling in metal.

The expression for the field emission current density in a one-dimensional conductor, derived from Equation (20), is presented as follows [18]:

$$J_{1D} = \frac{2e}{h} \int_0^{E_F} \frac{dE_x D(E_x)}{\exp\left(\frac{E_x - E_F}{k_B T}\right) + 1}, \tag{22}$$

where $D(E_x)$ denotes the probability of electron tunneling.

When considering the electron tunneling probability from Equation (21) and the limit at zero temperature, the expression for the one-dimensional current density in the field emission, represented by Equation (22), simplifies to [18]

$$J_{1D} = \frac{8e}{h} \int_0^{E_F} dE_x \left(\frac{E_F - E_x}{E_F}\right)\left(\frac{\sqrt{E_x}\sqrt{E_F + \Phi_w - E_x}}{E_F + \Phi_w}\right) e^{-4k(E_F + \Phi_w - E_x)^{3/2}/3F}. \tag{23}$$

By substituting E_x with $E_F - x$ and assuming prevalent electron tunneling in the vicinity of the Fermi energy, the approximated current density for the field emission, denoted as Equation (23), is obtained as [18]

$$J_{1D} = \frac{8e}{h} \frac{\sqrt{\Phi_w}}{(E_F + \Phi_w)} \frac{1}{\sqrt{E_F}} e^{-4k\Phi_w^{3/2}/3F} \int_0^{\infty} dx x e^{-2xk\sqrt{\Phi_w}/F}. \tag{24}$$

Using Equation (24), the expression for the current density in the case of one-dimensional field emission is derived as

$$J_{1D} = \frac{ehF^2}{4\pi^2 m (E_F + \Phi_w)\sqrt{E_F \Phi_w}} e^{-4k\Phi_w^{3/2}/3F}, \tag{25}$$

where m denotes the electron mass. Equation (25) illustrates the one-dimensional current density for field emission, indicating the number of quantum-tunneled electrons per unit time. This current increases with higher electric field strength and decreases with a reduction in the work function.

5. Comprehensive Electron Emission Theory in One Dimension

The generalized electron emission in one-dimensional materials can be computed for arbitrary electric fields and temperatures. The length of the wire is much larger than the mean free path of the electrons. Based on Equation (22), the current of one-dimensional field emission is approximately determined for diverse temperature conditions.

$$J_{1D} = \frac{8e}{h}\left[\int_0^{E_F + \Phi_W} dE_x \left(\frac{\sqrt{E_x}\sqrt{E_F + \Phi_w - E_x}}{E_F + \Phi_w}\right) \ln\left(1 + \exp\left(-\frac{E_x - E_F}{k_B T}\right)\right) e^{-4k(E_F + \Phi_w - E_x)^{3/2}/3F}\right]. \tag{26}$$

Combining Equations (18) and (26), the generalized one-dimensional current density for arbitrary accelerating fields and temperatures is represented as

$$J_{1DUF} = \frac{2e}{h}\left[k_B T e^{-\frac{\Phi_w}{k_B T}} + 4\int_0^{E_F + \Phi_W} dE_x \left(\frac{E_x - E_F}{E_F}\right)\left(\frac{\sqrt{E_x}\sqrt{E_F + \Phi_w - E_x}}{E_F + \Phi_w}\right) \ln\left(1 + \exp\left(-\frac{E_x - E_F}{k_B T}\right)\right) e^{-4k(E_F + \Phi_w - E_x)^{3/2}/3F}\right]. \tag{27}$$

The one-dimensional current density derived from the unified theory encompassing both field emission and thermionic emission can be computed using Equation (27). This comprehensive theory of electron emission serves to elucidate the emission behavior of electrons from one-dimensional conducting materials such as carbon nanotubes. It is noteworthy that the dimension of the one-dimensional current density is the electrical current (A), whereas for the three-dimensional current density, it is the electrical current divided by the area (A/m^2). The latter form (A/m^2) was used as the current density of nanotubes and nanowires in previous studies [27,28], while the one-dimensional current (A) was used for a carbon nanotube [15]. The electrical current (A), rather than the current density, is used for the one-dimensional current in our paper. Non-equilibrium Green's function with density function theory showed that carbon nanotubes allow the flow of currents of a few micro-amperes for a few volts [29].

6. Results and Discussion

Figure 1 depicts a schematic representation of one-dimensional electron emission. A robust local electric field is directed along the x-axis within the metal. At zero temperature, electrons occupy energy levels up to the Fermi energy. As the temperature rises, these electrons become excited, reaching energy levels exceeding the Fermi energy and, subsequently, surpassing the work function. Thermally excited electrons, whose energy levels exceed that of the vacuum, participate in thermionic emissions. On the other hand, under a strong electric field, electrons with energy levels below the vacuum energy engage in quantum tunneling, constituting the generalized field emission. The one-dimensional materials can be nanotubes or nanowires. For one-dimensional electron emission, the Fermi energy and work functions are important parameters. The interactions between electrons and phonons are considered small relative to the electron emission effect, so they are neglected in this research.

Figure 2 depicts the spectral radiance for various temperatures in one-dimensional materials, presented in terms of dimensionless photon energy. The spectral radiance, as defined in Equation (9), is expressed as dimensionless photon energy ($\frac{h\nu}{k_B T}$) for various temperatures in one-dimensional materials. The temperature sequence in Figure 2, progressing from bottom to top, corresponds to 2.7 K, 300 K, 1000 K, 3000 K, and 10,000 K. It is noteworthy that the unit of the one-dimensional energy density is joules per meter (J/m), while the unit for the three-dimensional energy density is joules per cubic meter (J/m^3).

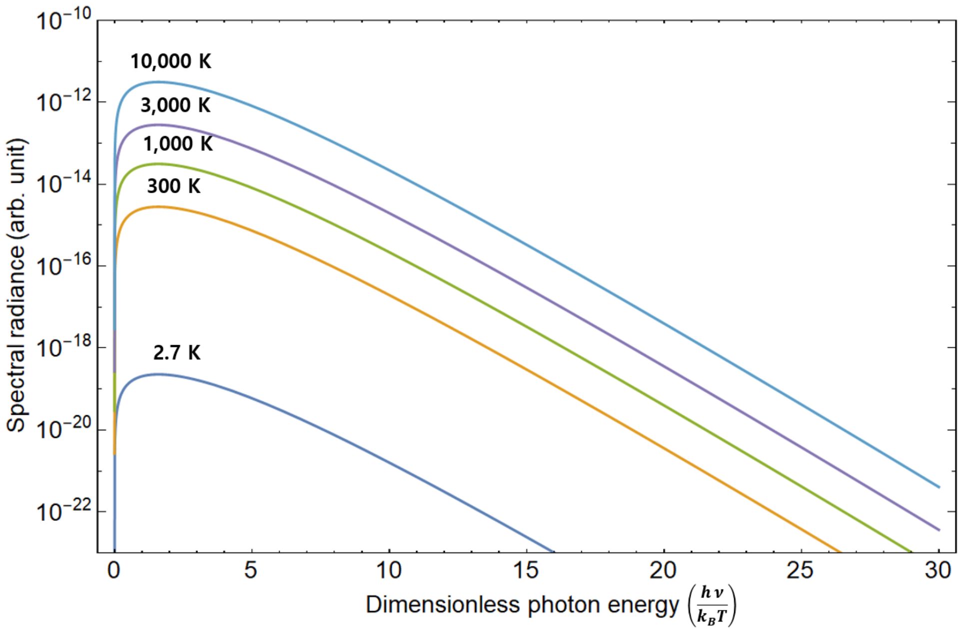

Figure 2. Spectral radiance presented as dimensionless photon energy for various temperatures in one-dimensional materials. The spectral radiance, as defined in Equation (9), is represented with dimensionless photon energy ($\frac{h\nu}{k_B T}$) across different temperatures in one-dimensional materials. The sequence of temperatures, from bottom to top, corresponds to 2.7, 300, 1000, 3000, and 10,000 K.

In Figure 3, the variation in the one-dimensional current with temperature is presented for different materials characterized by distinct work functions. The calculation of the one-dimensional current density was performed for materials with a Fermi energy of 4.57 eV and varying work functions from 2 to 5 eV. The observed trend indicates that the one-dimensional current density rises with increasing temperature and decreasing work function. This calculation for one-dimensional materials is consistent with well-known thermionic emission results.

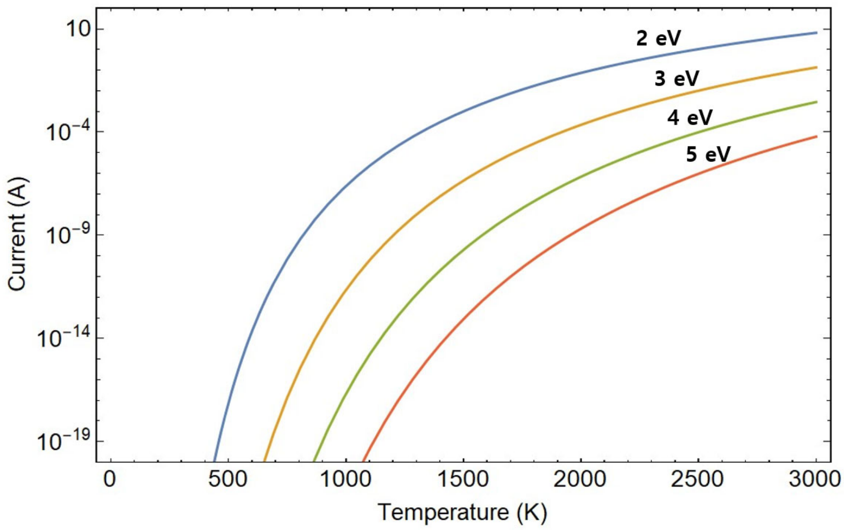

Figure 3. One-dimensional current with temperature for various materials characterized by distinct work functions. The electric field is zero, and only thermionic emission is considered. The calculation of the one-dimensional current was conducted for materials with a Fermi energy of 4.57 eV and various work functions, specifically 2, 3, 4, and 5 eV.

Figure 4 illustrates the one-dimensional current plotted against the electric field for various materials characterized by various work functions. The calculation of the one-dimensional current was conducted for materials with a fixed Fermi energy of 4.57 eV and varying work functions ranging from 2 to 5 eV. It was observed that the one-dimensional current of the conducting material increases with an increase in the electric field and a decrease in the work function. This calculation for one-dimensional materials is consistent with well-known field emission results.

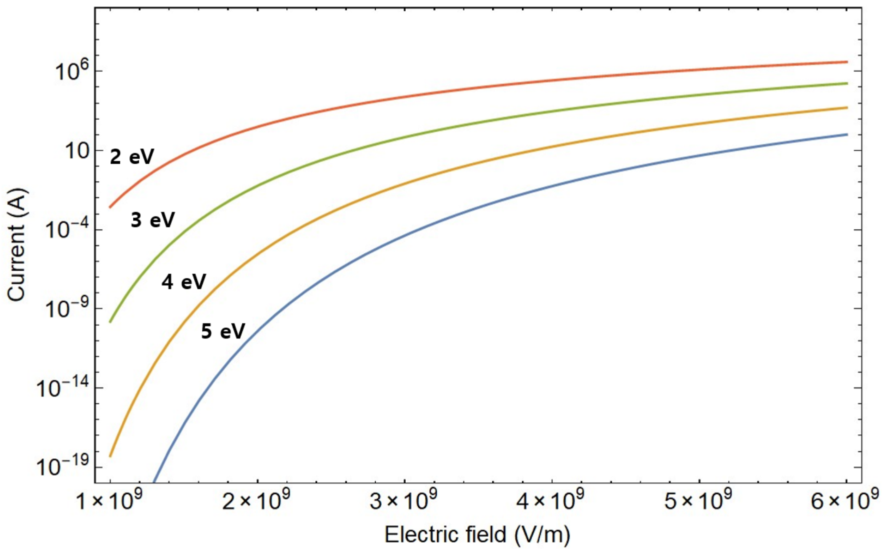

Figure 4. One-dimensional current plotted against the accelerating field for materials with varying work functions. The temperature is zero, and only field emission is considered. The calculation involved determining the one-dimensional current for materials with a common Fermi energy of 4.57 eV and different work functions, specifically 2, 3, 4, and 5 eV.

One-third of single-walled nanotubes (SWNTs) are good conductors because their armchair structure shows conducting properties, and the rest show semiconducting properties [19]. Multi-walled nanotubes (MWNTs), which consist of many SWNTs, are considered metals, because their forbidden band gap can be negligible. The field enhancement factor is defined as $\beta = \frac{F}{F_o}$, where F_o is the mean applied electric field and F is the electric field at the tip. The field enhancement factor of CNTs can be higher than 1000, so the applied voltage is much lower than that for the conventional field emitters such as tungsten tips. For a single nanotube, the electric field enhancement factor is $\beta = \frac{h}{d}$, where d and h are the diameter and height of the nanotube, respectively [19]. The electric field enhancement factor can be approximated to $\beta = \frac{h}{d}(1 + \frac{d}{D})$ when distance D between the nanotube tip and the anode surface is considered [19]. The electron emission experiments from CNTs basically follow the Fowler–Nordheim equation when considering logarithm data analysis [15,27].

In this paper, we consider one-dimensional field emission rather than three-dimensional field emission. We also consider the thermionic emission effect from the heating of CNTs. This research covers the generalized electron emission effects from arbitrary electric fields and arbitrary temperatures.

In Figure 5, the one-dimensional current of carbon nanotubes is presented as a function of the electric field. The work function and Fermi level function for carbon nanotubes are specified as 4.56 eV and 4.57 eV, respectively. The temperatures range from 500 K to 2500 K, progressing from bottom to top. The depicted trend reveals that the current emitted from carbon nanotubes rises with increases in both the temperature and the applied accelerating field.

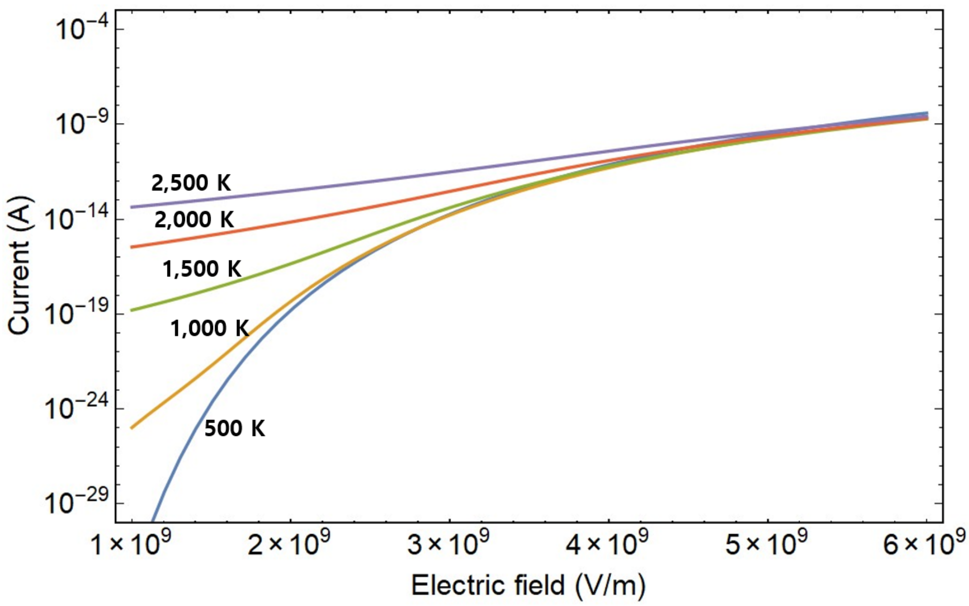

Figure 5. One-dimensional current of carbon nanotubes plotted against the accelerating field. In the case of carbon nanotubes, the work function and Fermi level are specified as 4.56 eV and 4.57 eV, respectively. The represented temperatures ascend from bottom to top, representing 500, 1000, 1500, 2000, and 2500 K.

Figure 6 illustrates the variation in the one-dimensional current with the applied electric field for various work functions, maintaining a constant temperature of 1000 K. The depicted graph presents a sequence of work functions ranging from 2 to 5 eV, from top to bottom. The observed trend indicates that the current increases with a rise in the applied accelerating field.

This research is valuable for understanding electron emissions originating from both electric potential energy and thermal energy within diverse one-dimensional materials. The calculation of the one-dimensional current elucidates thermionic emission, highlighting its temperature dependency, as well as field emission, emphasizing its sensitivity to strong electric fields. Additionally, it elucidates generalized electron emission, encompassing both effects and accommodating arbitrary variations in the temperature and electric field.

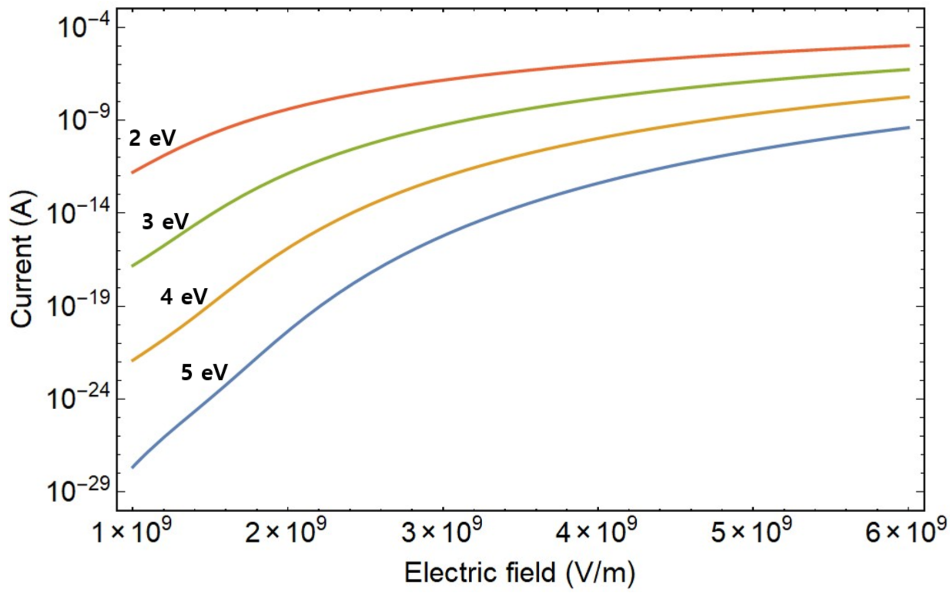

Figure 6. One-dimensional current plotted against the applied electric field for materials featuring various work functions, all at a constant temperature of 1000 K. In the sequence from top to bottom, the depicted work functions correspond to 2, 3, 4, and 5 eV.

7. Conclusions

We formulated a comprehensive electron emission theory that covers arbitrary thermal energy and electric potential energy across various one-dimensional materials. The analysis included the spectral radiance for one-dimensional thermal radiation, illustrated by the dimensionless photon energy across various temperatures in such materials. Additionally, calculations for the thermionic and field emission current densities were conducted for one-dimensional conductors. The generalized current densities, applicable at any temperature and under an applied electric field, were derived and computed for one-dimensional materials. The one-dimensional current for thermionic emission was computed as a function of the temperature across different materials with work functions of 2, 3, 4, and 5 eV. Similarly, the one-dimensional current for field emission was calculated as a function of the electric field for materials with the same range of work functions. The one-dimensional current of a carbon nanotube was displayed as a function of the electric field for temperatures spanning from 500 K to 2500 K. Furthermore, the one-dimensional current at a constant temperature of 1000 K was depicted as a function of the electric field for materials with work functions of 2, 3, 4, and 5 eV.

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Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

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Conflicts of Interest: The authors declare no conflicts of interest.

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