applied sciences

MDPI

Article

Evaluating Performance of the DGM(2,1) Model and Its Modified Models

Ying-Fang Huang, Chia-Nan Wang, Hoang-Sa Dang * and Shun-Te Lai

Department of Industrial Engineering and Management, National Kaohsiung University of Applied Sciences, No. 415, Chien Kung Road, Sanmin District, Kaohsiung 80778, Taiwan; winner@kuas.edu.tw (Y.-F.H.); cn.wang@kuas.edu.tw (C.-N.W.); joseph@kuas.edu.tw (S.-T.L.)

* Correspondence: hoangsa8181@gmail.com; Tel.: +886-7381-4526 (ext. 7104); Fax: +886-7392-3375

Academic Editor: Yang Kuang

Received: 27 November 2015; Accepted: 24 February 2016; Published: 4 March 2016

Abstract: The direct grey model (DGM(2,1)) is considered for fluctuation characteristics of the sampling data in Grey system theory. However, its applications are quite uncommon in the past literature. The improvement of the precision of the DGM(2,1) is only presented in few previous researches. Moreover, the evaluation of forecasted performance of the DGM(2,1) model and its applications was not conducted in previous studies. As the results, this study aims to evaluate forecasted performance of the DGM(2,1) and its three modified models, including the Markov direct grey model MDGM(2,1), the Fourier direct grey model FDGM(2,1), and the Fourier Markov direct grey model FMDGM(2,1) in order to determine the application of the DGM(2,1) model in practical applications and academic research. The results demonstrate that the DGM(2,1) model has lower precision than its modified models, while the forecasted precision of the FDGM(2,1) is better than that of MDGM(2,1). Additionally, the FMDGM(2,1) model presents the best performance among all of the modified models of DGM(2,1), which can effectively overcome the fluctuating of the data sample and minimize the predicted error of the DGM(2,1) model. The finding indicated that the FMDGM(2,1) model does not only have advantages with regard to the sample size requirement, but can also be flexibly applied to the large fluctuation and random sequences with a high quality of estimation.

Keywords: DGM(2,1) model; Fourier series; Markov chain; MDGM(2,1); FDGM(2,1); FMDGM(2,1) model

1. Introduction

The Grey prediction is an effective tool for understanding an uncertain environment with limited information [1]. These models consists of the basic model GM(1,1), the GM(2,1), Verhulst and direct grey model DGM(2,1), which are known as the common prediction models in Grey system theory [2]. However, some defects have been found in the performance of the grey forecasting model and can still be improved [3]. Among these, the DGM(2,1) is considered when the sampling data occurs the fluctuating phenomenon [4], but the accuracy of this model is not high and is difficult to apply directly to the actual estimation [5]. As a result, there has been much attention paid in recent years to studies on improving the precision of DGM(2,1). For example, Li [5] successfully utilized Lagrange's mean value theorem to optimize the existing DGM(2,1) model. Literature in [6] also proposed high accuracy for the optimized DGM(2,1) model, which has the white exponential superposition. Moreover, along with the enhancement of the performance of DGM(2,1), a new 2-order grey derivative expression deduct based on the solution structure of white differential equation of DGM(2,1) was also introduced by [7]. In the above discussions, the improved precision of DGM(2,1) mainly focused on modifying the whitening differential equation or optimizing the parameter of the grey model.

On the other hand, in order to increase the accuracy of the forecasting model, some combinations of forecasting approaches have been considered under special circumstances. Several integrated effective models have been introduced in recent research, including the novel dynamic grey model which integrates GM(1,1); the exponential smoothing technique and the Fourier series [8]; the modified grey Markov model [9]; and the grey fuzzy predictive model [10] or Genetic algorithms-based forecasting model [11], *etc*. Empirical testing or simulation has proven that these combined models dramatically improve the precision of GM(1,1). In addition, the Fourier Markov Grey forecasting model, or FMGM(1,1), which has been integrated into the GM(1,1), the Fourier series, and the Markov chain, is a combined grey model offering highly accurate predictions. This model has successfully forecast the typhoon rainfall system, the steel price system [3] and the turning time of the stock market [12]. The findings from these studies confirmed that the FMGM(1,1) model accurately predicted in unstable environments, with randomly fluctuating data, and limited data samples.

Although the combined models based on the GM(1,1) have been proposed in many previous works, the literature on improving the precision of DGM(2,1) through integration with other approaches is still limited in current studies. There are few combined DGM(2,1) forecasting models that have been introduced in recent studies. For instance, the study of [13] combined the Taylor approximation method and the DGM(2,1) model to predict the number of foreign students in Taiwan, and the results proved that the combined model has a higher precision than the original DGM(2,1). In the same manner as the FMGM(1,1) model, a combination of the DGM(2,1), the Fourier series and the Markov chain has also been developed, and has been successfully applied to forecast trends in Taiwan's electronic paper industry [14]; the results indicated that the FMDGM(2,1) model gave a better predicted precision than the FMGM(1,1) and the FM-Verhulst models. However, the evaluation of the forecast performance of the single DGM(2,1) model and its modified models has not been conducted in previous literature. Furthermore, its applications are quite uncommon in the past research. For this reason, this work aims to evaluate the forecast performance of the DGM(2,1) model and its modified models in order to determine the range of application of the DGM(2,1) models in practical applications and academic research.

Some of the modified models of the direct grey model DGM(2,1) in this work are the Markov direct grey model, or MDGM(2,1), the Fourier direct grey model, or FDGM(2,1), and the Fourier Markov direct grey model, or FMDGM(2,1). The effectiveness of the proposed models will be tested with the data of three case studies including the revenue of Thailand's medical tourism industry, the foreign direct investment (FDI) in Myanmar, and the total residential electricity consumption in Chong Qing. The precision of the proposed models will be evaluated through some statistical measurements. Then, the predicted accuracy of the modified models will be compared with that of the original DGM(2,1) model to assess the forecast performance.

Following this introduction, the rest of this paper presents the mathematical functions of grey forecasting model DGM(2,1) and its modified models. The empirical results and conclusions are addressed in Sections 3 and 4.

2. Methodology

2.1. Direct Grey Forecasting Model

The DGM(2,1) model is a single sequence, second-order linear dynamic model and filled by differential equations. The DGM(2,1) model is constructed by using the relationship between the original sequence and its inverse accumulating sequence. The formula of DGM(2,1) is presented as follows [1,4,15].

Step 1: Suppose an original series with n entries is as follows:

$$x^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(k), \ldots x^{(0)}(n) \right\} \quad (1)$$

From the original series $x^{(0)}$, a new series $x^{(1)}$ can be generated using a one-time accumulated generating operation (1-AGO), which is

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right\} \tag{2}$$

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)\ k = 1, 2, \ldots, n$.

The new sequence $x^{(1)}$ is used to construct the whitenization equation of the DGM(2,1) model by setting up a second-order differential equation directly.

$$\frac{d^2x^{(1)}}{dt^2} + a\frac{dx^{(1)}}{dt} = b \tag{3}$$

where a is the developing coefficient and b is the grey input coefficient. The coefficients a, b can be calculated using the least-squares method as shown below:

$$\hat{H} = [a, b]^T = (B^T B)^{-1} B^T Y \tag{3.1}$$

where

$$B = \begin{bmatrix} -x^{(0)}(2) & 1 \\ -x^{(0)}(3) & 1 \\ \cdots & \cdots \\ -x^{(0)}(n) & 1 \end{bmatrix} \tag{3.2}$$

$$Y = \begin{bmatrix} x^{(0)}(2) - x^{(0)}(1) & 1 \\ x^{(0)}(3) - x^{(0)}(2) & 1 \\ \cdots & \cdots \\ x^{(0)}(n) - x^{(0)}(n-1) & 1 \end{bmatrix} \tag{3.3}$$

According to Equation (3), the time response function can be given using

$$\hat{x}^{(1)}(k+1) = \left(\frac{\hat{b}}{\hat{a}^2} - \frac{x^{(0)}(1)}{\hat{a}}\right)e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}}(k+1) + (x^{(0)}(1) - \frac{\hat{b}}{\hat{a}})\frac{1+\hat{a}}{\hat{a}} \tag{4}$$

where

$$\begin{cases} \hat{x}^{(0)}(1) = \hat{x}^{(1)}(1) \\ \hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (\frac{b}{a^2} - \frac{\hat{x}^{(0)}(1)}{a})(1 - e^{a})e^{-ak} + \frac{b}{a} \end{cases} \tag{4.1}$$

From Equation (4), the predicted sequences of the DGM(2,1) can be obtained through the following:

$$\hat{x}^{(0)} = \left\{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \hat{x}^{(0)}(3), \ldots, \hat{x}^{(0)}(n)\right\} \tag{5}$$

2.2. Markov Direct Grey Model (the MDGM(2,1) Model)

The Markov chain is employed to find when random fluctuations occur in the sample data. This algorithm uses transition matrices to divide the state and to calculate the probability, so the future evolution of the conditional probability depends on the current state of the system, but not on its history. For this reason, the Markov chain plays a remedial role in overcoming the limitation of the grey model and has become an interesting component in improvements to the capability of the grey forecasting model in recent works [9,14,16–18]. This literature indicates that the Markov chain has significantly increased the accuracy of the grey forecasting model.

In this work, we first adopt predicted values of the DGM(2,1) model to conduct the Markov state transition. Then, transition matrices states are determined for each time step depending on the distribution of DGM(2,1) predicted series, and each state is an interval whose width is a fixed portion between the maximum and the minimum of the whole residual errors. The state partition is an important step in the Markov chain, however, there is no standard rule to divide these state intervals and, in general, the state partition is identified on the basis of the historical data and the research subject [19]. With those states, the state transition between two adjacent time steps can be defined. The transition probability from state i to state j after m step can be estimated using:

$$P_{ij}^{(m)} = \frac{M_{ij}(m)}{Mi}\ (i, j = 1, 2 \ldots m) \tag{6}$$

where $M_{ij}(m)$ is denoted by the number from state i to state m and M_i is the number of state i. Thus, the m step of transition matrix is given as:

$$R^{(m)} = \begin{bmatrix} p_{11}^{(m)} & p_{12}^{(m)} & \ldots & p_{1j}^{(m)} \\ p_{21}^{(m)} & p_{21}^{(m)} & \ldots & p_{21}^{(m)} \\ & & \ldots & \\ p_{i1}^{(m)} & p_{i1}^{(m)} & \ldots & p_{ij}^{(m)} \end{bmatrix} \tag{7}$$

Utilizing the $R^{(m)}$ probability matrix to calculate the future transition state, the state probability for the next step is defined by probability in π vectors, denoted as $\{e_i(t), i = 1, 2, \ldots \pi\}$ at time step t. Set the center of the π-state as $\{w_i, (i = 1, 2, \pi)\}$. Then, the forecast values for the Markov modified models are obtained as:

$$\tilde{x}^{(0)}(t+1) = \tilde{x}^{(0)}(t+1) + \sum_{i}^{\pi} e_i(t) w_i \tag{8}$$

where w_i is the corresponding weigh for the state i.

2.3. Fourier Direct Grey Model (the FDGM(2,1) Model)

The Fourier correction approach is one of the residual correction approaches, and it is used to transform the residual into frequency spectra and then select the low-frequency term [20]. In grey prediction, the Fourier series is commonly used to increase the precision of grey models when the data is fluctuating significantly [12].

Step 1: Where x is the actual series of n entries and $\hat{x}$ is the predicted series obtained from the DGM(2,1) model, the residual series named e is defined as follows:

$$e = \{e(2), e(3),, \ldots, e(k), \ldots, e(n)\} \tag{9}$$

where $e(k) = x(k) - \hat{x}(k) \quad with \quad (k = 2, 3, \ldots n)$.

Step 2: Expressing the $e^{(0)}(k)$ into the Fourier series, the error residual $e^{(0)}(k)$ is rewritten as follows:

$$e^{(0)}(k) = \frac{1}{2}a_0 + \sum_{i=1}^{M}\left[a_i \cos\left(\frac{2\pi i}{n-1}k\right) + b_i \sin\left(\frac{2\pi i}{n-1}k\right)\right] \tag{10}$$

where $M = [(n-1)/2 - 1]$ is called the minimum deployment frequency of the Fourier series and there is only a select integer number [21].

Transfer the Fourier series as:

$$e = AB \tag{11}$$

where

$$A = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \cdots & \cos\left(\frac{2\pi\times R}{n-1}\times 2\right) & \sin\left(\frac{2\pi\times R}{n-1}\times 2\right) \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \cdots & \cos\left(\frac{2\pi\times R}{n-1}\times 3\right) & \sin\left(\frac{2\pi\times R}{n-1}\times 3\right) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times n\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times n\right) & \cdots & \cos\left(\frac{2\pi\times R}{n-1}\times n\right) & \sin\left(\frac{2\pi\times R}{n-1}\times n\right) \end{bmatrix} \quad (12)$$

The parameters of B are obtained by using the ordinary least squares method (OLS), which results in the following equation:

$$B = \left(A^T A\right)^{-1} A^T [e]^T \quad (13)$$

From the predicted series $\hat{x}$ and $\hat{e}$, the Fourier modified series $\hat{x}_f$ is determined by:

$$\hat{x}_f = \left\{\hat{x}_f(1), \hat{x}_f(2), \hat{x}_f(3), \ldots, \hat{x}_f(n)\right\} \quad (14)$$

where $\begin{cases} \hat{x}_f(1) = x(1) \\ \hat{x}_f(k) = \hat{x}(k) + \hat{e}(k) \quad (k = 2, 3, \ldots n) \end{cases}$.

2.4. Fourier Markov Direct Grey Model (the FMDGM(2,1) Model)

In order to develop the FMDGM(2,1), the residual of the DGM(2,1) predicted series is adopted. The mathematical function of the proposed model is introduced as follows [3,12,14].

Step 1: Construct the FDGM(2,1) model.

From the original series $x^{(0)}$ in Equation (1), the predicted series $\hat{x}^{(0)}$ of the DGM(2,1) model is obtained in Equation (5), and then the residual series e is defined the Equation (11).

From Equations (10)–(13), the parameters of the Fourier series are calculated, and the predicted series $\hat{x}_f$ of the FDGM(2,1), as presented in Equation (13).

Step 2: Build up the Markov state transition matrix.

Based on the distribution of the predicted series FDGM(2,1) $\hat{x}_f$, and according to the Markov chain theory, states of stochastic process can be divided. Where $\hat{Y}_{fm}(k)$ is the Markov chain, the values of $\hat{Y}_{fm}(k)$ are distributed in the region of $\hat{x}_f$. This uses the same process of constructing the MDGM(2,1) model as in Section 2.2. The transition probability and each state of $\hat{Y}_{fm}(k)$ can be calculated using the formula of the Equations (6) and (7).

Step 3: Obtain predicted values of the FMDGM(2,1) model.

After determining the transition state and its probability by the Equations (6) and (7), we can utilize the $R(m)$ probability matrix to calculate the future transition state of a system. The state probability for the next step is defined by probability in π vectors, denoted as $\{r_i(t), i = 1, 2, \ldots \pi\}$ at time step t. Set the center of the π-state as $\{w_i, (i = 1, 2, r)\}$. Then, the forecast values for the modified FMDGM(2,1) model are obtained as:

$$\hat{x}_{fm}^{(0)}(t+1) = \hat{x}_f^{(0)}(t+1) + \sum_{i}^{\pi} r_i(t) w_i \quad (15)$$

where r_i is the corresponding weight for the state i.

2.5. Error Checking

In this work, some statistical measurements, including relative percentage error (RPE), root mean squared error ($RMSE$), mean of average percentage error ($MAPE$) and precision rate (p), were applied to measure the performance of the proposed models. The formula of these measurements is introduced as follows:

RPE reflects the difference between the actual value and the predicted value. It measures the size of error as a percentage of the actual value. The formula of *RPE* is expressed as follows [22]:

$$RPE = \frac{\left|x^{(0)}(k) - \hat{x}^{(0)}(k)\right|}{x^{(0)}(k)} \times 100\% \tag{16}$$

RMSE is part of a standard for evaluating forecasting accuracy that presents the sample standard deviation of the differences between actual values and predicted values. *RMSE* is calculated using the following equation:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(x^{(0)}(k) - \hat{x}^{(0)}(k)\right)^2}{n}} \tag{17}$$

MAPE is an accuracy measurement which is popularly applied in forecasting. *MAPE* denotes the average relative size of the predicted error [1]. *MAPE* is defined as follows:

$$MAPE = \left(\frac{1}{n}\sum_{k=1}^{n}\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{\hat{x}^{(0)}(k)}\right) \times 100\% \tag{18}$$

When *MAPE* is close to 0, the forecasting model is highly accurate and has provided good performance, and *vice versa*.

Besides this, in accordance with the value of *MAPE*, the precision rate of forecasting model can be classified into four levels: excellent, good, qualified and unqualified [23].

Precision rate (p), which measures the level of the closeness of the statement of forecast quantity and the actual value. p, is defined as follows:

$$p = 1 - MAPE \tag{19}$$

On the other hand, the developing coefficient a of the grey model is also used as a criterion to judge the forecasting capability of the grey model. The evaluative standard is described in Table 1 [24].

Table 1. Forecasting capability of the grey model.

Range of Developing Coefficient *a*		Forecasting Capability
1	$-a \leqslant 0.3$	The model can be used for medium and long-term forecasting
2	$0.3 < -a \leqslant 0.5$	The model is suitable for short-term forecasting
3	$0.5 < -a \leqslant 0.8$	The model is carefully employed in short-term forecasting
4	$0.8 < -a \leqslant 1$	The model should be modified with residual
5	$-a > 1$	The model is not suitable for forecasting

3. Experimental Results and Discussion

In this work, three case studies were utilized to validate the prediction accuracy and forecasting capability of the DGM(2,1) model and its modified models. First, data from each sample was used to establish the DGM(2,1) models as Equations (1)–(5). After this, the Fourier correcting approach and the Markov chain were employed to construct the modified models, including the MDGM(2,1), the FDGM(2,1) and the FMDGM(2,1), based on the predicted series of the DGM(2,1) model, as Equations (6)–(15). Then, the predicted series of the proposed models were used to determine the actual residual. Some evaluation standards as shown in Equations (16)–(19) were considered to test the accuracy of the proposed models. In addition, the developing coefficient, a, of the DGM(2,1) model was also used to assess the forecasting capability of the proposed models.

The empirical analysis of three samples is presented as follows.

3.1. Practical Sample 1: Forecasting the Revenue of Thailand's Medical Tourism Industry

The rapid development of the internet, low-cost outboard travelling, the globalization of health care services and the increase in private health care demand are the result of the booming of medical tourism industry around the world, especially in Asia. Over the last decades, promotional campaigns aimed at developing medical tourism as a major industry has been launched in Southeast Asian countries. With the encouragement of government and the efforts of policy-makers, and based on the competitive advantages of low-cost treatments, advanced technologies in medical specialties, flexible treatment procedures, less-stringent government regulations and effective marketing strategies, some countries such as Thailand, Singapore, Malaysia, India, Korea and Taiwan have successfully gained a remarkable position in global outbound medical services. Among these, Thailand, Singapore and India play a driving role in Asia with approximately 90% of Asia's medical tourism markets, followed by Malaysia, Korea and Taiwan [25]. Compared with other providers in Asia, Thailand's medical tourism industry has the highest annual growth rate with more than a million medical tourists per year. Recently, this nation has become a leading destination in the field of cosmetic surgery and sex change operations; therefore, understanding the trends in Thailand's medical tourism industry is important in evaluating the development of this industry in Asia. For this reason, in this work, forecasting the revenue in Thailand's medical tourism industry is used as a sample to test the effectiveness of the DGM(2,1) model and its modified models. To conduct this task, the annual data for the revenue in the period from 2005 to 2013 is used to construct the forecasting model. The empirical results are shown in Table 2 and Figure 1.

As shown in Table 2, the *RMSE* value of the DGM(2,1) model is 835.22, while that of the modified models of the MDGM(2,1), FDGM(2,1) and FMDGM(2,1) are 555.27, 169.92 and 196.09, respectively. Compared with the DGM(2,1) model, the *RMSE* values obtained from MDGM(2,1), FDGM(2,1) and FMDGM(2,1) are smaller. Furthermore, the *MAPE* values of the DGM(2,1), MDGM(2,1), FDGM(2,1) and FMDGM(2,1) models are 23.8%, 21.53%, 12.48% and 10.41% respectively. The value of *MAPE* also reveals that the value of *MAPE* of the DGM(2,1) model is largest in all of the considered models.

From the obtained results using *RMSE* and *MAPE*, the precision rate of each model is calculated. According to these results, the precision of the DGM(2,1), MDGM(2,1) and FDGM(2,1) has not yet significantly improved to a reliable forecasting level with the data for Thailand's medical tourism. However, the forecasting accuracy of the FMDGM(2,1) reached nearly 90%. According to the notation of [23], this value revealed that the FMDGM(2,1) model obtained a good forecast performance compared with other proposed models.

Table 2. Forecasting results for the revenues of Thailand's medical tourism industry.

Year	Actual Values *	Relative Percentage Error (%)			
		DGM(2,1)	**MDGM(2,1)**	**FDGM(2,1)**	**FMDGM(2,1)**
2005	754	0	0	0	0
2006	867	7.63	9.55	19.60	0.04
2007	978	6.48	8.74	17.38	34.72
2008	1014	5.08	19.76	16.76	0.03
2009	938	34.93	33.33	18.12	36.20
2010	1200	27.58	31.66	14.16	0.02
2011	3200	41.15	25.99	5.31	3.54
2012	3700	36.48	23.37	4.59	6.12
2013	4310	31.10	19.84	3.94	2.63
RMSE		835.22	555.27	169.92	196.09
MAPE (%)		23.80	21.53	12.48	10.41
Precision rate (%)		76.20	78.47	87.52	89.59
Forecasting grade		Unqualified	Unqualified	Qualified	Good

* Data from Ministry of Public Health, Thailand, KasiKon Research Center (USD million).

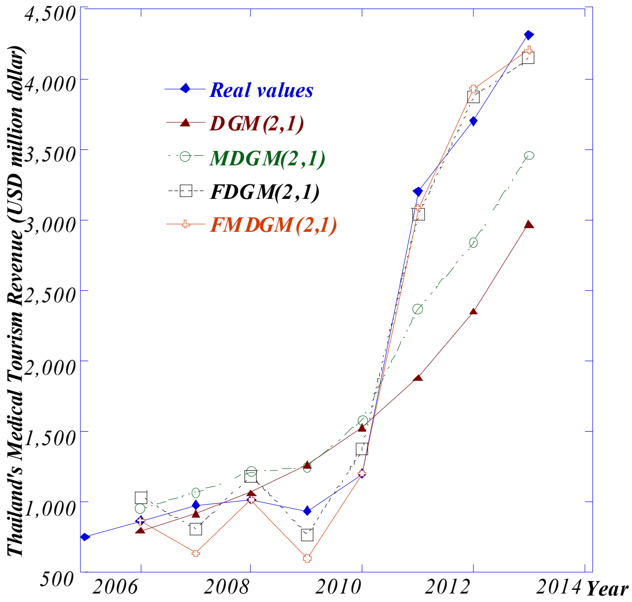

Figure 1. Comparison of the forecasting capability of different DGM(2,1) models for Thailand's medical tourism revenue.

3.2. Practical Sample 2: Forecasting Inbound FDI in Myanmar

After implementing an open-door policy, Myanmar has become an attractive destination for foreign direct investment (FDI) in recent years. The trend of the development of Myanma's FDI is interesting in academic research and business analysis. However, due to the unstable political situation in this nation, foreign investment is a high-risk business. Currently, Myanmar's statistical data is not fully significant and has a fluctuating characteristic. For these reasons, forecasting the trends in Myanmar's FDI is difficult using traditional forecasting techniques. Thus, this study aims to challenge the predictive capability of the DGM(2,1) and its modified models, and the available data for Myanmar's inbound FDI between 2007 and 2013 has been used to conduct performance testing of the proposed models. Empirical results are shown in Table 3 and Figure 2.

Table 3. Forecasting results for Myanmar's inbound FDI.

Year	Actual Values *	Relative Percentage Error (%)			
		DGM(2,1)	MDGM(2,1)	FDGM(2,1)	FMDGM(2,1)
2007	710	0	0	0	0
2008	863	8.92	1.62	12.33	10.53
2009	973	1.60	35.98	10.93	12.53
2010	1285	9.56	18.89	8.28	7.07
2011	2200	36.06	19.43	4.84	5.54
2012	2246	24.27	8.08	4.74	4.05
2013	2621	21.89	7.93	4.06	4.65
RMSE		461.8	271.08	105.9	107.9
MAPE (%)		17.05	15.32	7.53	7.4
Precision rate (%)		82.95	84.68	92.47	92.6
Forecasting grade		Unqualified	Unqualified	Good	Good

* Inbound foreign direct investment in Myanmar (USD million). Data taken from the website of the United Nations Conference on Trade and Development (UNCTAD).

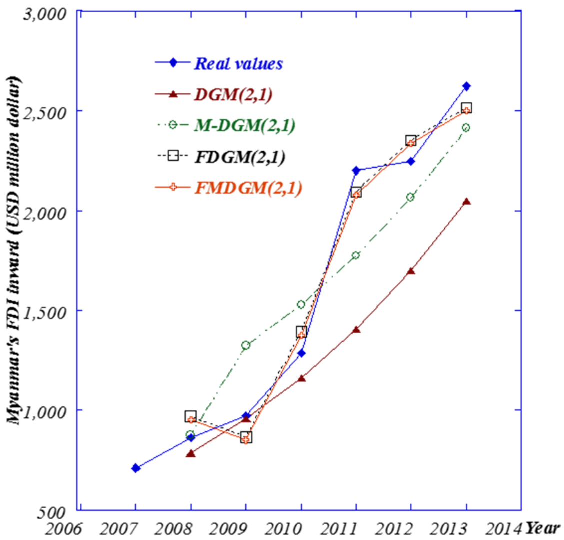

Figure 2. Comparison of the forecasting capability of different DGM(2,1) models for Myanmar's inbound FDI.

From the chart in Figure 2, it seems that the data for Myanmar's inbound FDI occurs with rapid fluctuation and the predicted curve of the DGM(2,1) does not fit with the line of actual values. The *RMSE* and *MAPE* values are described in Table 3. According to this, the *RMSE* values of the DGM(2,1), MDGM(2,1), FDGM(2,1), and FMDGM(2,1) models are 461.8, 271.08, 105.9 and 107.1, respectively. This means that the difference between predicted values and observed values of the DGM(2,1) is larger than that of MDGM(2,1), FDGM(2,1) and FMDGM(2,1). Additionally, the *MAPE* index of DGM(2,1) is 17.05% while that of MDGM(2,1), FDGM(2,1) and FMDGM(2,1) are 15.32%, 7.53% and 7.40%, respectively. It is claimed that the *MAPE* index of DGM(2,1) is the highest in four models. For this reason, it can be concluded that the DGM(2,1) has lower accuracy than the rest of the three modified models.

As shown in Table 3, the precision rate of the DGM(2,1) and MDGM(2,1) models does not reach the qualified level, which is 82.95% and 84.68%, respectively. However, the FDGM(2,1) and FMDGM(2,1) obtained a good level of forecasting, which were 92.47% and 92.6%, respectively.

3.3. Practical Sample 3: Forecasting the Residential Electricity Consumption in Chong Qing

In this practical sample, the total residential electricity consumption in Chong Qing is employed to construct proposed models. The evaluation of model performance is processed in the same manner as two previous samples. The sample data from 2000 to 2009 is adopted from [26].

Table 4 and Figure 3 presented the obtained results from practical sample 3. The value of *RMSE* and *MAPE* clearly indicated that the DGM(2,1) model turned out 11.18% for *RMSE* and 11.44% for *MAPE* . It has larger *RMSE* and *MAPE* values than the MDGM(2,1) (8.31% and 9.84%), the FDGM(2,1) (3.45% and 4.27%), and the FMDGM(2,1) (2.36% and 3.25%). The precision rate of DGM(2,1) (88.56%) is low compared with the MDGM(2,1) (90.16%), FDGM(2,1) (95.72%) and the FMDGM(2,1) (96.75%) models. Consequently, the accuracy of the DGM(2,1) model is the lowest. Furthermore, with the obtained results from the data of the total residential electricity consumption in Chong Qing, we can conclude that, in all of the modified of DGM(2,1) models, the FMDGM(2,1) model has a strong forecasting performance.

Table 4. Forecasting results for the residential electricity consumption in Chong Qing.

Year	Actual Values *	Relative Percentage Error (%)			
		DGM(2,1)	MDGM(2,1)	FDGM(2,1)	FMDGM(2,1)
2000	47.91	0	0	0	0
2001	51.02	4.96	5.13812	11.17	3.57
2002	53.12	6.19	3.50478	5.78	1.94
2003	48.44	6.31	16.9566	0.13	4.06
2004	53.56	0.04	9.67076	0.72	6.50
2005	60.9	7.73	0.72557	3.71	2.65
2006	76.47	22.27	15.5378	6.23	3.57
2007	74.99	15.34	8.47032	5.21	0.04
2008	84.99	19.36	13.2951	0.42	4.13
2009	94.41	20.73	15.2702	4.99	2.83
RMSE		11.18	8.31	3.45	2.36
MAPE (%)		11.44	9.84	4.27	3.25
Precision rate (%)		88.56	90.16	95.72	96.75
Forecasting grade		Qualified	Qualified	Good	Good

* Total residential electricity consumption (10^8 kWh). Data adopted from Hossain *et al.* [26].

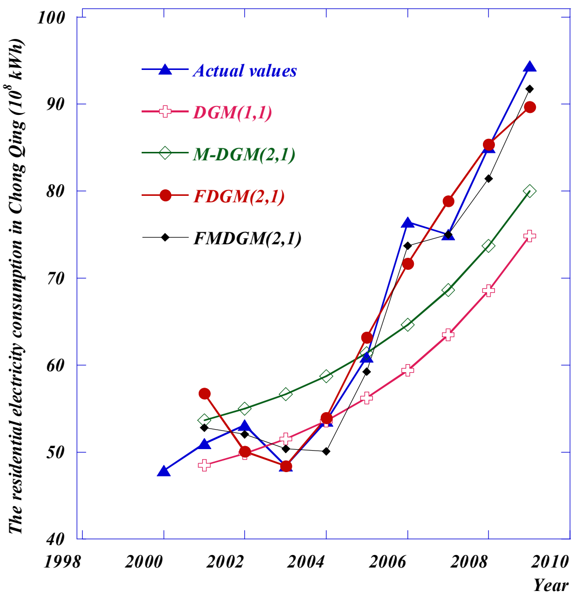

Figure 3. Comparison of the forecasting capability of different DGM(2,1) models for residential electricity consumption.

As noted in the introduction in Section 2.3, besides some statistical measurements, the developing coefficient, a, of grey models is also used as an evaluative standard to judge the forecasting capability of the proposed models. The coefficient, a, of the DGM(2,1) model for the three practical samples has been calculated and presented in Table 5. The a value of the three practical samples is 0.18369, -0.1775 and -0.221, respectively. According to the standard in Table 1, the a values of the three practice samples are placed in the first range of Table 1. This means that the DGM(2,1) model can be considered for middle and long-term forecasting [24].

Table 5. Coefficient *a* and *b* of DGM(2,1) in three practical samples.

Coefficients	Sample 1	Sample 2	Sample 3
a	−0.2825	−0.1775	−0.221
b	−127.851	17.041	−9.525

To sum up the above analyses, based on the obtained results from the data of three practical samples, we can recognize that the accuracy of the original DGM(2,1) model is significantly low compared to the MDGM(2,1), FDGM(2,1) and FMDGM(2,1) models. The precision rate of the DGM(2,1) falls from 82.95% to 88.56%. With this value, the forecast performance of the DGM(2,1) has not yet reached a good forecasting level [23] and needs to be enhanced when directly applied to real circumstances. However, for the modified models, after correcting the residual series by utilizing the Fourier series and the Markov chain, the forecast errors of the DGM(2,1) were clearly reduced. The *RMSE* and *MAPE* values of the MDGM(2,1),FDGM(2,1) and FMDGM(2,1) models are obviously smaller than that of original DGM(2,1) model. Therefore, the accuracy of the modified models is also increased compared with the original DGM(2,1) model. Referring the forecasting level of [23], the performances of the FDGM(2,1) and FMDGM(2,1) are classified placed at good and excellent forecasting levels, in which the precision rate falls from 92.47% to 98.46%. Although the modified FDGM(2,1) and FMDGM(2,1) have a higher accuracy than the original DGM(2,1), some accuracy measurements showed that the performance of the FDGM(2,1) and FMDGM(2,1) seem to be equal. It is difficult to confirm what is the best model. Despite this, the role of the Fourier correction approach is to increase the predictive capability of the considered input data set, and does not change the characteristic of grey prediction model [12]. For this reason, we can conclude that the FMDGM(2,1) is the most accurate model among all of the proposed models in this study.

4. Conclusions

The DGM(2,1) model is a type of grey model and is used for fluctuating and random sequences of data samples. Although the DGM(2,1) is capable of addressing some of the constrains of the traditional grey model, it still resulted in a worse performance. This study aimed to evaluate the forecasting performance of original DGM(2,1), and its modified models consisting of the MDGM(2,1), FDGM(2,1), and FMDGM(2,1) based on empirical analysis with data from three practical samples. The precision of the modified models was also compared with the original DGM(2,1). The results indicated that a combination of the Fourier series and the Markov chain in the modified models significantly improved the forecasting precision compared with the original DGM(2,1). The DGM(2,1) model had lower precision than the modified models, while the FDGM(2,1) had significantly better forecast precision than the MDGM(2,1). Furthermore, the results also showed that the FMDGM(2,1) had the best forecasting performance among all of the modified models and can be used for middle- and long-term forecasting.

For this finding, the modified model FMDGM(2,1) not only has advantages with regard to the sample size requirement, but can also be flexibly applied to large fluctuations and random sequences with a high quality of estimation. In future studies, the accuracy of grey forecasting model is expected to improve by combining other approaches.

Author Contributions: In this paper, Ying-Fang Huang contributed to the collection of data and pointed out analysis ideas, while Chia-Nan Wang contributed to designing the theoretical verifications. Hoang-Sa Dang analyzed data and prepared the manuscript. Shun-Te Lai was involved in results discussions. All authors have both read and approved the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Deng, J.-L. Introduction to grey system theory. *J. Grey Syst.* **1989**, *1*, 1–24.
2. Wen, K.-L. *Grey Systems: Modeling and Prediction*; Yang's Scientific Research Institute: Tucson, AZ, USA, 2004.
3. Lin, Y.-H.; Lee, P.-C. Novel high-precision grey forecasting model. *Autom. Constr.* **2007**, *16*, 771–777. [CrossRef]
4. Liu, S.; Dang, Y.; Fang, Z. *Grey System Theory and Its Application*; Science: Beijing, China, 2004.
5. Li, J. The optimization DGM(2,1) model and its application. *J. Grey Syst.* **2012**, *24*, 181–186.
6. Kong, X.-H.; Wei, Y. Optimization of DGM(2,1). *J. Grey Syst.* **2009**, *12*, 9–13.
7. Shao, Y.; Su, H.-J. On approximating grey model DGM(2,1). *AASRI Procedia* **2012**, *1*, 8–13. [CrossRef]
8. Lin, Y.-H.; Chiu, C.-C.; Lin, Y.-J.; Lee, P.-C. Rainfall prediction using innovative grey model with the dynamic index. *J. Mar. Sci. Technol.* **2013**, *21*, 63–75.
9. Chen, M.-Y.; Li, Z.; Wang, W. Improved grey-markov chain algorithm for forecasting. *Kybernetes* **2009**, *38*, 329–338. [CrossRef]
10. Chen, H.-W.; Chang, N.-B. Prediction analysis of solid waste generation based on grey fuzzy dynamic modeling. *Resour. Conserv. Recycl.* **2000**, *29*, 1–18. [CrossRef]
11. Wang, C.-H.; Hsu, L.-C. Using genetic algorithms grey theory to forecast high technology industrial output. *Appl. Math. Comput.* **2008**, *195*, 256–263. [CrossRef]
12. Hsu, Y.-T.; Liu, M.-C.; Yeh, J.; Hung, H.-F. Forecasting the turning time of stock market based on Markov-Fourier grey model. *Expert Syst. Appl.* **2009**, *36*, 8597–8603. [CrossRef]
13. Nguyen, P.-H.; Sheu, T.-W.; Nguyen, P.-T.; Pham, D.-H.; Nagai, M. Taylor approximation method in grey system theory and its application to predict the number of foreign students studying in taiwan. *Int. J. Innov. Sci. Res.* **2014**, *10*, 409–420.
14. Huang, Y.-F.; Wang, C.-N.; Dang, H.-S.; Lai, S.-T. Predicting the trend of taiwan's electronic paper industry by an effective combined grey model. *Sustainability* **2015**, *7*, 10664–10683. [CrossRef]
15. Xie, N.-M.; Liu, S.-F. Discrete GM(1,1) and mechanism of grey forecasting model. *Syst. Eng. Theory Pract.* **2005**, *1*, 93–99.
16. Mao, Z.; Sun, J. Application of Grey-Markov model in forecasting fire accidents. *Procedia Eng.* **2011**, *11*, 314–318.
17. Dong, S.; Chi, K.; Zhang, Q.; Zhang, X. The application of a grey markov model to forecasting annual maximum water levels at hydrological stations. *J. Ocean Univ. China* **2012**, *11*, 13–17. [CrossRef]
18. Lee, C.; Lin, C.-T.; Chent, L.H. Accuracy analysis of the Grey Markov forecasting model. *J. Stat. Manag. Syst.* **2004**, *7*, 567–580. [CrossRef]
19. Wang, X.-P.; Meng, M. Forecasting electricity demand using Grey-Markov model. In Proceedings of the 2008 International Conference on Machine Learning and Cybernetics, Kunming, China, 12–15 July 2008; pp. 1244–1248.
20. Tan, C.; Chang, S. Residual correction method of fourier series to GM(1,1) model. In Proceedings of the First National Conference on Grey Theory and Applications, Kauhsiung, Taiwan, 2 November 1996; pp. 93–101.
21. Guo, Z.; Song, X.; Ye, J. A verhulst model on time series error corrected for port throughput forecasting. *J. East. Asia Soc. Transp. Stud.* **2005**, *6*, 881–891.
22. Hsu, L.-C. Applying the grey prediction model to the global integrated circuit industry. *Technol. Forecast. Soc. Chang.* **2003**, *70*, 563–574. [CrossRef]
23. Ma, H.; Zhang, Z. *Grey Prediction with Markov-Chain for Crude Oil Production and Consumption in China*; Springer: Berlin, Germany, 2009; pp. 551–561.
24. Liu, S.; Deng, J. The range suitable for GM(1,1). *Syst. Eng. Theory Pract.* **2000**, *21*, 121–124.
25. NaRanong, A.; NaRanong, V. The effects of medical tourism: Thailand's experience. *Bull. World Health Organ.* **2011**, *89*, 336–344. [CrossRef] [PubMed]
26. Hossain, M.; Meng, L.; Farzana, S.; Thengolose, A. Estimation and prediction of residential building energy consumption in rural areas of chongqing. *Int. J. Eng. Trans. C Asp.* **2013**, *26*, 955–962. [CrossRef]