applied sciences

MDPI

Article

Research on the Robustness of Interdependent Networks under Localized Attack

Junde Wang [1,*], Songyang Lao [1], Yirun Ruan [1], Liang Bai [1] and Lvlin Hou [2]

[1] Science and Technology on Information Systems Engineering Laboratory, National University of Defense Technology, Changsha 410073, China; laosongyang@nudt.edu.cn (S.L.); ruanyirun@163.com (Y.R.); bailiang@nudt.edu.cn (L.B.)

[2] Department of logistics command, The Logistics Academy, Beijing 100858, China; houlvlin@gmail.com

* Correspondence: wangjunde@nudt.edu.cn; Tel.: +86-731-8457-4551

Academic Editor: Christos Verikoukis

Received: 18 April 2017; Accepted: 6 June 2017; Published: 9 June 2017

Abstract: Critical infrastructures (CI) are the cornerstone of modern society, and they are connected with each other through material, energy, or information. The robustness of interdependent CI systems under attack has been a hot topic in recent years, but previous studies mainly focused on malicious attacks or random failure. To analyze the impact of some natural disasters whose destructive force is mainly related to distance with respect to interdependent CI systems, we present a new localized attack mode considering destructive force decays with distance, and carry out simulations on several interdependent networks constructed by artificial and real world networks. Furthermore, this article analyzes the influence of coupling strength and coupling pattern on the robustness of interdependent system. The results show that dependency links between networks decrease the robustness of interdependence networks, but the robustness under failure probability degradation is not vulnerable like that under malicious attack or random failure. In addition, the coupling preference has little effect on the robustness of interdependent networks under the new localized attack strategy; when the average degree of subnetworks is large, the same conclusion can be obtained for the coupling strength.

Keywords: interdependent networks; critical infrastructures; robustness; localized attack

1. Introduction

Nowadays, critical networked infrastructures, such as power grids, water supply networks, transportation networks, and the Internet, play an increasingly important role in modern society. These infrastructures are not isolated, but interact with each other through connections, such as material, energy, and information [1–3]. Those connections keep interdependent systems functional, but increase the vulnerability of systems under operational errors, aging, and even intentional disruption. Considering the significant loss caused by infrastructure failure, it is of great practical significance to study the robustness of interdependent networks, and this issue has become a hot topic in recent years. After Buldyrev et al. [4] proposed fully coupled interdependent networks in 2010, many scholars have conducted research on this issue. The common research paradigm is to construct interdependent networks under different coupling patterns based on different subnetworks, then analyze the influence of different features on the robustness of interdependent networks under different attack strategies. There are two kinds of subnetworks usually used, namely, artificial networks, such as Watts-Strogatz (WS), Erdős-Rényi (ER), Random Regular (RR), Barabási-Albert (BA), and lattice networks [5–8], and realistic infrastructure networks, such as power grids, water supply networks, the Internet [9], etc.

According to the emphasis, the existing research can be divided into two aspects: one is to study the influence of the topology of subnetworks; the other is to analyze the influence of the dependency links between subnetworks on the robustness of interdependent networks. The topology of subnetworks, such as the degree of distribution [4,8], clustering coefficient [10–12], and assortative [13], is an important factor affecting the robustness of interdependent networks, and most recent works are carried out from this perspective. Moreover, these studies are often carried out in the context of different attack strategies and coupling preference [8,14–18]. Attack strategy is the method of selecting attack nodes, such as random attack (RA), targeted attack (TA) [16,19], and localized attack (LA) [20–24]. Coupling preference refers to the pattern that the coupling links are established between subnetworks [25,26], including assortative, disassortative, and random coupling, based on node degree or betweenness. The other factor affecting the robustness of interdependent networks is the dependency link, including coupling strength, link strength, and the direction of the link. First, interdependent networks can be divided into fully and partially-interdependent according to coupling strength, which means the partition of nodes with dependency links. This coupling strength leads to a change from a first-order to a second-order percolation transition at a critical threshold [27–29]. Next, strong and weak coupling can be identified according to the strength of the dependency link, or, for example, the probability of node failure after losing its dependency link, as there is a rich phase transition of interdependent networks when the strength of the dependency link changes [30]. Finally, the direction of the dependency link distinguishes directed and undirected interdependent networks [31,32]. A directed interdependent system has poorer robustness performance than an undirected system, the main reason being that there are more possibilities for the existence of longer dependency chains in directed interdependent systems than undirected ones.

Additionally, there are many other studies on this issue. Except the failure caused by a lack of a dependency link mentioned above, the failure caused by the excessive load of nodes or edges is also considered in the research of interdependent networks [17,33]. Similar to topology, studies based on load failure are often carried out considering networks, coupling preference, and attack strategies. Very recently, some researchers have studied the properties of interdependent networks from the point of view of recovery [34–36], propagation [37,38], and fuzzy information attack [39]. Moreover, the robustness of network of networks (NON), which is an extension of two-layer interdependent networks, has been examined. In particular, Gao and Dong et al. [40–45] systematically studied the percolation process of chain-like, star-like, and tree-like NON by analytical and numerical methods.

Although there are many studies on interdependent networks, most of them are based on random failure or malicious destruction, while very few articles focus on localized attacks, which are used to simulate the effect of natural disasters, such as earthquakes, on critical infrastructures [23]. In previous studies, all nodes in an attack area (or ratio of affected nodes, expressed with p) of a localized attack will fail, but the fact is not always like that. For example, the impact of earthquakes may be very large, but infrastructure nodes in the scope will not necessarily fail, and the failure probability of nodes should decrease with the distance from epicenter. Based on this consideration, this paper studies the robustness of interdependent networks under a new localized attack strategy, providing a reference for the design of a more robust critical infrastructure system.

The rest of this paper is organized as follows: The second section introduces cascading failures of interdependent networks, and the concept of a failure probability degradation model of localized attack. In the third part of the paper, simulation and analysis is conducted on four kinds of isolated networks and corresponding interdependent networks under the new localized attack, in addition, we analyze the effect of coupling preference and strength on new attack strategies. The last part summarizes the whole text and looks forward to the future work.

2. Models

2.1. Cascading Failures of Interdependent Networks

For simplicity and without loss of generality, we apply the model proposed by Buldyrev in this paper, as shown in Figure 1. In this model, two subnetworks A and B, with the same number of nodes and average degree, are coupled by random one-to-one dependency links, which means one node will fail when its dependency counterpart fails. Assume that only nodes belonging to the giant component remain functional. We define p_A and p_B as the probability that the node belongs to the larger cluster of subnetwork A and B, respectively; $\psi'_n(\phi'_n)$ is the fraction of the remaining nodes in subnetwork $A(B)$; ψ_n and ϕ_n are the giant cluster components; and $q_A(q_B)$ is the proportion of nodes in subnetwork $A(B)$ dependent on subnetwork $B(A)$. At step 1, a fraction p of nodes in subnetwork A are attacked [27,28], then the general form of the cascading failures can be described as shown in Table 1.

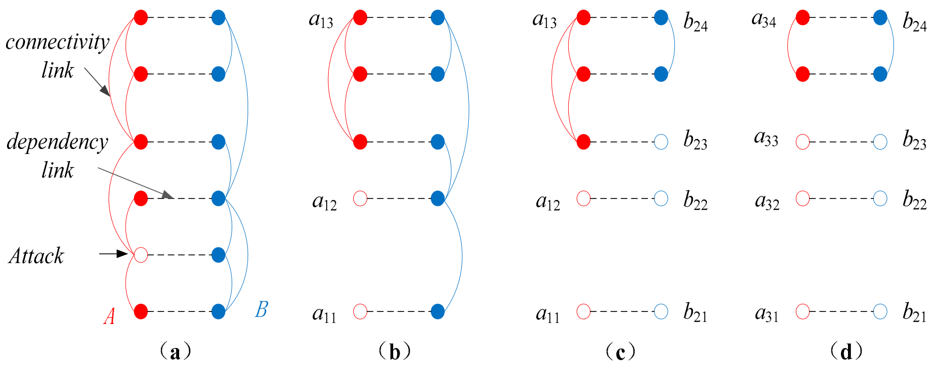


Figure 1. Cascading failures of interdependent networks [4]. Red and blue nodes represent nodes of subnetwork A and subnetwork B, respectively. Solid circles denote functional nodes, hollow circles denote failure nodes. (**a**) One node in sub network A is chosen to be attacked; (**b**) the attacked node in A and its dependency counterpart in B fail, and all of the links of the failure nodes, including connectivity and dependency links, are removed. Then A breaks into three clusters, and only nodes in cluster a_{13} remain functional according to the assumption; (**c**) because of the dependency and larger cluster, cluster b_{21}, b_{22}, and b_{23} in B are eliminated; and (**d**) failure of cluster b_{23} in turn leads to failure of some nodes in A, and cascading failures stop because no further failure occurs. Finally, only nodes in cluster a_{34} and b_{24} remain functional in the interdependent networks after only one node in subnetwork A is attacked.

Table 1. Cascading failures of interdependent networks.

Step	Subnetwork A		Subnetwork B	
	Remaining Fraction	**Giant Component**	**Remaining Fraction**	**Giant Component**
1	$\psi'_1 = 1 - p$	$\psi_1 = \psi'_1 p_A(\psi'_1)$	$\phi'_1 = 1 - q_B[1 - p_A(\psi'_1)p]$	$\phi_1 = \phi'_1 p_B(\phi'_1)$
2	$\Psi'_2 = (1-p)\{1 - q_A[1 - p_B(\phi'_1)]\}$	$\psi_2 = \psi'_2 p_A(\psi'_2)$	……	……
…	……	……	……	……
n	$\Psi'_n = (1-p)\{1 - q_A[1 - p_B(\phi'_{n-1})]\}$	$\psi_n = \psi'_n p_A(\psi'_n)$	$\phi'_n = 1 - q_B[1 - p_A(\psi'_n)p]$	$\phi_n = \phi'_n p_B(\phi'_n)$

In this paper, we use two indices to represent the robustness of interdependent networks. The first index is of relative size S of the giant components in subnetwork A and B when cascading failures end. For one-to-one correspondence and fully-coupled interdependent networks we can define S as shown in Equation (1); larger S indicates better robustness:

$$S = \frac{G_A + G_B}{N_A + N_B} = \frac{G_A}{N_A} = \frac{G_B}{N_B}, \tag{1}$$

The point where the giant component decreases to zero is typically referred to as the critical threshold p_c. It is also commonly used to characterize the robustness of interdependent networks. A larger threshold means more nodes need to be removed to make the interdependent systems collapse, and also shows that the robustness of the interdependent network is better.

2.2. Failure Probability Degradation Model of a Localized Attack

A localized attack is often used to simulate the impact of natural disasters on interdependent systems. In a localized attack scenario, nodes are chosen from a certain area, starting from a root node, then its nearest neighbors, then the next nearest neighbors, as shown as Figure 2, until the fraction of affected nodes reach a certain value, denoted by p. The traditional localized attack (TLA) assumes that all of these nodes will fail. However, the fact is not like that. It is a basic fact that destructive force usually decays with distance, and there is a distance threshold, L_0, beyond which the probability of node failure is 0. Additionally, for earthquakes, the closer to the epicenter, the slower the destructive force increases [46]. Simple and without generality, we assume the failure probability of a node at the epicenter is 1. Thus, we can define a new failure probability degradation model of a localized attack (PDLA), as shown as Figure 3, and the failure probability of one node with distance l from the attack center is as follows:

$$p(l) = \begin{cases} 1 - \frac{l^2}{(L_0+1)^2}, & 0 \le l \le L_0 \\ 0 & , \ l > L_0 \end{cases} , \ l = 0, 1, 2, \ \ldots \tag{2}$$

where L_0 represents the furthest distance from the affecting natural disaster. In this article, the length of the shortest path between nodes is regarded as distance for simplicity.

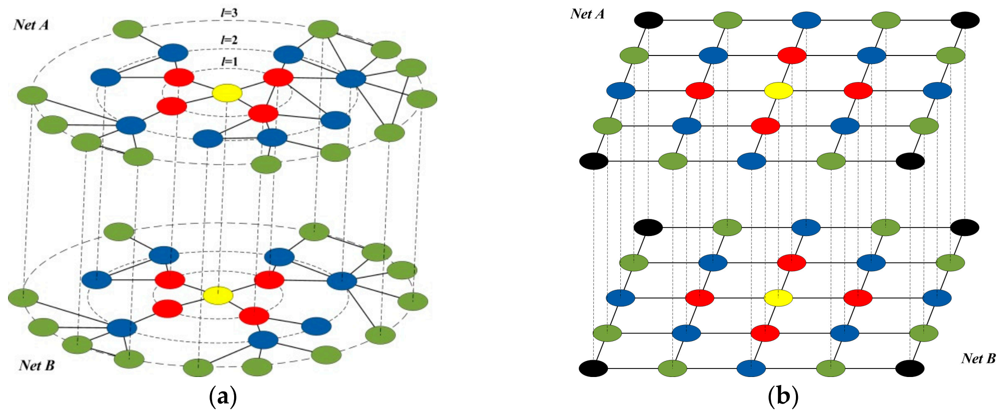


Figure 2. The yellow nodes represent attack center, the red, blue, green and black nodes denote nodes whose shortest distance to attack center is 1, 2, 3, 4, respectively. (**a**) Schematic illustration of a localized attack in partially-coupling interdependent networks; and (**b**) schematic illustration of a localized attack in fully-coupling interdependent lattice networks.

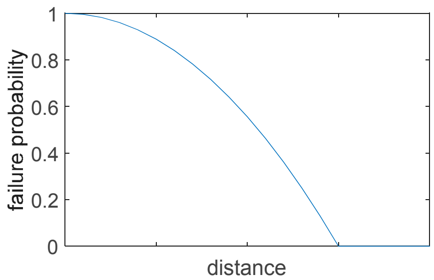


Figure 3. Illustration of the failure probability degradation model of a localized attack. The failure probability of a node within the area affected by a disaster decreases with its distance from the attack center.

3. Results and Analysis

To study the robustness of interdependent networks under PDLA, we use four kinds of network models to create interdependent networks, three of them are artificial networks, namely the WS small world network, BA scale-free network, and NN nearest-neighbor coupled network. The forth one is an infrastructure network, the western states power grid of the United States [47]. Then we construct WS-WS, BA-BA, NN-NN, and power-BA interdependent networks using the methods proposed in [4].

In order to compare the effect of PDLA, we also show the result of a targeted attack, a random attack, and a traditional localized attack.

3.1. Artificial Network

In this section, we focus on the robustness of artificial networks under different attack strategies, especially PDLA. First, we generate two WS networks with a network size of $N_{\mathrm{WS1}} = N_{\mathrm{WS2}} = 2000$, with an average degree of $<K_{\mathrm{WS1}}> = <K_{\mathrm{WS2}}> = 6$, the rewiring probability $p_{\mathrm{re}} = 0.2$; and generate BA and NN networks with $N_{\mathrm{BA1}} = N_{\mathrm{BA2}} = 2000$, $<K_{\mathrm{BA1}}> = <K_{\mathrm{BA2}}> \approx 6$; $N_{\mathrm{NN1}} = N_{\mathrm{NN2}} = 2000$, $<K_{\mathrm{NN1}}> = <K_{\mathrm{NN2}}> = 6$. Then we can create fully-coupled WS-WS, BA-BA, and NN-NN interdependent networks. When constructing interdependent networks, two WS or BA subnetworks are randomly coupled, but interdependent NN-NN networks are vulnerable under random coupling due to their special spatial distribution [7]. Therefore, the spatial location is considered when constructing interdependent NN-NN networks, as shown as Figure 2b. Finally, the robustness of interdependent networks is analyzed by numerical simulation under four attack strategies, namely TA, RA, TLA, and PDLA. Here TA means targeted attack on high degree nodes, RA denotes random attack on nodes during simulation.

Figure 4 shows the robustness of isolated WS, BA, and NN networks and interdependent WS-WS, BA-BA, NN-NN networks under four attack strategies. Critical thresholds p_c of interdependent networks are listed in Table 2. From the results of Figure 4 and Table 2, we can draw the following conclusions.

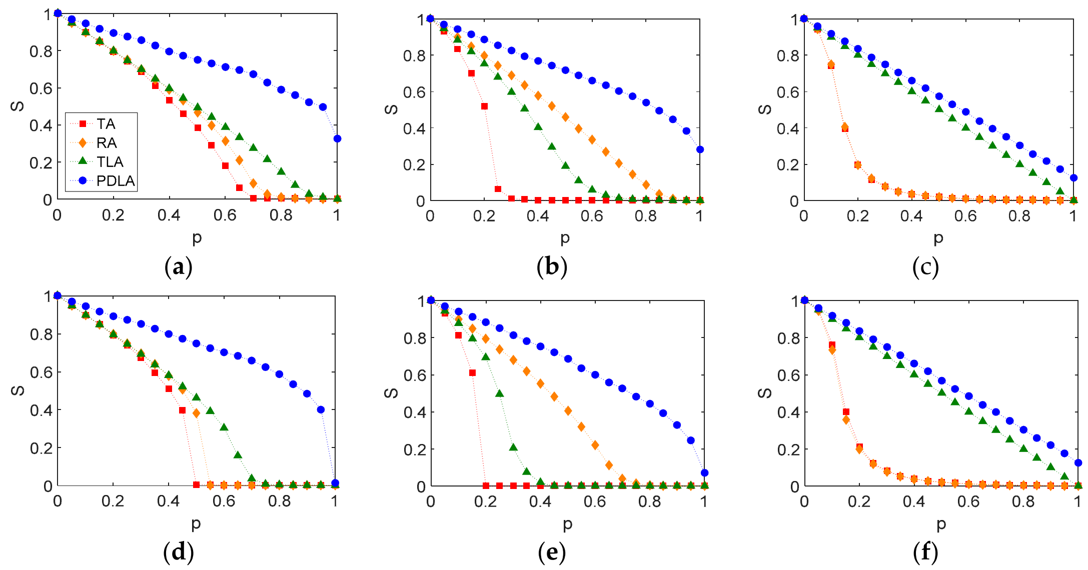


Figure 4. Cascading failures of isolated and interdependent networks. Each point is averaged over 100 independent realizations. One realization means a simulation using four attack strategies on one isolated network (or interdependent networks) under different p. (**a**) Isolated WS network; (**b**) isolated BA network; (**c**) isolated NN network; (**d**) interdependent WS-WS networks; (**e**) interdependent BA-BA networks; and (**f**) interdependent NN-NN networks.

Table 2. p_c of interdependent networks under different attack strategies.

Attack Strategy	TA	RA	TLA	PDLA
WS-WS	0.5	0.55	0.75	-
BA-BA	0.2	0.75	0.45	-
NN-NN	0.65	0.65	1	- [1]

[1] The symbol "-" indicates that there is no critical threshold where the network collapses completely.

3.1.1. The Existence of Dependency Links Makes Interdependent WS-WS and BA-BA Systems More Vulnerable, but Makes No Difference on Interdependent NN-NN Networks Coupled Based on Strict Spatial Position

Compared the results of isolated networks in Figure 4a,b with the results of interdependent networks in Figure 4d,e, we can see that, no matter under what type of attack strategies, the robustness of interdependent networks is worse than the isolated network. An example is that the robustness of interdependent WS-WS and BA-BA networks under TA shows a first-order phase transition, while the isolated network shows a second-order. In addition, the isolated WS network has a strong robustness under PDLA as shown in Figure 4a. When it comes to interdependent networks, the robustness of WS-WS is vulnerable. However, as shown in Figure 4c,f, interdependent NN-NN networks show the same failure process as isolated NN network, which is consistent with [7]. The reason for this phenomenon is that coupling strictly-considered space restrictions in interdependent NN-NN networks is equivalent to increasing the degree of each node by 1, making NN-NN essentially an NN network with average degree $<K> = 7$.

3.1.2. The Robustness of Network under PDLA Is Better Than That under TA, RA, or TLA

From Figure 4, we can see that networks, including isolated and coupled ones, are vulnerable under targeted attack, random attack, or traditional localized attack. The results under TLA and RA (Figure 4b) are consistent with [21,23], which show that the "localized attack has stronger attack power than random attack". However, these two papers do not consider the attenuation of destructive force, so the failure range under localized attack is larger than the actual situation.

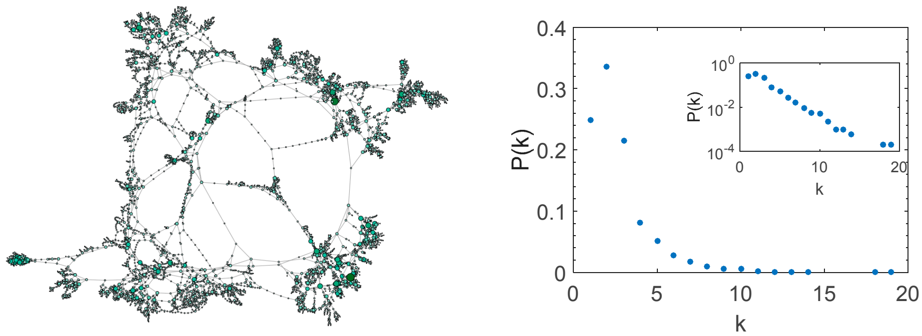
Actually, as shown in Figure 4, the robustness of a network under localized attack with failure probability degradation is relatively strong. For isolated networks, such as WS and BA networks, all nodes are affected by attack (the scale of the attacked area $p = 1$), there are still about 30% of nodes ($S_{\mathrm{WS}} \approx 0.33$, $S_{\mathrm{BA}} \approx 0.28$) that are functional after cascading failures stop, and the remaining functional nodes in an NN network are fewer, about 12.5%. Additionally, from Table 2 we can see that there is no critical threshold of interdependent networks under PDLA. In addition, it can be seen from Figure 4d,e that the failure process of interdependent networks under the targeted attack exhibits a first-order phase transition, while under PDLA, the process is second-order.

3.1.3. Degree Distribution Is an Important Factor Affecting Network Robustness

For WS networks, both isolated and coupled, the order of attack strategies according the destruction on networks is TA > RA > TLA > PDLA; but for BA networks, the order is TA > TLA > RA > PDLA; for NN networks, the order is TA = RA > TLA > PDLA. Obviously, the robustness under TA is the worst, and robustness under PDLA is the best. The differences only exist in the robustness under RA and TLA. The difference between WS and BA networks is due to the degree distribution: the degree distribution of the WS network is relatively uniform; this makes it possible for WS networks to break into more clusters under random attack, which means the interdependent WS-WS network is more vulnerable under RA than TLA. However, the degree distribution of the BA network is a power law distribution, which is to say, there are a few hub-nodes in this kind of network. In the TLA scenario, the probability of hub-nodes being attacked increases faster than that in the RA scenario with the expansion of the attack area, resulting in BA networks being more vulnerable under TLA than RA. This is consistent with the conclusion of [21] that "there is a higher probability that higher degree nodes will be within the attacked hole, which accelerates the fragmentation of the BA network". Note that both the isolated NN network and interdependent NN-NN network show the same failure process under TA and RA. This is because the degree of all nodes in the NN network is the same, so the failure possibility of them is identical under targeted attack and random attack.

3.2. Infrastructure Network

To analyze the robustness of interdependent infrastructure networks when facing disasters, such as earthquakes, the robustness of an interdependent cyber-physical network, consisting of an infrastructure network, the western states power grid network of the United States, as shown in Figure 5a, and its control network, are analyzed under PDLA in this section. The node number of the power grid network $N_{\text{power}} = 4941$, the edge number is 6594, the average degree $<K_{\text{power}}> \approx 2.67$, the clustering coefficient $C = 0.107$, and the average shortest path length $APL = 18.989$. Figure 4 shows the illustration and degree of distribution of the power grid network. Due to the lack of computer control network data, we use a BA scale-free network instead [15,48], with $N_{\text{BA}} = 4941$ and $<K_{\text{BA}}> \approx 4$, to construct power-BA interdependent networks.



(**a**) (**b**)

Figure 5. (**a**) Illustration of power grid, the node size represents degree of node, and the layout is not equivalent to the actual location of power grid nodes. (**b**) Degree of distribution of the power grid, the inset is the degree of distribution with the *y*-axis in the logarithmic scale.

Figure 6 shows the robustness of the power gird and power-BA under four attack strategies, and critical thresholds are listed in Table 3. Additionally, we can observe the following situations:

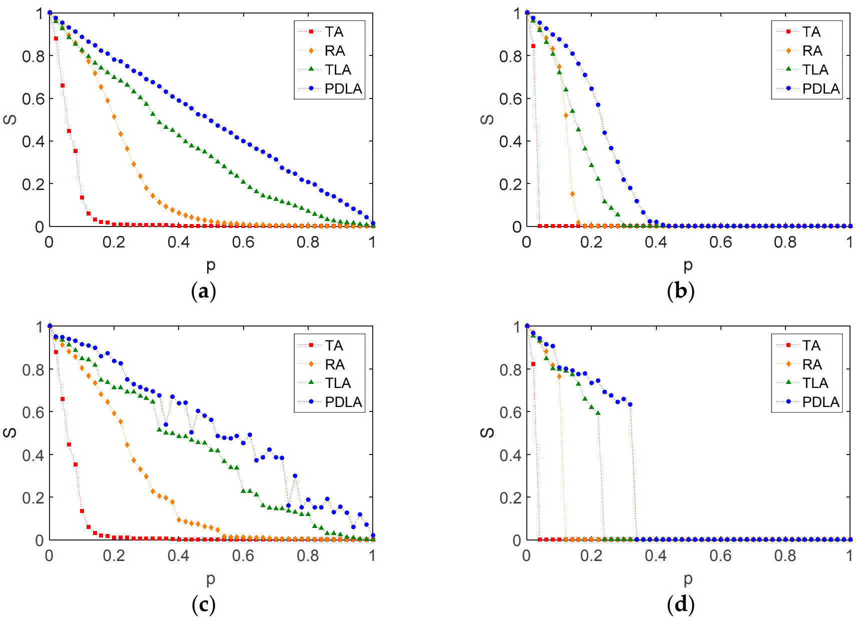


Figure 6. Cascading failures of an isolated power grid and an interdependent power-BA network under different attack strategies. (**a**) Cascading failures of the power grid averaged over 100 realizations; (**b**) cascading failures of power-BA averaged over 100 realizations; (**c**) cascading failures of the power grid under one realization; and (**d**) cascading failures of power-BA under one realization.

Table 3. Critical threshold p_c of power-BA under different attack strategies.

Attack Strategy	TA	RA	TLA	PDLA
power grid	0.2	≈0.6	≈1	1
power-BA	0.04	0.18	0.3	0.42

Firstly, because the degree of distribution and clustering coefficient of the power grid show small world characteristics, the order of attack strategies according to the damage of the power grid is the same as that of the WS network, TA > RA > TLA > PDLA, as shown in Figure 6a. The robustness of the power grid under TA is poor, very similar to the BA network, because of the existence of several hub nodes. Moreover, Figure 5a shows that the power grid has several clusters connected by many nodes arranged in lines. With the expansion of the attack range, once the connecting nodes are attacked, the power grid will break into several independent clusters and many nodes will fail, leading to a poor robustness of the power grid under RA.

Secondly, compared with the results of WS-WS and BA-BA in Figure 4c,f, the robustness of the interdependent power-BA networks in Figure 6b show the same, but more severe, decreasing trend with the increase of the attack area. In the scenario of a targeted attack, the entire interdependent networks will completely collapse with a critical threshold p_c around 0.04. Even in the PDLA scenario, the critical threshold p_c is 0.42, much less than that in WS-WS and BA-BA.

Lastly, from Figure 6c,d we can see that the percolation of the isolated power grid shows a second-order; however, the percolation of power-BA shows a first-order transition under all four attack strategies, and the critical threshold p_c becomes very small, as listed in Table 3. This shows that, like the artificial network in Section 3.1, the correlation between the control network and infrastructure network also increases the risk of interdependent systems under attack.

3.3. Effect of Coupling Preference and Coupling Strength

As mentioned in the introduction, the research on the robustness of interdependent networks usually takes into account different subnetworks, attack strategies, and coupling modes. Sections 3.1 and 3.2 have analyzed the robustness of several interdependent networks from the point of view of attack strategy. To ensure the integrity of the study, this section is carried out from the perspective of coupling, including the effect of coupling preference and coupling strength on interdependent networks under PDLA. Moreover, since many real networks show small-world or scale-free features [49], WS and BA network models are chosen to construct interdependent networks in this section, and analyze the impact of coupling preference and coupling strength on the robustness of interdependent networks under PDLA. Specifically, we use the same procedures as in Section 3.1 to analyze the robustness of fully-coupled WS-WS and BA-BA interdependent networks with assortative coupling (AC), disassortative coupling (DC), and random coupling (RC). Partially-interdependent system with random coupling, shown as Figure 2a, is also analyzed.

Figure 7 shows the cascading failures of interdependent networks with different coupling preferences. Table 4 lists the order of coupling preferences based on the robustness of the interdependent networks under four attack strategies. The results indicate that, in three kinds of attack strategies, LA, RA, and TLA, coupling preferences do have an impact on the robustness of interdependent networks. Such as in the case of targeted attack, both interdependent WS-WS and BA-BA networks show the best robustness under disassortative coupling; when facing a random attack, the robustness under assortative coupling is the best, while the robustness under disassortative coupling is the worst. Especially in BA-BA networks, the difference among different coupling preference is much more obvious, as shown in Figure 7b. However, in the PDLA scenario, the robustness of interdependent networks with different coupling preferences is very similar. This suggests coupling preference is not the main factor that affects the robustness of interdependent networks under PDLA, which is different from the other three attack strategies.

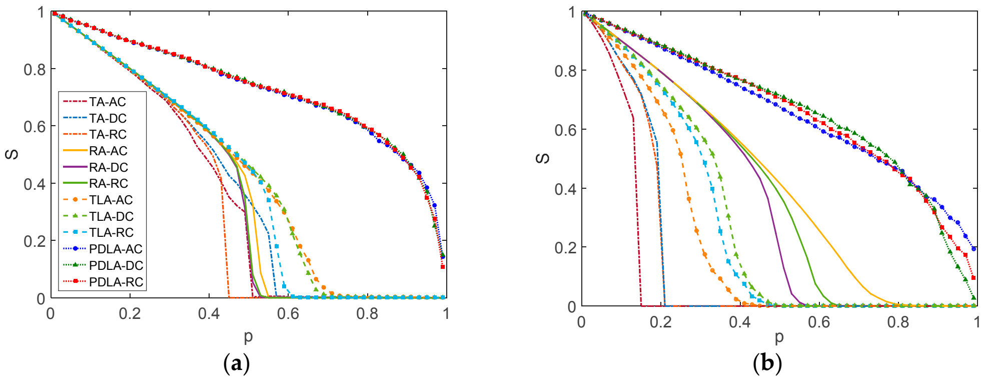


Figure 7. Cascading failures of interdependent networks with different coupling preference. The results of TA, RA, and TLA are also plotted for comparison. Every point is averaged over 100 independent realizations. (**a**) Robustness of WS-WS interdependent networks with $N_{\mathrm{WS1}} = N_{\mathrm{WS2}} = 2000$, $<K_{\mathrm{WS1}}> = <K_{\mathrm{WS2}}> = 6$, and rewiring probability $p_{\mathrm{re}} = 0.2$; and (**b**) robustness of interdependent BA-BA networks with $N_{\mathrm{BA1}} = N_{\mathrm{BA2}} = 2000$, $<K_{\mathrm{BA1}}> = <K_{\mathrm{BA2}}> \approx 6$.

Table 4. Order of coupling preference based on the robustness of interdependent networks.

Attack Strategy	TA	RA	TLA	PDLA
WS-WS	RC < AC < DC	DC ≈ RC < AC	RC < DC ≈ AC	RC ≈ DC ≈ AC
BA-BA	AC < RC < DC	DC < RC < AC	AC < RC < DC	RC ≈ DC ≈ AC

Moreover, Figure 8 shows the cascading failures of interdependent networks under different coupling strengths under PDLA. In each subgraph, we plot the failure curves with specific node sizes and average degree. Through the comparison of Figure 8a,b and Figure 8d,e, we see that node size of the subnetwork has little effect on the robustness, and there is a negative correlation between coupling strength and robustness: the stronger the coupling strength is, the worse the robustness is.

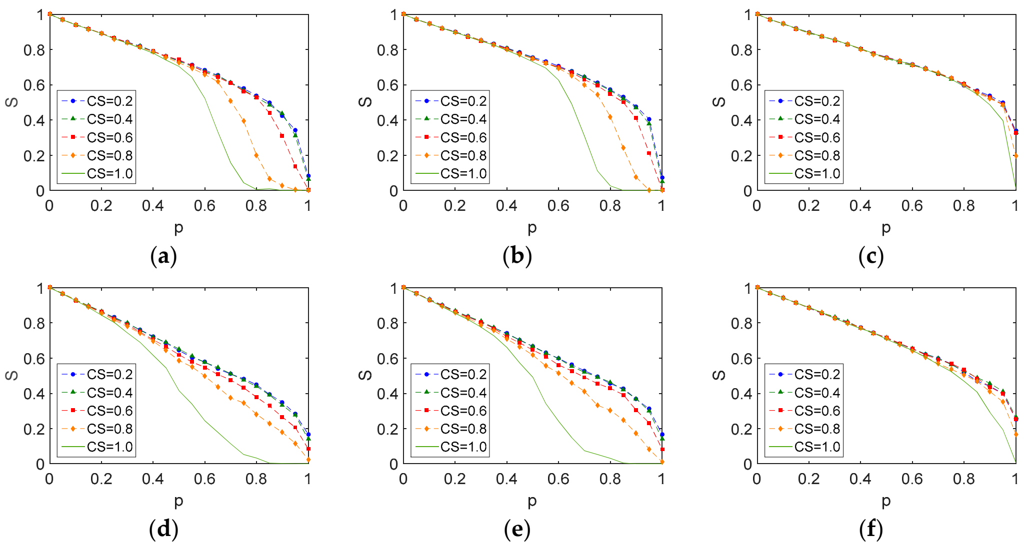


Figure 8. Cascading failure of interdependent networks with different coupling strengths under PDLA. Each point is averaged over 100 independent realizations. The relative size of giant component is defined here as $S = G_A/N_A$. (**a**) WS-WS with $N_{\mathrm{WS1}} = N_{\mathrm{WS2}} = 1000$, $<K_{\mathrm{WS1}}> = <K_{\mathrm{WS2}}> = 4$; (**b**) WS-WS with $N_{\mathrm{WS1}} = N_{\mathrm{WS2}} = 2000$, $<K_{\mathrm{WS1}}> = <K_{\mathrm{WS2}}> = 4$; (**c**) WS-WS with $N_{\mathrm{WS1}} = N_{\mathrm{WS2}} = 2000$, $<K_{\mathrm{WS1}}> = <K_{\mathrm{WS2}}> = 6$; (**d**) BA-BA with $N_{\mathrm{BA1}} = N_{\mathrm{BA2}} = 1000$, $<K_{\mathrm{BA1}}> = <K_{\mathrm{BA2}}> \approx 4$; (**e**) BA-BA with $N_{\mathrm{BA1}} = N_{\mathrm{BA2}} = 2000$, $<K_{\mathrm{BA1}}> = <K_{\mathrm{BA2}}> \approx 4$; and (**f**) BA-BA with $N_{\mathrm{BA1}} = N_{\mathrm{BA2}} = 2000$, $<K_{\mathrm{BA1}}> = <K_{\mathrm{BA2}}> \approx 6$.

In addition, by comparing Figure 8b,c and Figure 8e,f, we find that when the average degree of the subnetwork increases from four to six, the robustness of interdependent networks under different coupling strengths is very similar. It is suggested that the robustness of interdependent networks with different coupling strengths under PDLA is related to the average degree of the subnetworks. When the average degree is small, larger coupling strength makes interdependent networks more vulnerable; but when the average degree is large, the coupling strength has little effect on the robustness of interdependent networks. This is an interesting phenomenon worth further research.

4. Conclusions

To conclude, we embrace a failure possibility degradation model in the cascading failures of interdependent networks under localized attack. By comparing the robustness of several kinds of interdependent networks under this attack strategy with a targeted attack, random attack, and traditional localized attack, some conclusions are drawn which can provide references for the construction of more robust interdependent CI systems.

In the case of PDLA, the presence of a dependency link increases the risk of interdependent network collapse and leads to a poor robustness of interdependent networks than isolated network. For interdependent systems, the more nodes with a dependency link, the worse the robustness of the interdependent network. However, compared with other types of attack strategies, the impact of PDLA is relatively small, which shows that robustness of interdependent infrastructures under natural disaster is not as fragile as that under deliberate attack. Moreover, coupling preference does not significantly affect the robustness of the interdependent network, which is very different from TA, RA, and TLA. Beyond that, when the average degree of subnetworks is small, coupling strength has an effect on the robustness of interdependent networks, but when the average degree increases, this effect will gradually weaken.

Although some conclusions have been obtained, the research of this paper still has some shortcomings: First of all, when constructing the interdependent networks, we do not consider the geographical factor in order to facilitate the simulation, which can better reflect the impact of a localized attack. Secondly, there may be many forms of failure probability degradation of a localized attack, and only one possible form is analyzed in this article. Finally, during the analysis of coupling preference, this paper use strict assortative or disassortative coupling, which rarely appear in the real world, so designing a coupling method that is more practical still needs to be explored.

Acknowledgments: This work was supported by the National Natural Science Foundation of China under grant No. 61603408.

Author Contributions: Junde Wang and Songyang Lao conceived and designed the experiments; Junde Wang performed the experiments, Liang Bai and Lvlin Hou helped in the experiment; Junde Wang and Yirun Ruan analyzed the data; Junde Wang wrote the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Rinaldi, S.M.; Peerenboom, J.P.; Kelly, K. Identifying, understanding, and analyzing critical infrastructure interdependencies. *IEEE Control Syst.* **2001**, *21*, 11–25. [CrossRef]
2. Kim, Y.; Kang, W.H.; Song, J. Assessment of seismic risk and importance measures of interdependent networks using a non simulation-based method. *J. Earthq. Eng.* **2012**, *16*, 777–794. [CrossRef]
3. Dueñas-Osorio, L.; Craig, J.I.; Goodno, B.J. Seismic response of critical interdependent networks. *Earthq. Eng. Struct. Dyn.* **2007**, *36*, 285–306. [CrossRef]
4. Buldyrev, S.V.; Parshani, R.; Paul, G.; Stanley, H.E.; Havlin, S. Catastrophic cascade of failures in interdependent networks. *Nature* **2010**, *464*, 1025–1028. [CrossRef] [PubMed]
5. Martín-Hernández, J.; Wang, H.; Van Mieghem, P.; D'Agostino, G. Algebraic connectivity of interdependent networks. *Phys. A* **2014**, *404*, 92–105. [CrossRef]

6. Bashan, A.; Berezin, Y.; Buldyrev, S.V.; Havlin, S. The extreme vulnerability of interdependent spatially embedded networks. *Nat. Phys.* **2013**, *9*, 667–672. [CrossRef]
7. Li, W.; Bashan, A.; Buldyrev, S.V.; Stanley, H.E.; Havlin, S. Cascading failures in interdependent lattice networks: The critical role of the length of dependency links. *Phys. Rev. Lett.* **2012**, *108*, 228702. [CrossRef] [PubMed]
8. Buldyrev, S.V.; Shere, N.W.; Cwilich, G.A. Interdependent networks with identical degrees of mutually dependent nodes. *Phys. Rev. E* **2011**, *83*, 016112. [CrossRef] [PubMed]
9. Wang, S.; Hong, L.; Ouyang, M.; Zhang, J.; Chen, X. Vulnerability analysis of interdependent infrastructure systems under edge attack strategies. *Saf. Sci.* **2013**, *51*, 328–337. [CrossRef]
10. Li, R.Q.; Sun, S.W.; Ma, Y.L.; Wang, L.; Xia, C.Y. Effect of clustering on attack vulnerability of interdependent scale-free networks. *Chaos Solitons Fractals* **2015**, *80*, 109–116. [CrossRef]
11. Huang, X.; Shao, S.; Wang, H.; Buldyrev, S.V.; Eugene Stanley, H.; Havlin, S. The robustness of interdependent clustered networks. *EPL* **2013**, *101*, 18002. [CrossRef]
12. Shao, S.; Huang, X.; Stanley, H.E.; Havlin, S. Robustness of a partially interdependent network formed of clustered networks. *Phys. Rev. E* **2014**, *89*, 032812. [CrossRef] [PubMed]
13. Zhou, D.; Stanley, H.E.; D'Agostino, G.; Scala, A. Assortativity decreases the robustness of interdependent networks. *Phys. Rev. E* **2012**, *86*, 066103. [CrossRef] [PubMed]
14. Valdez, L.D.; Macri, P.A.; Stanley, H.E.; Braunstein, L.A. Triple point in correlated interdependent networks. *Phys. Rev. E* **2013**, *88*, 050803. [CrossRef] [PubMed]
15. Wang, J.; Jiang, C.; Qian, J. Robustness of interdependent networks with different link patterns against cascading failures. *Phys. A* **2014**, *393*, 535–541. [CrossRef]
16. Chen, Z.; Du, W.-B.; Cao, X.-B.; Zhou, X.-L. Cascading failure of interdependent networks with different coupling preference under targeted attack. *Chaos Solitons Fractals* **2015**, *80*, 7–12. [CrossRef]
17. Tan, F.; Xia, Y.; Wei, Z. Robust-yet-fragile nature of interdependent networks. *Phys. Rev. E* **2015**, *91*, 052809. [CrossRef] [PubMed]
18. Jun, T.; Piera, M.A.; Ruiz, S. A causal model to explore the acas induced collisions. *J. Aerosp. Eng.* **2014**, *228*, 1735–1748. [CrossRef]
19. Du, R.; Dong, G.; Tian, L.; Liu, R. Targeted attack on networks coupled by connectivity and dependency links. *Phys. A* **2016**, *450*, 687–699. [CrossRef]
20. Berezin, Y.; Bashan, A.; Danziger, M.M.; Li, D.; Havlin, S. Localized attacks on spatially embedded networks with dependencies. *Sci. Rep.* **2015**, *5*, 8934. [CrossRef] [PubMed]
21. Shao, S.; Huang, X.; Stanley, H.E.; Havlin, S. Percolation of localized attack on complex networks. *New J. Phys.* **2015**, *17*, 023049. [CrossRef]
22. Wu, B.; Tang, A.; Wu, J. Modeling cascading failures in interdependent infrastructures under terrorist attacks. *Reliab. Eng. Syst. Saf.* **2016**, *147*, 1–8. [CrossRef]
23. Dong, G.; Hao, H.; Du, R.; Shao, S.; Stanley, H.E.; Shlomo, H. Localized attack on clustering networks. *arXiv* **2016**.
24. Yuan, X.; Dai, Y.; Stanley, H.E.; Havlin, S. K-core percolation on complex networks: Comparing random, localized, and targeted attacks. *Phys. Rev. E* **2016**, *93*, 062302. [CrossRef] [PubMed]
25. Wang, S.; Hong, L.; Chen, X. Vulnerability analysis of interdependent infrastructure systems: A methodological framework. *Phys. A* **2012**, *391*, 3323–3335. [CrossRef]
26. Parshani, R.; Rozenblat, C.; Ietri, D.; Ducruet, C.; Havlin, S. Inter-similarity between coupled networks. *EPL* **2010**, *92*, 68002. [CrossRef]
27. Parshani, R.; Buldyrev, S.V.; Havlin, S. Interdependent networks: Reducing the coupling strength leads to a change from a first to second order percolation transition. *Phys. Rev. Lett.* **2010**, *105*, 048701. [CrossRef] [PubMed]
28. Zhou, D.; Gao, J.; Stanley, H.E.; Havlin, S. Percolation of partially interdependent scale-free networks. *Phys. Rev. E* **2013**, *87*, 052812. [CrossRef] [PubMed]
29. Dong, G.; Gao, J.; Tian, L.; Du, R.; He, Y. Percolation of partially interdependent networks under targeted attack. *Phys. Rev. E* **2012**, *85*, 016112. [CrossRef] [PubMed]
30. Liu, R.R.; Li, M.; Jia, C.X. Cascading failures in coupled networks: The critical role of node-coupling strength across networks. *Sci. Rep.* **2016**, *6*, 35352. [CrossRef] [PubMed]

31. Fu, G.; Dawson, R.; Khoury, M.; Bullock, S. Interdependent networks vulnerability analysis and strategies to limit cascading failure. *Eur. Phys. J. B* **2014**, *87*, 148. [CrossRef]
32. Liu, X.; Stanley, H.E.; Gao, J. Breakdown of interdependent directed networks. *Proc. Natl. Acad. Sci. USA* **2016**, *113*, 1138–1143. [CrossRef] [PubMed]
33. Qiu, Y. Optimal weighting scheme and the role of coupling strength against load failures in degree-based weighted interdependent networks. *Phys. A* **2013**, *392*, 1920–1924. [CrossRef]
34. Stippinger, M.; Kertész, J. Enhancing resilience of interdependent networks by healing. *Phys. A* **2014**, *416*, 481–487. [CrossRef]
35. Di Muro, M.A.; La Rocca, C.E.; Stanley, H.E.; Havlin, S.; Braunstein, L.A. Recovery of interdependent networks. *Sci. Rep.* **2016**, *6*, 22834. [CrossRef] [PubMed]
36. Hong, S.; Lv, C.; Zhao, T.; Wang, B.; Wang, J.; Zhu, J. Cascading failure analysis and restoration strategy in an interdependent network. *J. Phys. A* **2016**, *49*, 195101. [CrossRef]
37. Wang, H.; Li, Q.; D'Agostino, G.; Havlin, S.; Stanley, H.E.; Van Mieghem, P. Effect of the interconnected network structure on the epidemic threshold. *Phys. Rev. E* **2013**, *88*, 022801. [CrossRef] [PubMed]
38. Son, S.-W.; Bizhani, G.; Christensen, C.; Grassberger, P.; Paczuski, M. Percolation theory on interdependent networks based on epidemic spreading. *EPL* **2012**, *97*, 16006. [CrossRef]
39. Zhu, Q.; Zhu, Z.; Wang, Y.; Yu, H. Fuzzy-information-based robustness of interconnected networks against attacks and failures. *Phys. A* **2016**, *458*, 194–203. [CrossRef]
40. Gao, J.; Buldyrev, S.V.; Havlin, S.; Stanley, H.E. Robustness of a tree-like network of interdependent networks. *arXiv* **2011**.
41. Gao, J.; Buldyrev, S.V.; Havlin, S.; Stanley, H.E. Robustness of a network of networks. *Phys. Rev. Lett.* **2011**, *107*, 193903. [CrossRef] [PubMed]
42. Gao, J.; Buldyrev, S.V.; Havlin, S.; Stanley, H.E. Robustness of a network formed by n interdependent networks with a one-to-one correspondence of dependent nodes. *Phys. Rev. E* **2012**, *85*, 066134. [CrossRef] [PubMed]
43. Gaogao, D.; Ruijin, D.; Lixin, T.; Runran, L. Robustness of network of networks with interdependent and interconnected links. *Phys. A* **2013**, *424*, 11–18.
44. Dong, G.; Gao, J.; Du, R.; Tian, L.; Stanley, H.E.; Havlin, S. Robustness of network of networks under targeted attack. *Phys. Rev. E* **2013**, *87*, 052804. [CrossRef] [PubMed]
45. Dong, G.; Tian, L.; Zhou, D.; Du, R.; Xiao, J.; Stanley, H.E. Robustness of n interdependent networks with partial support-dependence relationship. *EPL* **2013**, *102*, 68004. [CrossRef]
46. Wang, G.Q.; Zhou, X.Y.; Zhang, P.Z.; Igel, H. Characteristics of amplitude and duration for near fault strong ground motion from the 1999 chi-chi, taiwan earthquake. *Soil Dyn. Earthq. Eng.* **2002**, *22*, 73–96. [CrossRef]
47. Watts, D.J.; Strogatz, S.H. Collective dynamics of small-world networks. *Nature* **1998**, *393*, 440–442. [CrossRef] [PubMed]
48. Nguyen, D.T.; Shen, Y.; Thai, M.T. Detecting critical nodes in interdependent power networks for vulnerability assessment. *IEEE Trans. Smart Grid* **2013**, *4*, 151–159. [CrossRef]
49. Wang, X.F.; Chen, G. Complex networks small-world scale-free and beyond. *IEEE Circuits Syst. Mag.* **2010**, *3*, 6–20. [CrossRef]