

Article

Methods for Grating Lobe Suppression in Ultrasound Plane Wave Imaging

Sua Bae and Tai-Kyong Song * 

Department of Electronic Engineering, Sogang University, Seoul 04107, Korea; suabae@sogang.ac.kr

* Correspondence: tksong@sogang.ac.kr; Tel.: +82-2-705-8907

Received: 29 August 2018; Accepted: 9 October 2018; Published: 11 October 2018



Abstract: Plane wave imaging has been proven to provide transmit beams with a narrow and uniform beam width throughout the imaging depth. The transmit beam pattern, however, exhibits strong grating lobes that have to be suppressed by a tightly focused receive beam pattern. In this paper, we present the conditions of grating lobe occurrence by analyzing the synthetic transmit beam pattern. Based on the analysis, the threshold of the angle interval is presented to completely eliminate grating lobe problems when using uniformly distributed plane wave angles. However, this threshold requires a very small angle interval (or, equivalently, too many angles). We propose the use of non-uniform plane wave angles to disperse the grating lobes in the spatial domain. In this paper, we present an approach using two uniform angle sets with different intervals to generate a non-uniform angle set. The proposed methods were verified by continuous-wave transmit beam patterns and broad-band 2D point spread functions obtained by computer simulations.

Keywords: ultrasonic imaging; beamforming; plane wave imaging; grating lobe suppression

1. Introduction

Plane wave imaging (PWI) has drawn a large amount of attention from researchers in the field of medical ultrasound imaging [1–3]. First, PWI can provide ultra-fast ultrasound imaging that is essential for a growing number of applications, such as the estimation of shear elasticity [1,4,5] and vector Doppler [6,7] as well as high-frame-rate B-mode imaging [8]. In PWI, plane waves (PWs) with different travelling angles are successively transmitted instead of traditionally focused ultrasound waves; after each firing, the returned ultrasound waves are received at all array elements. Synthetic transmit (Tx) focusing at each imaging point is achieved by compounding PWs with proper delays, while receive (Rx) focusing is performed in the conventional manner. As a result, ultrasound beams are focused at all imaging points for transmission and reception. Theoretically, the Tx beam pattern of PWI maintains the same main lobe width at all depths; the width is determined by the range of compounded PW angles.

When using a finite number of PWs with uniformly distributed steering angles, the synthetically focused beam has not only the main lobe, but also side lobes and grating lobes (GLs), which create artifacts in ultrasound image and deteriorate the image quality [2]. A large number of compounded PWs allow for the mitigation of the side lobe and GLs in the synthetic beam pattern and provide better image contrast, as illustrated in Figure 1. However, the frame rate of PWI decreases as the number of PWs increases. To reduce the side lobe without compromising the frame rate, various adaptive beamforming methods have been proposed. Austeng et al. proposed a minimum variance beamforming method for PWI [9]. In this method, the optimized weighting factors are applied when compounding the low-resolution images of different steered PWs. The joint Tx and Rx adaptive beamformer has also been proposed to apply the data-dependent weighting factors to both the receiving array domain and PW angle domain (i.e., frame domain) [10]. In addition, as the side lobes

from different angles are uncorrelated, some beamformers have been suggested to reject less coherent signals [11–13]. A global effective distance-based side lobe suppressing method has also been proposed to achieve high quality images of PWI with a small number of PWs [14].

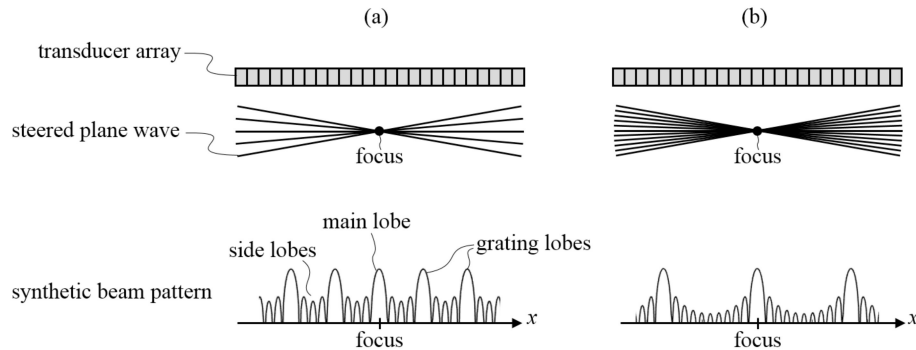


Figure 1. Schematic diagrams of a linear transducer array, transmitted plane waves (PWs) with uniformly distributed steering angles, and the synthetic transmit (Tx) beam pattern when (a) five PWs with a larger steering angle interval and (b) eleven PWs with a smaller angle interval are compounded. As the number of compounded PWs increases when the total range of the steering angle is fixed, the side lobe level decreases and grating lobes (GLs) occur less frequently.

Only a few studies, however, have been reported that consider GLs in PWI. Though PW angles with a constant angle interval are employed in most studies, they introduce uniformly spaced grating lobes (GLs), the interval of which is governed by the angle interval, as shown in Figure 1 [2]. If the angle interval is sufficiently small to locate GLs far from the main lobe, the GLs might have little effect on the image quality. However, to preserve the ultra-fast frame rate without compromising the resolution (i.e., to use a small number of PWs for the given PW angle range), the angle interval should be large, which introduces GLs close to the main lobe and deteriorates the image quality.

Here, we investigate the conditions of GL occurrence by analyzing the continuous wave (CW) synthetic transmit beam pattern. Based on the conditions of GL occurrence, the threshold of the angle interval for the elimination of GLs is presented with the use of uniformly distributed PW angles. In addition, we propose a method for GL level reduction using non-uniformly distributed PW angles, which consist of subsets of uniformly distributed angles, each of which has different angle intervals. To verify and evaluate the methods, simulation experiments are conducted using synthetic Tx beam patterns and round-trip point spread functions (PSFs).

2. Materials and Methods

2.1. GL Conditions in PWI

Let us consider N PWs with different steering angles, θ_n ($n = 1, 2, \dots, N$), in a range of $[\theta_{\min}, \theta_{\max}]$, which are selected in terms of $\alpha_n (= \sin(\theta_n))$ for the convenience of analysis such that $\alpha_n = \alpha_{\min} + (n - 1)d_\alpha$, $n = 1, 2, \dots, N$, where $\alpha_{\min} = -(N - 1)d_\alpha/2$ and d_α is a constant α -interval between successive α_n . Assuming that the monochromatic (i.e., CW) plane waves are emitted from an infinite-length transducer, PWI provides a synthetic Tx beam pattern given by:

$$\psi(x') = \sum_{n=1}^N \exp\{-jkx'(\alpha_{\min} + (n - 1)d_\alpha)\} = c_0 \frac{\sin(\pi x' d_\alpha N / \lambda)}{\sin(\pi x' d_\alpha / \lambda)}, \quad (1)$$

where $c_0 = \exp\{-jkx'(\alpha_{\min} + (N - 1)d_\alpha/2)\}$, $x' = x - x_f$, x_f is the lateral position of a focus, λ is the wavelength, and k is the wave number ($k = 2\pi/\lambda$) [3]. Note that almost the same beam pattern can be obtained from Equation (1) when using PW angles spaced by a constant θ -interval, as in [1,2], if the angles are sufficiently small such that $\sin \theta_n \approx \theta_n$ and $d_\alpha \approx d_\theta$. In the beam pattern, the main lobe is at

the focus, $x' = 0$ ($x = x_f$), and its amplitude is N . GLs must be observed for which both the numerator and denominator of Equation (1) have zeros except $x' = 0$, that occur at:

$$x' = \pm m\lambda/d_\alpha, m = 1, 2, 3, \dots \quad (2)$$

By substituting Equation (2) into Equation (1), the amplitude of the m -th GL can be expressed as:

$$\psi(x' = \pm m\lambda/d_\alpha) = \begin{cases} (-1)^m N & \text{for even } N \\ N & \text{for odd } N \end{cases} \quad (3)$$

It should be noted that the GLs predicted by Equations (2) and (3) arise when (1) all of the N PWs pass through the GL positions, preserving their linear wave-front (LWF) (LWF condition), and (2) PWs with a constant α -interval are employed (uniform d_α condition).

2.2. GL Suppression Method with Uniformly Distributed PW Angles

In practice, PWs are transmitted by a finite aperture (i.e., a finite-length transducer array), resulting in a finite collimated beam area. Figure 2 shows three PWs with different steering angles (black solid lines) and their collimated beam areas (gray shaded areas) with preserved LWFs. In Figure 2, the focus and the main lobe of the focused beam pattern are in the region where all the PWs preserve their LWFs (i.e., the darkest area in Figure 2). If a GL locates at this same region and the constant PW angle interval is employed (i.e., if both LWF and uniform d_α conditions are met), the amplitude of the GL would be as high as that of the main lobe because the number of compounded PWs should be the same at both the main lobe and GL locations. Note that by reducing d_α , one can move the GL away from the main lobe towards a region where the LWF condition is satisfied by fewer PWs. In such a case, the GL level would be lower than that of the main lobe because the number of PWs coherently compounded becomes smaller at the GL location; the PWs that do not maintain LWFs at the grating lobe locations are compounded with phase errors, leading to a decrease in the corresponding GL levels. This indicates that GL levels would decrease as GLs are moved farther from the main lobe.

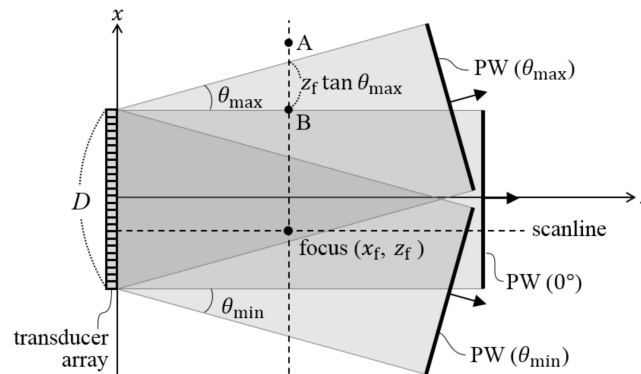


Figure 2. Collimated beam areas (gray shaded areas) of PWs with steering angles of θ_{max} , 0° , and θ_{min} (from top to bottom). The points, A and B, are located at the region where none of PWs pass and the region where half of transmitted PWs propagate, respectively.

One can expect that the GLs can be eliminated by locating all of the GLs in a region where none of the PWs preserve LWF (i.e., by completely violating the LWF condition). In Figure 2, for example, when the first GL position, $x = x_{GL,1} (= x_f + \lambda/d_\alpha)$, falls onto point A that is out of all the collimated beam areas of PWs, no GLs can be formed, even when a uniform d_α is employed. In this case, $x_{GL,1} > D/2 + z_f \tan \theta_{max}$. Consequently, the GL elimination requirement when using uniformly distributed PW angles can be defined as:

$$d_\alpha < \lambda / (D/2 + z_f \tan \theta_{max} - x_f), \quad (4)$$

where (x_f, z_f) represents the focal point and D is the transducer width. One can also observe that only the PWs with $\theta \geq 0^\circ$ pass through point B in Figure 2, preserving the LWF. Therefore, the GL level would be approximately halved at this point (i.e., reduced by -6 dB) if the angular interval is:

$$d_\alpha = \lambda / (D/2 - x_f), \quad (5)$$

when $\theta_{\max} = -\theta_{\min}$.

2.3. GL Suppression Method with Non-Uniformly Distributed PW Angles

The GL levels can also be reduced using non-uniformly distributed PW angles (i.e., by violating the uniform d_α condition). In this paper, to generate a non-uniform angle set, an approach using two uniform angle sets with different d_α values is presented.

We let the uniform angle set 1 and set 2 have N_1 PWs with an interval of $d_{\alpha,1}$ and N_2 PWs with an interval of $d_{\alpha,2}$, respectively. The GLs of set 1 would appear at integer multiples of $\lambda/d_{\alpha,1}$, while the GLs of set 2 would arise at integer multiples of $\lambda/d_{\alpha,2}$, according to Equation (2). To suppress the GL level, a non-uniform PW angle set can be obtained by combining the two uniform angle sets. The beam pattern of the non-uniform angle set is given by the sum of the beam patterns of the two uniform angle sets. Note that the main lobes of sets 1 and 2 are both centered at $x = x_f$. Therefore, after the field responses for two uniform sets are summed, the peak value of the resulting main lobe will always be larger than the main lobe peak of each angle set. When $d_{\alpha,1}$ and $d_{\alpha,2}$ are chosen to locate the GLs of two uniform sets in different locations (i.e., $m_1\lambda/d_{\alpha,1} \neq m_2\lambda/d_{\alpha,2}$ where m_1 and m_2 are integer numbers), the GLs of the non-uniform angle set will have smaller magnitudes than the main lobe. In addition, even when the GLs of two uniform sets overlap (i.e., $m_1\lambda/d_{\alpha,1} = m_2\lambda/d_{\alpha,2}$), they can also be reduced after they are combined if the two coincident GLs have opposite phases. For example, when N_1 and N_2 are both even, the GL at the same location in the beam pattern of the non-uniform set will have a magnitude of:

$$|\psi| = |(-1)^{m_1}N_1 + (-1)^{m_2}N_2|, \quad (6)$$

which can be derived by Equation (3). In this case, the magnitude will be decreased to $|N_1 - N_2|$ if m_1 is even and m_2 is odd, or vice versa.

Figure 3 shows an example of a non-uniform angle set that is obtained by combining two uniform angle sets. The angle distributions (left panels) and synthetic Tx beam patterns (right panels) of the uniform angle set 1 and set 2 are presented in Figure 3a,b, respectively. The number of PWs and angular intervals of set 1 and set 2 are $(N_1 = 6, d_{\alpha,1} = 0.069)$ and $(N_2 = 6, d_{\alpha,2} = 0.046)$, respectively. The combined non-uniform angle set and its synthetic beam pattern are shown in Figure 3c. The synthetic beam pattern was obtained by Equation (1), assuming a center frequency of 5.208 MHz and a sound speed of 1540 m/s (i.e., $\lambda = 0.296$ mm).

In Figure 3a,b, the main lobe is located at 0 and the GLs are repeated at a certain interval. The intervals between the GLs of set 1 and set 2 are 4.29 mm and 6.43 mm, respectively, according to Equation (2). The GLs of uniform sets 1 and 2 at different locations are halved in the beam pattern of the non-uniform angle set (see the right panel of Figure 3c). For the chosen parameters ($d_{\alpha,1} = 0.069$, $d_{\alpha,2} = 0.046$), $m_1\lambda/d_{\alpha,1}$ is equal to $m_2\lambda/d_{\alpha,2}$ when $m_1 = 3$ and $m_2 = 2$, which means that the third GL of set 1 coincides with the second GL of set 2, as in Figure 3a,b. As the two GL have the same magnitude with the opposite sign according to the Equation (3), they are canceled out in the combined transmit beam pattern (Figure 3c), as is expected from Equation (6). On the other hand, the next coincident GLs at $x = 25.6$ mm, corresponding to $m_1 = 6$ and $m_2 = 4$, have the same sign and thus have the same magnitude as the main lobe peak in the beam pattern of the non-uniform set. However, this theoretical beam pattern is calculated assuming an infinite aperture. Thus, when using a practically used finite-length transducer, these high GLs can also be removed by placing them in a region where fewer or no PWs pass through, which is shown in the Results Section, where the finite length of the array is considered.

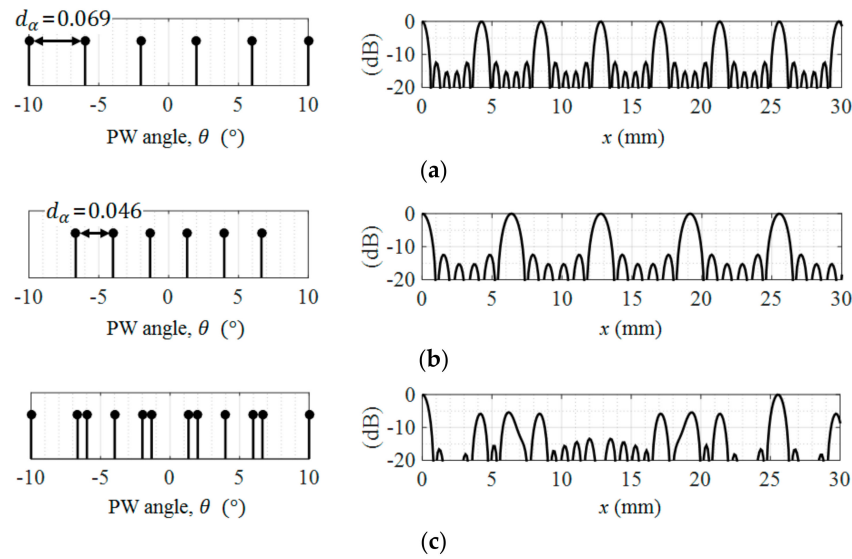


Figure 3. PW angles for synthetic focusing (left panels) and theoretical synthetic beam patterns (right panels) using three angle sets; (a) uniform angle set 1: $N_1 = 6$, $d_{\alpha,1} = 0.069$; (b) uniform angle set 2: $N_2 = 6$, $d_{\alpha,2} = 0.046$; (c) non-uniform angle set (combination of two uniform sets in (a,b)): $N = N_1 + N_2 = 12$.

3. Results and Discussion

Both of the GL suppression methods were verified through computer experiments by obtaining CW beam patterns and PSF images using a 128-element linear array transducer with a center frequency of 5.208 MHz and a pitch of 0.298 μm (length of the array = 38.1 mm). The CW beam pattern was calculated using MATLAB R2015a (MathWorks, Natick, MA, USA); acoustic field responses of plane waves with different angles, each with a frequency of 5.208 MHz generated from the linear array transducer, were calculated and then compounded with proper delays for a Tx focal point at ($x = 0$, $z = 30$ mm) [1]. The amplitude of each PW was set to 1.0, and the final synthesized beam pattern was normalized by its maximum value and displayed in logarithmic scale.

In the PSF experiments, a single pulse plane wave with a center frequency of 5.208 MHz was transmitted along different directions from the same array transducer. First, RF echo data sets of different plane wave angles reflected from a single point target at ($x = 0$, $z = 30$ mm) were generated using the Field II simulator [15]. Then, MATLAB was used to obtain the PSF images by reconstructing a B-mode image of the point target from the RF data sets using coherent plane wave beamforming [1], demodulation, and logarithmic compression. In the beamforming process, traditional dynamic Rx focusing was performed with an F-number of 1.0, and all the plane waves were coherently synthesized with proper delays for each pixel to obtain two-way (Tx and Rx) dynamic focusing. In the logarithmic compression, the PSF image was normalized by its maximum value and compressed with a dynamic range of 60 dB.

To validate the methods for GL suppression with a uniform d_α , CW Tx beam patterns were obtained, varying the interval, d_α , that is controlled by the number of PW angles within the range of $[-10^\circ, 10^\circ]$. The focus of the beam pattern was at a depth of 30 mm ($z_f = 30$ mm) on the center scanline ($x_f = 0$). Figure 4a presents the magnitude of the first GL relative to that of the main lobe as a function of normalized d_α , $\hat{d}_\alpha (= d_\alpha / (2\lambda / D))$; normalization was performed so that $\hat{d}_\alpha$ equals 1 when Equation (5) is satisfied. Figure 4b illustrates the PW propagating regions; region A is a region where no PWs pass through; region B is a region where more than one PW propagate, and region C is a region where all of the collimated beam areas of the PWs overlap. As mentioned above, the larger $\hat{d}_\alpha$ yields GLs closer to the main lobe. Sections A, B, and C shown in Figure 4a indicate that the first GL is located in regions A, B, and C in Figure 4b, respectively. Figure 4c shows the beam patterns of three values of $\hat{d}_\alpha$ from section A, B, and C (from left to right panels). The gray dash-dot line of each panel indicates

the theoretical first GL position ($x_{GL,1} = \lambda/d_\alpha$). From the left to right panels in Figure 4c, one can observe that the magnitude of GL increases as $\hat{d}_\alpha$ increases. Since none of the PWs propagate with the LWF in region A of Figure 4b (i.e., the GL elimination requirement in Equation (4) is satisfied), the GL level is sufficiently suppressed in section A of Figure 4a. In section B of Figure 4a, as $\hat{d}_\alpha$ increases, the first GL moves closer to the focus. As it is closer to the center scanline ($x = 0$) across region B of Figure 4b, the number of overlapped collimated beam areas (i.e., the number of PWs preserving the LWF) increases. Consequently, the GL level increases with $\hat{d}_\alpha$ in section B of Figure 4a. In section C of Figure 4a, the first GL has the same magnitude (0 dB) as the main lobe because the number of PWs passing through each point are the same over all of region C. Note that the GL can be suppressed below −6 dB if $\hat{d}_\alpha < 1$, as in Figure 4a.

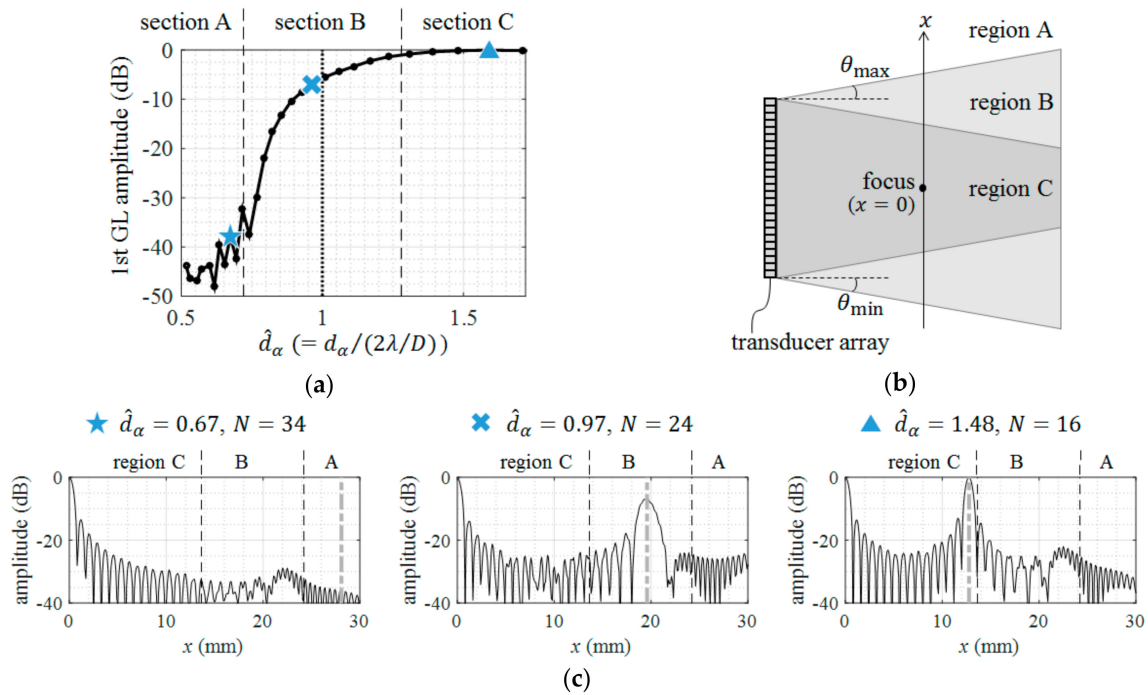


Figure 4. Magnitude of the first GL normalized by that of main lobe when the GL is located in regions A, B, and C. (a) Magnitude of the first GL in a simulated Tx beam pattern versus $\hat{d}_\alpha$. (b) Region where none of (region A), more than one of (region B), and all of (region C) the collimated beam areas of PWs overlap. (c) Simulated beam patterns when using $\hat{d}_\alpha$ (marked with star, cross, and triangle in a (from left to right panels)). The gray dash-dot line indicates the theoretical first GL position, $x_{GL,1}$, of each beam pattern.

The method using a non-uniform angle set was verified by the CW Tx beam pattern and PSF image simulations using the same PW sets used in Figure 3. Figure 5a shows the CW Tx beam patterns using the PW angle sets presented in the left panels of Figure 3a–c, from top to bottom. Since a finite-length transducer is used, regions where not all of the PWs pass through exist. In Figure 5a, region A and B indicate the region where none of the PWs pass and the region where more than one of the PWs propagate, respectively. The GL levels decrease in regions A and B; it is noteworthy that the GL at $x = 25.6$ mm in Figure 3c is greatly suppressed in the bottom left panel of Figure 5 when considering the finite aperture. The effect of GL reduction using the non-uniform angle set was also assessed with the 2D PSF images shown in Figure 5b. In the top and middle panels of Figure 5b, artifacts due to the first GL are observed, though the GL artifacts were reduced because of the Rx beamforming. The GL artifacts in the middle panel are located further from the point target than those in the top panel as the uniform PW set 2 has a smaller angular interval than that of the PW set 1 ($d_{\alpha,1} = 0.069$ vs. $d_{\alpha,2} = 0.046$).

When the non-uniform angle set is employed (bottom panel in Figure 5b), it is clearly observed that the GLs are successfully suppressed compared to those from the uniform PW sets.

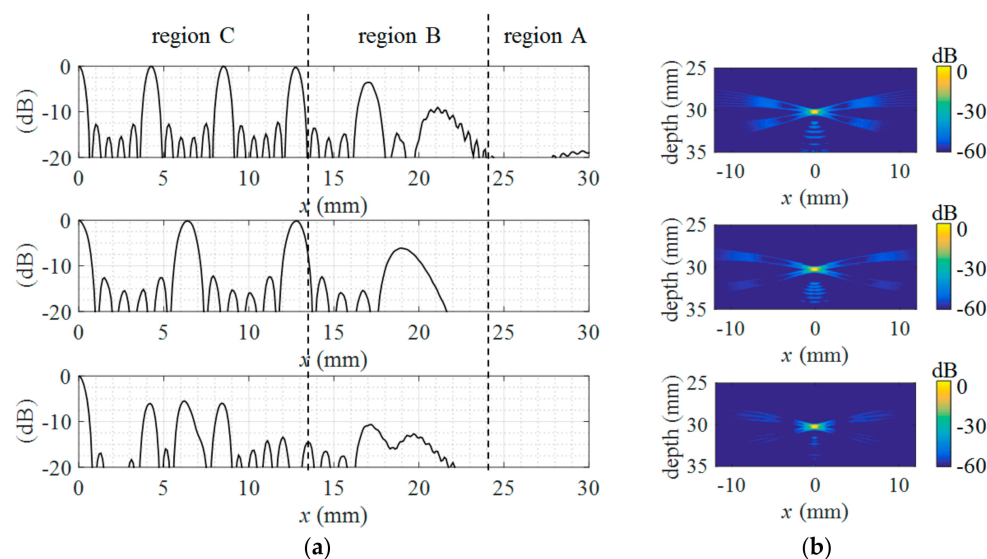


Figure 5. (a) Simulated Tx beam patterns and (b) 2D PSFs of round-trip focusing using the PW angle sets shown in Figure 3a–c (from top to bottom panels).

The proposed non-uniform angle is very easy to design as it is composed of two uniform angle sets with different angle intervals. However, there must be ways to discover other types of non-uniform PW angle distributions or to design an optimal PW angle set for a given imaging specification and the system requirements. One such approach might be to find a specific PW angle distribution that produces GLs where they can be further suppressed by the receive beam pattern, which may also have to be properly (or deliberately) designed. It is also worth noting that a deep neural network can be applied to find a method for the compressed sampling of PW angles [16].

In future studies, we should also consider ways to maintain or improve other important image qualities with a smaller number of PWs for fast imaging. The proposed methods in this paper can be combined with adaptive beamforming and sidelobe reduction techniques [9–14] that have recently been developed to improve the spatial and contrast resolutions of PW imaging. In addition, encoded PWs can be transmitted and compounded for a higher signal-to-noise ratio using Chirp, Barker code, or the Hadamard matrix [5,17,18].

4. Conclusions

In this paper, we describe two conditions for GL formation and present two methods for GL suppression. The first method is proposed for the case in which uniformly distributed angles are used. This method can completely eliminate GL problems, but may have to use a very small angle interval (or, equivalently, too many angles). In such a case, the second method using a non-uniform angle set could be a practical alternative solution for GL suppression. The two methods were verified by CW Tx beam patterns and broad-band 2D PSF images obtained by computer simulations. Future work needs to focus on designing other types of non-uniform PW angle sets to improve the overall image quality in combination with advanced receive beamforming and signal processing techniques.

Author Contributions: T.K.S. devised the algorithm, and S.B. contributed to the design of the experiments. All of the authors wrote and reviewed the manuscript.

Funding: This research was supported by the Next-generation Medical Device Development Program for Newly-Created Market of the National Research Foundation (NRF) funded by the Korean government, MSIP(NRF-2015M3D5A1065997).

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Montaldo, G.; Tanter, M.; Bercoff, J.; Benech, N.; Fink, M. Coherent plane-wave compounding for very high frame rate ultrasonography and transient elastography. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2009**, *56*, 489–506. [\[CrossRef\]](#) [\[PubMed\]](#)
2. Denarie, B.; Tangen, T.A.; Ekroll, I.K.; Rolim, N.; Torp, H.; Bjåstad, T.; Lovstakken, L. Coherent plane wave compounding for very high frame rate ultrasonography of rapidly moving targets. *IEEE Trans. Med. Imaging* **2013**, *32*, 1265–1276. [\[CrossRef\]](#) [\[PubMed\]](#)
3. Lee, J.P.; Song, J.H.; Song, T.-K. Analysis of ultrasound synthetic transmit focusing using plane waves. *J. Acoust. Soc. Korea* **2014**, *33*, 200–209. [\[CrossRef\]](#)
4. ZHao, H.; Song, P.; Urban, M.W.; Greenleaf, J.F.; Chen, S. Shear wave speed measurement using an unfocused ultrasound beam. *Ultrasound Med. Biol.* **2012**, *38*, 1646–1655. [\[CrossRef\]](#) [\[PubMed\]](#)
5. Song, P.; Urban, M.W.; Manduca, A.; Greenleaf, J.F.; Chen, S. Coded excitation plane wave imaging for shear wave motion detection. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2015**, *62*, 1356–1372. [\[CrossRef\]](#) [\[PubMed\]](#)
6. Udesen, J.; Gran, F.; Hansen, K.L.; Jensen, J.A.; Thomsen, C.; Nielsen, M.B. High frame-rate blood vector velocity imaging using plane waves: Simulations and preliminary experiments. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2008**, *55*, 1729–1743. [\[CrossRef\]](#) [\[PubMed\]](#)
7. Bercoff, J.; Montaldo, G.; Loupas, T.; Saverly, D.; Mézière, F.; Fink, M.; Tanter, M. Ultrafast compound Doppler imaging: Providing full blood flow characterization. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2011**, *58*, 134–147. [\[CrossRef\]](#) [\[PubMed\]](#)
8. Tong, L.; Gao, H.; Choi, H.F.; D’hooge, J. Comparison of conventional parallel beamforming with plane wave and diverging wave imaging for cardiac applications: A simulation study. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2012**, *59*, 1654–1663. [\[CrossRef\]](#) [\[PubMed\]](#)
9. Austeng, A.; Nilsen, C.I.C.; Jensen, A.C.; Næsholm, S.P.; Holm, S. Coherent plane-wave compounding and minimum variance beamforming. In Proceedings of the IEEE International Ultrasonics Symposium, Orlando, FL, USA, 18–21 October 2011; pp. 2448–2451.
10. Zhao, J.; Wang, Y.; Zeng, X.; Yu, J.; Yiu, B.Y.S.; Yu, A.C.H. Plane wave compounding based on a joint transmitting-receiving adaptive beamformer. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2015**, *62*, 1440–1452. [\[CrossRef\]](#) [\[PubMed\]](#)
11. Zimbico, A.J.; Granado, D.W.; Schneider, F.K.; Maia, J.M.; Assef, A.A.; Schiefler, N.; Costa, E.T. Eigenspace generalized sidelobe canceller combined with SNR dependent coherence factor for plane wave imaging. *Biomed. Eng. Online* **2018**, 1–23. [\[CrossRef\]](#) [\[PubMed\]](#)
12. Guo, W.; Wang, Y.; Yu, J. A sidelobe suppressing beamformer for coherent plane wave compounding. *Appl. Sci.* **2016**, *6*, 359. [\[CrossRef\]](#)
13. Wang, Y.; Zheng, C.; Peng, H.; Zhang, C. Coherent plane-wave compounding based on normalized autocorrelation factor. *IEEE Access* **2018**, *6*, 36927–36938. [\[CrossRef\]](#)
14. Guo, W.; Wang, Y.; Wu, G.; Yu, J. Sidelobe reduction for plane wave compounding with a limited frame number. *Biomed. Eng. Online* **2018**, *17*, 94. [\[CrossRef\]](#) [\[PubMed\]](#)
15. Jensen, J.A. Field: A program for simulating ultrasound systems. In Proceedings of the 10th Nordic-Baltic Conference on Biomedical Imaging, Lyngby, Denmark, 9–13 June 1996; Volume 34, pp. 351–353.
16. Gasse, M.; Millioz, F.; Roux, E.; Garcia, D.; Liebgott, H.; Friboulet, D. High-Quality Plane Wave Compounding Using Convolutional Neural Networks. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2017**, *64*, 1637–1639. [\[CrossRef\]](#) [\[PubMed\]](#)
17. Zhao, F.; Tong, L.; He, Q.; Luo, J. Coded excitation for diverging wave cardiac imaging: A feasibility study. *Phys. Med. Biol.* **2017**, *62*, 1565–1584. [\[CrossRef\]](#) [\[PubMed\]](#)
18. Tiran, E.; Deffieux, T.; Correia, M.; Maresca, D.; Osmanski, B.-F.; Sieu, L.-A.; Bergel, A.; Cohen, I.; Pernot, M.; Tanter, M. Multiplane wave imaging increases signal-to-noise ratio in ultrafast ultrasound imaging. *Phys. Med. Biol.* **2015**, *60*, 8549–8566. [\[CrossRef\]](#) [\[PubMed\]](#)

