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Theoretical and Numerical Estimation of Vibroacoustic Behavior of Clamped Free Parabolic Tapered Annular Circular Plate with Different Arrangement of Stiffener Patches

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Abstract: This paper compares the vibroacoustic behavior of a tapered annular circular plate having different parabolic varying thickness with different combinations of rectangular and concentric stiffener patches keeping the mass of the plate and the patch constant for a clamped-free boundary condition. Both numerical and analytical methods are used to solve the plate. The finite element method (FEM) is used to determine the vibration characteristic and both Rayleigh integral and FEM is used to determine the acoustic behavior of the plate. It is observed that a Case II plate with parabolic decreasing–increasing thickness variation for a plate with different stiffener patches shows reduction in frequency parameter in comparison to other cases. For acoustic response, the variation of peak sound power level for different combinations of stiffener patches is investigated with different taper ratios. It is investigated that Case II plate with parabolic decreasing–increasing thickness variation for an unloaded tapered plate as well as case II plate with 2 rectangular and 4 concentric stiffeners patches shows the maximum sound power level among all variations. However, it is shown that the Case III plate with parabolically increasing–decreasing thickness variation with different combinations of rectangular and concentric stiffeners patches is least prone to acoustic radiation. Furthermore, it is shown that at low forcing frequency, average radiation efficiency with different combinations of stiffeners patches remains the same, but at higher forcing frequency a higher taper ratio causes higher radiation efficiency, and the radiation peak shifts towards the lower frequency and alters its stiffness as the taper ratio increases. Finally, the design options for peak sound power actuation and reduction for different combinations of stiffener patches with different taper ratios are suggested.

Keywords: thick annular circular plate; Rayleigh integral; finite element modeling; rectangular and concentric stiffener patches; taper ratio; thickness variation

1. Introduction

Tapered annular circular plates with different combinations of rectangular and concentric patches has many engineering applications. They are used in many structural components i.e., building, design, diaphragms and deck plates in launch vehicles, diaphragms of turbines, aircraft and missiles, naval structures, nuclear reactors, optical systems, construction of ships, automobiles and other vehicles, the space shuttle etc. These tapering plates with different combinations of rectangular

and concentric patches are found to have greater resistance to bending, buckling and vibration in comparison to plates of uniform thickness. It is interesting to know that tapered plates with different thickness variation have drawn the attention of most of the researchers in this field. However, tapered plates with different combination of rectangular and concentric patches can alter the dynamic characteristic of structures with a change in stiffness. Hence, for practical design purposes, the vibration and acoustic characteristics of such tapered plates are equally important. In comparison to the present study, several existing works are presented where the researchers have investigated the vibration response [1–9] of circular or annular plates of tapered or uniform thickness. But in terms of acoustic behavior, many researchers have contributed most. Lee and Singh [10] used the thin and thick plate theories to determine the sound radiation from out-of-plane modes of a uniform thickness annular circular plate. Thompson [11] used the Bouwkamp integral to determine the mutual and self-radiation impedances both for annular and elliptical pistons. Levine and Leppington [12] analyzed the sound power generation of a circular plate of uniform thickness using exact integral representation. Rdzanek and Engel [13] determined the acoustic power output of a clamped annular plate using an asymptotic formula. Wodtke and Lamancusa [14] minimized the acoustic power of circular plates of uniform thickness using the damping layer placement. Wanyama [15] studied the acoustic radiation from linearly-varying circular plates. Lee and Singh [16] used the flexural and radial modes of a thick annular plate to determine the self and mutual radiation. Cote et al. [17] studied the vibro acoustic behavior of an un baffled rotating disk. Jeyraj [18] used an isotropic plate with arbitrarily varying thickness to determine its vibro-acoustic behavior using the finite element method (FEM). Ranjan and Ghosh [19] studied the forced response of a thin plate of uniform thickness with attached discrete dynamic absorbers. Bipin et al. [20] analyzed an isotropic plate with attached discrete patches and point masses with different thickness variation with different taper ratios to determine its vibro acoustic response. Lee and Singh [21] investigated the annular disk acoustic radiation using structural modes through analytical formulations. Rdzanek et al. [22] investigated the sound radiation and sound power of a planar annular membrane for axially-symmetric free vibrations. Doganli [23] determined the sound power radiation from clamped annular plates of uniform thickness. Nakayama et al. [24] investigated the acoustic radiation of a circular plate for a single sound pulse. Hasegawa and Yosioka [25] determined the acoustic radiation force used on the solid elastic sphere. Lee and Singh [26] used a simplified disk brake rotor to investigate the acoustic radiation through a semi-analytical method. Thompson et al. [27,28] analyzed the modal approach for different boundary conditions to calculate the average radiation efficiency of a rectangular plate. Rayleigh [29] determined the sound radiation from flat finite structures. Maidanik [30] analyzed the total radiation resistance for ribbed and simple plates using a simplified asymptotic formulation. Heckl [31] used the wave number domain and Fourier transform to analyses the acoustic power. Williams [32] determined the wave number as a series in ascending power to estimate the sound radiation from a planar source. Keltie and Peng [33] analyzed the sound radiation using the cross-modal coupling from a plane. Snyder and Tanaka [34] demonstrated the importance of cross-modal contributions for a pair of modes through total sound power output using modal radiation efficiency. Martini et al. [35] investigated the structural and elastodynamic analysis of rotary transfer machines by a finite element model. Croccolo et al. [36] determined the lightweight design of modern transfer machine tools using the finite element model. Martini and Troncossi [37] determined the upgrade of an automated line for plastic cap manufacture based on experimental vibration analysis. Pavlovic et al. [38] investigated the modal analysis and stiffness optimization: the case of a tool machine for ceramic tile surface finishing using FEM.

While reviewing the literature, this comes to a discussion at a common point that has inspired the present paper based on a comparison of vibroacoustic behavior of a parabolic tapered annular circular plate with attached rectangular and concentric patches at different positions. The paper is significant for the analysis of the comparison of vibroacoustic behavior of such clamped free tapered plate, which is done by keeping the mass of the plate and patch constant. Therefore, this paper is based on the vibroacoustic analysis of a clamped free parabolic tapered annular circular plate with different

attachments of rectangular and concentric stiffener patches for different positions with different taper ratios under time-varying harmonic excitations.

2. Mathematical Modeling and Analysis

2.1. Plate Free Vibration

Let us considered a plate with outer radius 'a' and inner radius 'b' as shown in Figure 1. In this paper, the modal analysis is performed to estimate the natural frequency and modes shape of the plate is given by the following equation:

$$([K] - \omega^2[M])\psi_{mn} = 0 \quad (1)$$

where $[M]$ is the mass matrix and $[K]$ is the stiffness matrix where as ψ_{mn} is the mode shape and ω is the respective natural frequency of the plate in rad/sec. The non-dimensional frequency parameter λ^2 is given by the following equation:

$$\lambda^2 = \omega a^2 \sqrt{\frac{\rho h}{D}} \quad (2)$$

where D , the flexure rigidity = $\frac{Eh^3}{12(1-\nu^2)}$, a = outer radius, E = Young's modulus of elasticity, ν = Poisson's ratio, h = thickness of the plate and ρ = density of plate.

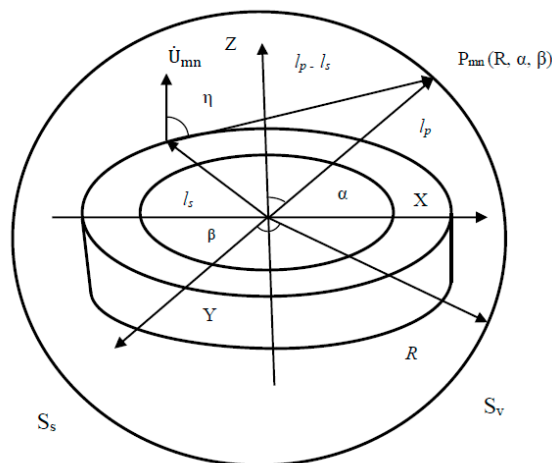


Figure 1. Sound radiation investigated for thick annular circular plate in Z direction enclosed in a sphere.

2.2. Analytical and Numerical Formulation for Acoustic Radiation from Tapered Annular Circular Plate

It is considered that an annular circular plate of inner radius 'b' and outer radius 'a' in flexural vibration is set on flat rigid baffle having infinite extent as reported in Figure 1. Acoustic scattering of the edges of a vibrating structure is neglected in this study. Let P be the sound pressure amplitude, S_s be the surface of the sound source, q be the Green methods function in free field, l_s and l_p be the position vectors of source and receiver and the surface normal vector at l_s be f ; then structure sound radiation can be obtained by the Rayleigh integral [10] as given by Equation (3):

$$P(l_p) = \int_{S_s} \left(\frac{\partial q}{\partial f}(l_p, l_s) P(l_p) - \frac{\partial P}{\partial f}(l_s) q(l_p, l_s) \right) ds(l_s) \quad (3)$$

The sound pressure, radiated from non-planar source in far and free field environment based on plane wave approximation can be expressed by Equation (4):

$$P(l_p) = \frac{\rho_0 c_0 B}{4\pi} \int_{S_s} \frac{e^{iB|l_p-l_s|} U(l_s)}{|l_p-l_s|} (1 + \cos \eta) dS \quad (4)$$

Let ρ_0 be the mass density of air, c_0 be the speed of sound in air, B be the corresponding acoustic wave number, and $\dot{U}$ and $\dot{u}$ be the corresponding vibratory velocity amplitude and spatial dependent vibratory velocity amplitude in the z direction at l_s , then from a normal plane [10], the modal sound pressure P_{mn} for an annular plate with $(m, n)^{th}$ mode is obtained from simplifying Equation (4) with Hankel transform and is expressed by Equations (5) and (6):

$$P_{mn}(R, \alpha, \beta) = \frac{\rho_0 c_0 B e^{iB_{mn} R_d}}{2R_d} \cos n\beta (-i)^{n+1} A_n \left[\dot{u}(l) \right] (1 + \cos \eta) \quad (5)$$

$$A_f \left[\dot{u}(l) \right] = \int_0^\infty u(l) J_n(B_l l) l dl, \quad B_l = B \sin \theta; \quad R_d = |l_p - l_s| \quad (6)$$

where X_n is Bessel function of order n , (α, β) are the cone and azimuthal angles of the observation positions, respectively, η is the angle between the surface normal vector and the vector from source position to receiver position, and A is the Hankel transform. According to the far field condition, R_d in the denominator is approximated by R where $R = |l_p|$ is considered to be radius of the sphere. Consider that on a sphere S_v the observation positions are represented by some points having equal angular increments $(\Delta\varphi, \Delta\alpha)$. If ' $\Delta\varphi$ ' represents the small increment in the circumferential direction of the plate, at all of the observation positions, the sound pressures is given by Equations (4)–(6). The modal sound power S_{mn} for the $(m, n)^{th}$ mode [10,16] from the far-field is given by Equation (7):

$$S_{mn} = (D_{mn} S_v)_s = \frac{1}{2} \int_0^{2\pi} \int_0^\pi \frac{P_{mn}^2}{\rho_0 c_0} R^2 \sin \alpha d\alpha d\beta \quad (7)$$

where the acoustic intensity is represented by D_{mn} and area of the control surface is represented by S_v . The radiation efficiency σ_{mn} of the plate [10] is given by Equation (8):

$$\sigma_{mn} = \frac{S_{mn}}{\left| \dot{u}_{mn} \right|_{ts}^2}, \quad \left| \dot{u}_{mn} \right|_{ts}^2 = \frac{1}{2\pi(a^2 - b^2)} \int_b^a \int_0^{2\pi} U^2 d\varphi dl \quad (8)$$

where, for the two normal surfaces of the plate the spatially average r.m.s velocity is represented as $\left| \dot{u}_{mn} \right|_{ts}^2$. Considering the plate thickness (h) effect, the sum of sound radiations [16] from two normal surfaces of the plate at $(Z = 0.5 h$ and $-0.5 h)$ will represent the modal sound power, which can be given by Equations (9)–(11):

$$P_{mn}(R, \alpha, \beta) = (1 + \cos \alpha) P_{mn}^s(R, \alpha, \beta) + (1 - \cos \alpha) P_{mn}^0(R, \alpha, \beta) \quad (9)$$

$$P_{mn}^s(R, \alpha, \beta) = \frac{\rho_0 c_0 B_{mn} e^{iB_{mn} R}}{2R} e^{-iB_{mn}(\frac{h}{2}) \cos \alpha} \cos n\beta (-i)^{n+1} A_n \left[\dot{U}(l) \right] \quad (10)$$

$$P_{mn}^0(R, \alpha, \beta) = \frac{\rho_0 c_0 B_{mn} e^{iB_{mn} R}}{2R} e^{-iB_{mn}(\frac{h}{2}) \cos \alpha} \cos n(\beta + \phi) (-i)^{n+1} A_n \left[\dot{U}(l) \right] \quad (11)$$

where, the corresponding acoustic wave number of the $(m, n)^{th}$ mode is represented by $B_{m,n}$, s and o in Equations (10) and (11) represent the source side and the opposite to the source side.

For numerical analysis, we have used ANSYS (ANSYS, Inc., Canonsburg, PA, USA) as a tool. The plates with rectangular and concentric patches are modeled in ANSYS with Plane 185 with 8 brick nodes and having three degrees of freedom at each node. The mesh is not exactly equal to all the cases of different thickness variation and stiffener. The number of element and nodes for uniform unloaded plates ends up being 5883 and 1664, respectively. For plates with different cases of thickness variation with different stiffener we tried to keep the mesh as close to the mesh of the unloaded plate. For vibration analysis and for a Case I plates with 1 rectangular stiffener, the modal structure consists of 5685 elements with 1638 nodes whereas for Case I plates with 1 concentric stiffener, it has 5524 elements and 1618 nodes. With other combinations of rectangular and concentric stiffener with different parabolic thickness, a variation of 5% of the mesh from that of the unloaded plate is considered. The numerical results obtained using FEM are compared with the existing literature. The structure is modeled as such that the total volume of the plate plus stiffeners is equal to the total volume of the uniform unloaded plate. As a result the whole mass of the plate plus stiffener is equal to the whole mass of the uniform unloaded plate. So for all the cases of plate, the mass will be constant. FLUID 30 and FLUID130 elements are used to create the acoustic medium environment around the plate. For fluid-structure interaction FLUID 30 is used. For the surface on outer sphere, FLUID 130 elements are created by imposing a condition of infinite space around the source and to prevent the back reflection of sound waves to the source. For acoustic analysis, the number of element and nodes for a uniform unloaded plate ends up being 14,680 and 3639, respectively. For a Case I plate with 1 rectangular stiffener, and after proper convergence of modeling, the numbers of elements and nodes ends up being 14,124 and 3465 respectively while for Case I plate with 1 concentric stiffener, the numbers of elements and nodes found to be 13,934 and 3345 respectively. Again, for other combination of rectangular and concentric stiffeners with different parabolic thickness, a variation of 5% of mesh from that of the unloaded plate is taken. Consider the air medium where the plate is vibrating with air density $\rho_0 = 1.21 \text{ kg/m}^3$. At 20°C , the speed of sound c_0 of air is taken as 343 m/s. The structural damping coefficient of the plate is assumed as 0.01.

2.3. Thickness Variation of the Plate

In this study, three different parabolic thickness variations of plates is considered for analysis and is reported in Figure 2. The radial direction is considered for thickness variation by keeping the total mass of the plate plus patch constant. In the radial direction the plate thickness is given by $h_x = h [1 - T_x \{f(x)\}^n]$, where 'h' is the maximum thickness of the plate where,

$$f(x) = \begin{cases} 0, & x=b \\ 1, & x=a \end{cases} \text{ and } f(x) = \frac{x-b}{a-b} \text{ where } b < x < a \quad (12)$$

The taper parameter or taper ratio (T_x) is given by the equation:

$$T_x = \left(1 - \frac{h_{\min}}{h}\right) \quad (13)$$

The Case I plate of (Figure 2 with parabolically decreasing thickness variation is given by the equation:

$$h_x = h \left\{1 - T_x \left(\frac{x-b}{a-b}\right)^n\right\} \quad (14)$$

The Case II plate (parabolically decreasing-increasing) and Case III plate (parabolically increasing-decreasing) thickness variation of (Figure 2) are given by the equations:

$$h_x = h \left\{1 - T_x \left(1 - \text{abs}\left(1 - 2\left(\frac{x-b}{a-b}\right)\right)\right)^n\right\} \quad (15)$$

$$h_x = h \left\{ 1 - T_x a b s \left(1 - 2 \left(\frac{x-b}{a-b} \right)^n \right) \right\} \quad (16)$$

where, $n = 2$ for parabolic thickness variation. The total volume of the plate plus patches as well as the unloaded plate is kept constant and is given by the equation:

$$\text{Volume} = \pi(a^2 - b^2)h = \int_b^a (a^2 - b^2)h_x dx \quad (17)$$

In this paper, a comparison for the effect of frequency parameters, effect of sound power levels, average radiation efficiency and peak sound power level is obtained for parabolic tapered plates. The out of plane $(m, n)^{\text{th}}$ modes in Z direction for the plate with different attachment of rectangular and concentric stiffener patches at different positions with different parabolically tapered varying thickness is considered. The plate is made tapered with different taper ratios of 0.25, 0.50 and 0.75. The mass of the plate plus rectangular or concentric patches are kept constant for this analysis. The inner clamped and outer free boundary condition is taken. Three arrangements of plates with different combinations of rectangular or concentric stiffener patches are considered as shown in Figure 3. The selection of different combinations of rectangular or concentric stiffener patches are such that the mass of the unloaded plate is equal to mass of the rectangular or concentric stiffener patches plus plate and in all three cases mass of the plate with stiffener patches remains constant. The specifications and the material properties of an annular circular plate with attached rectangular and concentric stiffener patches are reported in Table 1. Rayleigh integral has been used for sound power calculation and ANSYS has been used as a tool for computation.

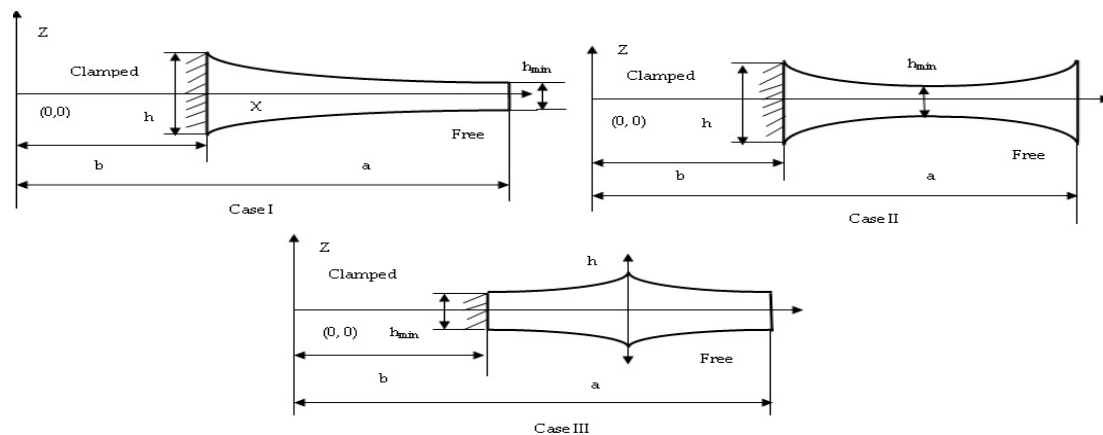
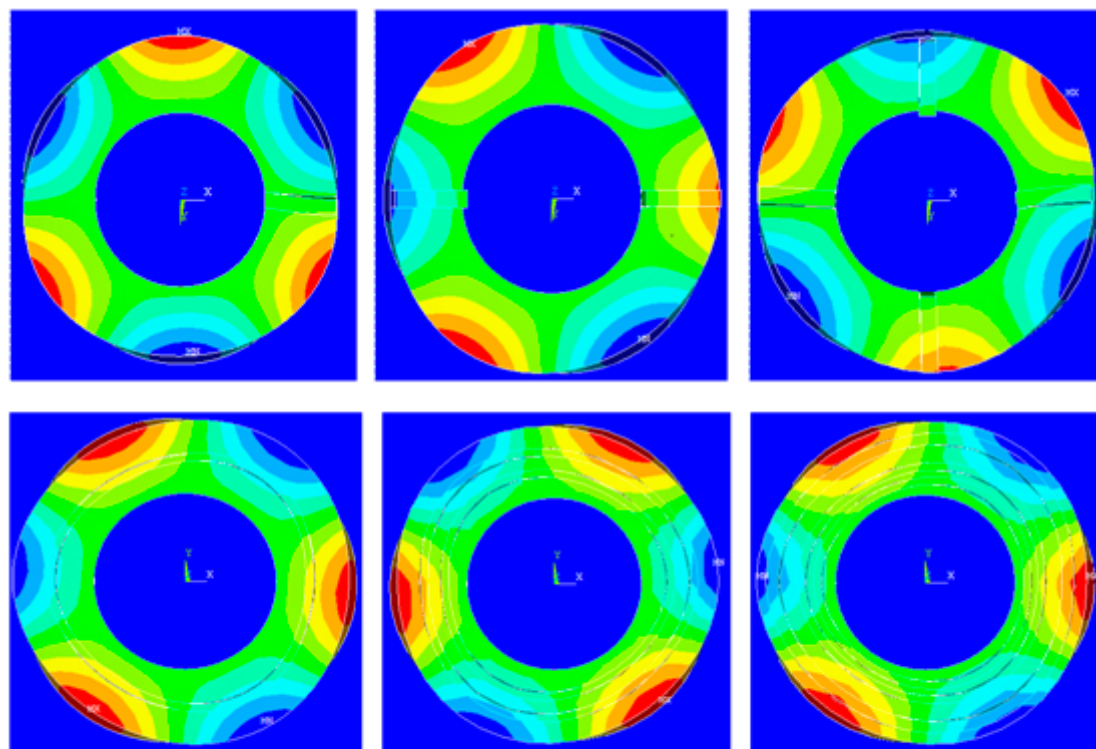


Figure 2. Plate with different parabolic varying thickness variations.

Table 1. The specifications and the material properties of an annular circular plate with different attachment of rectangular and concentric stiffener patches.

Dimension of the Plate with Different Stiffener Patches	Uniform Unloaded Plate	Plate with 1 Rectangular/Concentric Stiffener Patch	Plate with 2 Rectangular/Concentric Stiffeners Patches	Plate with 4 Rectangular/Concentric Stiffeners Patches
Outer radius (a) m	0.1515	0.1515	0.1515	0.1515
Inner radius (b) m	0.0825	0.0825	0.0825	0.0825
Radii ratio, (b/a)	0.54	0.54	0.54	0.54
Length of rectangular stiffener (m)	-	0.069	0.069	0.069
Width of rectangular stiffener (m)	-	0.0145	0.00828	0.00483
Thickness (h) of plate with rectangular stiffener (m)	0.0315	0.040	0.035	0.030
Density, ρ (kg/m ³)	7905.9	7905.9	7905.9	7905.9
Young's modulus, E (GPa)	218×10^9	218×10^9	218×10^9	218×10^9
Poisson's ratio, ν	0.305	0.305	0.305	0.305
Width of concentric stiffener (m)	-	0.0069	0.0069	0.0069
Thickness (h) of plate with concentric stiffener (m)	0.03150	0.007875	0.0039375	0.00196875



(a) Plate with 1 patch
Mass of patch = 0.1
times mass of the plate

(b) Plate with 2 patches
Mass of each patches = 0.05
times mass of the plate

(c) Plate with 4 patches
Mass of each patches = 0.025
times mass of the plate

Figure 3. Plate with different arrangements of rectangular and concentric stiffener patches with (0, 3) modes.

3. Results and Discussion

3.1. Validation of Natural Frequency Parameter and Acoustic Power Calculation

In this paper, the natural frequency parameter of a uniform unloaded annular circular plate is validated with the published result of Lee et al. [10] and is reported in Table 2. In reference [10], Lee et al. provide the solution for the natural frequency parameter of a uniform annular circular plate by Thick and thin plate theories. In our study we have calculated our result using FEM by taking the same dimension of plate as that of Lee et al. From Table 2, it is clearly understand that in this paper the results obtained are almost equal to the published results [10]. For the acoustic power calculation, the computed analytical, numerical and published experimental results [10] are considered as reported in Figure 4. From Figure 4, a good agreement of computed acoustic results is seen to be obtained analytically and numerically in line with published experimentally results [10].

Table 2. Validation and comparison of natural frequency parameter λ^2 of uniform clamped-free annular circular plate obtained in the present work with that of the published result of Lee et al. [10].

Plate	Mode	Non Dimensional Frequency Parameter, λ^2	
		H. Lee et al. [10]	Present Work
Uniform plate b/a = 0.54 h/a = 0.21	(0,0)	11.96	13.4929
	(0,1)	13.43	13.4946
	(0,2)	15.28	14.1185
	(0,3)	18.75	16.6681

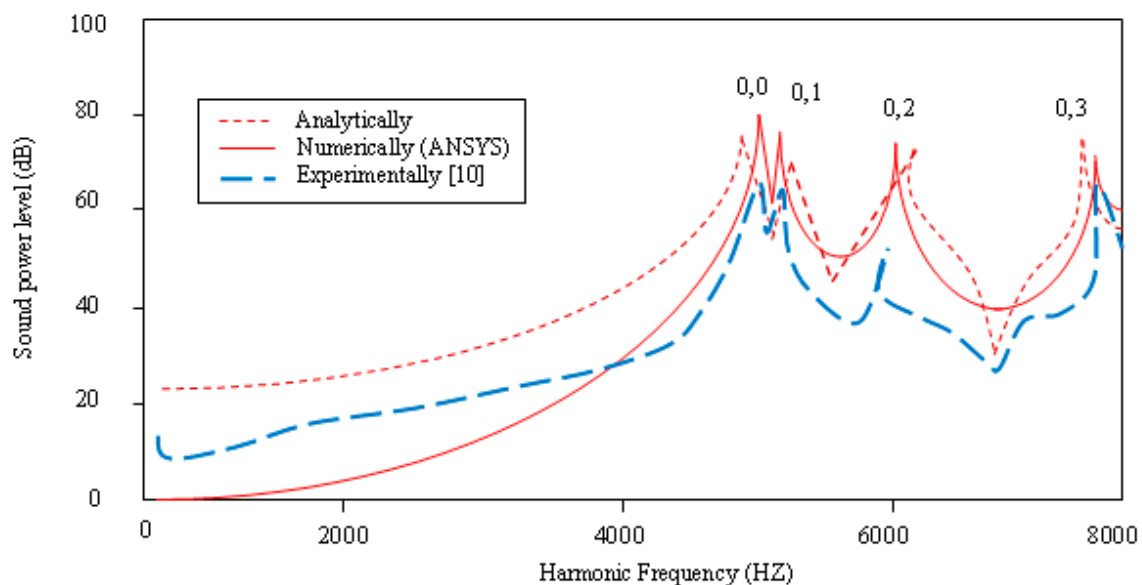


Figure 4. Comparison of sound power level analytically, numerically and experimentally for unloaded plate with uniform thickness for taper ratio $T_x = 0.00$.

3.2. Effect of Natural Frequency Parameter (λ^2) of Plate with Different Combinations of Rectangular and Concentric Stiffener Patches with Different Taper Ratios

In this paper, the effect of the natural frequency parameter (λ^2) is investigated for annular plates with different attachment of rectangular and concentric stiffener patches for different positions. The analysis is made for annular plates with different cases of parabolic thickness variations keeping the mass of the plate plus patches constant. Table 3 compares the first four natural frequency parameters numerically of a uniform unloaded plate for taper ratio, $T_x = 0.00$, with different attachments of rectangular and concentric stiffener patches along with the percentage variation of λ^2 . It is clear from Table 3 that the plate with different arrangements of rectangular stiffener patches has the same effect of natural frequency parameters as that of the unloaded plate for taper ratio, $T_x = 0.00$. However, for concentric stiffener patches, the natural frequency parameter decreases with addition of patches and minimum for 4 concentric stiffener patches. In Table 3, the negative variation of frequency parameter is calculated by $\frac{l^2(\text{stiffener}) - l^2(\text{original})}{l^2(\text{original})} * 100$. Figures 5 and 6 show the comparison of negative % variation of natural frequency parameter with different modes for a plate with taper ratio, $T_x = 0.00$ and with different attachment of rectangular and concentric stiffener patches. From Figure 5 it is shown that due to less stiffness associated with these modes, the (0, 2) mode of plate with 4 rectangular stiffener patches and (0, 0) mode of plate with 1 rectangular stiffener patch show the lowest percentage variation of λ^2 . However, due to greater stiffness associated with the (0, 1) mode of plate with 1 rectangular stiffener patch, it showed the highest percentage variation of λ^2 . Furthermore, from Figure 6 it is observed that for concentric stiffener patches the (0, 1) mode of plate with all combinations of patches shows the highest value of percentage variation of λ^2 due to greater stiffness associated with this mode. However, for all the remaining modes (0, 0), (0, 2) and (0, 3) of plates with concentric stiffener patches the stiffness decreases and as a result the percentage variation of λ^2 decreases associated with these modes. Figure 7 shows the numerical comparison of natural frequency parameters λ^2 with modes for an unloaded plate and for a plate with 4 rectangular and 4 concentric stiffener patches for taper ratio, $T_x = 0.00$. It is clear from Figure 7 that the effect of natural frequency parameter due to 4 rectangular stiffener patches is almost same as that of the unloaded plate. However, a plate with 4 concentric stiffener patches shows little decrease in the frequency parameter due to greater stiffness associated with this plate with concentric patch. Tables 4–6 numerically compare the first four natural frequency parameter λ^2 of a plate with different combinations of rectangular and concentric stiffener patches for different cases of thickness variation with different taper ratios. It is observed from Tables 4–6 that the

natural frequency parameter for a plate with concentric patches for all thickness variations reduces more in comparison to rectangular patches with increasing taper ratios. This may be due to the lower stiffness of the plate associated with concentric patches. Furthermore, it is observed that the frequency parameter for a Case II plate (parabolically decreasing–increasing thickness variation) for all cases of thickness variation with different combinations of rectangular and concentric stiffener patches reduces more in comparison to a Case I plate (parabolic decreasing thickness variation). This is due to the lower stiffness associated with the Case II plate than that of the Case I plate. It is further investigated that the effect of the frequency parameter for a Case III plate with different attachment of rectangular and concentric stiffener patches (parabolic increasing–decreasing thickness variation) is almost same as that of uniform unloaded plate due to more stiffness associated with the Case III plate. However, for all cases of different rectangular and concentric stiffener patches, plate with different parabolically thickness variations alters its modes at higher taper ratios.

Table 3. Numerical comparison of first four natural frequency parameter λ^2 of uniform unloaded plate with different attachment of rectangular and concentric stiffener patches for taper ratio, $T_x = 0.00$.

Type of Stiffener Patches	Mode	Uniform Unloaded Plate	Plate with 1 Stiffener Patch	% of Negative Variation in λ^2	Plate with 2 Stiffener Patches	% of Negative Variation in λ^2	Plate with 4 Stiffener Patches	% of Negative Variation in λ^2
Rectangular Stiffener	(0,0)	13.4929	13.4819	−0.0815	13.4862	−0.04960	13.4877	−0.0385
	(0,1)	13.4946	13.4942	−0.00296	13.4937	−0.00669	13.4914	−0.0237
	(0,2)	14.1185	14.1148	−0.0262	14.1099	−0.06090	14.1053	−0.0934
	(0,3)	16.6681	16.6638	−0.0258	16.6638	−0.02580	16.6658	−0.0258
Concentric Stiffener	(0,0)	13.4915	13.4223	−0.518	13.3615	−0.964	13.3242	−1.260
	(0,1)	13.5023	13.4842	−0.148	13.4427	−0.444	13.3614	−1.037
	(0,2)	14.1214	14.0815	−0.283	14.0235	−0.708	13.9212	−1.416
	(0,3)	16.6762	16.6014	−0.419	16.5056	−1.019	16.3843	−1.740

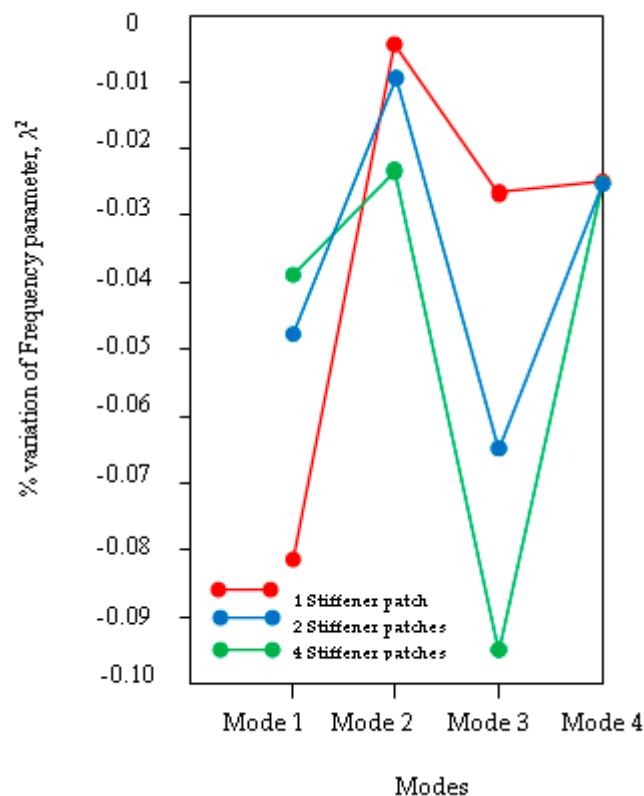


Figure 5. Comparison of % variation of natural frequency parameter with different modes for plate with taper ratio, $T_x = 0.00$ and with different attachments of rectangular stiffener patches.

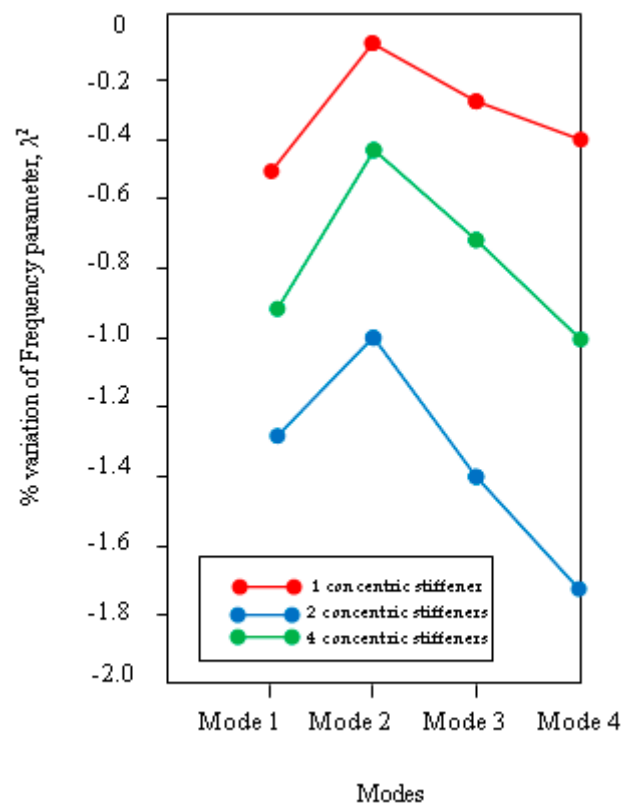


Figure 6. Comparison of % variation of natural frequency parameter with different modes for a uniform plate with taper ratio, $T_x = 0.00$ and with different attachment of concentric stiffener patches.

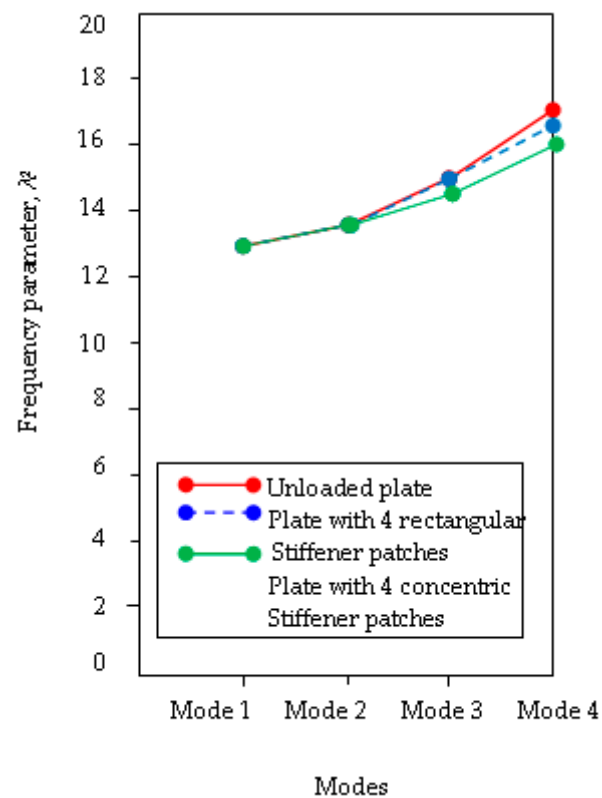


Figure 7. Comparison of variation of different frequency parameter with different modes for an unloaded plate and for a plate with 4 rectangular and 4 concentric stiffener patches.

Table 4. Numerical comparison of natural frequency parameter λ^2 of a plate with 1 rectangular and 1 concentric stiffener patch for different thickness variations and for different taper ratios T_x .

Case	Mode	Natural Frequency Parameter, λ^2							
		$T_x = 0.00$		$T_x = 0.25$		$T_x = 0.50$		$T_x = 0.75$	
		Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener
I	(0,0)	13.4819	13.4223	12.9912	12.9212	12.4711	12.2552	11.9311	11.8025
	(0,1)	13.4957	13.4842	12.9759	13.0540	12.4449	12.7821	11.8929	12.1634
	(0,2)	14.1148	14.0815	13.6148	13.8225	13.0942	13.3244	12.5513	13.0512
	(0,3)	16.6638	16.6014	16.0741	16.1024	15.4619	15.6012	14.8262	15.1020
II	(0,0)	13.4819	13.4223	12.8960	12.8021	12.2729	12.3230	11.6210	11.6254
	(0,1)	13.4957	13.4842	12.8799	12.9523	12.2424	12.5422	11.5764	11.7651
	(0,2)	14.1148	14.0815	13.5199	13.5535	12.8951	12.8641	12.2392	12.6821
	(0,3)	16.6638	16.6014	15.9622	16.0202	15.2286	15.4021	14.4611	14.9248
III	(0,0)	13.4819	13.4223	13.4891	13.3844	13.4917	13.4012	13.4914	13.4030
	(0,1)	13.4957	13.4842	13.4791	13.4214	13.4816	13.4414	13.4814	13.4342
	(0,2)	14.1148	14.0815	14.1116	14.0025	14.1142	14.0521	14.1139	14.0614
	(0,3)	16.6638	16.6014	16.6601	16.5254	16.6632	16.5528	16.6629	16.5403

Table 5. Numerical comparison of natural frequency parameter λ^2 of plate with 2 rectangular and 2 concentric stiffener patches for different thickness variations and for different taper ratios T_x .

Case	Mode	Natural Frequency Parameter, λ^2							
		$T_x = 0.00$		$T_x = 0.25$		$T_x = 0.50$		$T_x = 0.75$	
		Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener
I	(0,0)	13.4862	13.3625	12.9886	12.8938	12.4699	12.4829	11.9583	12.0021
	(0,1)	13.4937	13.4421	12.9782	12.9436	12.4463	12.5335	11.9217	12.0554
	(0,2)	14.1099	14.0275	13.6108	13.9637	13.0904	13.2724	12.5774	13.0068
	(0,3)	16.6638	16.5028	16.0744	15.9829	15.4625	15.5421	14.8610	15.0227
II	(0,0)	13.4862	13.3665	12.8937	12.7717	12.2720	12.2929	11.6478	11.7631
	(0,1)	13.4937	13.4485	12.8810	12.8342	12.2435	12.3227	11.6042	11.9229
	(0,2)	14.1099	14.0267	13.5156	13.8732	12.9216	12.7811	12.2648	12.6456
	(0,3)	16.6638	16.5057	15.9625	15.9018	15.2646	15.2882	14.4954	14.7824
III	(0,0)	13.4862	13.3619	13.4831	13.3228	13.5170	13.3415	13.5245	13.3238
	(0,1)	13.4937	13.4467	13.4860	13.4039	13.5248	13.4262	13.5168	13.402472
	(0,2)	14.1099	14.0228	14.1070	13.9442	14.1421	13.9594	14.1421	13.9684
	(0,3)	16.6638	16.5025	16.6603	16.4311	16.7020	16.4435	16.7017	16.4224

Table 6. Numerical comparison of natural frequency parameter λ^2 of plate with 4 rectangular and 4 concentric stiffener patches for different thickness variations and for different taper ratios T_X .

Case	Mode	Natural Frequency Parameter, λ^2							
		$T_X = 0.00$		$T_X = 0.25$		$T_X = 0.50$		$T_X = 0.75$	
		Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener	Rectangular Stiffener	Concentric Stiffener
I	(0,0)	13.4877	13.3214	12.9897	12.8342	12.4711	11.9235	11.9314	11.4862
	(0,1)	13.4914	13.3624	12.9788	12.8547	12.4463	11.9567	11.8937	11.5324
	(0,2)	14.1053	13.9232	13.6062	13.8615	13.0858	13.6681	12.5439	12.4683
	(0,3)	16.6658	16.3814	16.0761	15.8823	15.4639	14.9884	14.8279	14.3616
II	(0,0)	13.4877	13.3227	12.8948	12.7175	12.2732	11.7225	11.6216	11.1449
	(0,1)	13.4914	13.3612	12.8813	12.7619	12.2432	11.8034	11.5770	11.3425
	(0,2)	14.1053	13.9224	13.5110	13.7721	12.8876	12.1841	12.2320	12.0434
	(0,3)	16.6658	16.3825	15.9642	15.8015	15.2314	14.6248	14.4629	14.1828
III	(0,0)	13.4877	13.3234	13.4845	13.2837	13.4871	13.3016	13.4868	13.2926
	(0,1)	13.4914	13.3624	13.4883	13.3227	13.4908	13.3326	13.4908	13.3223
	(0,2)	14.1053	13.9285	14.1024	13.8212	14.1047	13.8418	14.1044	13.8434
	(0,3)	16.6658	16.3824	16.6624	16.2865	16.6652	16.2712	16.6649	16.2824

3.3. Acoustic Response Solution of Tapered Annular Circular Plate with Different Combination of Rectangular and Concentric Stiffener Patches with Different Taper Ratios

In this paper, the sound power level (dB, reference = 10^{-12} watts) of an annular circular plate with a different attachment of rectangular and concentric stiffener patches is estimated. The plate for the sound power level is analyzed for all cases of different parabolic thickness variation due to transverse vibration. The taper ratio is maintained from a range (0.00–0.75). The sound power level is investigated by applying 1 N concentrated load under time-varying harmonic excitations at different excitation locations at different nodes, and a harmonic frequency range of (0–8000) HZ is taken to determine the sound radiation characteristic. The Case I plate with parabolic decreasing thickness variation is taken as a convergence study. Figures 8 and 9 compare the sound power level for a Case I plate obtained analytically and numerically for the taper ratio $T_x = 0.75$ for 4 rectangular stiffener patches and 4 concentric stiffener patches, respectively, for different modes. A good agreement of computed results is seen in the comparison of sound power as depicted from Figures 8 and 9. Figures 10–12 shows the numerical comparison of sound power level for Case I plate with different combinations of rectangular stiffener patches for different taper ratios and for different modes under forced excitation. From Figure 10, a sound power level up to 30 dB is seen, and we do not get any broad range of frequencies for different taper ratios for plate with 1 rectangular stiffener patch. However, for a sound power level up to 40 dB, we get all taper ratios, $T_x = 0.00, 0.25, 0.50$ and 0.75 , with a broad range of frequencies in frequency band A only with 1 rectangular stiffener.

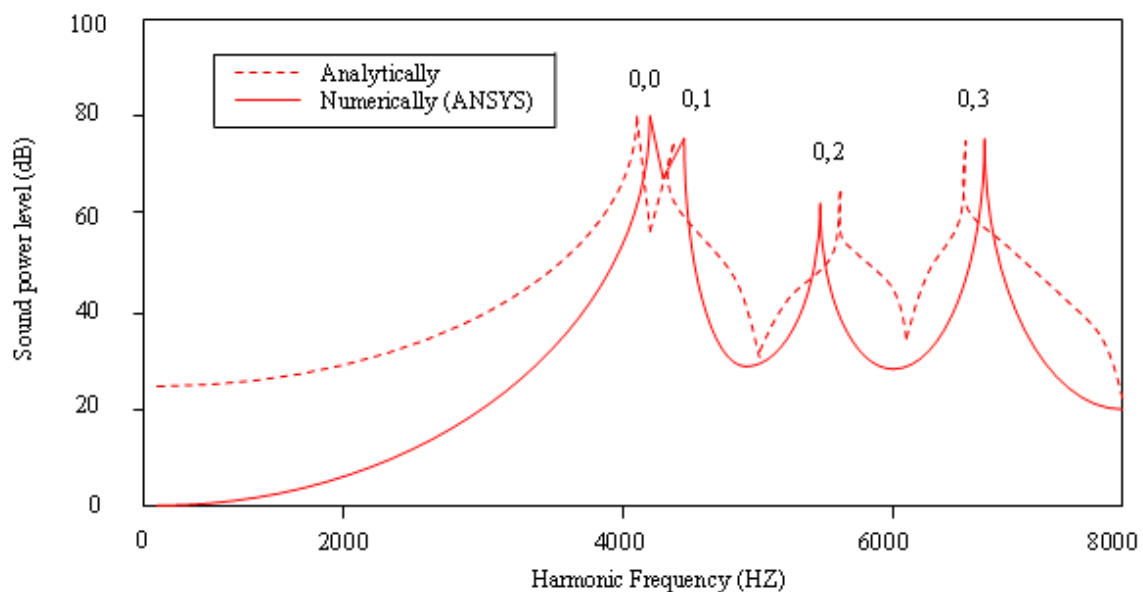


Figure 8. Comparison of sound power level analytically and numerically for annular plate attached with 4 rectangular stiffener patches and having parabolic decreasing thickness variations (Case I) for taper ratio $T_x = 0.75$.

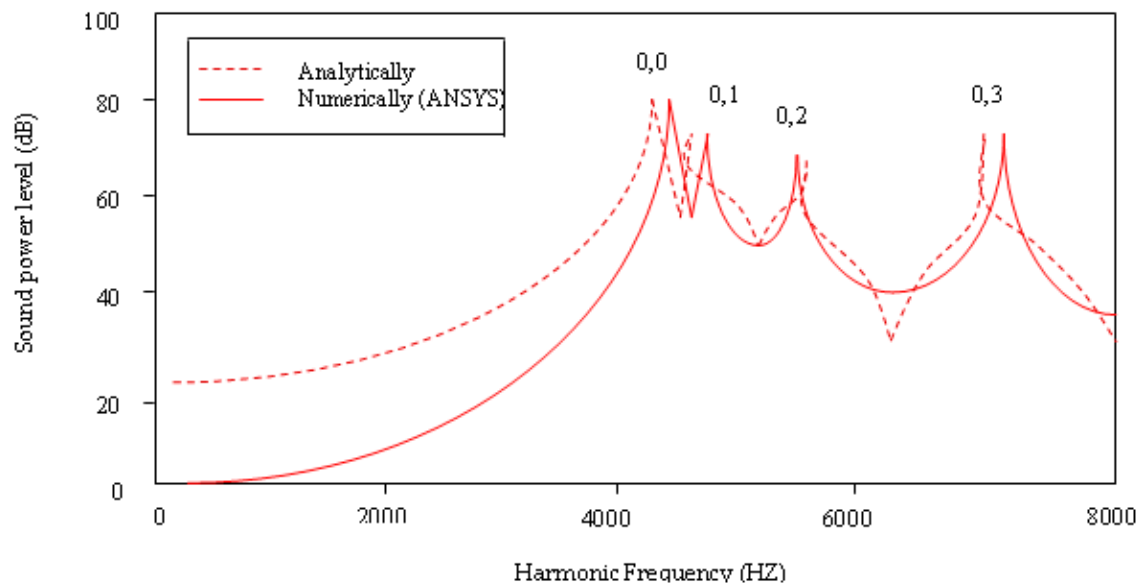


Figure 9. Comparison of sound power level analytically and numerically for annular plate attached with 4 concentric stiffener patches and having parabolic decreasing thickness variations (Case I) for taper ratio $T_x = 0.75$.

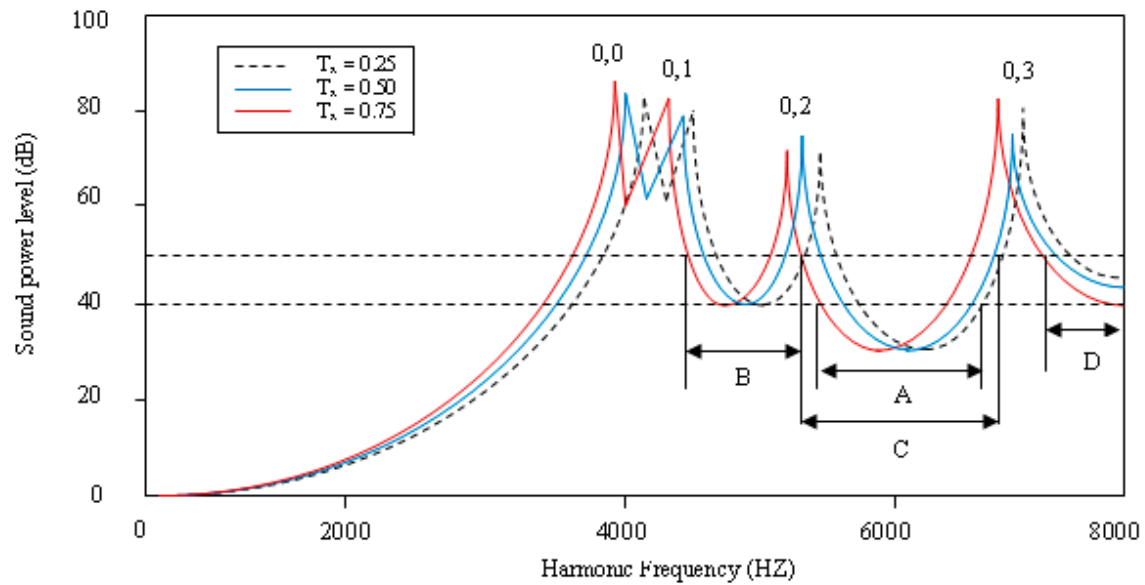


Figure 10. Numerical comparison of sound power level for annular plate attached with 1 rectangular stiffener patch and having parabolic decreasing thickness variations (Case I) for different taper ratios T_x .

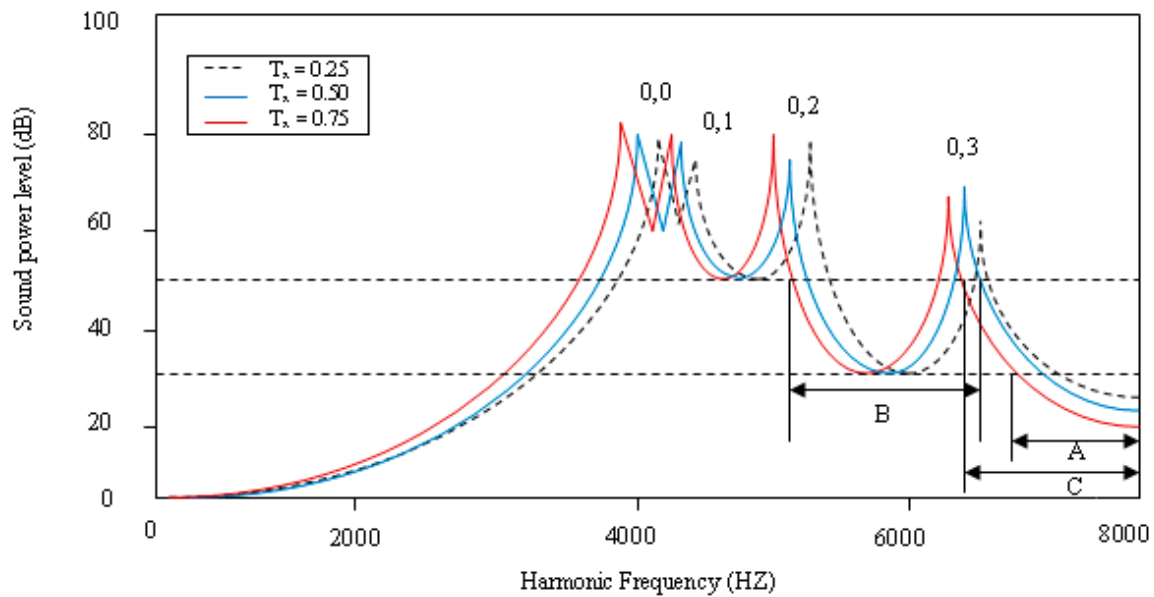


Figure 11. Numerical comparison of sound power level for annular plate attached with 2 rectangular stiffener patches and having parabolic decreasing thickness variations (Case I) for different taper ratios T_x .

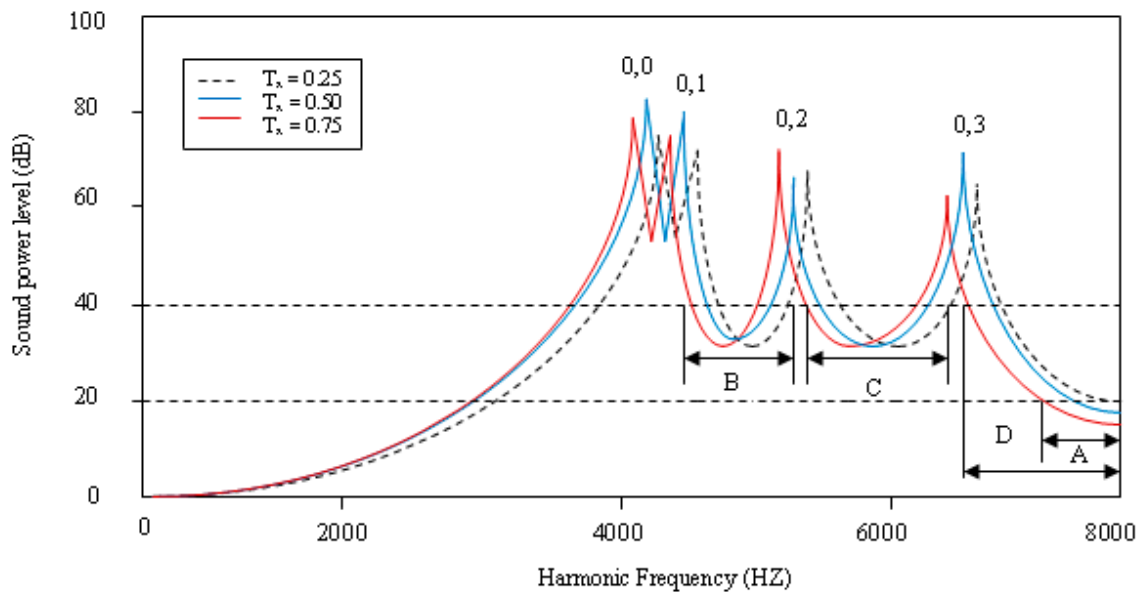


Figure 12. Numerical comparison of sound power level for annular plate attached with 4 rectangular stiffener patches and having parabolic decreasing thickness variations (Case I) for different taper ratios T_x .

Patches are as reported in Figure 10. It is noteworthy that for sound power level up to 50 dB, we get a broader range of frequencies for sound power level in different frequency bands, i.e., B, C and D, as reported in Figure 10. From Figure 11, it is apparent that for a sound power level up to 20 dB, we do not get any broad range of frequencies for plate with 2 rectangular stiffener patches. However for a sound power level up to 30 dB, we get the broad range of frequencies in frequency band A only with taper ratio, $T_x = 0.00, 0.25, 0.50$ and 0.75 as available design alternative. For a sound power level up to 50 dB, we get more broad range of frequencies for the sound power level in different frequency bands, i.e., B and C, as reported in Figure 11. From Figure 12, it is shown that for a sound power level up to 10 dB, we do not get any broad range of frequencies for plate with 4 rectangular stiffener patches. But for a sound power level up to 20 dB, we get the broad range of frequencies in frequency

bands A only with all taper ratios, $T_x = 0.00, 0.25, 0.50$ and 0.75 and, therefore, this is the available design alternative. However, for a sound power level up to 40 dB, we get more design options for the sound power level in different frequency bands, i.e., B, C and D as reported in Figure 12. Furthermore, Figures 13–15 show the numerical comparison of the sound power level for a Case I plate with different combinations of concentric stiffener patches for different taper ratios. From Figure 13, it is seen that for a sound power level up to 30 dB, we do not get any design options for different taper ratios for the plate with both 1 concentric stiffener patch and 4 concentric stiffener patches; and for 2 concentric stiffener patches, we do not find any sound power level upto 10 dB. However, for a sound power level up to 40 dB, we get all taper ratios, $T_x = 0.00, 0.25, 0.50$ and 0.75 as design options in frequency bands A and B for plate with 1 concentric stiffener patch combinations as reported in Figure 13. It is noteworthy that for a sound power level up to 50 dB, we get more design options for the sound power level in different frequency bands, i.e., C, D and E as reported in Figure 13. From Figure 14, it is apparent that for a sound power level up to 20 dB, then in frequency band A only taper ratio $T_x = 0.00, 0.25, 0.50$ and 0.75 are available design alternatives for a plate with the 2 concentric stiffener patches combination. But for sound power level up to 30 dB, we get wider frequency bands, B, C and D for different taper ratios as reported in Figure 14. From Figure 15, it is seen that for a sound power level up to 40 dB is possible only in frequency bands A only with all taper ratios, $T_x = 0.00, 0.25, 0.50$ and 0.75 and, therefore, this is the available design alternative for plate with 4 concentric stiffener patches combination. However, for a sound power level up to 60 dB, we get a broader range of frequency denoted as B and C for all taper ratios as reported in Figure 15.

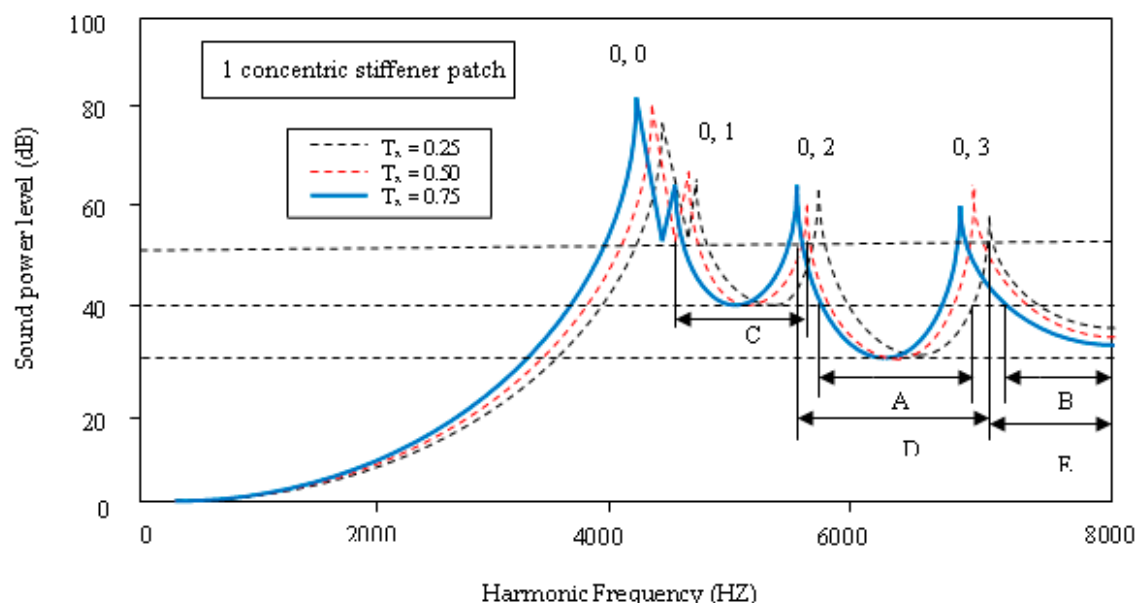


Figure 13. Numerical comparison of sound power level for annular plate attached with 1 concentric stiffener patch and having parabolic decreasing thickness variations (Case I) for different taper ratios T_x .

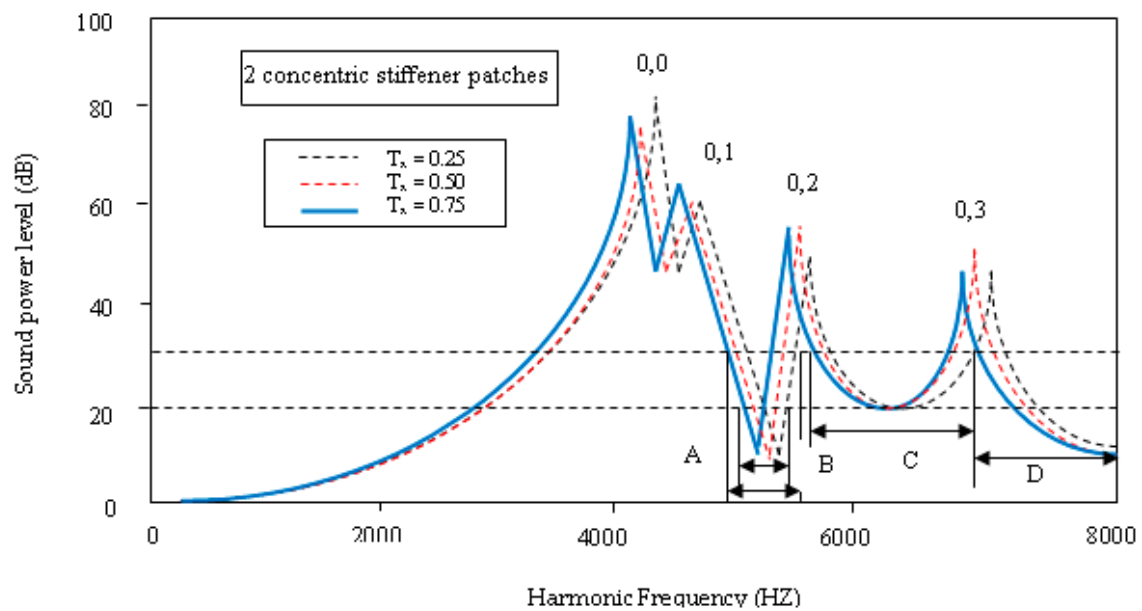


Figure 14. Numerical comparison of sound power level for annular plate attached with 2 concentric stiffener patches and having parabolic decreasing thickness variations (Case I) for different taper ratios T_x .

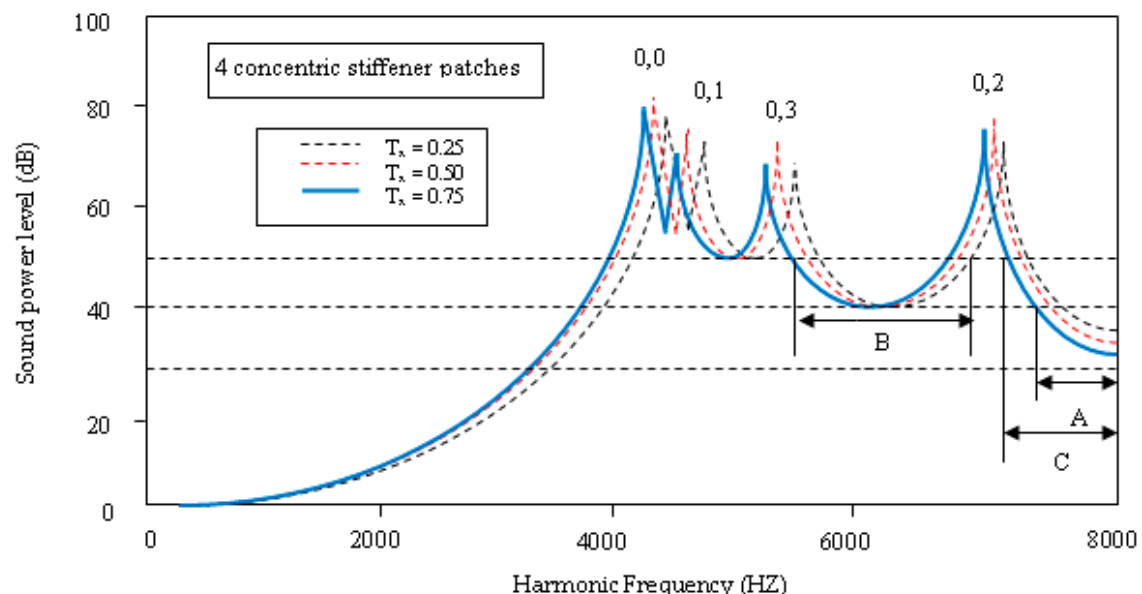


Figure 15. Numerical comparison of sound power level for annular plate attached with 4 concentric stiffener patches and having parabolic decreasing thickness variations (Case I) for different taper ratios T_x .

It may be inferred from Figures 10–15 that a plate with different combinations of rectangular and concentric stiffener patches plays a significant role in sound power reduction in different frequency bands. A plate with 4 rectangular stiffener patches combination causes maximum sound power level reduction in comparison to 1 rectangular stiffener patch and 2 rectangular stiffener patch combinations for a Case I plate; whereas, for a plate with 4 concentric stiffener patches the lowest sound power is observed in comparison to other combinations. However, the stiffness contribution due to various taper ratios has a very limited impact on sound power level reduction in comparison to that of modes and excitation locations of plate with different combinations of rectangular and concentric stiffener patches. Furthermore, from Figures 10–15, it is observed that for an excitation frequency

up to 2000 HZ, the effect of different combinations of rectangular and concentric stiffener patches and stiffness variation due to different taper ratios do not have a significant effect on sound power radiation for clamped-free boundary condition. However, when the excitation frequency increases beyond 2000 HZ and up to the first peak, the sound power level is higher for only higher taper ratios for a Case I plate with both 1 rectangular stiffener and 1 concentric stiffener patch, and variation of sound power level due to variation of peaks for different taper ratios is observed for a plate with both 2 rectangular stiffener and 2 concentric stiffener patches and for 4 rectangular stiffener and 4 concentric stiffener patch combinations. For a Case II plate, beyond a forcing frequency 2000 HZ, the highest sound power level is associated with plate for 2 rectangular stiffener patches combinations; while for a Case III plate, the sound power level is seen to decrease for all combinations of rectangular stiffener patches with increasing taper ratio. Similar effect is observed for plate with concentric stiffener patches where plate with 2 concentric stiffener patches is seen to have highest radiation power. Again for case III plate the sound power is seen to be decreased for all combination of concentric stiffener patches. Furthermore, different modes do influence the sound power peaks as evident from Figures 10–15. Sound power level peak obtained for different modes (0, 0) and (0, 1) almost remain same for different taper ratios. However, no such sound power similarity of modes (0, 0) and (0, 1) is observed for plates with concentric stiffener patches. The sound power level does shift towards a lower frequency with increasing taper ratio for all combinations of rectangular and concentric stiffener patches. For a higher frequency beyond 4000 HZ, it is observed that different taper ratios alter its stiffness at higher forcing frequency for different cases of thickness variation. It is noteworthy that for a higher frequency beyond 4000 HZ and up to 8000 HZ, a plate with different combination of rectangular and concentric stiffener patches alters its stiffness at higher forcing frequency and the acoustic power curve tends to intersect each other at this high forcing region. Table 7 compares the peak sound power level of a plate having different parabolically varying thickness with different combinations of rectangular and concentric stiffener patches for a taper ratio $T_x = 0.75$. It is interesting to note that a plate with 4 rectangular stiffener patches combination shows the lowest peak sound power level among all cases of thickness variations, and the lowest peak sound of 77 dB is obtained for Case III plate whereas the highest peak sound power level of 84 dB is obtained for a Case II plate with 2 rectangular stiffener patches combination. Similar effect is again observed for plate with concentric stiffener patch combination. The lowest sound power of 76 dB is observed for the plate with the 4 concentric stiffener patches combination, and the highest power of 83 dB is observed for plate with 1 concentric stiffener patch combination. Figures 16–21 shows the numerical comparison of sound power levels for Case I, Case II and Case III plates for different combinations of rectangular and concentric stiffener patches for taper ratio $T_x = 0.75$. From Figures 16–21, it is observed that for all cases of thickness variation and for excitation frequency up to 2000 HZ, different parabolic thickness variation does not have any significant effect on sound power radiation. Furthermore, from Figures 16–18 it is seen that beyond excitation frequency of 2000 HZ and up to the first peak, a Case II plate with 2 rectangular stiffener patches shows the highest radiation power of 84 dB in comparison to a radiation power of 82 dB for a Case I plate with 1 rectangular stiffener patch combination. However, at this forcing frequency of 2000 HZ case III plate remains unaffected and shows the lowest peak sound level for all cases of thickness variations and so it is suggested that Case III plate is the lowest sound power radiator among all cases of thickness variation with different combinations of rectangular stiffener patches. Again, a similar effect is observed for plate with the concentric stiffener patches combination. Beyond 200 HZ, Case II plate with 2 concentric stiffener patches is a very good sound radiator of sound power 83 dB in comparison to 82 dB of plate with 1 stiffener patch combination. A Case III plate with all combination of concentric stiffener patches is found to be a poor sound radiator.

Table 7. Numerical comparison of peak sound power level and radiation efficiency for annular plate having different parabolically thickness variations with different attachments of rectangular and concentric stiffener patches for $T_x = 0.75$.

Type of Stiffener Patches	Plate Thickness Variation for Taper Ratio $T_x = 0.75$	Unloaded Plate Thickness Variation for Taper Ratio $T_x = 0.75$		Plate with 1 Rectangular/Concentric Stiffener Patch		Plate with 2 Rectangular/Concentric Stiffener Patches		Plate with 4 Rectangular/Concentric Stiffener Patches	
		Sound Power Level (dB)	Radiation Efficiency (σ_{mn})	Sound Power Level (dB)	Radiation Efficiency (σ_{mn})	Sound Power Level (dB)	Radiation Efficiency (σ_{mn})	Sound Power Level (dB)	Radiation Efficiency (σ_{mn})
Rectangular Stiffener	Case I	83	1.079	82	1.058	81	1.048	79	1.015
	Case II	85	1.135	81	1.045	84	1.094	78	1.002
	Case III	79	1.020	79	1.020	78	1.007	77	0.994
Concentric Stiffener	Case I	83	1.079	82	1.058	81	1.048	77	0.994
	Case II	85	1.135	81	1.045	83	1.079	79	1.020
	Case III	79	1.020	78	1.020	78	1.007	76	0.935

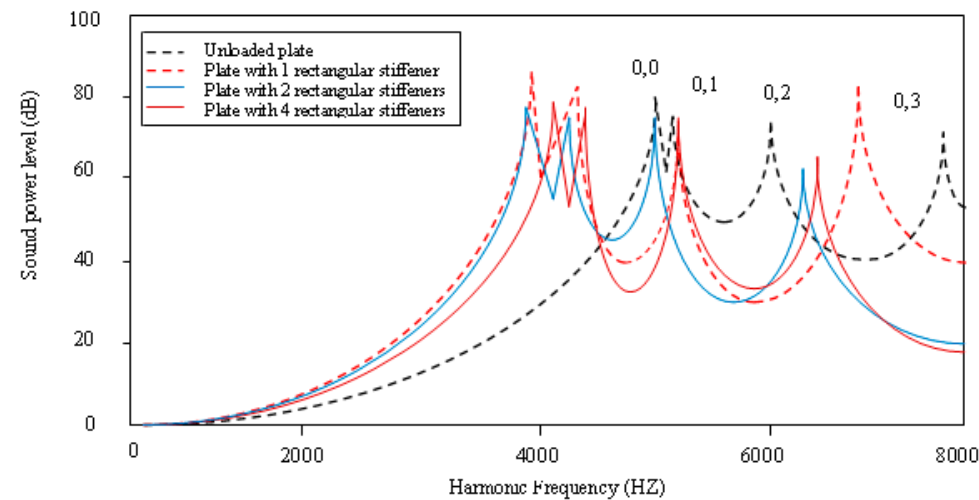


Figure 16. Numerical comparison of sound power level for annular plate having parabolic decreasing thickness variation (case I) for different attachments of rectangular stiffener patches for taper ratio $T_x = 0.75$.

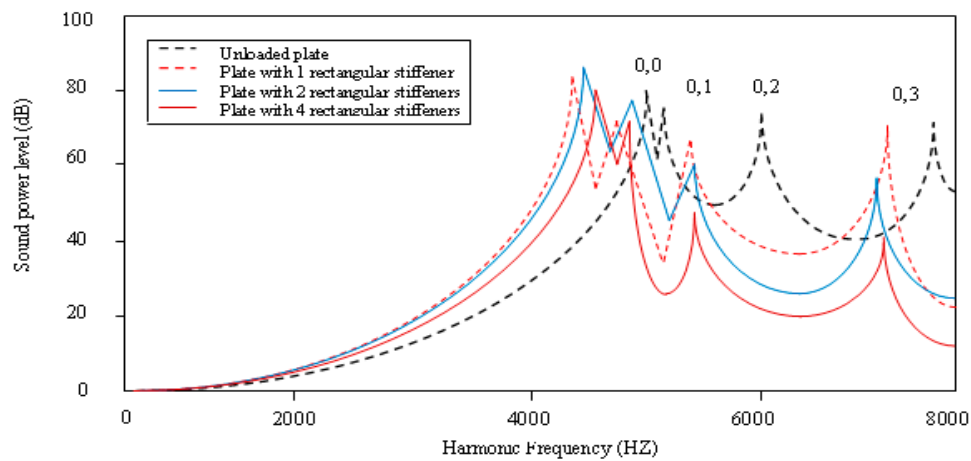


Figure 17. Numerical comparison of sound power level for annular plate having parabolic decreasing increasing thickness variation (Case II) for different attachments of rectangular stiffener patches for taper ratio $T_x = 0.75$.

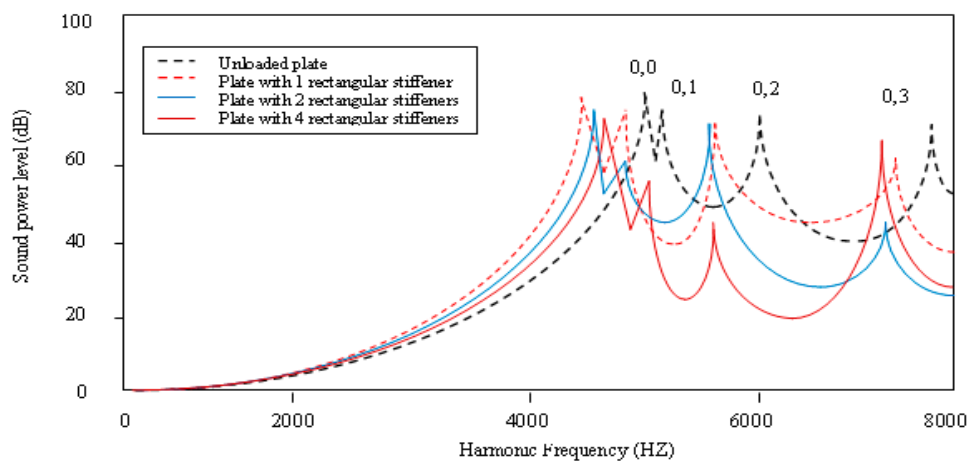


Figure 18. Numerical comparison of sound power level for annular plate having parabolic increasing decreasing thickness variations (Case III) for different combinations of rectangular stiffener patches for taper ratio $T_x = 0.75$.

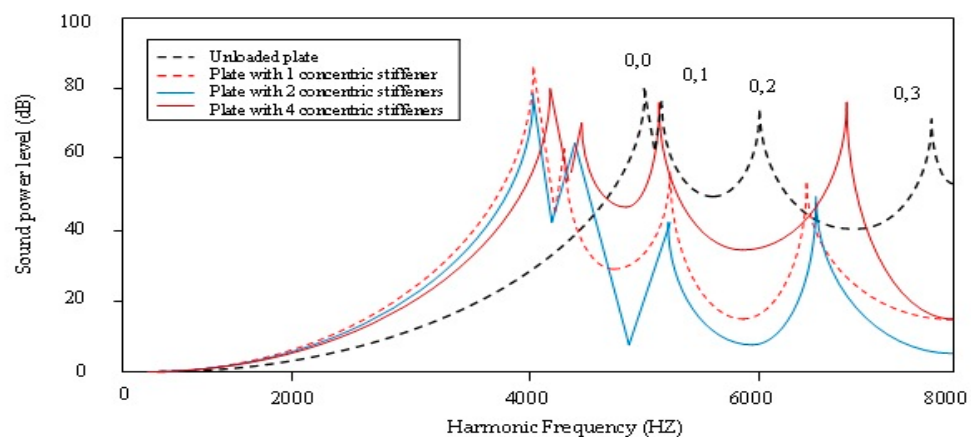


Figure 19. Numerical comparison of sound power level for annular plate having parabolic decreasing thickness variations (Case I) for different combinations of concentric stiffener patches for taper ratio $T_x = 0.75$.

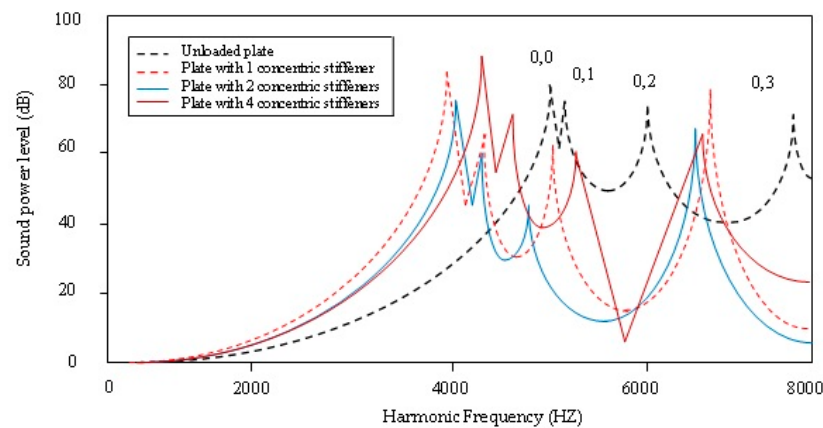


Figure 20. Numerical comparison of sound power level for annular plate having parabolic decreasing increasing thickness variations (Case II) for different combinations of concentric stiffener patches for taper ratio $T_x = 0.75$.

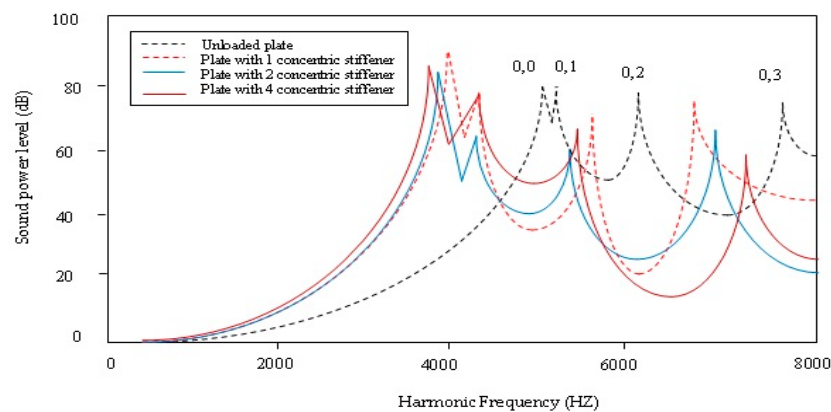


Figure 21. Numerical comparison of sound power level for annular plate having parabolic increasing decreasing thickness variations (Case III) for different combinations of concentric stiffener patches for taper ratio $T_x = 0.75$.

Figures 22 and 23 compare the analytical and numerical comparison of radiation efficiency (σ_{mn}) for a Case I plate with 4 rectangular stiffener patches and 4 concentric stiffener patches respectively having parabolically decreasing thickness variation for taper ratio $T_x = 0.75$. A good agreement of results is seen in the comparison of radiation efficiency as reported in Figures 22 and 23. Figures 24 and 25 show the variation of radiation efficiency with different taper ratios T_x for different combination of rectangular and concentric stiffener patches for a Case I plate with parabolically decreasing thickness variation. It is seen that for all combinations of rectangular and concentric stiffener patches, the effect of radiation efficiency due to different taper ratios is independent of exciting frequency up to 1000 HZ, but at a given forcing frequency a higher taper ratio causes higher radiation efficiency beyond 1000 HZ. However, sound power level peaks do shift towards a lower frequency as taper ratio increases. For higher frequency beyond 2000 HZ, different taper ratios alter its stiffness at higher forcing frequency and the radiation efficiency curves tend to intersect each other at this high forcing region. It is interesting to note that the radiation curve tends to unity in the frequency band 6800–7200 HZ and a clear peak is seen at this frequency band for all combination of rectangular and concentric stiffener patches. Furthermore, from Figures 24 and 25 it is seen that the radiation efficiency increases with the taper ratio for all combinations of rectangular and concentric stiffener patches. Out of these combinations, the Case II plate with 2 rectangular stiffener patches and 2 concentric stiffener patches delivers the highest radiation efficiency whereas Case I plate with both 1 rectangular and 1 concentric stiffener patch and 4 rectangular and 4 concentric stiffener patches is seen to be a moderate radiator as depicted in Table 7. However, at higher forcing frequency, it is seen

that both a plate with 4 rectangular and 4 concentric stiffener patches with all cases of thickness variation (Cases I, II and III) shows the least radiation efficiency for all combinations of rectangular stiffener patches, as evident from Table 7. Therefore, it is interesting to mention that a Case III plate shows the lowest radiation efficiency (σ_{mn}) for all cases of thickness variations and is a poor radiation emitter among all the thickness variations with different combinations of rectangular and concentric stiffener patches. Figure 26 shows the numerical comparison of radiation efficiency for a plate with both 4 rectangular and 4 concentric stiffener patches for taper ratio $T_x = 0.75$. It is found that the plate shows almost the same radiation efficiency as depicted from Figure 26. Figure 27 shows the numerical comparison of the sound power level for a plate with 4 rectangular and 4 concentric stiffener patches for taper ratio $T_x = 0.75$. It is observed that both the plates show almost the same peak for taper ratio $T_x = 0.75$. Hence, the effect of stiffness variation along with the modes has negligible effect for both the combinations.

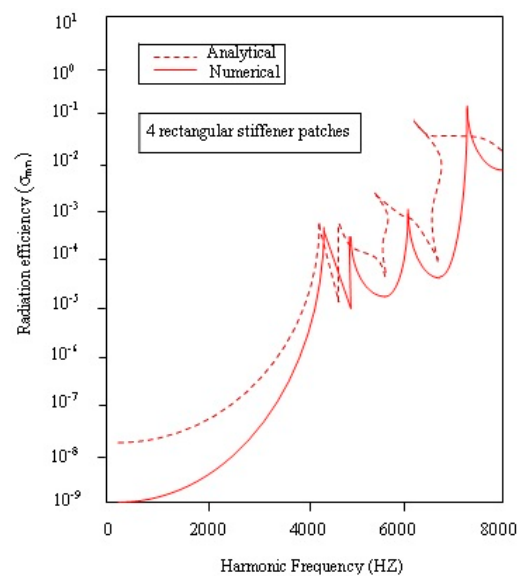


Figure 22. Comparison of radiation efficiency (σ_{mn}) analytically and numerically for annular plate attached with 4 rectangular stiffener patches and having parabolic decreasing thickness variations (Case I) for taper ratio $T_x = 0.75$.

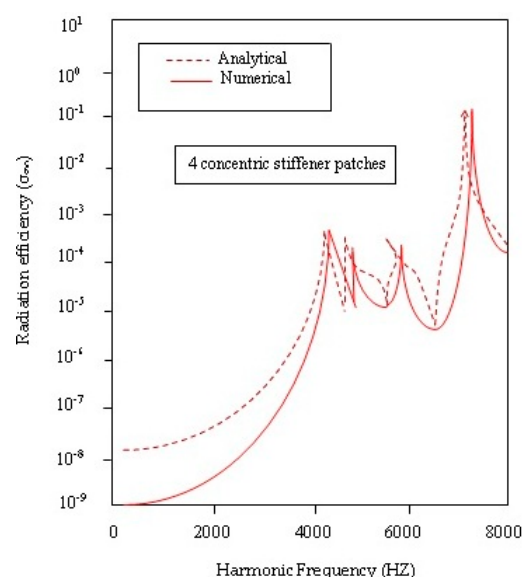


Figure 23. Comparison of radiation efficiency (σ_{mn}) analytically and numerically for annular plate attached with 4 concentric stiffener patches and having parabolic decreasing thickness variations (Case I) for taper ratio $T_x = 0.75$.

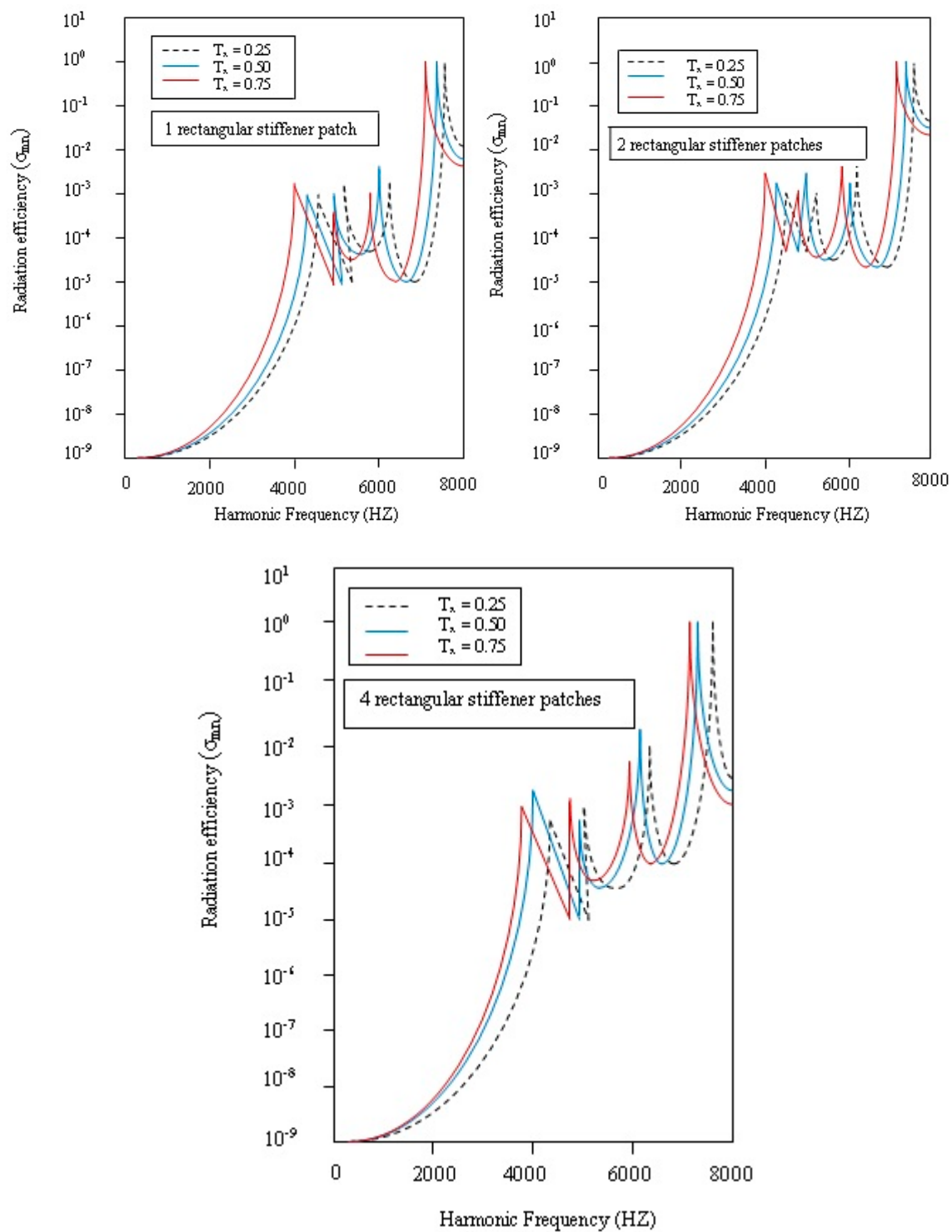


Figure 24. Numerical comparison of radiation efficiency (σ_{mn}) for annular plate having parabolically decreasing thickness variations (Case I) with different attachment of (a) 1 rectangular stiffener patch (b) 2 rectangular stiffener patches (c) 4 rectangular stiffener patches for taper ratio $T_x = 0.75$.

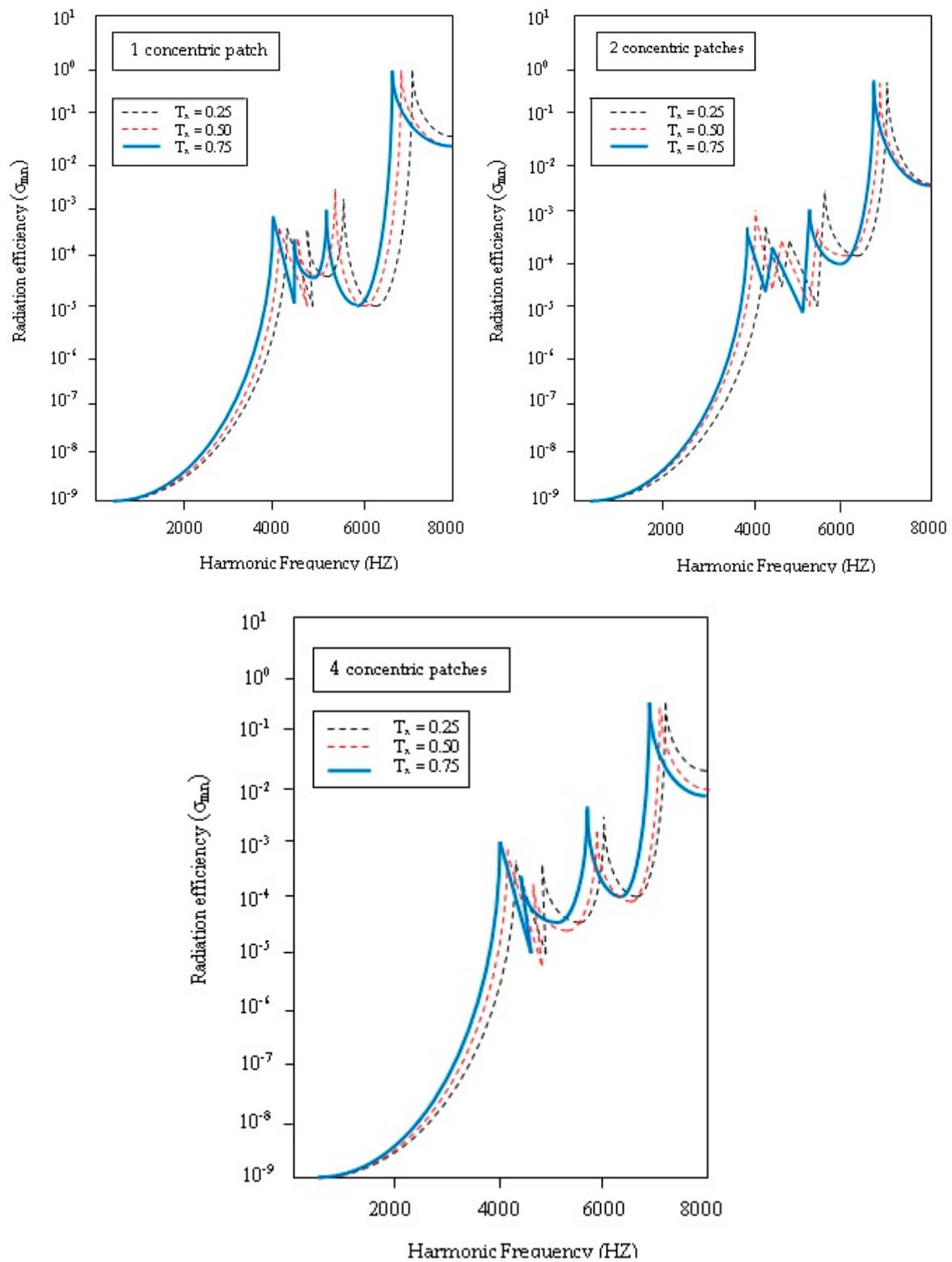


Figure 25. Numerical comparison of radiation efficiency (σ_{mn}) for annular plate having parabolically decreasing thickness variations (Case I) with different attachment of (a) 1 concentric patch (b) 2 concentric patches and (c) 4 concentric patches for taper ratio $T_x = 0.75$.

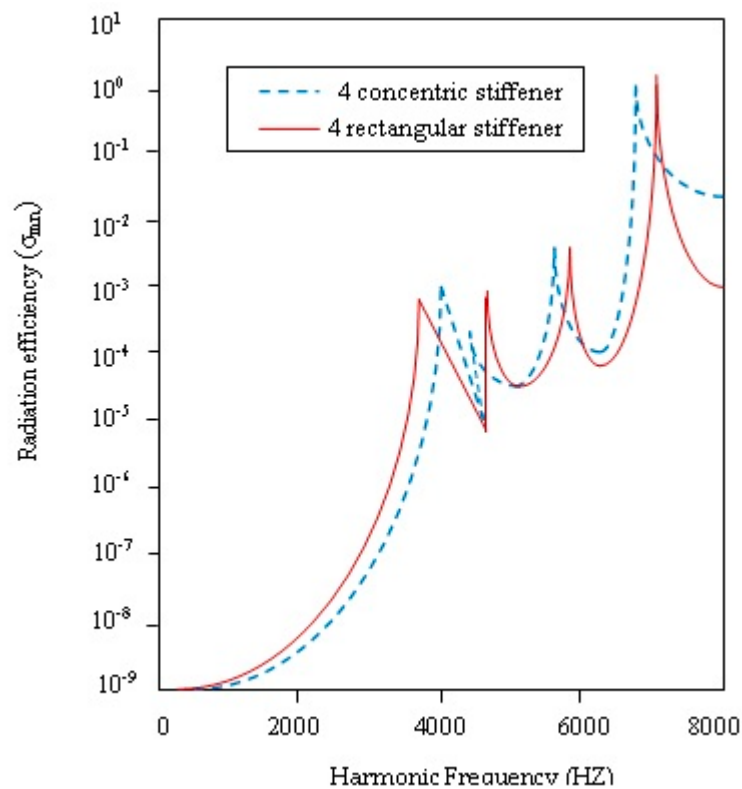


Figure 26. Numerical comparison of radiation efficiency (σ_{mn}) for annular plate attached with 4 rectangular and 4 concentric stiffener patches and having parabolic decreasing thickness variations (Case I) for taper ratio $T_x = 0.75$.

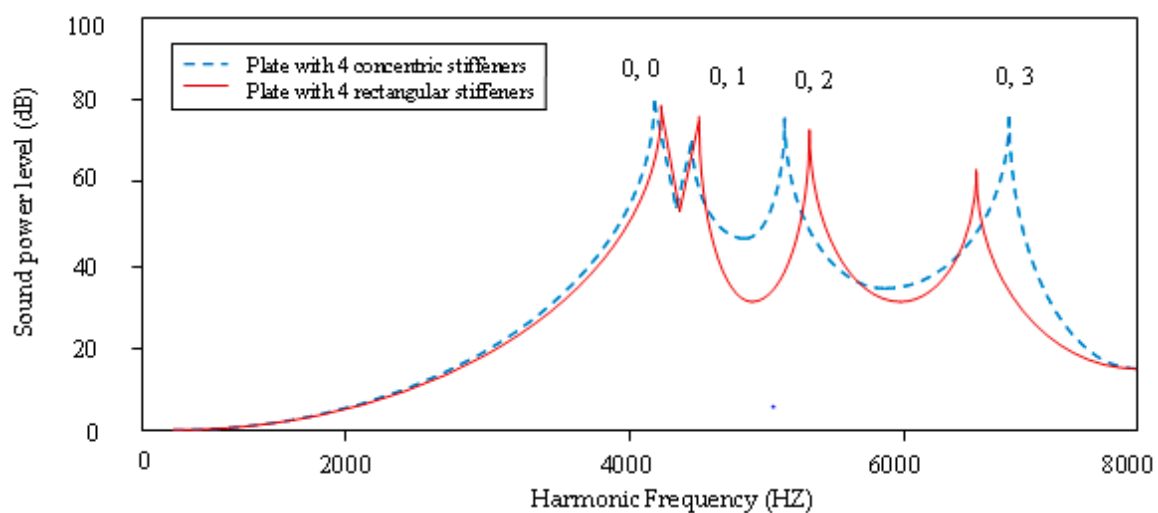


Figure 27. Numerical comparison of sound power level (dB) for annular plate attached with 4 rectangular and 4 concentric stiffener patches having parabolic decreasing thickness variations (Case I) for taper ratio $T_x = 0.75$.

3.4. Peak Sound Power Level Variation with Different Taper Ratios for All Combinations of Rectangular and Concentric Stiffener Patches Attached to a Plate

Peak sound power level for a plate was estimated for annular plates with different attachment of rectangular and concentric stiffener patches. The peak sound was considered for plates with different parabolically varying thickness. The different taper ratios were taken as reported in Figures 28 and 29, respectively. Furthermore, peak sound power level for different attachments of rectangular and

concentric stiffener patches attached to a plate is reported at the first peak which corresponds to (0, 0) mode of the plate. From Figure 28 it is seen that for a Case I plate with 1 rectangular stiffener patch combination, peak sound power level increases for increasing value of taper ratio whereas for 2 rectangular stiffener patches and 4 rectangular stiffener patches combinations, variations of peak sound power levels are observed for an increasing value of taper ratio. For a Case I plate, the maximum peak sound power level is obtained for taper ratio $T_x = 0.75$ for plate with 1 rectangular stiffener patch combination. Furthermore, it is seen that peak is minimum for taper ratio, $T_x = 0.25$ and maximum for taper ratio, $T_x = 0.50$ for plate with 4 rectangular stiffener patches combination, whereas for a plate with 2 rectangular stiffener patches combination, the peak is at a minimum for taper ratio $T_x = 0.25$ and maximum for taper ratio $T_x = 0.75$. Similarly, for a Case II plate, the maximum peak sound power level is obtained for taper ratio $T_x = 0.75$ and minimum peak is seen for taper ratio $T_x = 0.50$ for a plate with 2 rectangular stiffener patches combination. Also, for a Case II plate, it is interesting to note that peak sound power level increases for increasing value of taper ratio for plate with 1 rectangular stiffener patch combination and the peak is seen to be maximum for a taper ratio $T_x = 0.75$. However, for a plate with 4 rectangular stiffeners patches combination, the peak is seen to be at a minimum for taper ratio $T_x = 0.75$ and maximum for taper ratio $T_x = 0.50$. Furthermore, it is investigated that for case III plate, peak sound power level decreases for increasing value of taper ratio for all combinations of rectangular stiffener patches attached to a plate. For a Case III plate, the maximum peak of the sound power level is obtained for taper ratio $T_x = 0.25$ for a plate with 2 rectangular stiffener patches combination and minimum peak is observed for taper ratio $T_x = 0.75$ for a plate with 4 rectangular stiffener patches combination. From Figure 29, it is observed that for a Case I plate peak is maximum for taper ratio, $T_x = 0.75$ for plate with 1 concentric stiffener patches and minimum for taper ratio $T_x = 0.50$ for 2 concentric stiffener patches. For a Case II plate the highest peak is seen for taper ratio $T_x = 0.75$ for 2 concentric stiffener patches. Similarly, for case III plate lowest peak is observed for taper ratio, $T_x = 0.75$ for plate with 4 concentric stiffener patches. However, from Figures 28 and 29, it is necessary to mention that for a Case II unloaded tapered plate the highest peak sound power level is seen for taper ratio $T_x = 0.75$. Furthermore, it is also observed that the peak sound power level increases for case I plate for taper ratio $T_x = 0.75$ and peak sound power level decreases for a Case III plate for taper ratio, $T_x = 0.75$.

It is thus quite obvious that different combinations of rectangular and concentric stiffener patches have a significant impact on peak sound power level corresponding to the (0, 0) mode. Furthermore, different combinations of rectangular and concentric stiffener patches with different taper ratios provide us design options for peak sound power level. For example, for peak sound power reduction, taper ratio $T_x = 0.75$ with 4 rectangular stiffener patches and 4 concentric stiffener patches combination, as well as taper ratio $T_x = 0.50$ with 2 rectangular stiffener patches combination, for a Case III plate may be the options. Similarly, for sound power actuation, taper ratio $T_x = 0.75$ with 1 rectangular stiffener patch and 1 concentric stiffener patch combination for a Case I plate and 2 rectangular stiffener patches and 4 concentric stiffener patches combination for a Case II plate may be the alternative solution. However, for an unloaded tapered plate, it can be added that for taper ratio $T_x = 0.75$ for a Case III plate may be considered as a poor sound emitter and a taper ratio $T_x = 0.75$ for a Case I and Case II plate may be considered as the highest sound emitter.

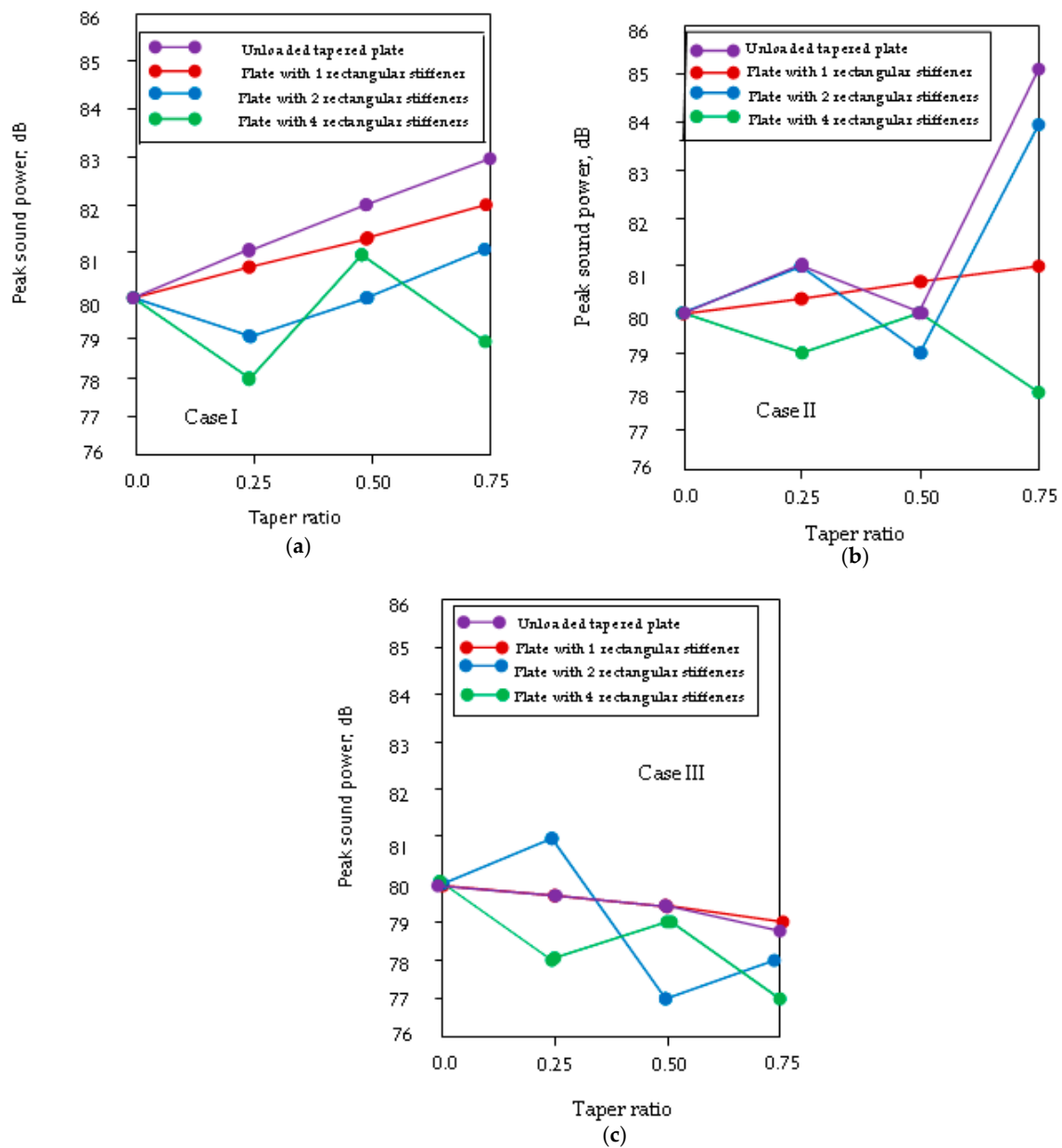


Figure 28. Comparison of peak sound power level (dB) for (a) Case I, (b) Case II, and (c) Case III plates having different parabolic thickness variations with different attachments of rectangular stiffener patches.

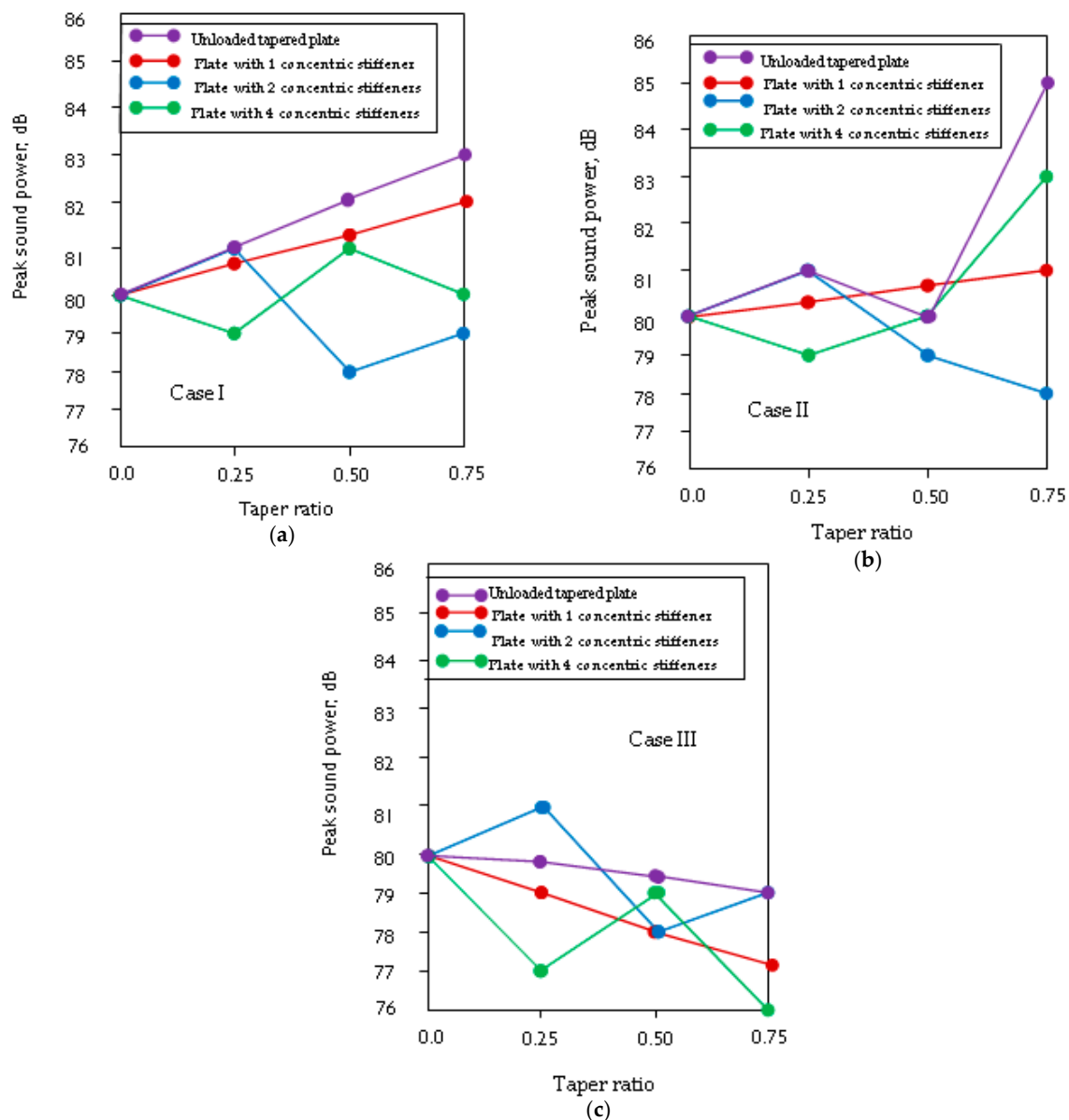


Figure 29. Comparison of peak sound power level (dB) for (a) Case I, (b) Case II, and (c) Case III plates having different parabolic thickness variations with different combinations of concentric stiffener patches.

4. Conclusions

A comparison is made of vibroacoustic behavior of tapered annular circular plates having different parabolically varying thickness with different combinations of rectangular and concentric stiffeners patches keeping the mass of the plate plus patch constant for a clamped-free boundary condition. It is observed that due to lower stiffness associated with the Case II plate, the non-dimensional frequency parameter of a Case II plate reduces more in comparison to a Case I plate. However, for a Case III plate, the non-dimensional frequency parameter is same as that of unloaded plate. In response to acoustic behavior, it is observed that different combinations of rectangular and concentric stiffener patches and modes variation have significant impacts on the sound power level in comparison to the stiffness variation due to the taper ratio. It is observed that for a sound power level up to 50 dB, and for a plate with different parabolically varying thickness, we get all taper ratios, $T_x = 0.00, 0.25, 0.50$ and 0.75 , with a broad range of frequencies as design options in different frequency bands for different combinations of rectangular and concentric stiffener patches. It is further shown that a plate

with 4 rectangular and 4 concentric stiffener patches combination shows the minimum sound power level for all cases of thickness variation, whereas the highest power is obtained for a Case II plate with 2 rectangular and concentric stiffener patches combination. It is interesting to note that a Case III plate has the lowest sound power level among all variations and is seen to be the lowest sound radiator. Further different combinations of rectangular and concentric stiffener patches with different taper ratios provide us design options for peak sound power level. For example, for peak sound power reduction, taper ratios, $T_x = 0.75$ with a 4 rectangular stiffener patches 4 concentric stiffener patches combination, and taper ratio $T_x = 0.50$ with a 2 rectangular stiffener patches combination for a Case III plate may be the options. Similarly, for sound power actuation, a taper ratio $T_x = 0.75$ with 1 rectangular stiffener patch and 1 concentric stiffener patch combination for a Case I plate and 2 rectangular stiffener patches and 4 concentric stiffener patches combination for a Case II plate may be an alternative solution. Furthermore, for unloaded tapered plates, it can be added that taper ratio $T_x = 0.75$ for a Case III plate may be considered as a poor sound emitter and a taper ratio $T_x = 0.75$ for Case I and Case II plates may be considered as the highest sound emitter.

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