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# DCA-Based Collision Avoidance Path Planning for Marine Vehicles in Presence of the Multi-Ship Encounter Situation

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**Abstract:** The problem of ship collision avoidance path planning is one of the key problems in the ship motion control field. Aiming at the high computational time problem of path planning in multi-ship encounter situations and the impact of the target ship's action changes on path planning, this paper proposes a dynamic path-planning method based on dynamic cluster analysis (DCA), which is used to dynamically cluster target ships with similar attributes into a group ship, reducing the number of calculated targets and improving the efficiency of path planning. Taking into full consideration the action requirements of the International Regulations for Preventing Collisions at Sea (COLREGs), the course alteration action matrix (CAAM) for collision avoidance is established to limit the space of candidate solutions. On the basis of the rapid optimization capability of the deterministic optimization algorithm (DOA), a dynamic monitoring mechanism is introduced to establish a multi-ship encounter intelligent collision avoidance decision-making model that meets the needs of real-time collision avoidance. The simulation results showed that the method can obtain a dynamic collision avoidance path that is safe and feasible.

**Keywords:** multi-ship encounter; collision avoidance; path planning; dynamic cluster analysis



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## 1. Introduction

Multi-ship encounter collision avoidance path planning is a hot research topic in the field of ship motion control. Many methods have been used to solve this problem. As early as the 1980s and 1990s, a knowledge-based system was used for multi-ship encounter collision avoidance path planning [1,2]. The problems of decision-making blind spots and low decision-making efficiency are affected by the collision avoidance knowledge base and reasoning mechanism.

Analytical geometry methods are also used for collision avoidance path optimization. In [3], a cooperative path planning algorithm was proposed, and the concept of ship priority was used in the algorithm. However, only three typical encounter situations (the overtaking, head-on, and crossing situations) were considered, and Rule 18 and 19 of the COLREGs were not considered. For the multi-ship encounter situation, the uncertainty of the target ship's action makes this cooperative collision avoidance difficult to achieve in the practice of ship collision avoidance. In [4], a deterministic collision avoidance path optimization algorithm TBA (Trajectory Base Algorithm) was proposed, which effectively shortened the calculation time but did not consider the action changes of the target ship. The CCDWA (COLREG-compliant dynamic window approach) was used to solve the collision avoidance problem in ref. [5]. However, this method only considers three typical encounter situations between the two ships and does not consider the changes in the action of the target ship.

Control theory methods are also used for collision avoidance path optimization [6–8]. In [9], a concept of a ship collision avoidance system based on model predictive control was proposed. This algorithm considers the action changes of the target ship and the constraints

of the COLREGs. With an increase in the number of target ships, the computational complexity of the algorithm increases, which affects the decision-making time. A combined nonlinear model predictive control (NMPC) method was used for trajectory tracking and collision avoidance [10]. However, this approach only considers three typical encounter situations and does not take into account the collision-avoidance action requirements of the COLREGs.

Fuzzy logic methods are used for collision avoidance path optimization. Aiming at the problem of collision avoidance in complex multi-ship encounter situations, ref. [11] proposed a Fuzzy–Bayesian ship intelligent collision avoidance decision-making model to achieve continuous collision avoidance actions. In addition, this method was upgraded in ref. [12], but the action changes of the target ship were not considered. In [13–15], Fuzzy theory was employed to infer the collision risk index for collision avoidance, but Rules 18 and 19 of the COLREGs were not considered.

Artificial Potential Field (APF) has been used for collision avoidance path optimization in some studies [16–18]; the principle is to guide the target to avoid obstacles through the action rules of Virtual Field Force (VFF). Ref. [19] proposed a multi-vessel cooperative path planning method based on APF, which makes collision avoidance decisions according to the priority of target vessels. Additionally, the uncertainty of the action of the target ship makes it difficult to realize the coordinated path planning scheme. Ref. [20] proposed a COLREGs-constrained real-time path-planning method for autonomous ships using modified artificial potential fields; this method has high decision-making efficiency, but it only considers three typical encounter situations.

Evolutionary algorithms are also used for multi-ship encounter collision avoidance path optimization. In [21], the decision to avoid the four target ships took about 900 s. A multi-vessel path planning method based on an evolutionary algorithm was proposed [22,23] called ESoSST (Evolutionary Sets of Safe Ship Trajectories). Subsequently, the method was used in TSS (Traffic Separation Scheme) waters [24], as well as in waters with restricted visibility [25]. ESoSST is a cooperative path planning method, and the computational time is about 10–30 s.

The swarm intelligence optimization algorithm has been used for collision avoidance path optimization. A method of ship collision avoidance path planning based on the ant colony optimization (ACO) algorithm has been proposed [26,27], which assumes that target ships maintain their course and speed. The simulation results show that the decision-making time to avoid four target ships and eight target ships can reach up to 29 s and 57 s, respectively. In [28], particle swarm optimization (PSO) was used for the path planning problem of unmanned surface vehicles with currents effects, but the targets were static.

The differential game theory method is also used for collision avoidance path optimization [29,30]. The method takes into account the action changes of the target ship. However, the drawback is that the high computational complexity in multi-ship encounters reduces the decision-making efficiency.

Distributed decision-making methods are used for collision avoidance path optimization [31,32]. These methods rely on information exchange and negotiation between ship agents to make collision avoidance decisions, but they are not applicable to non-agent ships.

The velocity obstacle (VO) method, based on the relative motion principle in physics, is also used for ship collision avoidance path optimization. In [33], VO was used for avoiding dynamic targets, but only three typical encounter situations were considered. Ref. [34] used VO to avoid target ships with predictable trajectories, but the constraints of the COLREGs were not considered.

With the rapid development of artificial intelligence technology, there has been related research on the application of machine learning in the field of ship collision avoidance [35,36]. In [37], a concise deep reinforcement learning obstacle avoidance method was presented, but the target ships were static. Ref. [38] proposed an automatic collision avoidance of multiple ships method based on deep Q-learning, but only three typical encounter situations were considered. In [39], deep reinforcement learning was used for

AUV (autonomous underwater vehicle) obstacle avoidance planning, but the constraints of the COLREGs were not considered. Moreover, when machine learning deals with complex problems, it is often inefficient because of the high amount of computation.

In view of the above literature review, this paper presents a DCA-based collision avoidance path planning method for marine vehicles in presence of a multi-ship encounter situation. The proposed method fully takes into account the constraints of the COLREGs and the changes in the action of the target. The purpose of the method is to give an optimal collision avoidance path quickly and dynamically. The rest of the paper is organized as follows. In Section 2, a simplification of an encounter situation based on DCA is presented. The multi-ship encounter intelligent collision avoidance decision-making method is given in Section 3. Simulation results are shown in Section 4, and conclusions are given in Section 5.

## 2. Simplification of Encounter Situation Based on DCA

In practice at sea, in fishing areas, in TSS waters, or in habitual waterways, for a group of ships with a similar course, speed and position are often encountered (e.g., a fleet of fishing vessels). In the process of collision avoidance, the officer on watch (OOW) usually considers such a group of ships as one target. On the basis of this collision avoidance logic, it is possible to simplify the multi-ship encounter situation. In this paper, a dynamic cluster analysis (DCA) method is used to classify the target ships with similar attributes as group ships.

### 2.1. Cluster Analysis Method

In this paper, the Hierarchical Clustering Method (HCM) is used. The method divides the  $n$  samples into  $n$  classes, and then, the two classes with the closest attributes are merged into a new class by calculating the distance  $D_C$ . After multiple clustering, all samples are merged into one class, and then a dendrogram is drawn. Finally, the number of classes and the sample composition of classes are determined.

#### 2.1.1. Calculation of Distance between Samples

Suppose there are  $n$  samples  $X_i$  ( $i = 1, 2, \dots, n$ ), and each sample has  $m$  attributes;  $x_{ij}$  ( $j = 1, 2, \dots, m$ ) represents the  $j$ -th attribute value of the  $i$ -th sample.

$d_{ij}$  is the Euclidean distance between  $X_i$  and  $X_j$ .

$$d_{ij} = \left[ \sum_{k=1}^m (x_{ik} - x_{jk})^2 \right]^{1/2} \tag{1}$$

$\bar{x}_j$  and  $s_j$  represent the sample mean and standard deviation of the  $j$ -th attribute value, respectively. The calculation formula is as follows.

$$\bar{x}_j = \frac{1}{n} \sum_{i=1}^n x_{ij} \tag{2}$$

$$s_j = \left[ \frac{1}{n-1} \sum_{i=1}^n (x_{ij} - \bar{x}_j)^2 \right]^{1/2} \tag{3}$$

$x'_{ij}$  represents the normalized data of each attribute value.

$$x'_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j} \tag{4}$$

The standardized Euclidean distance  $dt_{ij}$  can be obtained by calculation Formula (5).

$$dt_{ij} = \left[ \sum_{k=1}^m (x'_{ik} - x'_{jk})^2 \right]^{1/2} \tag{5}$$

### 2.1.2. Calculation of the Distance between Classes

In this paper, the Group Average Method was used to calculate the distance between classes. There are two classes,  $G_{p1}$  and  $G_{p2}$ , containing  $q_{p1}$  and  $q_{p2}$  samples, respectively. The distance  $D_G$  between two classes is calculated as follows.

$$D_G(p_1, p_2) = \frac{1}{q_{p1} \cdot q_{p2}} \sum_{i=1}^{q_{p1}} \sum_{j=1}^{q_{p2}} d_{ij} \tag{6}$$

### 2.2. Group Ship Model

Suppose the target ship's course is  $C_{ts}$ , the speed is  $V_{ts}$ , the distance to own ship (OS) is  $D_{ts}$ , and the true bearing is  $TB_{ts}$ . Take each target ship (TS) as the sample of cluster analysis, choose  $C_{ts}$ ,  $V_{ts}$ ,  $D_{ts}$ , and  $TB_{ts}$  as characteristic indices, and multiple target ships with a close course, speed, and position are classified into one group ship (GS).

The constraints of the cluster analysis are as follows. If the course difference of the target ships is more than  $E_c$  degrees, the speed difference of the target ships is more than  $E_v$  knots, or the distance between the target ships is greater than  $2SD$  (safe distance), these three cases cannot be classified as one class.  $E_c$ ,  $E_v$ , and  $SD$  are not fixed and can be determined according to specific encounter situations.

$N$  target ships ( $TS_1$ – $TS_n$ ) are classified into  $K$  ( $K \leq N$ ) group ships ( $GS_1$ – $GS_k$ ) on the basis of the HCM.  $GS_p$  ( $p = 1, 2, \dots, k$ ) contains  $q_p$  ( $1 \leq q_p \leq N$ ,  $q_1 + \dots + q_k = N$ ) target ships. The domain of  $GS_p$  is a circle, the central position is  $(x_{gsp}, y_{gsp})$ , the radius is  $R_{gsp}$ , the course is  $C_{gsp}$ , the speed is  $V_{gsp}$ ,  $DCPA_{gsp}$  is the distance to the closest point of approach (DCPA) from the center of  $GS_p$  to OS, and the passing distance to OS is  $SD_{gsp}$ . The calculation formula of each variable of  $GS_p$  is as follows.

$$\left\{ \begin{array}{l} x_{gsp} = (\max(x_{tsqp}) + \min(x_{tsqp}))/2 \\ y_{gsp} = (\max(y_{tsqp}) + \min(y_{tsqp}))/2 \\ R_{gsp} = ((\max(x_{gsp}) - \min(x_{gsp}))^2 + (\max(y_{gsp}) - \min(y_{gsp}))^2)^{1/2} \\ C_{gsp} = \text{average}(C_{tsqp}) \\ V_{gsp} = \text{average}(V_{tsqp}) \\ SD_{gsp} = \text{abs}(DCPA_{gsp}) - R_{gsp} \end{array} \right. \tag{7}$$

where  $x_{tsqp}$  is the X coordinate value of each target ship in  $GS_p$ ;  $y_{tsqp}$  is the Y coordinate value of each target ship in  $GS_p$ ;  $C_{tsqp}$  is the course of each target ship in  $GS_p$ ;  $V_{tsqp}$  is the speed of each target ship in  $GS_p$ .

Two samples of the group ship are shown in Figure 1.

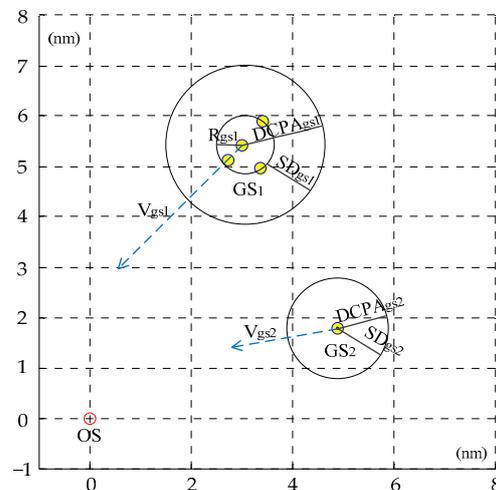


Figure 1. Samples of the group ship.

### 2.3. Dynamic Cluster Analysis

The cluster analysis method was introduced for the collision avoidance problem [40], but it does not consider the action changes of the target ship. Actually, when the action of the target ship changes, it may be necessary to re-cluster the target ships to simplify the encounter situation. In order to realize the dynamic cluster analysis of the target ships, it is necessary to detect the action changes of the target ship, and the detection period is set as  $T_c$ . When it is detected that the course difference between the target ships exceeds the threshold  $E_c$  or the speed difference exceeds the threshold  $E_v$ , the clustering analysis is performed again. A flow chart of the DCA of the target ships is shown in Figure 2.

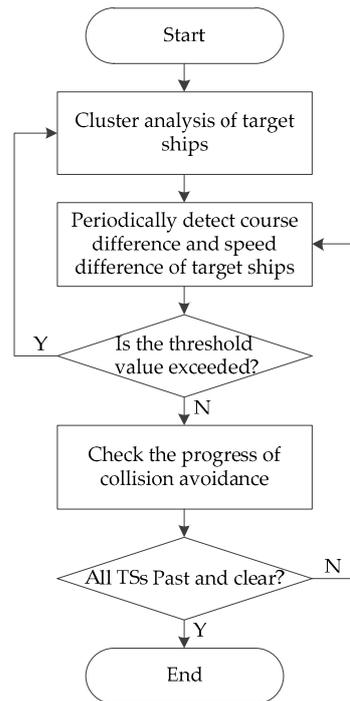


Figure 2. Flow chart of the DCA of target ships.

## 3. Multi-Ship Encounter Intelligent Collision Avoidance Decision-Making

### 3.1. Course Alteration Action Matrix

In order to determine the course-altering direction of the collision avoidance path, this paper establishes the course alteration action matrix (CAAM) model  $AM$ .

$$AM = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad (8)$$

where the rows represent the relative bearing (RB) of the target ship, the columns represent the type of the target ship,  $a_{mn}$  is action value, and the range of action values is  $(-1, 0, 1)$ . A value of  $-1$  indicates that OS should change course to the port side,  $0$  means that either a change to the port side or starboard is possible, and  $1$  means that OS should change course to starboard.

When encountering multiple ships, according to the RB of each target ship and the ship type, the action vector  $AT$  of the ship's course alteration to avoid collision is constructed.

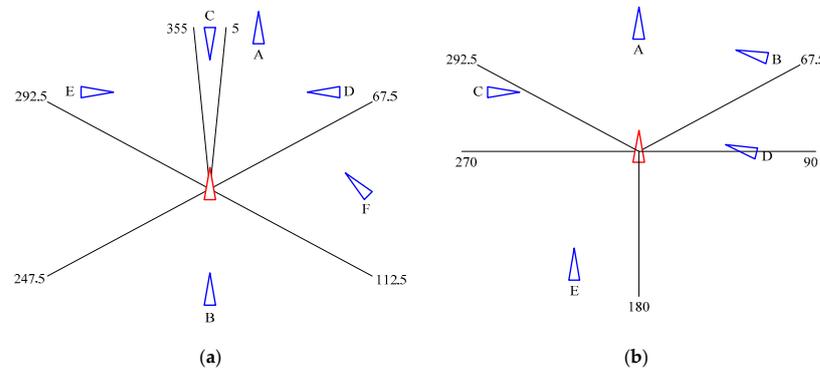
$$AT = [AM_1, AM_2, \cdots AM_N] \quad (9)$$

The priority of the action value is from high to low:  $1, -1, 0$ . If a value of  $1$  appears in  $AT$ , it is limited to altering the course to starboard. Then, whether there is a  $-1$  value is

judged, and if so, it is limited to altering the course to the port side. If the values in *AT* are all 0, it means that OS can change course to the port side or to starboard.

### 3.1.1. CAAM in Sight of One Another

In sight of one another, considering the constraints of the COLREGs (Rules 13, 14, 15, 17, and 18) and good seamanship, the encounter situation is divided as shown in Figure 3a.



**Figure 3.** Encounter situation division (A, B, C, D, E, F represent the target ships): (a) Encounter situation division in sight of one another; (b) encounter situation division not in sight of one another.

OS is a regular power-driven vessel (PDV). For a vessel being overtaken (A), the action value is 0. For an overtaken vessel (B), the action value is  $-1$ . For a head-on vessel (C), the action value is 1. For a crossing vessel from starboard (D), the action value is 1. For a crossing vessel from the port side (E), the action value is 1. For the other types of vessels in Rule 18, the action value is 0. For an abeam vessel (F), OS should avoid altering course toward it. Based on these, the CAAM in sight of one another is shown in Table 1.

**Table 1.** CAAM in sight of one another.

RB\TS	NUC	RAM	VEF	SV	PDV1	PDV2
[355.0, 005.0]	0	0	0	0	0	1
[005.0, 067.5]	0	0	0	0	0	1
[067.5, 112.5]	$-1$	$-1$	$-1$	$-1$	0	$-1$
[112.5, 247.5]	$-1$	$-1$	$-1$	$-1$	0	$-1$
[247.5, 292.5]	1	1	1	1	0	1
[292.5, 355.0]	0	0	0	0	0	1

NUC is a vessel not under command, RAM is a vessel restricted in her ability to maneuver, VEF is a vessel engaged in fishing, SV is a sailing vessel, PDV1 is overtaken PDV, and PDV2 is not overtaken PDV.

### 3.1.2. CAAM Not in Sight of One Another

Not in sight of one another, considering constraints of the COLREGs (Rule 19) and good seamanship, the encounter situation is divided as shown in Figure 3b.

OS is a regular power-driven vessel (PDV). For a vessel being overtaken (A), the action value is 0. For vessels forward of the beam (B, C), the action value is 1. For a vessel abeam (D) or abaft the beam (E), OS should avoid altering course towards it. Based on these, the CAAM not in sight of one another is shown in Table 2.

## 3.2. Deterministic Optimization Algorithm

### 3.2.1. Problem Description

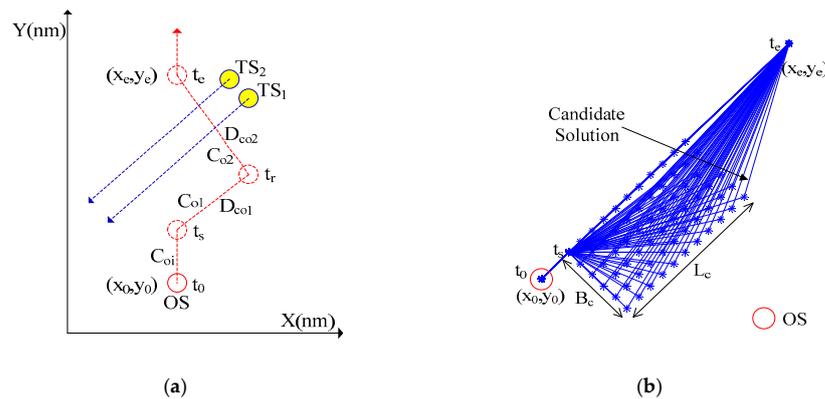
At time  $t_0$ , OS is at position  $(x_0, y_0)$  with initial course  $C_{oi}$  and speed  $V_{oi}$  encountering  $N$  target ships ( $TS_1$ – $TS_n$ ), and a risk of collision (ROC) exists. To pass these target ships at

a safe distance ( $SD$ ), OS intends to take action to avoid a collision at time  $t_s$  taking steer course  $C_{o1}$  to sail  $D_{co1}$  n miles and steer course  $C_{o2}$  at time  $t_r$  to sail  $D_{co2}$  n miles to the goal position  $(x_e, y_e)$  and return to the initial course  $C_{oi}$  at time  $t_e$ . The values of  $t_0, (x_0, y_0), C_{oi}$ , and  $V_{oi}$  can be obtained from OS. The data for the target ship's  $C_{ts}, V_{ts}, D_{ts}$ , and  $TB_{ts}$  are available from radar (these data are assumed to be known in this paper). The value of  $t_s$  and  $(x_e, y_e)$  can be preset according to a specific encounter situation.

**Table 2.** CAAM not in sight of one another.

RB\TS	Overtaken Vessel	Other Vessel					
[292.5, 067.5]	0	1					
[067.5, 090.0]	0	1					
[090.0, 180.0]	0 </tr <tr> <td>[180.0, 270.0]</td> <td>0</td> <td>1</td> </tr> <tr> <td>[270.0, 292.5]</td> <td>0</td> <td>1</td> </tr>	[180.0, 270.0]	0	1	[270.0, 292.5]	0	1
[180.0, 270.0]	0	1					
[270.0, 292.5]	0	1					

The problem of collision avoidance path planning can be presented by an optimization problem, namely searching for the optimal  $(C_{o1}, t_r, C_{o2})$  among the candidate solutions, which make OS pass target ships at a safe distance, and voyage losses are minimal under the constraints of COLREGs. Figure 4a shows the process of path planning for collision avoidance.



**Figure 4.** Deterministic optimization algorithm: (a) Process of path planning for collision avoidance; (b) candidate solutions for collision avoiding path.

### 3.2.2. Determination of Candidate Solutions

The side to be steered can be determined on the basis of CAAM. A rectangular area on the side to be steered is defined, and the length ( $L_c$ ) and width ( $B_c$ ) of the area are determined according to the specific encounter situation. Candidate waypoints are determined at equal intervals along the length and width of the rectangular area. The value of the interval ( $D_{wp}$ ) is determined according to own ship's length overall (LOA); the larger the LOA, the larger the interval, and vice versa. Each candidate waypoint corresponds to a  $(C_{o1}, t_r, C_{o2})$ , which is a candidate solution for the collision avoidance path, as shown in Figure 4b.

### 3.2.3. Objective Function

In the algorithm, each group ship is used as the calculation target, and a global search is performed in the candidate solutions. The optimal collision avoidance path should satisfy the following conditions.

- The  $SD_{gsp}$  of each group ship is not to be less than  $SD$
- The course change is at least 30 degrees
- The sum of  $D_{co1}$  and  $D_{co2}$  takes the minimum value

### 3.2.4. Steps of the Algorithm

Pseudocode for collision avoidance path planning on the basis of the deterministic optimization algorithm (DOA); see Algorithm 1.

<b>Algorithm 1:</b> Pseudocode for collision avoidance path planning on the basis of DOA.	
<b>Input:</b>	$C_{oi}, V_{oi}, C_{ts}, V_{ts}, D_{ts}, TB_{ts}, SD_{gsp}$
<b>Process:</b>	
1:	Initialization: $(x_0, y_0), t_0, t_s, (x_e, y_e), L_c, B_c$
2:	Calculate the sum of $D_{co1}$ and $D_{co2}$
3:	If $SD_{gsp} \geq SD$ && course change is at least 30 degrees
4:	keep the candidate solution
5:	Else
6:	abandon the candidate solution
7:	End if
8:	Search for a solution in which the sum of $D_{co1}$ and $D_{co2}$ goes to the minima
<b>Output:</b>	$C_{o1}, t_r, C_{o2}$

### 3.3. Collision Avoidance Decision-Making Model

Aiming at the action changes of the target ship, this paper introduces a dynamic monitoring mechanism to realize dynamic collision avoidance path planning. When the action of the target ship leads to a new ROC, the collision avoidance decision needs to be updated, that is, the collision avoidance path of OS needs to be re-planned. If the action change of the target ship does not form a new ROC, the current collision avoidance path will be maintained.

The multi-vessel encounter intelligent collision avoidance decision-making model established in this paper includes three modules: the encounter situation simplification module, the ship collision avoidance decision-making module, and the dynamic monitoring and evaluation module. The model is shown in Figure 5.

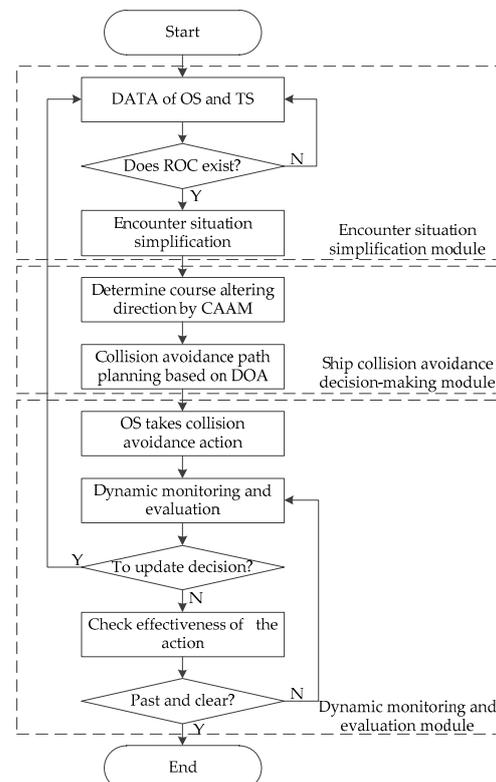


Figure 5. Model of collision avoidance decision-making.

#### 4. Simulation and Analysis

To verify the validity of the decision-making model, two simulation cases were implemented using MATLAB software; one case is for in sight of one another, and the other is for not in sight of one another.

##### 4.1. Experimental Conditions and Explanations

The simulation tests were conducted on the basis of a computer with an Intel(R) Core (TM) i7-3770 3.4GHz processor, 4G RAM, and a 32-bit Windows 7 Professional system. The simulation results are presented in the form of data tables and ship trajectory graphs.

The following parameter values were used in the simulation:  $E_c = 1^\circ$ ,  $E_v = 0.5$  kn,  $T_c = 5$  s,  $SD = 1$  nm,  $LOA = 116$  m,  $L_c = 4$  nm,  $B_c = 2$  nm, and  $D_{wp} = 0.2$  nm. In the simulation cases, the initial data of OS and target ships were considered to be known, and the process of acquiring data from the radar is not considered in this paper.

In the simulation cases, DCPA and TCPA (time to the closest point of approach) were used to determine the ROC. When an ROC exists, all target ships with approximate TCPA values were considered to give the optimal collision avoidance path.

##### 4.2. Path Planning in Sight of One Another

###### 4.2.1. Initial Situation of the Encounter

In sight of one another, OS's course is 000, the initial speed is 13.2 kn, the initial position is (0,0), and the initial time is 0000. OS encounters 10 target ships,  $TS_1$ – $TS_{10}$ ;  $TS_1$ – $TS_6$  are fishing vessels propelled by machinery but not engaged in fishing. The data of the target ships at 0000 are known, as shown in Table 3. It can be determined that a ROC exists, and the TCPA values for each TS are approximate.

**Table 3.** Data of target ships at 0000.

Data	$C_{ts}$ (°)	$V_{ts}$ (kn)	$D_{ts}$ (nm)	$TB_{ts}$ (°)	$DCPA_{ts}$ (nm)	$TCPA_{ts}$ (h)	Type
$TS_1$	270	7.80	4.20	028	0.1890	0.2737	PDV
$TS_2$	270	7.80	4.50	026	0.3593	0.2926	PDV
$TS_3$	270	7.80	4.40	032	−0.1091	0.2869	PDV
$TS_4$	270	7.80	4.70	030	0.0475	0.3065	PDV
$TS_5$	270	7.80	4.60	036	−0.4346	0.2987	PDV
$TS_6$	270	7.80	4.90	034	−0.2924	0.3190	PDV
$TS_7$	180	4.70	6.10	003	0.3192	0.3403	VEF
$TS_8$	180	4.70	5.80	001	0.1012	0.3240	VEF
$TS_9$	160	4.60	4.50	325	−2.2411	0.2218	VEF
$TS_{10}$	160	4.60	4.50	320	−2.5727	0.2099	VEF

###### 4.2.2. Simulation Process and Analysis

A cluster analysis was performed, and target ships ( $TS_1$ – $TS_{10}$ ) were classified into three group ships ( $GS_1$ – $GS_3$ ). The distribution of target ships and group ships at 0000 is shown in Figure 6.

According to the CAAM in sight of one another, OS should alter course to starboard. The candidate solution area is on OS's starboard. The action time  $t_s$  is 0002, and the goal position is (0, 7). The optimal collision avoidance decision is to steer course 032 at 0002 ( $C_{0002} = 032$ ), 329.2 at 0019 ( $C_{0019} = 329.2$ ), and 000 at 0037 ( $C_{0037} = 000$ ).

At 0005,  $TS_5$  and  $TS_6$  change their course to 245, and their speed remains unchanged. The trajectories of each ship from 0000 to 0005 are shown in Figure 7a.

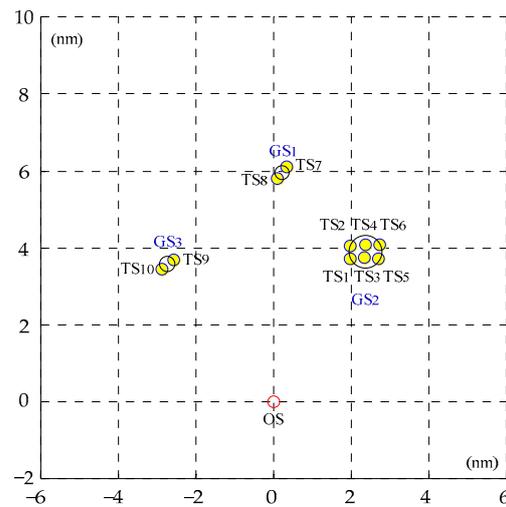


Figure 6. Distribution of target ships and group ships at 0000.

The course changes of TS<sub>5</sub> and TS<sub>6</sub> lead to a new ROC, so OS should update the collision avoidance decision. The data of the target ships at 0005 are shown in Table 4.

Table 4. Data of target ships at 0005.

Data	$C_{ts}$ (°)	$V_{ts}$ (kn)	$D_{ts}$ (nm)	$TB_{ts}$ (°)	$DCPA_{ts}$ (nm)	$TCPA_{ts}$ (h)	Type
TS <sub>1</sub>	270	7.8	2.8820	19.7558	1.5752	0.1301	PDV
TS <sub>2</sub>	270	7.8	3.2007	17.7361	1.8428	0.1411	PDV
TS <sub>3</sub>	270	7.8	3.0434	25.9978	1.3765	0.1463	PDV
TS <sub>4</sub>	270	7.8	3.3586	23.7444	1.6356	0.1581	PDV
TS <sub>5</sub>	245	7.8	3.2155	32.0471	0.6739	0.1557	PDV
TS <sub>6</sub>	245	7.8	3.5268	29.6075	0.8852	0.1691	PDV
TS <sub>7</sub>	180	4.7	4.7054	359.6280	-1.9233	0.2473	VEF
TS <sub>8</sub>	180	4.7	4.4197	356.7759	-2.0050	0.2268	VEF
TS <sub>9</sub>	160	4.6	3.6436	309.7770	-3.4124	0.0777	VEF
TS <sub>10</sub>	160	4.6	3.7496	303.9154	-3.6276	0.0577	VEF

After the re-clustering analysis, the target ships (TS<sub>1</sub>–TS<sub>10</sub>) were classified into four group ships (GS<sub>1</sub>–GS<sub>4</sub>). The distribution of target ships and group ships at 0005 is shown in Figure 7b.

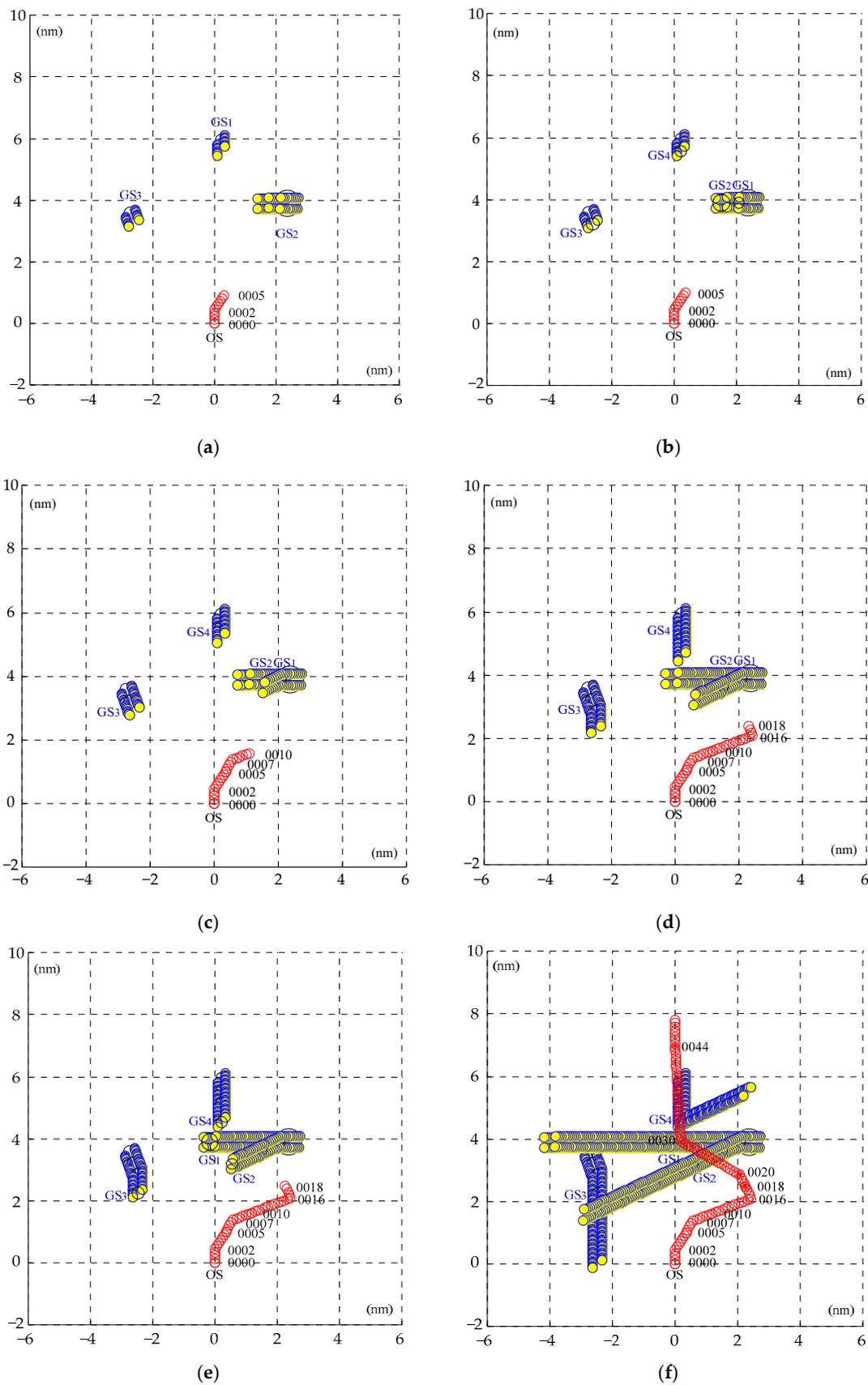
According to the CAAM in sight of one another, OS should alter course to starboard. The candidate solution area is on OS’s starboard. The action time  $t_s$  is 0007, and the goal position is (0, 7). The optimal collision avoidance decision is to steer course 068.9 at 0007 ( $C_{0007} = 068.9$ ), 333.5 at 0016 ( $C_{0016} = 333.5$ ), and 000 at 0041 ( $C_{0041} = 000$ ).

At 0010, TS<sub>9</sub> and TS<sub>10</sub> change their courses to 180, and their speed remains unchanged. The trajectories of each ship from 0000 to 0010 are shown in Figure 7c.

The course changes of TS<sub>9</sub> and TS<sub>10</sub> do not lead to a new ROC, so OS should maintain the current collision avoidance decision.

At 0018, TS<sub>7</sub> and TS<sub>8</sub> change their course to 065, and their speed remains unchanged. The trajectories of each ship from 0000 to 0018 are shown in Figure 7d.

The course changes of TS<sub>7</sub> and TS<sub>8</sub> lead to a new ROC, so OS should update the collision avoidance decision. The data of the target ships at 0018 are shown in Table 5.



**Figure 7.** Simulation results: (a) Ship trajectories (0000–0005); (b) distribution of target ships and group ships at 0005; (c) ship trajectories (0000–0010); (d) ship trajectories (0000–0018); (e) distribution of target ships and group ships at 0018; (f) ship trajectories (0000–0044).

**Table 5.** Data of target ships at 0018.

Data	$C_{ts}$ (°)	$V_{ts}$ (kn)	$D_{ts}$ (nm)	$TB_{ts}$ (°)	$DCPA_{ts}$ (nm)	$TCPA_{ts}$ (h)	Type
TS <sub>1</sub>	270	7.80	2.8973	295.4838	2.7809	0.0679	PDV
TS <sub>2</sub>	270	7.80	3.0563	301.1898	2.8336	0.0957	PDV
TS <sub>3</sub>	270	7.80	2.5883	299.3745	2.4293	0.0746	PDV
TS <sub>4</sub>	270	7.80	2.7554	305.7158	2.4653	0.1029	PDV
TS <sub>5</sub>	245	7.80	1.8101	287.5982	1.7628	0.0271	PDV
TS <sub>6</sub>	245	7.80	1.9084	297.7328	1.7532	0.0497	PDV
TS <sub>7</sub>	65	4.70	2.9474	318.9801	0.2520	0.2079	VEF
TS <sub>8</sub>	65	4.70	2.8918	311.8991	−0.1098	0.2045	VEF
TS <sub>9</sub>	180	4.60	4.5747	268.6760	4.3404	0.0829	VEF
TS <sub>10</sub>	180	4.60	4.8970	265.9638	4.7142	0.0760	VEF

After the re-clustering analysis, target ships (TS<sub>1</sub>–TS<sub>10</sub>) were classified into four group ships (GS<sub>1</sub>–GS<sub>4</sub>). The distribution of target ships and group ships at 0018 is shown in Figure 7e.

According to the CAAM in sight of one another, OS altering course to starboard or the port side is acceptable. To pass the stern of TS<sub>7</sub> and TS<sub>8</sub>, OS chooses to alter course to the port side, and the candidate solution area is on OS’s port side. The action time  $t_s$  is 0020, and the goal position is (0, 7). The optimal collision avoidance decision is to steer course 299.8 at 0020 ( $C_{0020} = 299.8$ ), 356.6 at 0030 ( $C_{0030} = 356.6$ ), and 000 at 0044 ( $C_{0044} = 000$ ).

From that point to time 0044, the actions of each TS remain unchanged. The trajectories of each ship from 0000 to 0044 are shown in Figure 7f.

During the period from 0000 to 0044, OS changes its course to  $C_{0002}$ ,  $C_{0007}$ ,  $C_{0016}$ ,  $C_{0020}$ , and  $C_{0030}$  successively. The DCPA values of target ships corresponding to different courses are shown in Table 6. As shown in the table, the absolute value of all DCPA is greater than the SD value, indicating that all target ships can pass OS at a safe distance.

**Table 6.** DCPA of target ships.

DCPA	$C_{0002}$	$C_{0007}$	$C_{0016}$	$C_{0020}$	$C_{0030}$
TS <sub>1</sub>	1.5738	2.1656	2.7917	1.9259	2.0922
TS <sub>2</sub>	1.8413	2.4926	2.8444	1.7616	2.0825
TS <sub>3</sub>	1.3750	2.1052	2.4401	1.6003	1.7320
TS <sub>4</sub>	1.6342	2.4308	2.4761	1.4194	1.7078
TS <sub>5</sub>	1.1425	−1.5959	1.7736	1.5948	1.8884
TS <sub>6</sub>	1.3925	−1.8967	1.7639	1.4232	1.5925
TS <sub>7</sub>	−1.9216	−3.4719	1.0840	1.3743	1.3631
TS <sub>8</sub>	−2.0033	−3.3727	1.3876	1.0325	1.0443
TS <sub>9</sub>	−3.4105	3.4895	3.9984	3.5225	3.5229
TS <sub>10</sub>	−3.6257	3.6497	4.3814	3.9110	3.9127

### 4.3. Path Planning Not in Sight of One Another

#### 4.3.1. Initial Situation of the Encounter

Not in sight of one another, OS’s course is 000, the initial speed is 12.8 kn, the initial position is (0,0), and the initial time is 0000. OS encounters 10 target ships, TS<sub>1</sub>–TS<sub>10</sub>; TS<sub>1</sub>–TS<sub>5</sub> are fishing vessels propelled by machinery but not engaged in fishing. The data of the target ships at 0000 are known, as shown in Table 7. It can be determined that an ROC exists, and the TCPA values for each TS are approximate.

**Table 7.** Data of target ships at 0000.

Data	$C_{ts}$ (°)	$V_{ts}$ (kn)	$D_{ts}$ (nm)	$TB_{ts}$ (°)	$DCPA_{ts}$ (nm)	$TCPA_{ts}$ (h)	Type
TS <sub>1</sub>	249	8.20	4.20	035	−0.6615	0.2370	PDV
TS <sub>2</sub>	249	8.20	4.40	037	−0.8442	0.2467	PDV
TS <sub>3</sub>	249	8.20	4.10	028	−0.1475	0.2341	PDV
TS <sub>4</sub>	249	8.20	4.30	030	−0.3046	0.2451	PDV
TS <sub>5</sub>	249	8.20	4.50	032	−0.4752	0.2557	PDV
TS <sub>6</sub>	181	4.60	5.40	356	0.4015	0.3095	VEF
TS <sub>7</sub>	181	4.60	5.50	359	0.1214	0.3160	VEF
TS <sub>8</sub>	181	4.60	5.20	349	1.0157	0.2931	VEF
TS <sub>9</sub>	181	4.60	5.00	353	0.6322	0.2851	VEF
TS <sub>10</sub>	135	14.1	4.40	330	−0.4869	0.1759	PDV

#### 4.3.2. Simulation Process and Analysis

A cluster analysis was performed, and target ships (TS<sub>1</sub>–TS<sub>10</sub>) were classified into three group ships (GS<sub>1</sub>–GS<sub>3</sub>). The distribution of the target ships and group ships at 0000 is shown in Figure 8a.

According to the CAAM not in sight of one another, OS should alter course to starboard. The candidate solution area is on OS’s starboard. The action time  $t_s$  is 0002, and the goal position is (0, 7). The optimal collision avoidance decision is to steer course 048 at 0002 ( $C_{0002} = 048$ ), 337.3 at 0015 ( $C_{0015} = 337.3$ ), and 000 at 0039 ( $C_{0039} = 000$ ).

At 0004, TS<sub>1</sub> and TS<sub>2</sub> change their course to 220, and their speed remains unchanged. The trajectories of each ship from 0000 to 0004 are shown in Figure 8b.

The course changes of TS<sub>1</sub> and TS<sub>2</sub> lead to a new ROC, so OS should update the collision avoidance decision. The data of the target ships at 0004 are shown in Table 8.

**Table 8.** Data of target ships at 0004.

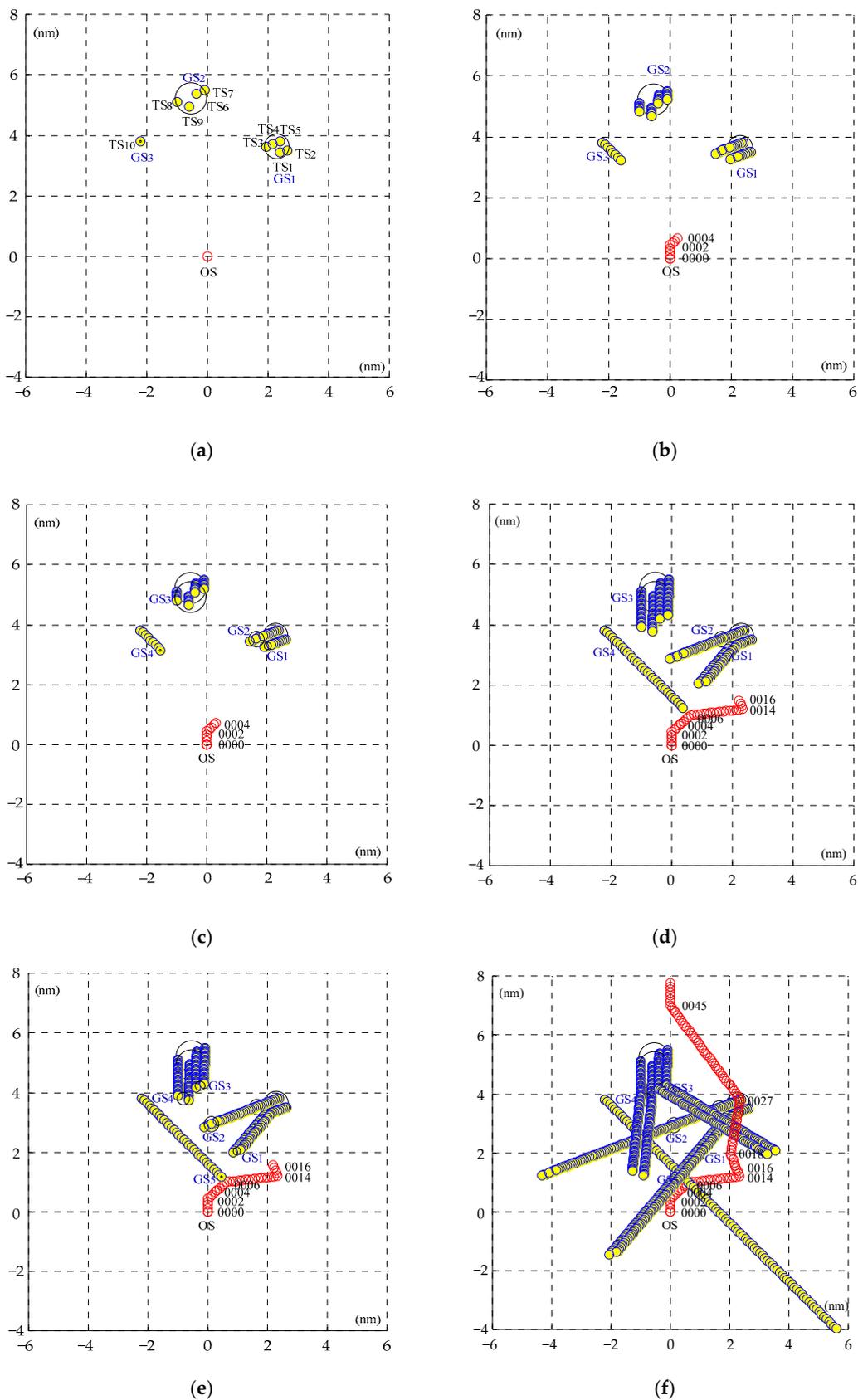
Data	$C_{ts}$ (°)	$V_{ts}$ (kn)	$D_{ts}$ (nm)	$TB_{ts}$ (°)	$DCPA_{ts}$ (nm)	$TCPA_{ts}$ (h)	Type
TS <sub>1</sub>	220	8.20	2.9905	31.9752	−0.6682	0.1391	PDV
TS <sub>2</sub>	220	8.20	3.1837	34.9236	−0.5508	0.1497	PDV
TS <sub>3</sub>	249	8.20	2.9305	22.0352	1.6450	0.1174	PDV
TS <sub>4</sub>	249	8.20	3.1159	25.1585	1.6059	0.1292	PDV
TS <sub>5</sub>	249	8.20	3.3037	28.1626	1.5520	0.1411	PDV
TS <sub>6</sub>	181	4.60	4.4285	350.916	−3.1409	0.1917	VEF
TS <sub>7</sub>	181	4.60	4.5048	354.6700	−2.9802	0.2074	VEF
TS <sub>8</sub>	181	4.60	4.2967	342.1831	−3.4720	0.1554	VEF
TS <sub>9</sub>	181	4.60	4.0573	346.7227	−3.0791	0.1622	VEF
TS <sub>10</sub>	135	14.10	3.0653	322.7536	−1.7946	0.1340	PDV

After the re-clustering analysis, target ships (TS<sub>1</sub>–TS<sub>10</sub>) are classified into four group ships (GS<sub>1</sub>–GS<sub>4</sub>). The distribution of target ships and group ships at 0004 is shown in Figure 8c.

According to the CAAM not in sight of one another, OS should alter course to starboard. The candidate solution area is on OS’s starboard. The action time  $t_s$  is 0006, and the goal position is (0, 7). The optimal collision avoidance decision is to steer course 083.6 at 0006 ( $C_{0006} = 083.6$ ), 338.0 at 0014 ( $C_{0014} = 338.0$ ), and 000 at 0043 ( $C_{0043} = 000$ ).

At 0016, TS<sub>6</sub> and TS<sub>7</sub> change their course to 121, their speed increases to 7.8 kn, and they are not engaged in fishing; TS<sub>8</sub> and TS<sub>9</sub> change their courses to 186, and their speeds remain unchanged. The trajectories of each ship from 0000 to 0016 are shown in Figure 8d.

The course changes of TS<sub>8</sub> and TS<sub>9</sub> do not lead to a new ROC, but the course and speed changes of TS<sub>6</sub> and TS<sub>7</sub> lead to a new ROC, so OS should update the collision avoidance decision. The data of the target ships at 0016 are shown in Table 9.



**Figure 8.** Simulation results: (a) Distribution of target ships and group ships at 0000; (b) ship trajectories (0000–0004); (c) distribution of target ships and group ships at 0004; (d) ship trajectories (0000–0016); (e) distribution of target ships and group ships at 0016; (f) ship trajectories (0000–0045).

**Table 9.** Data of target ships at 0016.

Data	$C_{ts}$ (°)	$V_{ts}$ (kn)	$D_{ts}$ (nm)	$TB_{ts}$ (°)	$DCPA_{ts}$ (nm)	$TCPA_{ts}$ (h)	Type
TS <sub>1</sub>	220	8.20	1.4046	287.5424	1.3501	0.0214	PDV
TS <sub>2</sub>	220	8.20	1.2074	294.3046	1.1132	0.0258	PDV
TS <sub>3</sub>	249	8.20	2.6274	298.9221	2.4994	0.0538	PDV
TS <sub>4</sub>	249	8.20	2.4886	303.5264	2.2981	0.0634	PDV
TS <sub>5</sub>	249	8.20	2.3531	308.5622	2.0853	0.0724	PDV
TS <sub>6</sub>	121	7.80	3.6635	315.1114	−0.5767	0.1846	PDV
TS <sub>7</sub>	121	7.80	3.5559	319.5975	−0.2834	0.1809	PDV
TS <sub>8</sub>	186	4.60	3.9493	305.8532	2.5107	0.1794	VEF
TS <sub>9</sub>	186	4.60	3.5576	307.6149	2.1762	0.1656	VEF
TS <sub>10</sub>	135	14.10	1.7820	256.6961	−1.6666	0.0240	PDV

After the re-clustering analysis, the target ships TS<sub>1</sub>–TS<sub>10</sub> were classified into five group ships (GS<sub>1</sub>–GS<sub>5</sub>). The distribution of target ships and group ships at 0016 is shown in Figure 8e.

According to CAAM not in sight of one another, OS should alter course to starboard. The candidate solution area is on OS’s starboard. The action time  $t_s$  is 0018, and the goal position is (0, 7). The optimal collision avoidance decision is to steer course 010 at 0018 ( $C_{0018} = 010$ ), 323.4 at 0027 ( $C_{0027} = 323.4$ ), and 000 at 0045 ( $C_{0045} = 000$ ).

From then to time 0045, the actions of each TS remained unchanged. The trajectories of each ship from 0000 to 0045 are shown in Figure 8f.

During the period from 0000 to 0045, OS changed its course to  $C_{0002}$ ,  $C_{0006}$ ,  $C_{0014}$ ,  $C_{0018}$ , and  $C_{0027}$  successively. The DCPA values of the target ships corresponding to different courses are shown in Table 10. As shown in the table, the absolute value of all DCPA is greater than the SD value, indicating that all target ships can pass OS at a safe distance.

**Table 10.** DCPA of target ships.

DCPA	$C_{0002}$	$C_{0006}$	$C_{0014}$	$C_{0018}$	$C_{0027}$
TS <sub>1</sub>	1.2238	−1.4464	1.3535	1.3676	3.8540
TS <sub>2</sub>	1.1519	−1.4198	1.1166	1.1217	3.6474
TS <sub>3</sub>	1.6425	−2.1703	2.5029	2.5182	4.1397
TS <sub>4</sub>	1.6035	−2.2246	2.3015	2.3421	3.8922
TS <sub>5</sub>	1.5496	−2.2655	2.0887	2.1664	3.6401
TS <sub>6</sub>	−3.1381	−3.9930	1.7759	−1.5933	−1.3650
TS <sub>7</sub>	−2.9774	−3.9748	1.4752	−1.2931	−1.0871
TS <sub>8</sub>	−3.4691	−4.0002	2.4454	−3.2954	3.4485
TS <sub>9</sub>	−3.0763	−3.7086	2.1162	−2.8951	3.1648
TS <sub>10</sub>	−1.7920	−2.2157	−1.6690	1.6848	4.4668

### 5. Conclusions

In this paper, a method to simplify the encounter situation based on dynamic cluster analysis was proposed to improve the efficiency of collision avoidance path planning. Target ships with similar properties were dynamically classified into one group ship, the model of the group ship was given, and the number of computational targets was effectively reduced. Fully considering the constraints of the COLREGs, a course alteration action matrix model (CAAM) was established to determine the side of course changing, which narrows the range of candidate solutions and helps improve the efficiency of the decision-making. In order to realize dynamic collision avoidance path planning, a dynamic monitoring mechanism was introduced, and a multi-ship encounter intelligent collision avoidance decision-making model was established. The simulation results show that the dynamic path planning method in this paper is safe and feasible. However, in the simulation tests, the ship’s maneuverability was not considered; this will be taken into account in subsequent studies.

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