Information **2012**, *3*, 256-277; doi:10.3390/info3030256

OPEN ACCESS

information

ISSN 2078-2489

www.mdpi.com/journal/information

Review

A Review on the Interpretability-Accuracy Trade-Off in Evolutionary Multi-Objective Fuzzy Systems (EMOFS)

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Received: 16 June 2012; in revised form: 21 June 2012 / Accepted: 29 June 2012 / Published: 12 July 2012

Abstract: Interpretability and accuracy are two important features of fuzzy systems which are conflicting in their nature. One can be improved at the cost of the other and this situation is identified as "Interpretability-Accuracy Trade-Off". To deal with this trade-off Multi-Objective Evolutionary Algorithms (MOEA) are frequently applied in the design of fuzzy systems. Several novel MOEA have been proposed and invented for this purpose, more specifically, Non-Dominated Sorting Genetic Algorithms (NSGA-II), Strength Pareto Evolutionary Algorithm 2 (SPEA2), Fuzzy Genetics-Based Machine Learning (FGBML), (2 + 2) Pareto Archived Evolutionary Strategy ((2 + 2) PAES), (2 + 2) Memetic- Pareto Archived Evolutionary Strategy ((2 + 2) M-PAES), *etc*. This paper introduces and reviews the approaches to the issue of developing fuzzy systems using Evolutionary Multi-Objective Optimization (EMO) algorithms considering 'Interpretability-Accuracy Trade-off' and mainly focusing on the work in the last decade. Different research issues and challenges are also discussed.

Keywords: Evolutionary Multi-Objective Fuzzy System (EMOFS); Evolutionary Multi-Objective Optimization (EMO); Genetic Fuzzy Systems (GFS); Interpretability-Accuracy Trade-Off; Genetic Algorithm (GA); Multi-Objective Evolutionary Algorithms (MOEA)

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1. Introduction

Interpretability [1–3] and accuracy [4] are the two important features of a fuzzy system developed for a specific application. The term 'interpretability' describes the capability of a model that allows a human being to understand its behavior by inspecting its functioning or its rule base. On the other hand, "accuracy" is the feature of the system that shows its capability to faithfully represent the real system. It can also be defined as the quantification of closeness between real system and its modeled fuzzy system.

Interpretability and accuracy are contradictory issues in the design of a fuzzy system. An increment in either feature can only be done at the cost of the other. This situation is called Interpretability-Accuracy (I-A) Trade-Off [5] and currently is a challenging research issue. Through this, various degrees of interpretability and accuracy of fuzzy systems are obtained and either one of them may be selected depending on the user's needs and the requirements of application.

The identification of fuzzy systems from data samples for specific functions associates different tasks, like input selection, rule selection, rule generation, fuzzy partition, membership function tuning *etc*. These tasks can be implemented as an optimization or search process using Evolutionary Algorithms (EAs), more specifically Genetic Algorithms (GA) [6,7]. The above-discussed integration of GA in the design of Fuzzy Systems results in the evolution of a special research area called 'Genetic Fuzzy Systems' (GFS) [8–11]. Genetic fuzzy systems have been proven to be capable of building compact and transparent fuzzy models while maintaining a very good level of accuracy, [12,13].

To deal with Interpretability-Accuracy Trade-Off in Fuzzy Systems, Multi-Objective Evolutionary Algorithms (MOEAs) have been used, leading to the next generation of GFSs named Evolutionary Multi-Objective Fuzzy Systems (EMOFS) [14–18]. A list of references in this field has been given in [19]. A Multi-Objective Fuzzy Modeling is used in [20] to deal with Interpretability-Accuracy Trade-Off. These EMOFS may be any rule-based system [21], classification system [22], *etc*.

A recent discussion and review on the existing approaches of EMOFS has been given in [23]. A taxonomy on existing proposals in EMOFS has been carried out, focusing mainly on Interpretability-Accuracy Trade-Off, multi-objective control problems and fuzzy association rule mining. We have focused on only the first issue, Interpretability-Accuracy Trade-Off, in this paper. This paper continues the work of [23], mainly considering the issue of Trade-Off in EMOFS.

The paper is divided into four sections. In Section 2, EMO is introduced briefly. In Section 3, a vast discussion has been carried out on the application of EMO in the design of fuzzy systems covering the issues related to the interpretability as well as accuracy and their trade-off. In Section 4, the recent research issues related to EMOFS are discussed. Conclusions and the future scope are given in Section 5.

2. Evolutionary Multi-Objective Optimization

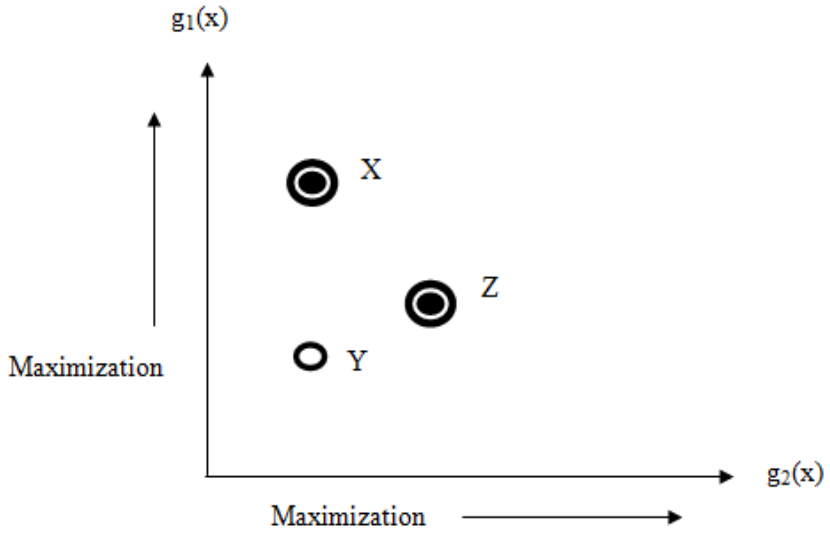
Evolutionary algorithms are stochastic optimization techniques, which simulate the concept of natural evolution. Evolutionary approaches consist of methodologies, genetic algorithms, evolutionary programming and evolutionary strategies. These techniques have been proven to be a robust and powerful search mechanism. In Evolutionary Multi-Objective Optimization, the objectives conflict with each other. These approaches are capable of tackling the problems of (1) large, complex and high

dimensional search space, and (2) multiple conflicting objectives. In these techniques, no optimal, ideal and single solution can be derived, instead, a set of solutions are produced because the improvement in one objective leads to degradation in the remaining objectives. These solutions are called Pareto-Optimal Solutions. These Pareto Optimal Solutions in terms of objective function are called Pareto Front.

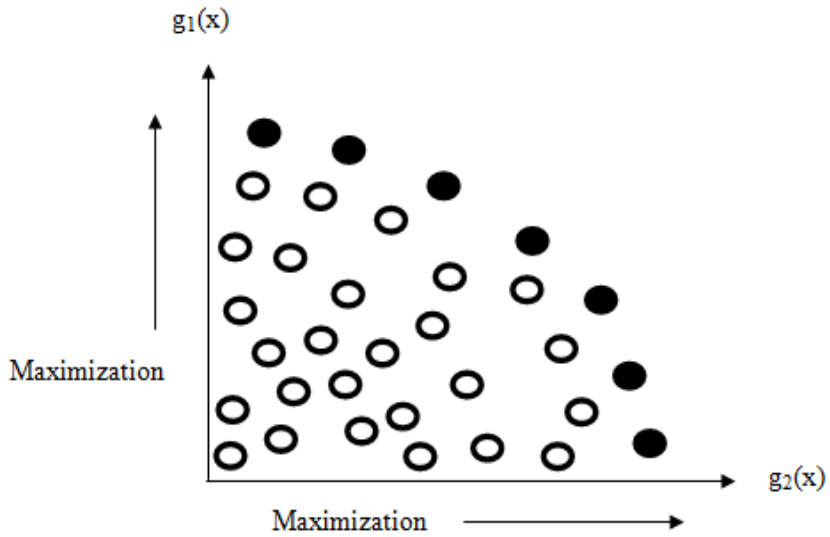
For example, two objective maximization problems can be formulated as: maximize $g(x) = (g_1(x), g_2(x))$.

In Figure 1(a), solution X dominates Y or Y is dominated by X. It can be concluded that X is better than Y. But, X and Z are non-dominated by each other. A Pareto-Optimal Solution is a solution that is not dominated by any other solutions and Pareto Front (Figure 1(b)) of any problem is the set of all Pareto-Optimal Solutions in terms of objective functions.

Figure 1. (**a**) Dominated and non-dominated solutions; (**b**) Two-Objective Problem and Pareto Front.



(**a**)



(**b**)

Multi-objective optimization problems are solved by using evolutionary algorithms, like Genetic Algorithms (GA) and result in a new area called EMO [24–29]. The introduction and review of EMO is very well discussed in [30,31].

The conflicting nature of the objectives leads to many problems, like dominance resistance and speciation. In [32], two diversity management mechanisms are introduced using Non-Dominated Sorting Genetic Algorithms (NSGA-II). Handling large numbers of objectives in multi-objective optimization is a very critical research issue. To deal with this issue, an approach is discussed in [33]. A similar issue is discussed in [34] for handling many conflicting objectives using standard Pareto ranking and diversity promoting selection mechanism. Regarding the issue of constraint handling, an approach is developed for nonlinear constrained optimization-problems with fuzzy costs and constraints in [35].

3. Handling Interpretability-Accuracy Trade-Off using MOEAs in Fuzzy Systems

In the early 1990s, the work in the area of EMOFS was oriented towards the development of accurate fuzzy systems, with less concentration on interpretability. However, in the late 1990s, the interpretability became an important issue along with accuracy. Table 1 summarizes most of the work on the issues discussed above in the decade after 1990.

Table 1. Interpretability and Accuracy Related Work in the 1990s.

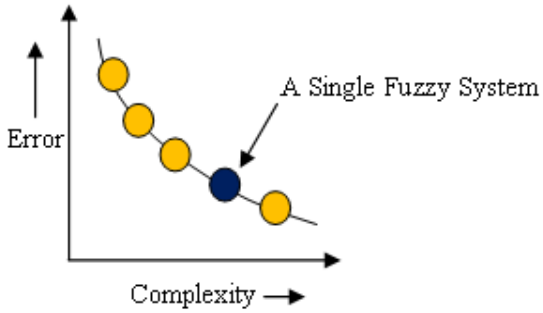
Approaches developed	Focus	References
Maximization of the number of correctly classified patterns along with minimization of the number of rules and fuzzy rule selection represented as a combinatorial optimization problem	Accuracy improvement & complexity minimization	[36]
Association of rule weights in rules also called certainty factor	Accuracy improvement	[37,38]
Multiple consequents in a rule	Accuracy improvement	[39]
Use of fine fuzzy partition (over-fitting), multiple fuzzy grid approach	Accuracy Improvement	[40]
Applying independent membership functions	Accuracy & Scalability improvement	[41]
Use of multi-dimensional fuzzy membership function	Accuracy and scalability improvement	[42,43]
Use of tree-type fuzzy partitions	Accuracy & Scalability Improvement	[44,45]
Scalability and hierarchical fuzzy systems	Accuracy & Scalability Improvement	[46]
Use of don't care conditions/ scalability improvement/input selection for each rule (rule wise input selection)	Complexity minimization	[47]

3.1. MOEAs with Two Objectives

Many non-dominated fuzzy systems can be obtained along the trade-off surface (Figure 2) by a single run of a MOEA, in which the user can select one, depending on the situation and requirements.

The most commonly used MOEAs are NSGA-II [48], Strength Pareto Evolutionary Algorithm (SPEA) [49], Strength Pareto Evolutionary Algorithm 2 (SPEA2) [50] and Pareto Archived Evolutionary Strategy (PAES) [51].

Figure 2. Pareto front non-dominated fuzzy systems.



During the search for non-dominated fuzzy systems in an EMO environment, accuracy and complexity are the two important factors to be considered with the objectives of accuracy maximization and interpretability maximization (complexity minimization). Initially, an aggregation approach is used for this purpose.

After that the MOEAs are well adapted for this issue and it is represented as,

$$\{minimize(f_{error}(M), f_{complexity}(M)\}$$

To obtain these objectives, several criteria, like number of selected fuzzy rules, number of correctly classified rules, tuning of membership, granularity of the uniform partition, *etc*., are considered.

A two objective approach, considering maximization of the number of correctly classified training patterns and minimization of the number of selected fuzzy rules, is proposed in [52]. In this, a hybrid algorithm is also proposed by the integration of a learning method of classification rules and a multi-objective genetic algorithm.

A multi-objective genetic procedure has been proposed in [53] with the objectives of feature selection and granularity learning. The approach is used in a fuzzy rule-based classification system to automatically learn the knowledge base.

Interpretability-accuracy trade-off analysis was done in [54] with the objectives of classification accuracy and number of rules. The approach is discussed for a classification problem. Initially, this approach introduces the extraction of rules from numerical data using a heuristic rule criterion approach.

In [55], a multi-objective genetic algorithm (MOGA) is used to obtain Fuzzy Rule Based Systems with a better trade-off between interpretability and accuracy in linguistic fuzzy modeling. A new post-processing approach is developed to get the desired goal which is based on the selection of rules along with the tuning of membership functions. The developed approach uses MOEA, SPEAII.

A Pareto based multi-objective evolutionary approach has been proposed in [56] to generate a set of Mamdani fuzzy systems from numerical data. A variant of (2 + 2) Pareto Archived Evolutionary Strategy has been used for this approach. The objectives concerned are root mean squared error for accuracy and sum of conditions which compose the antecedents of rules for complexity. Finally, the goal is to find the right trade-off between accuracy and complexity.

A rule selection and a tuning of the membership functions of an initial set of candidate linguistic fuzzy rules were performed in [57] by minimizing the number of rules and the system error in a multi-objective environment. This leads to improvement in the complex trade-off between accuracy and interpretability.

A new post processing method is developed in [58] for maintaining a good interpretability-accuracy trade-off in linguistic fuzzy systems, which performs rule selection and membership function tuning by focusing on the Pareto zone having most accurate solutions but the least number of possible rules. SPEA2 algorithm has been utilized for this approach.

A multi-objective genetic algorithm is proposed in [59] to generate Mamdani Fuzzy Rule Based Systems with trade-off between complexity and accuracy. In this approach, both rule base and granularity of the uniform partitions defined on the input and output variables are learned concurrently.

A brief review on the state of the art on the use of multi-objective genetic algorithms to obtain the compact fuzzy rule-based systems under rule selection and parameter tuning has been done in [60]. A linguistic model with improved accuracy and smallest number of possible rules are proposed.

A set of linguistic fuzzy rule-based systems with different trade-offs between accuracy and interpretability has been generated in [61] using multi-objective evolutionary approach. Accuracy is measured by approximation error and interpretability is quantified by rule base complexity. It learns rule base and parameters of the membership functions of associated linguistic labels concurrently. A modeling linguistic 2-tuple representation has been used and it uses (2 + 2) Pareto Achieved Evolutionary Strategies (PAES), Non-Dominated Sorting Genetic Algorithms (NSGA-II) and evolutionary driven single objective Evolutionary Algorithm (EA).

A multi-objective evolutionary algorithm for tuning fuzzy rule-based systems has been proposed in [62], considering the two objectives, accuracy and interpretability. An interpretability index is proposed based on the three metrics, membership displacements, membership function symmetry, and membership function area similarity.

A Mamdani fuzzy rule-based system with different good trade-offs between complexity and accuracy has been developed by using multi-objective evolutionary algorithm in [63]. In this approach, both rule base and granularity of uniform partitions defined on the input and output variables are learned concurrently.

Six different MOEA are used to obtain simpler and still accurate linguistic fuzzy models by performing rule selection and tuning of membership functions in [64,65]. These algorithms are NSGA-II [48], SPEA2 [49], SPEA2A_{CC} [66], SPEA2$_{ACC}^2$, NSGA-II$_A$, NSGA-II$_U$. These algorithms use two objectives, systems error and number of rules. A new post processing approach has been developed in [66] which considers the selection of rules together with the tuning of membership functions to get the right trade-off between accuracy and interpretability.

A deep-tuned fuzzy rule-based classifier system (FRBCS) from examples has been designed in [67]. The approach is based on rule learning and membership function tuning. The algorithm used in this approach is SPEA2, generating interpretable and accurate systems.

A method for generating single granularity based fuzzy classification rules and lateral tuning of membership functions has been proposed in [68] in a multi-objective genetic fuzzy environment. The NSGA-II algorithm is used in the practical implementation of this method.

A multi-objective evolutionary framework applied to regression problem has been proposed in [69]. In this framework, a two-level rule selection (2LRS) and learning of membership function parameters are introduced. Also, different trade-offs between accuracy and Rule Base (RB) complexity were obtained for a Mamdani Fuzzy Rule Based Systems.

A post processing approach is developed to reduce complexity of data-driven linguistic fuzzy models in [70]. The purpose is to get sufficient accuracy and better fuzzy linguistic performance with respect to their initial values. The basis of this approach lies on rule selection by formulating the bi-objective problem with objective accuracy and interpretability. Data sets from the KEEL project repository are used for evaluating this approach.

In [71], a multi-objective evolutionary algorithm is proposed to deal with two conflicting issues, complexity of the problem and the approximation error. The proposal focuses on the function of approximation problems.

The Pareto optimum set of fuzzy systems with different I-A trade-off has been generated in [72] using a multi-objective evolutionary approach. The two objectives taken in this approach are fuzzy rule parameter optimization and identification of system structure in terms of number of membership functions and fuzzy rules. The modification of NSGA-II algorithm is presented for modeling of a fuzzy system for function approximation from a set of training data.

The MOEA for searching the Pareto optimal fuzzy rules are discussed in [73] to get the Pareto optimal fuzzy system.

An approach for an evolutionary training set selection in the framework of multi-objective evolutionary learning of Mamdani fuzzy rule-based systems (MFRBS) has been proposed in [74,75]. A modified version of PAES (2 + 2) M-PAES is used in this approach. The objectives are system accuracy and rule base complexity. The rule base and parameters of membership functions are concurrently learned, and selected reduced training sets are used to compute the fitness of each individual, which leads to saving considerable execution time.

A MOEA has been proposed in [76] for improving the complexity in accuracy–complexity trade-off using adaptive defuzzification.

3.2. MOEA with Three Objectives

Several approaches have used three objectives to deal with the interpretability and accuracy trade-off issue in developing fuzzy systems.

A three-objective approach is proposed in [77] for the extraction of interpretable fuzzy rules from numerical data. The objectives are maximization of the number of correctly classified training patterns, minimization of the number of selected fuzzy rules and minimization of the total number of antecedent conditions (total rule length). The approach is developed for high-dimensional pattern classification problems. Also, a hybrid fuzzy GBML algorithm has been proposed for getting non-dominated rule sets from proposed three objective optimization problems.

In [78], the fuzzy modeling is presented as a multi-objective problem, taking consideration of the goals, accuracy, interpretability and autonomy. It is assumed to handle all these issues via a single objective ε-constrained decision-making problem the solution of which is provided by a hierarchical evolutionary process. The resulting fuzzy models are discussed as a classification problem.

A rule selection criterion for prescreening a candidate as fuzzy has been proposed in [79]. This task is completed in two steps. In the first step, candidate rules are generated by two rule evaluation measures, which are confidence and support, and in a second step multi-objective evolutionary algorithms are used for rule selection. The objectives of multi-objective optimization are classification error for measuring accuracy, the number of rules and conditions within a fuzzy classification rule system to measure its comprehensibility or complexity, respectively.

A multi-objective evolutionary algorithm (MOEA) is proposed in [80] to generate a Mamdani fuzzy rule-based system with a different accuracy-complexity trade-off. It is done by concurrently learning the granularities of input and output partitions, membership function parameters and rules. The concept of virtual and concrete partition is introduced as well. The proposed MOEA is tested over three real world regression problems.

The NSGA-II algorithm has been used to create multiple Pareto optimal fuzzy systems in [81]. The three objectives used are the precision performance, number of fuzzy rules and number of fuzzy sets. A modified fuzzy clustering algorithm is used to identify the antecedents of the fuzzy rule, while the consequents are designed separately to reduce the computational burden.

An approach for improving the interpretability of linguistic fuzzy rule-based systems has been proposed in [82]. In this approach, adaptive defuzzification improves the system accuracy. The proposed approach is based on the three objectives, (i) reduction in the number of total rules which considers that rules with weights close to zero should be removed; (ii) reduction in rules which have rule weights one and do not need any weight; and (iii) reduction of rules which are triggered jointly. Also, MOEA is utilized to get a set of solutions with a trade-off between accuracy and complexity.

A three-objective evolutionary algorithm has been proposed in [83,84] to generate a set of Mamdani FRBS with different trade-offs among accuracy, complexity and partition integrity. Accuracy is measured in terms of mean squared error, complexity is estimated by the number of conditions in the antecedents of the rules and integrity is defined by a proposed index. In this approach, Rule Base and MF parameters are learned concurrently.

A multi-objective genetic fuzzy system has been proposed in [85] to learn the granularities of the fuzzy partitions, tune the membership functions and learn the fuzzy rules. The fuzzy model is initialized by an integrated approach of the Wang-Mendal (WM) method and decision-tree algorithms. Also, dynamic constraints are proposed to improve the accuracy by 3-parameter MF tuning.

HILK (Highly Interpretable Linguistic Knowledge) in [86] is a fuzzy modeling approach dedicated to design the interpretable FRBS. It is integrated with a three-objective evolutionary algorithm (HILKMO) for performing genetic feature selection and fuzzy partition learning.

An index is proposed to preserve the semantic interpretability of linguistic fuzzy models in [87,88]. Also, a post processing multi-objective evolutionary algorithm is proposed which performs rule selection and tuning of fuzzy rule-based systems with three objectives: accuracy maximization, semantic interpretability maximization and complexity minimization.

Three types of interpretability measures are introduced in [89], which include semantic quality measures, rule base quality measures and model dimension measures. Also, a new alteration measure is proposed for fuzzy partition tuning.

A Pareto Multi-Objective Cooperative Co-Evolutionary Algorithm (PMOCCA) is proposed in [90] for constructing interpretable and precise fuzzy systems. PMOCCA is used to optimize the number of rules, antecedents of the rules and parameters of antecedents simultaneously. The initial fuzzy system is initialized by the fuzzy clustering algorithm.

A multi-objective fuzzy genetics-based machine-learning (GBML) algorithm is developed for fuzzy rule-based classifiers for examining the Interpretability-Accuracy Trade-Off in [91]. This approach is the amalgam of the Michigan and Pittsburgh approach. The accuracy is measured by correctly classified training patterns and the complexity is measured by the number of fuzzy rules and/or total number of antecedent conditions of fuzzy rules.

3.3. Improving the Search Ability of the MOEAs

The capability of MOEA to find a variety of FRBS with different trade-offs between complexity and interpretability is called search ability. The improvement in the search ability of any MOEA is a critical research issue.

In [92] the search ability of NSGA-II algorithm was improved by solving the issues of removal of overlapping solutions, recombination of similar patterns and selection of extreme and similar patterns.

An improvement in the search ability has been proposed in [93] by using multiple weighted sums with different weight vectors instead of original objectives in a fuzzy classifier. The idea is implemented on the classification problem.

In MOGFS approaches, a set of non-dominated solutions has been generated, which makes it very difficult to choose one. A double cross-validation approach is used to do this task in [94] for a fuzzy classifier.

The comparison between GBML and Genetic Rule Selection has been done in [95] in terms of their search ability to efficiently find compact fuzzy rule-based classification systems with high accuracy.

The search ability of MOEA in Pareto-optimal or near Pareto optimal fuzzy rule-based systems for classification problems has been discussed in [96,97]. NSGA-II (Non-dominated Sorting Genetic Algorithm) and MOEA/D (Multi-Objective Evolutionary Algorithm based on Decomposition) [98] are used in MoFGBML (Muti-objective Fuzzy Genetics Based Machine Learning) algorithm under various settings of computational load, finer fuzzy partitions and granularity of fuzzy partition.

3.4. MOEA to Design Ensemble Classifiers

The design of reliable classifiers by integrating multiple classifiers into a single one resulted in the development of ensemble classifiers. The generation of ensemble classifiers with high diversity using MOEA is an important research issue.

In [99], a MOEA is examined to develop an ensemble classifier by different non-dominated fuzzy rule-based classifiers with different accuracy-complexity trade-off. Accuracy is measured by the number of correctly classified training patterns while its complexity is measured by the number of fuzzy rules and the total number of antecedents' conditions.

Three objective-based multi-objective formulations of fuzzy rule selection have been discussed in [100] for a fuzzy rule-based ensemble classifier design. The multi-objective interpretation of the fuzzy rule selection is discussed with two objectives, accuracy maximization and complexity

minimization. A number of non-dominated rule sets for fuzzy classifiers are produced along with an interpretability-accuracy trade-off curve.

3.5. MOEA for Scaling Functions and Fine Fuzzy Partition

Optimization of scalarizing functions and fine fuzzy partitions in EMOFRBS is a crucial research issue.

An approach to optimize scalarizing functions using EMO has been developed in [101]. The effectiveness of the approach is achieved by the computational experiments using NSGA-II.

The fine fuzzy partitions are used in the evolutionary multi-objective optimization for designing the fuzzy rule-based classifiers in [102]. It is concluded that the application of fine fuzzy partition enhances the number of obtained non-dominated fuzzy rule-based classifiers. The relationship between granularity of fuzzy partitions and number of antecedent conditions are examined.

3.6. Approaches Related to User Preferences

User preferences [103] can be integrated in the MOEA for searching the Pareto optimal fuzzy systems.

An iterative fuzzy modeling has been performed in [104] by MOEA with user's preferences. User preferences are represented by several satisfaction level functions, which can be interactively modified by users. In [105,106], the user preference is integrated with a multi-objective genetic fuzzy rule selection. A preference function is proposed based on the satisfactory function of six objectives: average confidence, average coverage, number of used attributes, maximum number of used granularity, classification accuracy and number of rules.

3.7. Approaches Related to High Dimensional Problems

High dimensionality in fuzzy systems can be handled by using Evolutionary Multi-Objective Optimization (EMO), and it is an important research issue. High dimensional and large data sets lead to expansion in search space and affect the performance of evolutionary algorithms in the form of solution quality and convergence.

A MOEA is proposed for knowledge extraction from numerical data for high dimensional pattern classification problems with many continuous attributes in [107]. The three objective rule selection problem is discussed. The objectives are the number of correctly classified training patterns by the rule set, the number of rules and the total length of rules. Many rule sets with different accuracy-complexity trade-off have been generated.

In [108], an approach is proposed to deal with high-dimensional and large data sets in a multi-objective evolutionary framework for Mamdani Fuzzy Rule Based Systems (MFRBS). The proposed algorithm is based on a co-evolutionary approach that allows concurrent evolutionary training set selection (TSS) and multi-objective evolutionary learning of the RB and membership function parameters. The approach is tested on the high dimensional and large regression data sets.

A MOEA has been proposed in [109] for the learning of linguistic Knowledge Base (KB) in high dimensional regression problems. This approach is based on embedded genetic database learning involving variables, granularities and slight fuzzy partition displacement.

3.8. Semantic Co-intension Approach

Explicit semantics (fuzzy sets, operators, inference engine) and implicit semantics (knowledge gathered by user) are compared using a co-intension approach called Semantic Co-intension. A novel index has been proposed in [110] for designing highly interpretable rule-based classifiers, based on Semantic Co-intension.

3.9. Context Adaptation

Context adaptation is the approach to develop context-free models for creating context adapted FRBS so as to increase the accuracy. In [111], a novel index based on fuzzy ordering relations has been proposed for the quantification of interpretability. This proposed index and mean square error are used as the goal of the MOEA.

3.10. EMO Approaches for Data Mining Applications

The EMO has been used in developing data mining approaches addressing different issues, like sub-group discovery, rule mining *etc*.

A three-objective based multi-objective genetic rule selection has been introduced in [112] for pattern classification problems and it finds Pareto optimal rules and Pareto optimal rule sets in a data mining application. Similar work has been done in [113,114] introducing the concept of rule discovery and selection in an EMO based environment.

In [115], a non-dominated MOEA is proposed for extracting fuzzy rules in subgroup discovery (NMEEF-SD). The approach is based on the NSGA-II. In [116], a post processing approach for improving the results of algorithm NMEEF-SD in a sub group discovery is proposed. It allows the partitions to be adapted in the context of variables.

3.11. Other Specific Applications Developed Using EMO

Using EMO in fuzzy systems, several applications have been developed.

A genetic fuzzy framework has been proposed for financial prediction in [117] in multi-objective evolutionary algorithms. In this contribution, the relationship between predictive capability and interpretability of FRBS obtained by MOEA is studied.

A fine tuned fuzzy logic controller for heating, ventilating and air conditioning systems has been developed using multi-objective evolutionary algorithms in [118]. The two objectives considered are maximizing system performance and minimizing number of rules obtained. The proposed algorithm is based on SPEA2 algorithm.

The accuracy-complexity relationship has been analyzed in [119] for fish habitat modeling using a Genetic Takagi-Sugeno Fuzzy Model called Fuzzy Habitat Preference Model (FHPM).

4. Burning Research Issues

Several research issues have been identified in EMOFS while considering the issue of Interpretability-Accuracy Trade-Off. Some of these are listed below:

1. Formulation and quantification of interpretability along with the identification of its global definition [2,20,120–123] in EMO framework is an important research issue because interpretability is the subjective feature of any system, which is not easy to quantify.
2. Improvement in the interpretability of a system by selecting parameters like number of inputs, number of rules, rule length, fuzzy partition granularity, membership function separability, linguistic modifiers, linguistic hedges *etc*. Choosing these parameters may be considered in order to develop new interpretability indexes.
3. Handling Interpretability-Accuracy (I-A) Trade–Off using EMO [5,72,124] is a critical issue because interpretability and accuracy are the features conflicting with each other. One can be improved at the cost of the other, which leads to generation of multiple sets of solutions instead of any single solution.
4. An increment in the number of objectives degrades the performance of any EMO algorithm. Hence, improvement of the performance of MOEA when the numbers of objectives are high is a big research line. It helps to deal with the High Dimensional Problems [125], leading to the development of Hierarchical Fuzzy Systems.
5. Integration of user preferences [103–106,126] that facilitate to focus on a specific zone of Pareto Front to get the desired solution more efficiently.
6. Handling large and multi-dimensional data sets [127] by EMO algorithms.
7. Improvement in the search ability [92–98] of the MOEA and dealing with exponentially increased solutions approximating the Pareto Front.
8. Generation of mechanisms for interpretable explanations for fuzzy reasoning and inference mechanism, quantification of explanation ability of FRBS [128].

5. Conclusion and Future Scope

The EMO algorithms applied in developing Fuzzy Systems need improvement in order to deal with problems like high dimensionality, exponentially populated solutions, Interpretability-Accuracy Trade-Off, quantification of interpretability and explanation ability of the fuzzy systems, *etc*. This paper introduces and reviews such problems and their recent solutions in the capacity of different EMO algorithms, listed in Table 2.

Table 2. Evolutionary Multi-Objective Optimization (EMO) Algorithms used in Multi-objective Fuzzy Systems.

S. No.	EMO Used	References
1	SPEA2	[55,57,58,60,62,65,85,88,109,117,118]
2	NSGA-II	[55–57,60,61,65,68,72,76,81,85,89,92,93,96,101, 105–107,110,111,112,115,117]
3	$SPEA2_{ACC}$	[64–66]
4	(2 + 2) PAES	[56,61,63]
5	(2 + 2) M-PAES	[59,69,74,75,82–84,108]
6	HILK EMO	[86,110]
7	Fuzzy GBML	[94,95,97,102]
8	PMOCCA	[90]

Many types of problems are also considered for fuzzy systems with their multi-objective development, listed in Table 3.

Table 3. Types of problems identified and discussed in the literature.

S. No.	Type of the problem identified	References
1	Classification of Problems	[52–54,67,68,77–79,90-97,99–102, 104–106,107,110, 112-116,117]
2	Regression	[69,85,87,88,109]
3	Linguistic FRBS	[55–65,74–76,80,82,83,86,89,108,111,118]
4	Function Approximation Problems	[71,72]
5	TS Type FRBS	[119]

In the future, the authors are interested to develop efficient and robust MOEA, applicable for the development of accurate and interpretable fuzzy systems. Focus would also be dedicated to invent new indexes for measuring the interpretability of EMOFS and new EMO approaches for managing Interpretability-Accuracy Trade-off.

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