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Article

Equivalence of the Symbol Grounding and Quantum System Identification Problems

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Abstract: The symbol grounding problem is the problem of specifying a semantics for the representations employed by a physical symbol system in a way that is neither circular nor regressive. The quantum system identification problem is the problem of relating observational outcomes to specific collections of physical degrees of freedom, *i.e.*, to specific Hilbert spaces. It is shown that with reasonable physical assumptions these problems are equivalent. As the quantum system identification problem is demonstrably unsolvable by finite means, the symbol grounding problem is similarly unsolvable.

Keywords: decoherence; decompositional equivalence; physical symbol system; observable-dependent exchange symmetry; semantics; tensor-product structure

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1. Introduction

The *symbol grounding problem* (SGP) was introduced by Harnad [1] as a generalization and clarification of the semantic under-determination issues raised by Searle [2] in his famous "Chinese room" argument. With reference to a "physical symbol system" model of cognition that posits purely syntactic operations (e.g., [3–6]), Harnad stated the SGP as follows:

> How can the semantic interpretation of a formal symbol system be made intrinsic to the system, rather than just parasitic on the meanings in our heads? How can the meanings of the meaningless symbol tokens, manipulated solely on the basis of their (arbitrary) shapes, be grounded in anything but other meaningless symbols?
>
> [1] (p. 335)

Here "the meanings in our heads" refers to the semantics assigned to a symbol system by the theorists who devised it; the SGP can thus be seen as the problem of freeing semantics from the need for a third-party interpreter, or as Harnad put it, making the semantics "intrinsic" to the symbol system. Taddeo and Floridi [7] emphasized this aspect of the SGP by reformulating it as a requirement, the "zero semantical commitment condition," that must be satisfied by any theory purporting to explain how the representations employed by any system, whether or not they satisfy the strict conditions on "symbols" and "symbol processing" specified in Harnad's original formulation, obtain their semantics. The zero semantical commitment condition rules out any purported explanations of the semantics of a symbol system that either tacitly assume the semantics in question and hence are circular, or that tacitly base the semantics on some deeper assumed semantics and hence are regressive.

From a purely formal, model-theoretic perspective, the SGP might be rejected out of hand as not just unsolvable but ill-conceived. One might, in particular, argue from this perspective that essentially-arbitrary stipulation is the *only* way any symbol or collection of symbols receives any semantics, and hence that the idea of an "intrinsic" semantics of any symbol or symbol system is incoherent. If this is the case, the SGP is clearly moot: any possible semantics is wholly dependent on some stipulating agent, and hence is either circular or regressive. From a cognitive science perspective, however, the SGP appears not just solvable but urgent. Human mental states, in particular, surely refer to something or other independently of 3rd-party stipulations. The "meanings in our heads" that Harnad worried about theories "parasitizing" are, after all, *our* meanings; they must be grounded somehow. The SGP has, accordingly, been ranked as one of the major open problems in the philosophy of information [8,9], and has received considerable attention from researchers in both artificial intelligence and cognitive robotics (reviewed in [7]) and embodied and situated cognition (reviewed in [10,11]). Recent approaches to the SGP have focussed on grounding semantics in motor actions, and transferring these grounded semantics to symbolic representations via machine-learning algorithms that optimize the action-symbol mapping based on feedback from the environment (e.g., [12–14]). The extent to which such feedback from the environment embodies implicit semantic assumptions, for example in choices of training sets, and hence the extent to which such machine-learning approaches satisfy the zero semantical commitment condition remains open to question.

The present paper challenges all claimed solutions of the SGP by showing that under reasonable physical assumptions the SGP is equivalent to the *quantum system identification problem* (QSIP), the problem of determining which quantum system a given experimental outcome characterizes. The quantum system identification problem is a generalization to quantum information theory of the system identification problem for finite-state machines formulated within classical automata theory [15,16], which itself is a formalized version of the well-known "blind men and the elephant" point that finite observations are insufficient to fully and precisely characterize an observed object. What quantum theory adds to the classical system identification problem is entanglement, and hence an in-principle inability to observationally track even stipulated boundaries separating collections of physical degrees of freedom, and hence even stipulated boundaries separating physical systems characterized by such degrees of freedom, through time. The "reasonable physical assumptions" under which the SGP and the QSIP are claimed to be equivalent are, briefly, that the non-symbols available as potential grounds of symbols are *physical* entities, and that the physics that describes the dynamical behavior of these

entities satisfies a fundamental symmetry, decompositional equivalence [17–19], that is satisfied by minimal quantum theory, *i.e.*, by quantum theory with no *physical* "collapse of the wave function." As all available experimental evidence indicates that minimal quantum theory—together with its extension to the relativistic domain, quantum field theory—correctly describes the physical world (e.g., [20]), the "reasonable physical assumptions" can be simplified to "symbols are grounded, if at all, by quantum systems" where a "quantum system" is a physical system correctly described by minimal quantum theory. It is worth emphasizing that the assumption that minimal quantum theory correctly describes physical systems is an *assumption* that could, despite the evidence supporting it, eventually be shown to be wrong; the correct description of physical systems could turn out to be Bohmian mechanics [21], a stochastic collapse theory (e.g., [22]), or some other theory mathematically distinct from but currently experimentally indistinguishable from minimal quantum theory. What is proposed here is that the increasingly tight bounds being placed by experiments on any deviations from the mathematical structure of minimal quantum theory render it a *reasonable* assumption. It has previously been shown that, under this assumption, the QSIP is unsolvable by finite observational means [18,19,23]. Showing that the SGP and the QSIP are equivalent shows, therefore, that subject to this assumption the SGP is unsolvable by finite observational means.

That the QSIP and the SGP should be closely-related problems is, from a certain point of view, completely straightforward. Experimental outcomes must be represented symbolically to be recorded in a classical memory. Any such recording links a symbol representing the outcome value—e.g., "α_i"—to a symbol representing the quantum system of interest, e.g., "$\mathbf{S}$" or "$\mathcal{H}_{\mathbf{S}}$". This linkage may be indirect—for example, an outcome value may be linked to a measurement operation that is itself linked to a specific system by being defined mathematically as an automorphism of the Hilbert space of that system—but the structure of the quantum formalism guarantees that such a link will exist. Determining which quantum system a given experimental outcome characterizes requires determining what the symbol representing the quantum system of interest refers to. The QSIP thus requires that a non-circular, non-regressive, *operational* semantics be provided for the formal expressions that refer to "systems" in quantum theory; it requires that the systems to which such symbols refer be *identifiable* in the laboratory. Solving the QSIP involves, therefore, solving at least a broad instance of the SGP, the instance in which the symbols of interest refer to systems under investigation in the laboratory. The SGP, in turn, requires at least an in-principle ability to explicitly identify whatever non-symbols serve as "grounds" for physical symbol systems; without an ability to identify the claimed grounds, the claim that they serve as grounds is effectively empty. If such non-symbols are physical entities correctly described by minimal quantum theory, identifying them requires solving the QSIP. The present paper makes this straightforward but informal relationship precise. In so doing, it shows that no semantics of symbols that refer to quantum systems, which if minimal quantum theory correctly describes the physical world includes *all* physical systems, can be "intrinsic" to the symbols employed.

2. Preliminaries

2.1. Symbols and Grounds

What does it mean to "ground" a symbol? In the context of a physical symbol system as defined by Newell and Simon [3], in which symbols are processed purely syntactically, "grounding" a symbol requires relating it to something that is not a symbol, and hence not part of the network of purely-syntactic relationships that define the symbol system. This grounding relation must be non-arbitrary, in the sense of being independent of 3rd-party observers or theorists: that the grounding relation holds must, in other words, be an objective, observer-independent fact about the world. Harnad, for example, proposed that the symbols composing human or successful robot symbol systems are grounded in "nonsymbolic representations" such as sensory transducer outputs that are related to external objects solely by physical laws and physical facts about the structure of the perceiving system's body [1]; Pylyshyn's earlier proposal that cognitive symbol systems get their semantics-bearing inputs from "non-cognitively-penetrable" modules [6] expresses a similar criterion for "non-symbolic" representations.

With the development of mobile robots capable of exploring open environments, motor actions can be combined with sensory inputs to form coupled, multi-modal "nonsymbolic" representations. Recent work in cognitive robotics has focussed on relating these multimodal non-symbols to symbols in a way that optimizes some measure of the success of actions performed by the robot (e.g., [13,14]). In this work, "grounding" becomes a functional measure: a symbol is grounded to the extent that its use contributes to successful outcomes. As what counts as "success" in such scenarios is determined by the experimenter, one could argue that even this action-based sense of grounding fails to achieve the zero semantical commitment required by Taddeo and Floridi [7]. However, one could also construct situations in which "success" was defined strictly by the environment —e.g., on Mars—with unsuccessful performance being quickly fatal; such situations plausibly model those faced by organisms anywhere, and render the derived semantics as naturalistic as may be achievable.

The key idea underlying both the early, largely philosophical concepts of grounding and the more recent, robotic implementations is that the representational roles played by the non-symbols are fixed by mechanical, physiological, or other non-cognitive regularities. Setting aside fine distinctions between the languages and explanatory projects of the various non-cognitive sciences, these can be characterized as *physical* regularities that characterize *physical* systems. All going solutions to the SGP can, therefore, be characterized as grounding systems of syntactically-related symbols on regularities in the behavior of physical systems; in information-theoretic terms, they propose that the information specifying the symbol-ground relation is encoded not in "the meanings in our heads" but in the structure of the physical world. To avoid circularity or regress, any such solution must assume that the relevant physical regularities, and hence the physical systems that they characterize, can be identified independently of the symbols whose semantics they encode. An arm moving along a certain trajectory, for example, can be identified independently of the goals of the agent that is moving the arm; a red block sitting on a table can be identified independently of an agent's word for or experience of redness. This assumption of independent identifiability is effectively an assumption of objectivity, and is most easily understood in a

philosophically realist context in which "the world" is clearly distinguished from "the theorist's mind." It is, moreover, only meaningful if "identifiability" includes *re*-identifiability over extended periods of time. A red block that only lasts an instant cannot ground the symbol "red block", especially if grounding is taken to involve successful manipulative action; for an object or process to be a semantic ground, it must both persist through time and be re-identifiable over time. In information-theoretic terms, the encoding of the symbol-ground relation by the world must be considered to be both time-persistent and repeatedly effective in enabling grounded symbol use by the relevant agent.

2.2. Systems, States and Observables

Regularities in the behavior of a physical system are regularities in the patterns of state transitions executed by the system; observing such regularities involves observing the states of the physical system over an extended period of time. Grounding a symbol on an observable physical regularity is, therefore, grounding it on observations of physical state transitions; any instance of a process word such as "move", for example, is grounded on an observation—perhaps just a remembered observation—of something moving. One can, therefore, describe grounding without loss of generality in terms of a dyadic relation between a symbol and a subset of the states of a physical system. Hence the question of identifying the ground of a symbol becomes the question of identifying a collection of states of a physical system, something that can only be accomplished through observation.

Even in classical physics, observing the state of a physical system requires interacting with it; one must bounce photons off of a red block, for example, to observe either its position or its redness visually. The representation of observation by the action of a mathematical operator that is employed within quantum theory is, therefore, completely general; classical observation is distinguished from quantum observation by the mathematical structure of the state space on which such operators act, and by the claim that all such operators commute. The outcome of an observation is, in either the classical or the quantum case, a real (*i.e.*, not complex) value, typically accompanied by a unit of measurement, for example, "2 m", "37 ns" or "15 kg". A critical feature of such outcomes, often overlooked, is that they must be *recordable* in a persistent memory, and to be in any way useful to either the observer or any 3rd party, actually *recorded* in a persistent memory. This requirement for persistent recording places a powerful and principled constraint on both the outcome values themselves and any associated units: both must be encodable as finite bit strings, and both must be actually encoded, by the observer, in a memory device that can be accessed at a later time. While this requirement for finite encoding of outcomes is so obvious as to appear trivial, it will prove below to have significant consequences for understanding symbol grounding.

Because observations can only yield information about the current state of a physical system, in the form of outcome values for the physical degrees of freedom—position, mass, electric charge, *etc*.—that are probed by a sequence of measurement operations, it is natural to represent the physical system itself by the abstract space comprising all of its physically-allowed states. The action of a measurement operator on a system may change its current state—for example, bouncing sufficient photons off of an object may change its temperature or even its position—but such an action does not change what states are allowed for the system by the laws of physics; measurement operators are, therefore,

automorphisms on physical state spaces. In classical physics, each physical state comprises one real value for each of the system's physical degrees of freedom; the assumed mutual commutativity of classical measurement operators assures that all such values can, at least in principle, be measured simultaneously. Quantum theory replaces this classical state space with a Hilbert space comprising all linear combinations, with complex numbers as coefficients, of the allowed values of the system's physical degrees of freedom. Because not all quantum-theoretic measurement operators commute, it is not possible to obtain simultaneous outcome values for all of the physical degrees of freedom of a quantum system.

Here another fundamental assumption must be made explicit. Every meaningful observable is an operator defined on and hence specific to a *particular* physical state space, that is, the state space comprising the degrees of freedom of a particular physical system. It is convenient to write expressions such as "$\hat{x} = i\hbar(\partial/\partial p)$" to represent observables (here, the quantum position observable in momentum space) without specifying the system on which they act, and hence to assume implicitly that the operator acts on whatever state space is designated by the observer. Such expressions are, however, formally ill-defined, and this shorthand notation systematically hides the location of the information needed to specify the physical system being operated upon. Taken literally, a position operator without a system specification is like a student who reports a list of position values but is unable to characterize the objects in those positions, or even to say whether the same object was observed in each position. Observing the position of a particular system $\mathbf{S}$ requires telling the student how to distinguish $\mathbf{S}$ from other things; in quantum theory, it requires using the particular observable $\hat{x}_{\mathbf{S}} = i\hbar(\partial/\partial p_{\mathbf{S}})$ that acts on the particular Hilbert space $\mathcal{H}_{\mathbf{S}}$ of $\mathbf{S}$ and on no others. Hence the question of indentifying the system $\mathbf{S}$ - for quantum systems, an instance of the QSIP - can also be posed as the question of identifying the Hilbert space $\mathcal{H}_{\mathbf{S}}$, the position operator $\hat{x}_{\mathbf{S}} = i\hbar(\partial/\partial p_{\mathbf{S}})$ that acts specifically on $\mathcal{H}_{\mathbf{S}}$, or any other observable defined specifically on $\mathcal{H}_{\mathbf{S}}$. Associating an outcome value α_i with a quantum system $\mathbf{S}$ requires knowing that α_i was obtained as an outcome value by the application of an observable defined over $\mathcal{H}_{\mathbf{S}}$, and not by the application of a different observable defined over some other Hilbert space and hence some other system. As with the requirement for finite encoding, this apparently trivial point that operators are defined on and hence specific to particular state spaces will prove to have significant consequences for symbol grounding.

2.3. Quantum and Classical

As noted earlier, all experimental evidence to date supports the correctness of *minimal* quantum theory [20]; indeed quantum entanglement is now routinely observed over mesoscopic and macroscopic spatial (e.g., [24–26]) and temporal (e.g., [27–29]) scales. This growing body of evidence renders the existence of a domain in which physical dynamics are *actually* classical, as opposed to just approximately and apparently classical, increasingly unlikely. In particular, it renders the existence of any *physical* process of quantum state collapse that generates irreversibly classical physical states increasingly unlikely. Thus while classical physics remains an obviously useful *description* of physical dynamics for some systems at some scales, experimental evidence increasingly indicates that no physical systems are actually classical. Even superselection "rules" that appear to restrict quantum systems to particular

values of some degrees of freedom, for example to charge sectors, can be considered consequences of a choice of quantum reference frame, and hence as not imposing *actual* classicality [30]. Classical physics is, in other words, strictly an approximation: all *physical* systems are *quantum* systems. Hence it is consistent, and in a strict sense only correct, to treat any physical system $\mathbf{S}$ as a quantum system and to formally represent its states by a Hilbert space $\mathcal{H}_{\mathbf{S}}$.

Again, as noted earlier, the idea that all physical systems are quantum systems is most straightforwardly interpreted from a realist perspective, *i.e.*, one that acknowledges that there are real physical systems that have real physical degrees of freedom, and that all such systems behave as described by quantum, as opposed to classical, physical theory. In particular, all ordinary macroscopic systems such as tables and chairs and laboratory apparatus are composed of more fundamental physical systems, the degrees of freedom of which are all correctly described by quantum, not classical, physical theory. Observers interact with collections of these real physical degrees of freedom, including the macroscopic collections that characterize macroscopic systems, via real physical interactions representable as Hermitian operators defined over Hilbert spaces. It is difficult to fully abandon such a realist stance and maintain contact with experimental practice. "Non-ontic" approaches to quantum theory typically reject the existence of quantum states; Fuchs, for example, insists that "QUANTUM STATES DO NOT EXIST" ([31] (p. 4), emphasis in original) and regards quantum theory not as a literal description of the world but as a "users manual" for making probability judgments. Fuchs does not, however, deny the existence of either physical degrees of freedom or physical systems. Indeed while he regards physical systems as "autonomous agents" capable of surprising behavior, he requires that they have well-defined, finite Hilbert-space dimensions that effectively limit their autonomy. He also does not deny the physicality of measurement interactions; indeed he treats measurements as *causing* the "experiences" of observers, which are limited in every case to the finite sets of outcomes allowed by the Born rule applied to finite-dimensional Hilbert spaces ([31], Figure 1 and caption). Anti-realism can, clearly, be taken farther than Fuchs takes it. On one possible reading of Wheeler's "it from bit" proposal [32] or even of Floridi's "informational structural realism" [33], symbols constitute fundamental reality and so cannot be "grounded" in anything non-symbolic even in principle (however see [34] for Floridi's own objection to this reading). If this anti-realist reading is accepted, there are by definition no non-symbolic grounds for any symbol, so the SGP is clearly moot. In this case, moreover, quantum "systems" themselves comprise symbols and symbols only and "observation" becomes a mapping from symbols to other symbols. Here, the idea of doing an *experiment* in which non-symbolic entities are manipulated is altogether lost; "observations" are simply manipulations of symbols and as such are indistinguishable from theoretical calculations.

2.4. Summary

With these preliminary considerations in place, the informal relationship between the SGP and the QSIP noted above can be rendered somewhat more precise. Solving the SGP requires providing a general method for repeatably identifying the non-symbol(s) that serve, individually or collectively, as the ground for a symbol or symbol system. These non-symbols can, without loss of generality, be considered to be physical systems representable as Hilbert spaces. Solving the SGP, therefore, requires the ability to

repeatably identify such physical systems. It therefore requires solving the QSIP. The QSIP, however, can only be solved by *observation*. Solving the QSIP by observation requires acquiring, recording, and comparing observational outcomes. Recorded observational outcomes are encodable as finite bit strings and are therefore symbols. Comparing observations over time, therefore, requires grounding symbols, *i.e.*, it requires solving the SGP. The following two sections work through these steps in detail.

3. Solving the QSIP Requires Solving the SGP

Quantum theory is a formal, mathematical theory of the dynamical behavior of the physical world. The original axioms laid down by von Neumann [35] have, since the development of decoherence theory from the 1970s onward (e.g., [36–40]), been largely supplanted by axiomitizations that build both real-valued observational outcomes and the Born rule into a single postulate regarding measurement. Such axiomitizations avoid all mention of "collapse" and therefore yield the minimal quantum theory assumed here. The recent textbook *Quantum Computation and Quantum Information* [41] provides a particularly clear formulation of axioms of this kind:

(1) The state $|\mathbf{U}\rangle$ of any isolated quantum system $\mathbf{U}$ may be represented as a unit vector in a Hilbert space $\mathcal{H}_{\mathbf{U}}$.

(2) The time evolution of $|\mathbf{U}\rangle$ is unitary, and may be represented by a propagator $e^{-(i/\hbar)H_{\mathbf{U}}t}$ where $H_{\mathbf{U}}$ is the Hamiltonian operator characterizing $\mathbf{U}$.

(3) Measurements of $|\mathbf{U}\rangle$ may be represented as actions by a positive operator-valued measure (POVM), a collection $\{E_i^{\mathbf{U}}\}$ of positive semi-definite Hilbert-space automorphisms that sum to the Identity, on $\mathcal{H}_{\mathbf{U}}$.

(4) The components of an isolated composite system $\mathbf{U}$ may be represented by a tensor-product structure (TPS) of $\mathcal{H}_{\mathbf{U}}$.

As shown in [41], the POVM formalism called for by axiom (3) generalizes the traditional formalism of Hermitian observables and their associated von Neumann projections; in particular, any POVM component E_i can be written as $M_i^\dagger M_i$ for some Hermitian operator M_i. There has been increasing interest over the past decade in providing alternative, typically information-theoretic axioms for minimal quantum theory (e.g., [42–46] among others); such alternatives must, clearly, be provably equivalent to the standard axioms in their empirical predictions.

The QSIP arises as a practical problem whenever quantum theory is applied to make predictions about observable outcomes. For example, if a prediction has been made that the position degrees of freedom of electrons prepared in some particular way will have some particular statistical distribution of values—e.g., be distributed as a double-slit interference pattern—testing this prediction requires an ability to *identify* electrons that have been prepared in the particular way called for. Often this is a matter of identifying an apparatus that reliably prepares electrons in the called for way, and confirming that it is working correctly; in other cases, it is a matter of identifying an apparatus that detects electrons that have been prepared in the called for way, and confirming that it is working correctly. Altering the interpretation of the formalism does not obviate this requirement for identification. If the Heisenberg "picture" of quantum theory, in which the idea that quantum states evolve over time as in axiom (2)

above is replaced by the idea that observables—Hermitian operators or POVM components—evolve over time, one is faced with the task of identifying, again as a practical matter in the laboratory, the required time-varying observable, typically by identifying a detector that physically implements that observable and confirming that it is working correctly. In every case, the apparatus or detector that must be identified is a *physical* and therefore *quantum* system, however well its appearance or workings may be described by classical physics. Also in every case, the apparatus or detector must be repeatably re-identifiable over time; otherwise the replication of experiments is impossible.

The QSIP can now be stated explicitly: given a specification of a Hilbert space $\mathcal{H}_{\mathbf{S}}$, a specification of a POVM (*i.e.*, a normalized collection of Hermitian operators) defined on $\mathcal{H}_{\mathbf{S}}$, or a finite collection of observational outcomes obtained with such a POVM, by what finite operational means can an observer identify $\mathbf{S}$? By what finite operational means, for example, can an experimenter determine whether some system in front them has all and only the physical degrees of freedom, and all and only the dynamically allowed values of those physical degrees of freedom, that are specified by $\mathcal{H}_{\mathbf{S}}$? By what finite operational means can an experimenter determine that an apparatus or a set of laboratory procedures implements all and only the operations specified by a given POVM? As with classical system identification problems, the QSIP can also be given a "reverse engineering" formulation: given a physical system, how can an observer determine its Hilbert space? Given an apparatus or a set of laboratory procedures, how can she determine the POVM that it implements?

Implicit in the statement of the QSIP is a requirement that multiple observers can use the same means of identifying $\mathbf{S}$, and that they can use these means at multiple times. To satisfy this requirement, any means of identifying quantum systems must be both memorable and communicable; it must, therefore, be expressible using a finite set of symbols, and hence a finite string of bits. The QSIP is, therefore, the problem of identifying a physical system given a finite string of bits, and can be considered, without loss of generality, to be the problem of identifying a physical system given a particular finitely-encoded symbol such as "$\mathcal{H}_{\mathbf{S}}$" or even "that voltmeter". An *uninterpreted* symbol, however, is useless for identifying a physical system; one can only employ "$\mathcal{H}_{\mathbf{S}}$" to identify $\mathbf{S}$, for example, if one knows what "$\mathcal{H}_{\mathbf{S}}$" means. Solving the QSIP, therefore, requires that the symbols employed to specify quantum systems, write down POVMs, and record observational outcome values have semantics, and requires moreover that the semantics of these symbols relates them not just to other symbols, but to collections of physical degrees of freedom accessible to laboratory manipulation. Solving the QSIP, in other words, requires having a *grounded* semantics for the symbols employed to specify physical systems and record observational outcomes, and so requires solving the SGP.

It may be objected at this point that the symbols employed to specify physical systems and record observational outcomes acquire their semantics not through some special solution of the SGP for the language of physics or the language of non-cognitive science in general, but rather through the semantics of ordinary, non-technical, natural language. All students of physics, for example, learn the language of physics after they have learned a non-technical natural language. This objection carries weight, however, only if symbol grounding is not problematic in such non-technical natural languages. This is not the case: the SGP was formulated as a problem precisely because symbol grounding is problematic in ordinary natural languages, and is studied by both cognitive scientists and robotics researchers in a natural-language context. Hence solving the QSIP requires solving the SGP even if the semantics of

symbols employed to specify physical systems and record observational outcomes are entirely derived from the semantics of an ordinary natural language.

It may also be objected that the semantics of symbols such as "that voltmeter" can be understood entirely in demonstrative terms, and that this demonstrative understanding of semantics can be extended to cover all terms used to refer to physical systems and physical processes. Such an objection may be pursued by noting that embodied, embedded agents are also able to manipulate objects in the environment, and hence able to resolve potential ambiguities in the demonstration of an object by manipulation followed by further demonstration. Such a process of demonstration accompanied by manipulation in fact characterizes much of language learning by human infants (e.g., [47]), as well as forming the basis of the robotic symbol-grounding approaches noted earlier. Extending this process to a principled, as opposed to a "for all practical purposes" solution of the QSIP, however, requires demonstrating that it yields all and only the required degrees of freedom in all cases, or at least in all cases outside of some circumscribed set of exceptions. The insufficiency of finite observations for classical system identification demonstrated by Ashby [15], Moore [16] and others suggests that no such demonstration is possible; that this suggestion is correct is shown in Section 5 below.

4. Solving the SGP Requires Solving the QSIP

With the above considerations in mind, let us now examine the SGP. As noted earlier, solving the SGP requires relating symbols to non-symbols, such as sensory transducer outputs or executable representations of bodily motions, that obtain their semantics from observer-independent regularities in the behaviors of physical systems. Establishing such relations between symbols and non-symbols requires an ability to identify both the behavioral regularities and the physical systems in question by observational means. The physical systems to be identified, however, are all quantum systems; hence solving the SGP requires observationally identifying quantum systems. Solving the SGP requires, therefore, solving the QSIP.

If solving the SGP requires solving the QSIP, one would expect that systematic failures to solve the QSIP would cause, and therefore in practice correspond to, systematic failures to solve the SGP. In particular, one would expect systematic failures to solve the QSIP underlying the failures to solve the SGP due to circularity or regression that the zero semantical commitment condition is designed to prevent. Such failures to solve the QSIP are in fact commonplace and have a specific form: they are attempts to identify a quantum system that either assume classical properties of the system being identified and are therefore circular, or assume classical properties of some system in interaction with the system being identified and are therefore regressive. Indeed the cases characterized as "typical" above are failures to solve the QSIP due to circularity: classical properties such as size, shape and color are *assumed* in these cases to pick out a particular quantum system. Stated in terms of the SGP, in these cases "nonsymbolic" sensory transducer outputs are *assumed* to be causal consequences of particular objects in the world. Any such assumption violates the zero semantical commitment condition, as testing it requires precisely the semantic assumption in question.

Circular and regressive assumptions are built deeply into the quantum-theoretic formalism, and the ability to perform repeatable experiments arguably depends upon them. This is nowhere clearer

than in decoherence theory, the main current approach to explaining the "emergence of classicality" within minimal quantum theory (for textbook-length reviews, see [38,40]). Decoherence calculations require both the specification of a TPS that divides the universe into a "system of interest" **S** and its "environment" **E** and the specification of an interaction Hamiltonian $H_{\mathbf{SE}}$. They also require a classical assumption: that the observer observes, and hence obtains classical information about, *only* the system of interest. In particular, the observer does not obtain classical information about the state of the environment, a condition that is often represented formally by assuming that the environment can be represented as a classical statistical ensemble. This classical assumption is critical, as it is what allows tracing out the off-diagonal terms in the interaction Hamiltonian. Classicality thus "follows" from decoherence theory only because it is built in to decoherence calculations from the beginning.

This circularity at the heart of decoherence theory is amplified by attempts to provide decoherence with a physical interpretation that ties it more directly to laboratory practice. The first question that arises in any decoherence calculation is that of how to define the TPS that separates the degrees of freedom composing **S** from those composing **E**. The standard answer is that the observer *decides* how to structure this TPS by deciding what degrees of freedom are "relevant" or "accessible" in some particular situation (e.g., [48,49]). Any such decision is clearly based on classical criteria, and so obviates any claim that decoherence explains classicality. Zurek recognized that such dependence on decisions made by observers rendered decoherence non-objective, noting that "it is far from clear how one can define systems given an overall Hilbert space "of everything" and the total Hamiltonian" ([37] p. 1794) and that "a compelling explanation of what the systems are—how to define them given, say, the overall Hamiltonian in some suitably large Hilbert space—would undoubtedly be most useful" (p. 1818). The "environment as witness" formulation of decoherence theory [50,51] is an attempt to provide such an explanation by shifting the "decision" process from the observer to the environment; in this formulation, observers interact not with **S** itself but with an encoding of the state of **S** in the state of **E**. This scenario is clearly realistic as a description of laboratory practice: as Zurek [39] and others point out, observers typically obtain information about objects by interacting with photons, phonons and other ambient fields. It does not, however, solve the problem of objectively defining a TPS. The reason is that quantum theory satisfies a fundamental symmetry, decompositional equivalence, that forbids physical dynamics from depending on the TPS chosen to describe it. Decompositional equivalence is the symmetry that allows the Hamiltonian of a closed system **U** to be written, if 3^{rd} and higher-order terms are neglected, as a sum $H_{\mathbf{U}} = \sum_{ij} H_{ij}$, where H_{ij} describes the pairwise interaction between two physical degrees of freedom i and j of **U**, and that allows alternative TPSs $\mathcal{H}_{\mathbf{S}} \otimes \mathcal{H}_{\mathbf{E}}$ and $\mathcal{H}_{\mathbf{S}'} \otimes \mathcal{H}_{\mathbf{E}'}$ to describe the same universe [18,19]. Any assumption that particular systems, and hence particular TPSs, are "preferred" by physical dynamics violates decompositional equivalence. It cannot, therefore, be assumed that the "environment" only encodes information about the states of particular systems; if the environment is assumed to encode information about the states of systems embedded in it, it must be assumed to encode information about the states of *all* such systems. In this case, however, observers must be regarded as choosing which encoded information to extract from the environment, which is precisely the assumption of relevance that the environment as witness formulation was designed to avoid [52].

The above analysis suggests that decompositional equivalence explains why circularity and regression are such common points of failure for proposed solutions of the SGP: such solutions presume a mapping

from symbols to non-symbols that cannot, in any universe satisfying decompositional equivalence, be well-defined. Faced with this intrinsic ambiguity in the physical world, semantics has no choice but to fall back on stipulation. Any such stipulation assumes the use of a language with well-defined semantics, and hence succumbs to regression and, eventually, to circularity.

5. The Unsolvability of the QSIP Renders the SGP Unsolvable

Let $\mathcal{H}_{\mathbf{U}} = \mathcal{H}_{\mathbf{S}} \otimes \mathcal{H}_{\mathbf{E}}$ be a TPS defined on a closed system $\mathbf{U}$, $\{E_i\}$ be a POVM defined on $\mathcal{H}_{\mathbf{S}}$ and $\{\alpha_i\}$ be a finite set of real, finitely-encoded outcome values. The QSIP can then be stated as the question of whether $\{E_i\}$ can be identified as the source of the values contained in $\{\alpha_i\}$, and in particular, whether all alternative sources $\{E'_i\}$ defined on alternative TPSs $\mathcal{H}_{\mathbf{U}} = \mathcal{H}_{\mathbf{S}'} \otimes \mathcal{H}_{\mathbf{E}'}$ can be conclusively ruled out. That $\mathbf{U}$ satisfies decompositional equivalence is assumed by writing the TPSs as equalities $\mathcal{H}_{\mathbf{S}} \otimes \mathcal{H}_{\mathbf{E}} = \mathcal{H}_{\mathbf{U}} = \mathcal{H}_{\mathbf{S}'} \otimes \mathcal{H}_{\mathbf{E}'}$.

Consistent with the physical picture assumed by the environment as witness formulation of decoherence theory, consider an observer embedded in $\mathbf{E}$, or alternatively in $\mathbf{E}'$, in such a way that the observer's local interactions with the environment have no effect on the $\mathbf{S}$-$\mathbf{E}$ interaction $H_{\mathbf{SE}}$ (alternatively, on $H_{\mathbf{S}'\mathbf{E}'}$) and hence no effect on the action of the POVM $\{E_i\}$ (alternatively, on the action of $\{E'_i\}$). The observer is free to collect a finite number of additional outcome values following a finite number of non-destructive experimental manipulations of $\mathbf{S}$ (alternatively, of $\mathbf{S}'$); such values will be considered to be incorporated into the set $\{\alpha_i\}$.

Under these circumstances, the sequence of outcome values $\alpha_j ... \alpha_k$ obtained by the observer can be considered to be a sequence of discrete states of a classical finite-state machine (FSM). Theorem 2 of Moore ([16] (p. 140)) then applies, showing that no finite sequence of observations of the states of any classical FSM is sufficient to identify the FSM. Hence no finite sequence of outcome values $\alpha_j ... \alpha_k$ obtained by the action of a POVM is sufficient to identify the POVM. An inability to identify the POVM being employed to make observations implies, however, an inability to identify the Hilbert space on which the POVM being employed is defined, and hence an inability to identify the physical system being examined. The QSIP is, therefore, unsolvable by finite observational means [23].

When considered in this way, as the task of inferring a unique system identification from a finite number of discrete observations, the insolubility of the QSIP becomes obvious. Treating the $(j+1)^{st}$ observation as an observation of the same system that produced the j^{th} observation, for example, already involves an assumption that the system has maintained its identity as $\mathbf{S}$, with no changes in physical composition and hence in component degrees of freedom, between the two observations. The *a priori* nature of this assumption and the difficulty of maintaining it in the face of a dynamic and uncontrollable physical world have been known since Heraclitus (ca 500 BCE). Consideration of the processes by which human beings re-identify perceived objects as the same individuals across gaps in observation only reinforces the *a priori* nature of assumptions of object identity over time (e.g., [53]).

The unsolvability of the QSIP does not, of course, imply that observations are insufficient to distinguish physical systems across the board. It rather implies the existence of a symmetry, called "observable-dependent exchange symmetry" in [23], under which physical systems indistinguishable by finite actions of some set of Hermitian operators that share eigenvalues form an equivalence class.

Choice of a different observable may permit the observational differentiation of systems within a previously-characterized equivalence class, by no finite combination of observables is sufficient to demonstrate that any such equivalence class has only a single member, *i.e.*, to uniquely identify a quantum system.

The unsolvability of the QSIP by finite means renders the SGP unsolvable by finite means; indeed, it shows that the SGP can only be solved to within an equivalence class under observable-dependent exchange symmetry. What does it mean, then, to say that one can establish the referent of 'that voltmeter' by pointing to a voltmeter? If the reasoning above is accepted, it can only mean that speaker and hearer agree that what *appears* to be a bounded enduring object of reference may be assumed, in context, to *be* a bounded enduring object of reference. This is a semantic assumption that directly violates the zero semantical commitment condition. It is good enough for all practical purposes, including all practical scientific purposes, but it does not solve the SGP.

6. So What?

It is fair, at this point, to ask the question that can be asked of any meta-theoretical result: so what? In the present case, this question can take at least two forms. First, the unsolvability of the QSIP has been demonstrated in the context of minimal quantum theory, and minimal quantum theory is, as noted in the Introduction, only an empirically reasonable assumption. What happens to the QSIP if this assumption is wrong, and some theory that is empirically similar to but mathematically distinct from minimal quantum theory turns out to be correct? Second, what does it mean to say that assuming that what appear to be enduring objects of reference are enduring objects of reference is "good enough for all practical purposes"? Does the unsolvability of the QSIP have any consequences in practice?

The answer to the first "so what?" question turns, as suggested at the end of Section 4, on whether the physical theory that correctly describes the world satisfies decompositional equivalence. Finite observers in any universe that satisfies decompositional equivalence are faced with observable-dependent exchange symmetry [23]; hence finite observers in any such universe cannot solve the QSIP. Any theory that treats a TSP of a state space as *equal to* that state space—*i.e.*, any theory in which "tensor product" has its usual meaning—satisfies decompositional equivalence. Determining whether the informally-stated ontology of a theory is consistent with the usual tensor-product formalism, and hence with decompositional equivalence, is however not straightforward. Bohmian mechanics, for example, treats "particles" as classical entities and achieves "quantum-ness" by giving these particles highly-nonclassical trajectories. Determining whether a Bohmian electron seen now is the same entity as a Bohmian electron seen previously would require knowing its trajectory, which in Bohmian mechanics depends on the trajectories of all other particles in the universe and is therefore unknowable by any finite observer. In the formulation of [21], decompositional equivalence is assumed when it is allowed that particles can be grouped into many-body systems in any arbitrary way. Bohm and Hiley infer the consequence of this explicitly: separability of the quantum state of the multi-particle universe obtains only "in the classical limit" and only in this limit can the universe be considered to have "relatively independent parts that interact mechanically" ([21] (p. 332)). As any Bohmian observer is embedded within this Bohmian universe and hence subject to observable-dependent exchange symmetry, such an observer cannot determine by

observation whether the classical limit in fact obtains. As in minimal quantum theory, classicality in Bohmian mechanics is at best an *assumption* that may serve for all practical purposes.

The second "so what?" question can be viewed as the question of whether observers may be missing something of consequence by assuming separability, individual re-identifiability, or other characteristics of the "classical limit" of minimal quantum theory or any other theory that satisfies decompositional equivalence. It is difficult to address this question from a meta-theoretical perspective that assumes classicality, which as Bohr [54] pointed out must be assumed, at least for laboratory apparatus and observers, for experiments to be considered repeatable. It is clear, moreover, that the human cognitive architecture enforces this classicality assumption pre-consciously whenever it re-identifies an object as "the same thing" after a period of non-observation [53]. From this perspective, our "practical purposes" cannot help but be ones for which an assumption of classicality is "good enough." Whether we can devise experimental manipulations that demonstrate violations of this assumption on ever-larger scales—for example, experiments that demonstrate quantum entanglement at macroscopic spatial and temporal scales simultaneously—only the future can decide. The last two decades of experimental physics at least provide cause for optimism.

7. Conclusions

Given the equivalence and finite unsolvability of the QSIP and the SGP demonstrated here, the symbols that represent observational outcomes have at best indeterminate reference. Each such symbol, e.g., "α_i" or "5 volts" can be associated with an equivalence class, under observable-dependent exchange symmetry, of physical systems, but the boundaries of this equivalence class in the space of all TPSs of $\mathcal{H}_{\mathbf{U}}$ cannot be determined by finite means. Whether any such symbol refers to the *same* member of any such equivalence class in different instances of its use, *i.e.*, as a recording of the outcome of a temporally- or spatially-distinct experiment, likewise cannot be determined by finite means. Indeed the phrase "the *very same physical system*" has no operational meaning; experiments can at best be replicated within an empirically unbounded equivalence class of systems. This absence of meaningful individual identity over time is familiar and well-accepted in the case of elementary particles ([55] provides a recent review); the assumption of decompositional equivalence extends it to all physical systems.

The in-principle referential ambiguity of all symbols referring to physical systems demonstrated here vindicates the model-theoretic view of semantics as fundamentally stipulative. It shows, in particular, that the semantics of a symbol cannot be "intrinsic" to either the symbol or the symbol system in which it is embedded as desired by Harnad [1] or Searle [2]; the apparently intrinsic semantics of human symbol systems can only, on this reasoning, be an artifact of how their use is experienced. The current result is, moreover, consistent with the claims of in-principle referential ambiguity of terms in natural languages advanced by Quine [56,57] and others, and with the pragmatic approach to natural language semantics that has been dominant since the later Wittgenstein (see [58] for review).

The present result is, finally, consistent with a view of quantum measurement as a semantic process and of POVMs as semantic mappings that assign real values to states of $\mathbf{U}$, and with the consequent view of the "classical world" as a purely semantic construct as opposed to a physically-emergent collection of objective entities [17–19]. This purely semantic view of classicality is an instance of what Landsman [59]

calls "stance 1" regarding quantum theory, *i.e.*, the stance that views quantum theory as literally correct as a description of the physical world. The semantic view differs from the standard Everettian view [60] in that it rejects the idea that "branches" or "worlds" characterize alternative states of a fixed collection of systems, *i.e.*, a fixed TPS of $\mathcal{H}_{\mathrm{U}}$. It raises an obvious question: what must be true about the dynamics of U for a consensus semantics on the part of multiple observers to be possible even just "for all practical purposes"? This is a fundamental question that physics has yet to answer.

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Conflict of Interest

The author declares no conflict of interest in connection with this work.

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