

Article

Complexity and Dynamical Depth

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Abstract: We argue that a critical difference distinguishing machines from organisms and computers from brains is not complexity in a structural sense, but a difference in dynamical organization that is not well accounted for by current complexity measures. We propose a measure of the complexity of a system that is largely orthogonal to computational, information theoretic, or thermodynamic conceptions of structural complexity. What we call a system's *dynamical depth* is a separate dimension of system complexity that measures the degree to which it exhibits discrete levels of nonlinear dynamical organization in which successive levels are distinguished by local entropy reduction and constraint generation. A system with greater dynamical depth than another consists of a greater number of such nested dynamical levels. Thus, a mechanical or linear thermodynamic system has less dynamical depth than an inorganic self-organized system, which has less dynamical depth than a living system. Including an assessment of dynamical depth can provide a more precise and systematic account of the fundamental difference between inorganic systems (low dynamical depth) and living systems (high dynamical depth), irrespective of the number of their parts and the causal relations between them.

Keywords: complexity; dynamical system; self-organization; machine; organism; teleodynamics; morphodynamics; non-linear; computation; information

1. Introduction

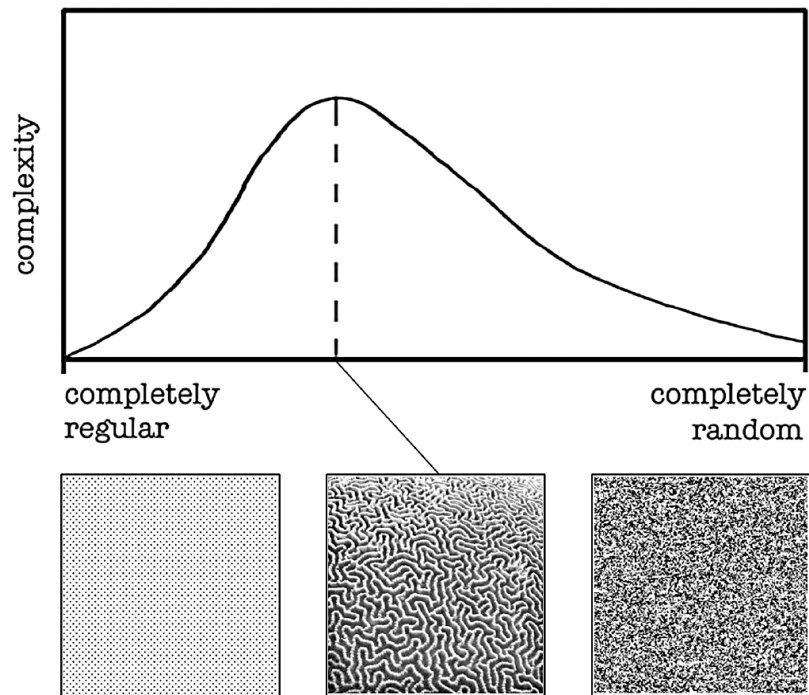
In his influential essay “The Architecture of Complexity,” published in 1962, Herbert Simon provides an admirably plain and seemingly self-evident view of complexity: “Roughly, by a complex system I mean one made up of a large number of parts that interact in a non-simple way [1] (p. 468).” Because of the non-simplicity of their interactions “given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole [1] (p. 468).” Simon’s approach to complexity has been implicitly accepted by almost all attempts to define complexity in different sciences. Since the 1960s many analytic and quantitative definitions of complexity have been offered. There are two main categories: The first category includes methods developed by information theorists and mathematicians who understand complexity in terms of a measure of the number of distinguishable components times some measure of the number of possible non-redundant (e.g., random) arrangements of these components with respect to one another. Members of the second category try to capture a system’s complexity in terms of the number of its physical features or the ways these features could have been produced. Members of this second category are more diverse, and may involve considering the work and time involved in these processes. These theories also tend to be responses to certain apparent paradoxes that arise when the assessment is limited to information theoretic approaches.

For example, intuition suggests that a living frog is far more complex than if those same molecules were mixed up in a solution where they are randomly distributed. Using a commonly accepted measure of information capacity of the first category—information entropy, developed by Claude Shannon in 1948 [2]—it would take less information to describe the details of the living frog than the “frog soup.” That is because there is considerable redundancy in the organization of molecules in the living frog, so that a vast number of correlated features do not need to be repeated again and again in the description, whereas in solution these correlations have been eliminated.

This suggests that one should rank the complexity of both fully ordered and nearly random phenomena near zero and rank the organization of living phenomena to be high. This is exemplified by the often-reproduced graph depicting this qualitative ideal, shown in Figure 1. The peak in the curve representing maximum complexity presumably marks an optimal mix of ordered and disordered relationships. But exactly how to justify what constitutes this optimal mix is a matter of considerable debate, as is the justification for calling this complexity. Other than merely conforming to intuition and subjective sensibility, no widely accepted clear and simple criterion for judging this to exemplify maximal complexity is currently available.

In response to this and related challenges, many alternative measures of complexity in the second category have been developed. In the following discussion we will review some of the general features of both general categories of complexity measures, discuss their limitations, and introduce an additional complementary form of complexity unique to organisms that appears not to be accounted for in any of the current approaches.

Figure 1. Graphic depiction of the intuitively ideal measure of complexity that treats both completely random and completely ordered distributions as minimally complex, but treats organism complexity (coral surface; middle image) as highly complex.



1.1. Complexity as Randomness

In his *Mathematical Theory of Communication* Claude Shannon demonstrated that the information capacity of a communication channel could be assessed in terms of the degree of unpredictability of any potential message it could convey. In the decades that followed a number of theorists, including Ray Solomonoff [3,4], Andrej N. Kolmogorov [5], and Gregory J. Chaitin [6], independently developed ways to define and measure the complexity of sequences of symbols that were analogous to Shannon's approach to measuring information capacity. Measures of this kind of complexity came to be variously named *algorithmic information content* (AIC), *Kolmogorov complexity*, or simply *algorithmic complexity*. In these approaches the complexity or information content of a string of symbols is equated with the length of the *shortest* computer program or algorithm that can generate this string (typically defined in terms of implementation on a universal Turing machine for maximum generality). The algorithm is considered to be a compressed description of the string. If the string consists of values that describe states of a physical system (e.g., measure values) the algorithm is a measure of the *compressibility* of an abstract description of that physical phenomenon.

Charles Bennett [7] developed a related measure of complexity, which was also independently developed in a paper by Chaitin [8]. Bennett called it *logical depth* [7]. He defines logical or algorithmic depth as the *time* that a universal Turing machine requires to execute the shortest algorithm that generates a given sequence of symbols. The logical depth of such a sequence is directly related to its Kolmogorov complexity, but additionally it takes into account the work involved in the generation of that sequence. There are many other algorithmic definitions of complexity [9] (pp. 63–65), but

almost all are based on related assumptions and so for the purpose of the present paper these will be considered variants of the general AIC paradigm.

The concepts of complexity developed by Solomonoff, Kolmogorov, Chaitin, and Bennett are very similar, and are also related to Shannon's measure of entropy. They treat complexity as the inverse of compressibility and identify ways to compress a signal or its description (such as a character string) by virtue of discovering redundancies. However, whereas Shannon entropy only takes into account simple statistical redundancy, the various algorithmic approaches can additionally take into account more cryptic redundancies due to recursive aspects of the computational process capable of generating that description. In effect, AIC approaches seek to identify the most compressed, *i.e.*, shortest possible description of a given phenomenon.

The AIC approach to complexity was implicit in Herbert Simon's [1] (p. 478) idea that "a system can be described economically" if it has redundancy. So although Simon did not explicitly define complexity as a measure of non-redundancy in the description of a physical phenomenon, his approach anticipated this later development. The question that remains is whether the complexity of a description and the complexity of the phenomenon described are equivalent.

So the question is begged whether or not these methods which apply to abstract descriptions (typically treated as binary strings) are also adequate to assess the complexity of real physical systems. We discern five weaknesses or limitations in the use of AIC approaches to measure physical complexity. First, even if physical systems are understood as spatial or temporal sequences of physical differences they "do not come readily encoded as numbers [10] (p. 189)". Second, as Chaitin [6] showed, it is impossible to prove that a certain algorithm is indeed the shortest possible description of a given sequence. Third, like Shannon's concept of information, AIC and logical depth, does not attempt to assess the meaning or function of the descriptions they analyze but only their syntactic aspects. Any assessment of the complexity of physical phenomena needs to also consider any semantic (referential) and pragmatic (functionally relevant) aspects of these descriptions, especially since these aspects are critically relevant to biological and cognitive systems. Fourth, information theoretical concepts of complexity only characterize *descriptions* of physical systems, not how they came about or how they are causally organized. In this sense they refer to the "map" and not to the "territory." This may or may not reflect the causal relations inside a dynamical system. Fifth—and this is the biggest problem for all methods that connect complexity to non-redundancy or randomness—the concept of complexity underlying AIC and logical depth is *counterintuitive*. The information theoretical concepts of complexity attributes higher complexity to an entirely chaotic phenomenon than to an ordered one. Whereas the former cannot be described by a description that is shorter than a full list of attributes because "no aspect of its structure can be inferred from any other" and therefore "it is its own simplest description [1] (p. 478)," the latter can be compressed because of its internal regularities. Thus, the information-theoretical and mathematical approach to complexity *only* identifies complexity with incompressibility in one dimension of organization, whereas real physical systems may involve multiple dimensions, including their generative history and functional attributes.

This understanding of complexity therefore produces a paradoxical problem in that it effectively treats a maximally disordered system and the random string of characters that describes it as more complex than ones that exhibit interesting and/or unprecedented properties, such as being alive or

being conscious. Intuition suggests instead that a thoroughly random and maximally unpredictable, *i.e.*, maximally incompressible, sequence (state to state, character to character) is simple in its organization.

Clearly, the related definitions of complexity of Kolmogorov, Chaitin, Solomonoff, and Bennett are well-suited for analyzing the complexity of descriptions that can be rendered in abstract sequences of symbols, numbers, or computational steps, but will not be able to account for features that are not distinguishable and countable in the immediately present physical object of analysis. Thus they often lead to counterintuitive results when applied to dynamical systems. In order to understand the complexity of such systems we need a more subtle measure that more accurately reflects features of a system's causal properties, not just the complexity of its description (whether formal or algorithmic).

1.2. Complexity and Physical Structure

In the last three decades natural scientists, especially physicists and biologists, have introduced concepts of complexity that focus on the organization of natural entities and not merely on the regularities and irregularities of their formal descriptions. These attempts to assess the relative complexity of physical structures are diverse and often specially formulated with respect to a particular disciplinary context. As a result, they do not exhibit clear analytic parallels but only parallels in the problems they address. We class them together based on their inclusion of physical, not just symbolic, attributes into the analysis. Here, we survey a few of these measures in order to exemplify their commonalities and their disciplinary diversity.

The simplest way of quantifying complexity on the basis of structure is to count the number of a system's elements and/or interactions between them [11]. For example, some biologists [12] offer definitions of organism complexity based on the number of different cell types, the number of components, and the organism's morphology, *i.e.*, the number of structural elements and their combinations, but generally agree that an increase in number of identical components should not count toward an increase in complexity [9] (p. 74). Others define biological complexity "related to the diversity or lack of self-similarity" of the interactions within a hierarchically organized system [13] (p. 130), see also [14]. Such approaches rely on the way that the system components are categorized with respect to similarities and differences in their identified properties. And what constitutes a proper part is not always obvious. This inevitably introduces a subjective factor into the assessment and requires that comparisons be based on identical categorization schemes. This is not a problem for AIC measures based upon symbol counts or algorithm steps.

Perhaps the physical measure of complexity that is most similar to AIC measures and yet does not reduce to a measure of randomness is the concept of *effective complexity*. This concept was introduced by Murray Gell-Mann and Seth Lloyd [15–17] in order to better take into account the organized heterogeneity of natural systems. Their analysis splits the Kolmogorov complexity or algorithmic information content describing an object into two separate categories: its regularities and its random features. In contrast to AIC measures, however, an object's effective complexity is defined as the AIC of the *regularities* alone, *i.e.*, the length of the shortest algorithm that can generate the regularities of an empirical system or the sequence of symbols describing them [18]. This measure has the advantage of conforming to the intuition that highly regular and highly random structures are judged minimally complex while heterogeneously ordered structures intermediate between these extremes are more

complex. The concept of effective complexity has the disadvantage of being too sensitive to subjective criteria, since the assessment of regularities depends on an external observer who decides which parts of the system are to be considered regular *versus* irregular.

Adami and Cerf developed a related concept they called *physical complexity*. Their approach was developed specifically for application in biology, though it can be applied to “any sequence of symbols that is about a physical world or environment [11].” Physical complexity is “an instance of effective complexity [19] (p. 1087)”, but additionally considers functional relationships with respect to the environment in which the system exists. It is defined as the Kolmogorov complexity (AIC) of the bio-molecular sequences (e.g., genomes) of a population that encode the adaptations of the organisms to specific features of their environments. The physical complexity of a genetic sequence is the amount of information coded in the genetic material that can be mapped to specific features of the environment to which it is adapting. This information is “given by the difference between the entropy of the population in the absence of selection, and the entropy of the population given the environment, that is, given the selective forces that the environment engenders”. (ibid.) All entropies are calculated using the Shannon entropy of this distribution. It is an attempt to measure that fraction of the information encoded in a genome that enables an organism to make predictions about its environment and hence increases its chance of survival (ibid.). Though counting coding genes appears less subjective than counting structural regularities, the complex nonlinear relationships between genes and phenotypes and between organisms and their environments depends on some difficult to defend simplifying assumptions.

Following a different logic, Bialek and Tishby [20] developed a concept that they called *predictive information*. It is based on the principle of the extensivity of entropy [11]. Systems are called “extensive” if their entropy grows linearly with the increase of the number of their elements. Since systems that contain elements whose causal interactions differ at different levels of scale their statistical entropy will tend to be a nonlinear function of their size. So the degree to which a system deviates from extensivity can be used to measure the degree of the interaction diversity of its elements. In this respect predictive information is a measure of what might also be described as network complexity and is particularly sensitive to the presence of local inhomogeneities and long-distance interactions. This makes it particularly well-suited for analyzing structures such as brains where there are complicated scale-dependent connection differences.

Another approach applicable to brain complexity called *neural information* was developed by Tononi and colleagues [21]. It is related to predictive information. Despite the name, neural information is applicable to all kinds of empirical systems not just brains. “One of its building blocks is integration” which “is computed as the difference between the sum of the component’s individual entropies and the joint entropy of the system as a whole [11].” The concept of integration (like non-extensivity) also measures the degree to which a system’s entropy deviates from additivity (*i.e.*, linearity).

Finally, Seth Lloyd and Heinz Pagels [10] introduced the concept of *thermodynamic depth* in an effort to overcome the lack of causal relevance of the concept of logical depth and of AIC conceptions of complexity in general. Thermodynamic depth was conceptualized as a quantity that characterizes “the evolution of a state and not the state itself [10] (p. 187).” Lloyd and Pagels [10] proceed from the fact that there are many possible trajectories, which can lead a system to a given macroscopic state in

its state-space. They define the depth of a system's macroscopic state as "the amount of information required to specify the trajectory that the system has followed to its present state [10] (p. 190)." For example "the genetic complexity [thermodynamic depth] of an organism is proportional to the amount of genetic information tried out and discarded by the process of natural selection on the ancestors of the organism [10] (p. 209)" because of the number of possible ways this process could have occurred. In this way they claim that [thermodynamic depth] "assigns low complexity to random and highly ordered but regular states [10] (p. 208)" so that a crystal and a gas at equilibrium will both have low thermodynamic depth because of the close values of their course-grained and fine-grained entropies [10] (p. 190). Thermodynamic depth is correlated with the uncertainty of specifying the possible trajectories by which a system could have arrived at its current state. This is effectively a measure of the entropy of possible trajectories, whereas other measures of complexity are based on possible configurations of a system's macroscopic state. Thermodynamic depth can be as high in non-living systems, such as computational devices or nonlinear processes, as in organisms.

These approaches were developed in order to overcome the shortcomings of the information theoretical approaches to complexity, and they succeed in two critical respects. Since they have been designed to characterize real material systems they are sufficient to describe features of their physical structure. They also avoid some of the counterintuitive assessments of a system's complexity because, in opposition to the information theoretical approaches, they *identify a system's complexity with the diversity of its component physical relationships*. Since they consider the regularities of a system and not merely its irregularities they reflect aspects of what might be described as the inner coherence of a system's causal organization as reflected in its spatiotemporal relations. We ascribe inner coherence to a system which allows an observer of a part of it to anticipate to a certain degree other aspects of the same system. The parts of internally coherent systems contain information, although often vague, about the whole system. Inner coherence enhances the compressibility of a system's description because it increases the ability to predict its organization from partial knowledge of it. But structural redundancies are judged less complex by AIC standards.

Despite their advances over AIC measures of complexity, physical approaches still suffer from a serious limitation. Their main common characteristic is that they are designed to quantify the complexity of phenomena by counting structural features and/or the amount of work that produced them. They analyze a system's phenomenal regularities and/or entropy but do not attempt to characterize the qualitative differences of their causal organization. To be able to identify the forms of complexity that make living and mental processes so causally distinctive we need to be able to additionally assess these qualitative differences in causal organization. Unlike non-living processes a living system is dynamically organized to do work to modify its own organization with respect to interactions with its environment. So not only does this require an assessment of its composition and the amount of work required to produce or describe it, it also requires an assessment of this dynamical self-organization.

2. Open Systems: Self-Organization and Organisms

The different ways that compressibility is used to assess complexity in information-theoretical as compared to physical approaches, demonstrates both the relevance of this property and also the way

that it applies differently to descriptions as compared to actual physical structures. The role of compressibility in assessing complexity is further complicated with respect to what are often called complex dynamical systems. Interest in dynamical systems and “chaotic” processes developed along with major advances in non-linear mathematics and an explosive increase in computing power (which made realistic simulations of complex physical processes possible). As dynamical systems theories have developed over the years to be able to more precisely describe open systems and their coupling with the environment, research interest has focused on dissipative systems. These are thermodynamic systems that are maintained far-from-equilibrium for some period of time. Of particular interest are thermodynamic systems that are being constantly perturbed away from equilibrium. In the process of continually dissipating this disturbance they can develop toward highly organized dynamical configurations. These so-called “self-organized” systems have provided important models for exploring the difference between simple mechanistic systems and biological systems.

The most essential feature of self-organized processes is that their local statistical entropy decreases which correspondingly increases their inner coherence and thus their predictability. In physical terms we can say that they *compress* the trajectories of their future development into small fractions of their state-space; regions called “attractors.” We will refer to this real compressibility as *dynamical compressibility* because it is a consequence of the activity of the system itself. Dynamical compressibility allows an external observer to create highly compressed abstract descriptions of such systems. So, there is an intrinsic relation between the dynamical compressibility of self-organized dynamic systems and the compressibility of the formal descriptions of those systems. As we will show below, physical processes that spontaneously develop toward a more compressed state pose some interesting challenges for measures of complexity that are based on the compressibility of the formal descriptions.

Self-organization is a technical term. Invoking the term “self” is intended to signal that the increase of a dynamic system’s orderliness is the result of interactions among its components and not the result of the imposition of this form of organization from some extrinsic source. However, less precise usage often invokes a more expansive conception of self, which includes biological agency. This confuses two very distinctive causal dynamics—those that are associated with inorganic processes that speed their dissolution by maximizing entropy production and those associated with living processes that preserve their organization by acting to counter degrading influences. Failure to distinguish between these two very different classes of dynamical processes ignores a significant difference in their causal organization: they abide by two essentially different logics of constraints. This distinction will be the focus of the final sections of this paper.

It is widely accepted that living systems depend on cellular and chemical dynamics that exhibit self-organization. There are, however, good reasons to question the ability of dynamical systems models to adequately assess the complexity of organism dynamics. In artifacts the target states that are invariably approached by such processes play no role in the constitution of the physical systems that produce them. In contrast this role is fundamental to the forms of causality characterizing organisms. Self-organized processes develop in response to persistent gradients of energy and/or material that maintain them away from thermodynamic equilibrium, but this resulting regularization maximizes the reduction of the very gradient that produces it. In other words: *Each self-organized system tends to evolve toward a dynamical form that would return the system to equilibrium in the most efficient way*

possible were the external gradients to become exhausted. This essential property of dissipative dynamic systems is known as the principle of *maximum entropy production* (MEP) [22]. According to the second law of thermodynamics all physical processes produce entropy. Self-organization is characteristic of dynamical thermodynamic systems maintained far from equilibrium that not only increase global entropy but also increase the rate of entropy production in a way that compensates for any imposition of additional energy and/or material. The order that emerges inside such a system enables it to offload the destabilizing influence of an energy/material gradient more quickly and with less work than if its dynamics were not regularized.

So, self-organization is paradoxically self-undermining in this respect. There is nothing internal to a self-organized dynamical system to maintain this order in the absence of external gradients, nor any mechanism to compensate for changes in this external support. In this respect it is not accurate to claim that self-organized systems are the sources of their own organization, and so the term “self” is ultimately a metaphoric use in this context. Koutroufinis [23] describes this kind of dynamics as “self-organization without self.”

One reason to be suspicious of the completeness of any theory of complexity based on some measure of compressibility is that it will fail to adequately account for ongoing physical processes that involve the sort of dynamical compression that characterized self-organized processes. Superficially the structure of a self-organized dissipative structure (e.g., a whirlpool or a growing snow crystal) can be highly regular and therefore algorithmically compressible, and yet this is because the process that produces this regularity has reduced the irregularity and entropy within that structure. A descriptive compression of an already physically compressed state eliminates critical information about the stochastic nonlinear dynamics that produced and maintains that compressed physical form.

This problematic situation is resolved by recognizing that there may be two complementary aspects of complexity that are relevant to the assessment of dynamically compressed systems: one associated with the structure of its physical features and one associated with how the regularities of its dynamical features are generated and actively maintained.

Below we will argue that organisms add an additional level of dynamical compression beyond what is present in non-organic self-organizing dynamic processes. This is a consequence of the way that living processes invert three defining features of self-organization. First, organisms do not maximize entropy production but rather tend to more efficiently organize entropy production with respect to the work they do to persist and reproduce. Second, because of this, their dynamical organization is not self-undermining, even though it incorporates features of self-organization. Third, organism dynamics is specifically self-beneficial and depends on system-internal rather than system external determinants of its organization. These attributes cannot be a consequence of simply additively combining self-organized processes. Together these dynamical differences contribute to the added level of complexity that distinguishes living from non-living systems irrespective of their relative degrees of algorithmic or physical-structural complexity.

For these reason we require a new concept of complexity that emphasizes those characteristics of organism dynamics that make them unique among dynamically compressed systems. It must both account for the dynamical compression that occurs via self-organization and also the ways that living processes diverge from all non-living processes.

3. Dynamical Depth: An Orthogonal Form of Complexity

In an effort to clearly distinguish living dynamics from nonliving dynamics, Deacon [24] identifies three modes of system dynamics that are distinguished by their hierarchic (*i.e.*, nested) dependencies and their reversals of spontaneous global dynamical tendencies to reach different kinds of stable end-states (or attractors), if they are provided with the required time to do so. These dynamical modes include *homeodynamics* (e.g., processes at or near thermodynamic equilibrium), *morphodynamics* (e.g., non-chaotic dissipating processes such as exemplified by self-organizing systems), and *teleodynamics* (e.g., self-preserving processes such as exemplified by living systems). These dynamical modes define distinctive levels of what we will call *dynamical depth*, because they involve the nesting of stochastic dynamical processes within one another. In this section we show how this measure of complexity is distinguished from the various complexity measures discussed above.

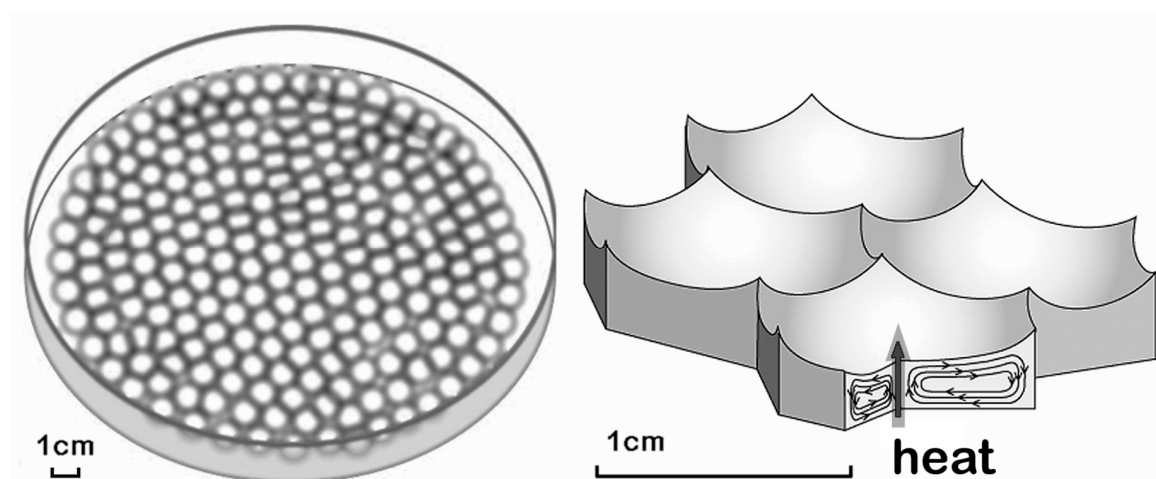
These three dynamical modes are also distinguished by differences in the ways they eliminate, introduce, or preserve *constraints*. The term “constraints” refers to all factors that reduce the number of the possible states of a system, so that its behavior resides only in a limited part of its state space. Since entropy increases with the number of a system’s possible states, it follows that constrained systems are not in the state of their maximum possible entropy and therefore are able to perform work. Constraints can be imposed from outside on a system or can be generated internally, *i.e.*, by a system’s own dynamics. We describe the former as *extrinsic* and the latter as *intrinsic* constraints.

Linear thermodynamic processes, which characterize homeodynamic systems, epitomize the first level of dynamical depth. They are defined by their spontaneous tendency to *eliminate constraints*, and thus increase entropy. The natural end-state of a homeodynamic system is the state of maximum *possible* entropy, which is also the state of maximum *possible* elimination of constraints. An isolated thermodynamic system (*i.e.*, a multi-particle system which is energetically and materially closed, like a gas in a fixed size container) moves always towards the state of the system’s maximum *possible* entropy, which is also the state of *zero entropy production* or thermodynamic equilibrium. Chemical reactions in an isolated system also tend to exhaust their “free energy” and thus their potential to asymmetrically change from one state of chemical relationships to another. At this point the system has reached chemical and thermodynamic equilibrium *i.e.*, the end-state in which the concentrations of the reactants and products do not change. Simple non-idealized mechanical systems, such as a harmonic oscillator or a pendulum with friction, as well as compound mechanical (e.g., clockwork) systems occupy the same level of dynamical depth because they do not generate any new constraints but only eliminate externally imposed constraints (*i.e.*, energetic gradients). A mechanical clockwork degrades the potential energy which is stored in its spring. It could only generate new constraints if it were able to reposition its own gears into other configurations as a consequence of its own internal movements. The essential common feature of non-idealized (e.g., frictional) mechanical systems and isolated multiple particle systems (e.g., gases) is that they both reach the state of maximum *possible* elimination of constraints. At this point a homeodynamic system has exhausted its capacity to do further work (whether mechanical, chemical, or thermodynamic). Finally, we also class among the first level of dynamical depth all current forms of computational devices so far constructed or conceptualized. It is easy to understand this conclusion if one considers that, in principle, all operations of electronic computers can also be implemented (at least in principle) by mechanical devices.

A second deeper level of dynamical depth is produced by self-organization. As noted above, self-organization arises in the special case where an externally imposed energetic and/or material gradient (*i.e.*, extrinsic constraints) are introduced into a partially open dissipative system at a rate that exceeds the rate at which that system can spontaneously dissipate them. As a result the system's dynamics changes in a way that allows it to increase dissipation of externally introduced energy and/or matter, so that the rate of their dissipation matches the rate of their imposition. This is achieved when the system's dynamics reorganizes itself and becomes spatiotemporally regularized (*i.e.*, dynamically compressed) in ways that compensate for the effect of being continually driven away from equilibrium by the external gradient. Deacon [24] terms such processes *morphodynamic* processes because of the way that they internally generate these regularities; *i.e.*, spatiotemporal patterns. So, a critical feature that distinguishes morphodynamic processes from merely homeodynamic processes is that morphodynamic processes generate new constraints (*i.e.*, spatiotemporal patterns) locally. Thus morphodynamic systems develop *intrinsic* constraints that dissipate the imposed extrinsic constraints.

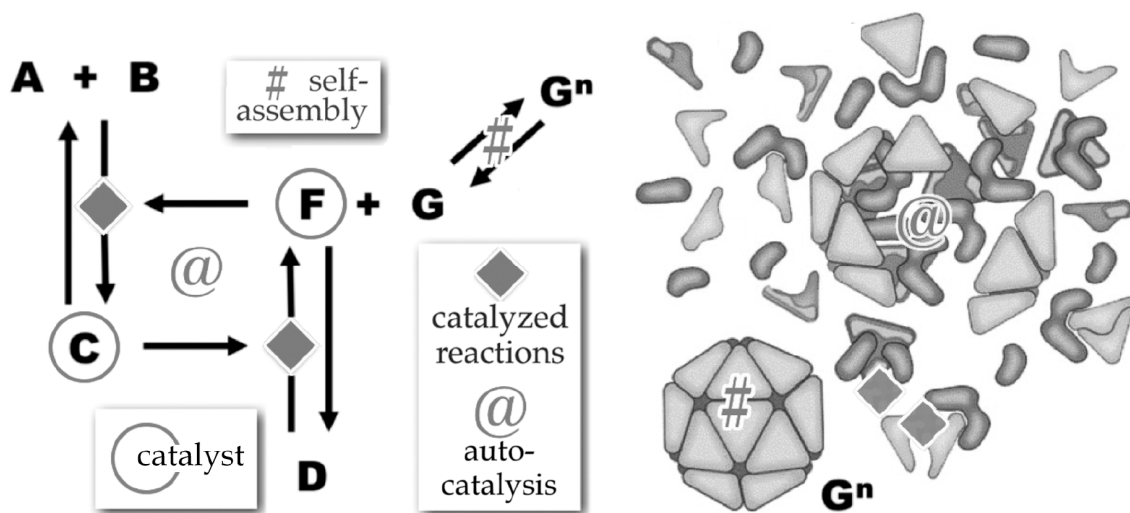
A well known example of intrinsic constraint generation is the pattern of hexagonal convection cells (Bénard cells) that emerge when a thin layer of liquid (e.g., oil) is heated from below to a temperature beyond which simple conduction can dissipate this imposed heat (see Figure 2). Morphodynamic processes will tend to spontaneously regenerate their intrinsic constraints if random perturbations damage their pattern so long as the supportive externally imposed gradients persist within certain limits. Intrinsic constraints in a morphodynamic process persist because they efficiently degrade extrinsic constraints. Since they depend on persistent extrinsic constraints intrinsic constraints cannot contribute to their own persistence. They can only degrade these extrinsic constraints. This special correspondence between intrinsic and extrinsic constraints does not, however, support the system's self-preservation. It is effectively organized in a way that would hasten its self-destruction, were extrinsic supports removed.

Figure 2. Depiction of Rayleigh-Bénard convection cells formed in a thin volume of liquid (e.g., oil) heated from the bottom and cooled from above. Correlated movement of fluid orthogonal to the temperature gradient is driven by density differences due to heat content and the hexagonal regularity emerges as the closest-packed tessellation of units per surface area. The result is maximization of the diffusion of heat through the liquid.



Finally, and most critical for this analysis, at a third level of dynamical depth are what Deacon [24] terms *teleodynamic* processes. Teleodynamic processes emerge from precisely complementary, interdependent morphodynamic processes. This complementary relationship is such that the supportive boundary conditions for each component morphodynamic process are generated by one or more of the other morphodynamic processes in the system. As a result the otherwise highly improbable boundary conditions that make the component morphodynamic processes possible can be preserved to become highly probable. Such a combination of boundary constraints would otherwise be astronomically improbable to occur spontaneously. Deacon [24] describes a molecular model system called an autogen (or autocell [25]), which precisely exemplifies an empirically testable form of this distinctive dynamical organization and its unique systemic attributes (see Figure 3).

Figure 3. Left: The reaction network of a simple autogenic process. **Right:** Idealized depiction of an autogenic complex in formation. An autogen is formed when a reciprocally autocatalytic set of molecules (C and F) modifies available substrate molecules (A, B & D) to produce (along with additional Cs and Fs) a side product (G) that tends to self-assemble (#) into an impermeable container (a process analogous to the formation of a virus capsule). Such a system will tend to reconstitute itself if disrupted and reproduce duplicate systems if sufficiently disrupted. Modified with permission from Deacon [24].



This complementarity and interdependence constitutes a yet *higher order form of constraint* on the constraint-generation of the component morphodynamic processes. The most essential characteristic of any teleodynamic system is the internal generation of such a higher order constraint. In other words, teleodynamics consists in a *higher order intrinsic constraint* that constitutes the synergy of the lower order morphodynamic processes. This higher order constraint increases the probability that these same component morphodynamic processes will persist despite changes in extrinsic conditions. The result is a tendency to behave in a way that makes this distinctive form of self-reconstituting dynamics highly likely to persist, re-form if disrupted, and even become reproduced if the basic ingredients for the formation of the autogen are available.

A critical criterion for any means of quantifying relative complexity is that there must be discrete countable features or elements. The difficulty of identifying “proper parts” or clearly distinguishable

relations has often been a problem for measures of complexity that have been developed in order to compare biological and non-biological phenomena. This arises in part from the fact that the relationships between organisms structures and functions (e.g., genes and adaptations) is seldom simple and linear and because biological molecules co-produce one another. The concept of “dynamical depth” avoids those difficulties because it does not depend on the identification of “proper parts.” Instead, it focuses on discrete inversions of thermodynamic trends that are qualitative features of these three forms of dynamics. Therefore, one distinctive characteristic of dynamical depth is that it does not increase in a continuous transition, but rather in quantized increments. The transitions from homeo- to morpho- and from morpho- to teleodynamics always occur abruptly, roughly comparable to first order phase transitions in physics. These levels of dynamical depth refer to fundamentally discontinuous “qualities” of systemic organization; between them there are no intermediate stages. This discontinuity is defined by the distinct inversions of certain dynamical system tendencies occurring at these transitions. Thus for example, isolated homeodynamic systems tend to develop to a point where there is no further entropy production (equilibrium), morphodynamic systems tend to increase their entropy production, and teleodynamic systems tend to develop toward *minimizing* the amount of entropy production necessary to remain far-from-equilibrium (see for example, Falkner and Falkner [26]). So, although dynamical depth does not provide a large range in magnitude, it has the virtue of unambiguously delineated countable thresholds marking increasing constraint of lower order dynamical processes by higher order dynamical processes.

The progressive internalization of constraint generation with increasing levels of dynamical depth exemplifies another defining attribute: an increase in what can be described as *organizational autonomy*. Thus the organization of homeodynamic processes is entirely dependent upon extrinsically imposed conditions, whereas the organization of morphodynamic (e.g., self-organized) processes is additionally a function of the ways that the components of the medium in question tend to interact. As a result morphodynamic processes generate intrinsic constraints whose forms are independent of the form of the extrinsic constraints that drive the system to generate them. Teleodynamic systems demonstrate an even greater degree of organizational autonomy and are the only systems that can be described as *self-organized* in a non-metaphoric sense of *self*.

There are several reasons to ascribe selfhood to teleodynamic systems:

- (1) The end-directedness of a teleodynamic system cannot be adequately described as an “attractor,” as this concept is understood in dynamical systems theories. The constrained dynamics that constitutes a morphodynamic attractor develops as a consequence of externally imposed environmental influences (*i.e.*, extrinsic constraints). In contrast, the state toward which a teleodynamic system develops tends to resist environmental influences. This is because its higher order intrinsic constraint prevents the disruption of the synergy between the component morphodynamic processes that determines its unity. The teleodynamic target state—*i.e.*, the self-preservation of this synergy—is in this respect a *higher order attractor*.
- (2) Teleodynamic systems are organized in a way that preserves and re-generates their own intrinsic constraints. A teleodynamic system interacts with its environment in a way that sustains supportive relationships with its environment and compensates for unsupportive or destructive relationships, e.g., by repairing or reproducing itself [24]. A teleodynamic system

- can therefore be said to include an implicit *self-representation* that persists despite a partial loss of system coherence, and with respect to which coherent organization can be re-achieved.
- (3) In contrast to morphodynamic attractors, the higher order attractor of a teleodynamic system plays a critical role in its own generation. Since attractor generation is a form of dynamical compression teleodynamic systems can be considered *self-compressing systems*. This is a new type of compression that might be described as the compression of the relationships between compression processes. Above we have noted that complexity theories treat description as a form of compression, so it is not unreasonable to describe this higher order form of compression as information that represents and regulates the dynamics that maintains system integrity with respect to forces that tend to disrupt it. In this respect we might be justified in saying that organisms are literally self-referring. Because of this self-referentiality teleodynamic organization may not be adequately describable by formalisms that are adequate to describe non-living dynamical systems (see Koutroufinis [23,27]).
 - (4) A teleodynamic system implicitly incorporates within its organization information about those aspects of its environment that are specifically relevant to its self-maintenance and reproduction. This information is embodied in the specific organization of the set of compression processes that maintain system integrity with respect to potentially beneficial or harmful aspects of its environment. Teleodynamic systems thus have an *Umwelt* and not just “surroundings” as do merely reactive morphodynamic systems. In 1909 Jakob von Uexküll introduced the term *Umwelt* referring to those features of a living being’s environment to which they are sensitive. *Umwelt* and self are two sides of the same. This is characterized by biosemiotician Kalevi Kull’s [28] translation of *Umwelt* as “self-centered world.” In simple terms, this it is the creation of a self/other boundary, which implicitly incorporates a representation of this *Umwelt*.
 - (5) The distinction between self-compression, self-representation, and representation of the environment does not imply three different activities of the system but only three aspects of the system’s integrated dynamics. The higher order intrinsic constraint organizes processes distributed over the entire system in such a coherent way that one and the same material-energetic process might contribute to all three activities.

Self in this sense is a dynamical process organized in such a way that it minimizes the probability that its organization will be lost. Inevitably, the conditions conducive to generating this distinctive dynamical configuration are far less probable than those producing self-organized dynamics alone. The fundamental behavioral tendency of living systems is to maintain these rare supportive conditions by internally generating and preserving those that are least likely to arise spontaneously. This curious causal-circularity linking living processes to the production of their own preconditions is why living complexity is not adequately described by non-dynamical conventional measures of complexity.

What we term *dynamical depth* then, is this hierarchic complexity and irreducibility of constraint-generating dynamics, such as distinguishes teleodynamics from morphodynamics and morphodynamics from homeodynamics. Each of these transitions is characterized by the *generation of intrinsic constraints* on the relationships between processes at lower levels and as a result with increasing autonomy from extrinsically imposed constraints. Since constraints are a prerequisite for

producing physical work, the increasing autonomy of constraint generation with dynamical depth also corresponds to an increasing diversity of the capacity to do work. Thus the flexibility with which a dynamical system can interact with its environment also increases with dynamical depth.

Finally, dynamical depth is an orthogonal measure of complexity with respect to algorithmic measures of complexity. This defines two distinct dimensions of complexity that can vary independently. Algorithms and descriptions of considerable AIC complexity lack dynamical depth. Algorithms and descriptions are symbolic structures. As such they are non-physical processes but timeless abstract entities. So, they can neither be described in terms of entropy or entropy production nor in terms of degradation or generation of constraints. In comparison any electronic/mechanical implementation of the corresponding computation processes would exhibit a first level of dynamical depth due to its thermodynamics. And even though the AIC complexity of a living frog may be lower than that of its molecules homogenized in a solution, its dynamical depth will be many levels deeper.

4. Conclusions

Throughout this discussion we have discovered again and again that the concept of compression is critical to the definition and assessment of complexity. The concept of compression is intimately related to both constraint and description.

In the AIC conception of complexity, more compressible equals less complex and less compressible equals more complex. The compressibility of a description or algorithm is in effect a measure of constraint exhibited by the physical system being described. As Shannon's original analysis demonstrated, the measure of information of a received message is proportional to the constraint exhibited in that sign/signal medium compared to its possible prior Shannon entropy. The degree of compressibility is then related to how much information is provided by a given signal. This fits with the common intuitive sense that the power of a description or explanation is related to its compactness with respect to the amount of detail it accounts for.

Analogously, a physical system that is more constrained in its dynamics than another occupies a smaller fraction of the phase space of the system's possible states. In this respect, the system can be described as dynamically compressed. Since dynamical compression of a self-organized or living system develops over time there is a progressive reduction in local entropy up to the point that it settles into a quasi-stable low-entropy dynamical behavior (often described as an attractor). To measure some version of the system's AIC complexity, then, will fail to assess the amount of system entropy that was initially reduced and is being constantly offloaded by system dynamics.

For this reason, AIC measures of complexity, which work well for analyzing complexity in terms of the compressibility of an adequate description, algorithm, or construction process, tend to yield paradoxical results when applied to living processes. Above, we have surveyed a few of the many attempts to develop a measure of physical complexity that rank both highly disordered and highly ordered structures as low complexity and rank a mix of ordered and disordered structures as highly complex. The difficulty with these approaches is finding a way to objectively justify which features to count and what amounts to an optimal combination of order and randomness. Additionally, we argue that they confound two distinct and opposed applications of the concept of compression.

We approach this problem by paying attention to the ways that the stochastic processes in self-organized and living systems are themselves dynamically compressed. This avoids problems that arise due to applying an AIC compression analysis to the results of a physical compression process. A physical system, that has developed to a state that continually minimizes its local entropy, such as occurs in morphodynamic processes, has a low apparent AIC complexity. Therefore, assessing only its AIC complexity effectively ignores this dynamical compression and the local system entropy that is thereby actively kept low. At the higher levels of dynamical organization that characterizes living organisms there are additional constraints that maintain interdependence between their component morphodynamic compression processes. This introduces a higher order level dynamical compression over and above these component compression processes. Since this higher order compression is what maintains organism integrity it is in effect a form of *self-compression*. In order to accurately model organism complexity, it is necessary to strictly separate the description of these dynamical compression processes from the description of their consequent effect. Both are relevant dimensions of organism complexity.

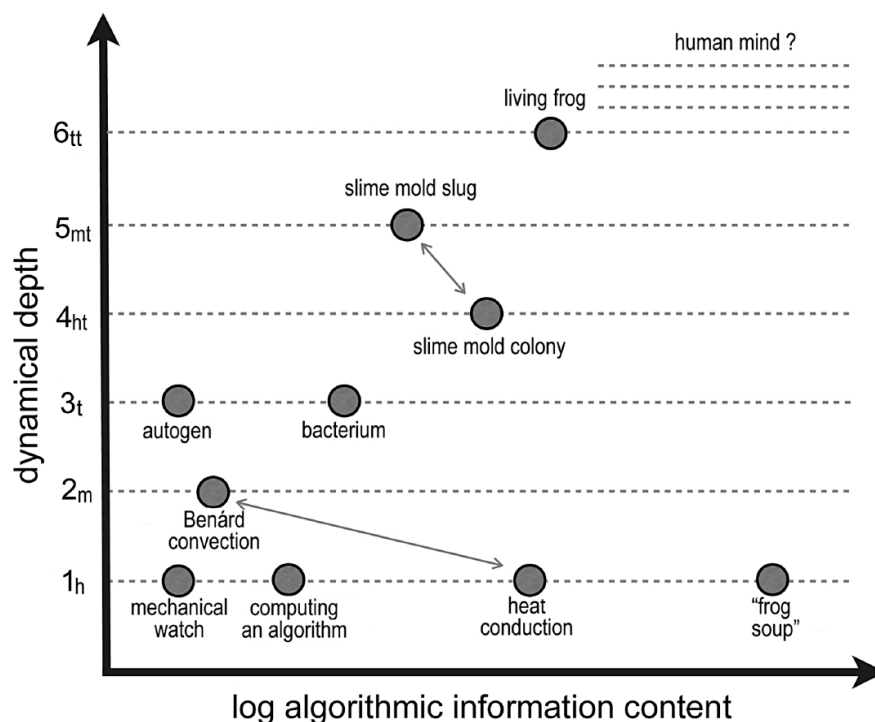
The concept of dynamical depth provides an accounting of each distinct level of suppressed dynamical complexity even though it does not assess the actual reduction of entropy at that level. With each additional increment of dynamical depth beyond morphodynamics it is no longer possible to preserve the strict distinction between dynamical compression and analytical compression; physical organization and algorithmic description. This is because living (teleodynamic) processes limit the variety of possible *forms* of constraint-generating processes, rather than any specific physico-chemical process. In this respect they are like descriptive compressions or representations. This is, of course, characteristic of the role of DNA in living organisms and is what warrants describing DNA structures as containing information contributing to the functional design of the organism. This implicit incorporation of a compressed self-representation explains why assessing the complexity of organisms requires a subtler concept of complexity than is required for assessing the complexity of non-living processes. Organisms decompress (re-expand) this information during their development by virtue of the way this information constrains the organization of morphodynamic epigenetic processes. So to ignore this dynamical infrastructure is to ignore the complexity of life's complexity-generating process.

Although in this paper we have only applied the concept of dynamical depth to three levels of nested dynamics, it should be obvious that there can be additional higher order dynamics layered on top of what is characteristic of simple organisms. Complex organisms may occupy different levels of dynamical depth during different adaptive phases or life stages and may even subdivide into components of different dynamical depth.

For example, consider slime molds (e.g., the Dictyostelids, or cellular slime molds). Large undifferentiated colonies of these amoeboid protists can form large gelatinous sheets of individuals, but under stressed conditions individual amoeba secrete cAMP which acts to cause the surrounding amoebae to aggregate toward its highest concentrations and to form into a slug-like mobile superindividual. This aggregation is a morphodynamic process in which differential cAMP concentration within the colony progressively constrains the uncorrelated behaviors of individual amoebae. It is in this respect a higher-order homeodynamic-to-morphodynamic transition occurring among individual single cell organisms that are themselves characterized by three levels of dynamical depth. So this might be characterized as a transition from a fourth to a fifth level of dynamical depth.

This relationship is depicted in an idealized graph in Figure 4, along with a number of other phenomena in order to illustrate progressively deeper levels of dynamical depth. The graph extends beyond inorganic morphodynamic (*i.e.*, self-organized) processes, protolife (autogenesis), simple single cell organisms, such as bacteria (teleodynamic systems), to include slime molds in different organizations and even a frog, as an exemplar of a teleodynamic system composed of interacting teleodynamic components (cells).

Figure 4. Idealized graphic illustration of the complexity relationships between representative systems exhibiting different algorithmic information content (AIC) and dynamical depths. AIC is depicted on a log scale due to the immense values involved. Dynamical depth is linear, but since each level reflects an immense reduction in entropy at a lower level each increment likely also involves a vast increase in the amount of reduced entropy. The integer values of dynamical depth are assigned subscripts to indicate levels of homeodynamic (h), morphodynamic (m), and teleodynamic (t) patterns of organization, and the double subscripts indicate higher order (recursive) dynamical depth in which teleodynamic unit systems (e.g., organisms) are themselves involved in homeodynamic (ht), morphodynamic (mt), or teleodynamic (tt) patterns of interaction. Each system's value of AIC is only meant to show rank order assuming that in the ideal case they could be measured with the same metric. Both dimensions lack upper limits. "Frog soup" is intended to describe a well-mixed solution containing all the molecules making up an adult frog. It is uncertain how to rank the dynamical depth of human minds since they are embedded in elaborate spatially and temporally extended symbolic social networks, though their dynamical depth will inevitably be considerably greater than the other systems depicted.



The dynamical depth of the global signaling dynamics within brains (whose neuronal components are themselves teleodynamic units) may involve additional levels of higher order homeo-, morpho- and

even teleodynamic communicative interactions. The brain's capacity to produce level-upon-level of increasing dynamical depth is a consequence of the potential for massively non-linear recurrent signal processing within a highly interconnected web of billions of incessantly active neurons. Paying attention to this substantial increase in dynamical depth may help explain many of the striking differences distinguishing biological cognition from computation.

These relationships between entropy and complexity, compression and constraint, and their counterparts in processes of description, explanation, and representation are fundamental to the complexity that we recognize in living and mental processes. This makes the concept of dynamical depth a critical missing link in the chain of concepts required to build a complete theory of complexity. By ignoring the multiple levels whereby life's compression processes give rise to dynamical and functional organization, we exclude from analysis a vast fraction of the complexity that distinguishes living and mental processes from inorganic mechanical and computational processes. Indeed, the very concept of information (*i.e.*, compressed description) is implicit in the higher order form of constraint that characterizes teleodynamics. So a formal analysis of living and mental phenomena is fundamentally incomplete unless it captures this semiotic-physical interwovenness created by life's dynamical depth.

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Author Contributions

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Conflicts of Interest

The authors declare no conflict of interest.

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