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Article

Controlled Remote State Preparation via General Pure Three-Qubit State

Yuebo Zha [1,*], Zhihua Zhang [2], Yulin Huang [1] and Jianyu Yang [1]

[1] School of Electronic Engineering, University of Electronic Science and Technology of China, 2006 Xiyuan Road, Gaoxin Western District, Chengdu 611731, China; E-Mails: yulinhuang@uestc.edu.cn (Y.H.); jyyang@uestc.edu.cn (J.Y.)

[2] Department of Mathematics, Faculty of Science, JiangSu University, 301 Xuefu Road, Jingkou District, Zhenjiang 212013, China; E-Mail: zhihuamath@aliyun.com

* Author to whom correspondence should be addressed; E-Mail:zhayuebo@163.com; Tel.:+86-28-6183-1369; Fax:+86-28-61830064.

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Abstract: The protocols for controlled remote state preparation of a single qubit and a general two-qubit state are presented in this paper. The general pure three-qubit states are chosen as shared quantum channel, which are not Local operations and classical communication (LOCC) equivalent to the mostly used GHz state. This is the first time that general pure three-qubit states have been introduced to complete remote state preparation. The probability of successful preparation is presented. Moreover, in some special cases, the successful probability could reach a unit value.

Keywords: controlled remote state preparation; pure three-qubit state; generalised schmidt-decomposition

1. Introduction

Quantum teleportation (QT for short) is the first quantum information processing protocol presented by Bennett [1] to achieve the transmission of information contained in quantum state determinately. Many theoretical schemes have been proposed since then [2–6]. It has also been realized experimentally [7–14]. To save resources needed in the process of information transmission, Lo put forward a scheme for remote preparation of quantum state (RSP for short) [15]. Compared with QT, in RSP the sender does not own the particle itself but owns all the classical information of the state he or she wants to prepare for the receiver, who is located separately from the sender. The resource consumption is reduced greatly in RSP, as the sender do not need to prepare the state beforehand. The RSP has already attracted much attention. A number of RSP protocols were presented, such as RSP with or without oblivious conditions, optimal RSP, RSP using noisy channel, low-entanglement RSP, continuous variable RSP, and so on [16–26]. Experimental realization was also proved [27,28].

Besides the usual RSP, sometimes controllers are introduced to the process of remote state preparation, which is the so called controlled remote state preparation (CRSP for short), and it has drawn the attention of many researchers. In contrast to the usual RSP, the CRSP needs to incorporate a controller. The information could be transmitted if and only if both the sender and receiver cooperate with the controller or supervisor. CRSP for an arbitrary qubit has been presented in a network via many agents [29]. A two-qubit state CRSP with multi-controllers using two non-maximally GHz states as a shared channel is shown in [30]. CRSP with two receivers via asymmetric channel [31], using POVM are presented [32,33]. The five-qubit Brown state as quantum channel to realize the CRSP of three-qubit state is elaborated in [34]. Recently, the joint CRSP [37] and CRSP with partially entangled quantum channel [38] are also presented. Most of the existing schemes chose to use the GHz-type state, W-type state, Bell state or the composite of these states as the shared quantum channel. However in this paper, we choose the general pure three-qubit state as quantum channel, which is not LOCC equivalent to the GHz state. For some special cases, the probability for successful CRSP can reach a unit value.

In [35], the authors proved that for any pure three-qubit state, there exists a local base, which allows one to express a pure three-qubit state in a unique form using a set of five orthogonal state. It is the called generalised Schmidt-Decomposition for three-qubit state. Using the generalised Schmidt-Decomposition, Gao *et al.* [36] proposed a controlled teleportation protocol for an unknown qubit and gave analytic expressions for the maximal successful probabilities. They also gave an explicit expression for the pure three-qubit state with unit probability of controlled teleportation [36]. Motivated by the ideas of the two papers, we try to investigate the controlled remote state preparation using the general pure three-qubit states and their generalised Schmidt-Decomposition.

The paper is arranged as follows. In Section 2, the CRSP for an arbitrary qubit is elucidated in detail. We find that the successful probability is the same as that of controlled teleportation for qubits with real coefficients. In Section 3, the CRSP for a general two-qubit state is expounded. For a two-qubit state with four real coefficients, the corresponding successful probability is the same as that of controlled teleportation of a qubit. In Section 4, we conclude the paper.

2. CRSP for an Arbitrary Qubit

Suppose that three separated parties Alice, Bob and Charlie share a general pure three-qubit state $|\Phi\rangle_{cab}$, the particle a belongs to Alice, b to Bob and c to Charlie, respectively. The distribution of the three particles are sketched in Figure 1.

In Figure 1, the small circles represent the particles, the solid line between two circles means that the corresponding two particles are related to each other by quantum correlation. According to [35], the general pure three qubit state has a unique generalised Schmidt-Decomposition in the form

$$|\Phi\rangle_{cab} = (a_0|000\rangle + a_1 e^{i\mu}|100\rangle + a_2|101\rangle + a_3|110\rangle + a_4|111\rangle)_{cab} \tag{1}$$

where $a_i \geq 0$ for $i = 0, \cdots, 4$, $0 \leq \mu \leq \pi$, $\sum_{i=0}^{4} a_i^2 = 1$. The a_i and μ in Equation (1) are decided uniquely with respect to a chosen general pure three qubit state.

Now Alice wants to send the information of a general qubit

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1 \tag{2}$$

to the remote receiver Bob under the control of Charlie. Alice possesses the classical information of this qubit, *i.e.*, the information of α and β, but does not have the particle itself. Next, we make three steps to complete the CRSP for $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$.

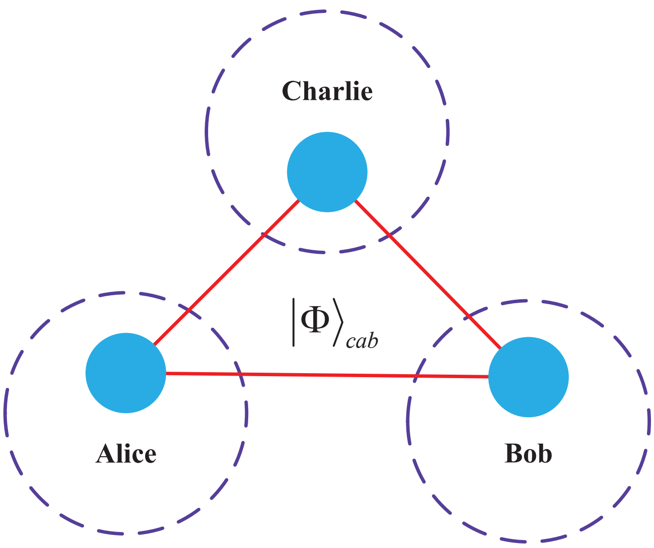

Figure 1. The diagram of scanning radar for aircraft landing.

Step 1: The controller Charlie firstly makes a single qubit measurement under the base

$$|\varepsilon_c^0\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\eta}\sin\frac{\theta}{2}|1\rangle, \quad |\varepsilon_c^1\rangle = \sin\frac{\theta}{2}|0\rangle - e^{i\eta}\cos\frac{\theta}{2}|1\rangle \tag{3}$$

where $\theta \in [0, \pi]$, $\eta \in [0, 2\pi]$. The choice of θ and η could be flexible according to the need of the controller. If $\theta = \pi$ and $\eta = 0$, $|\varepsilon_c^0\rangle$ and $|\varepsilon_c^1\rangle$ will be the $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$ base. Then Charlie broadcasts his measurement outcomes publicly to Alice and Bob using one classical bit. Using Equation (3), the quantum channel can be rewritten as

$$|\Phi\rangle_{cab} = \sqrt{p_0}|\varepsilon_c^0\rangle|\Omega_0\rangle_{ab} + \sqrt{p_1}|\varepsilon_c^1\rangle|\Omega_1\rangle_{ab} \tag{4}$$

where

$$p_0 = \sin^2\frac{\theta}{2} + a_0^2\cos\theta + a_0 a_1\cos(\mu - \eta)\sin\theta \tag{5}$$

$$p_1 = \cos^2\frac{\theta}{2} - a_0^2\cos\theta - a_0 a_1\cos(\mu-\eta)\sin\theta \tag{6}$$

$$|\Omega_0\rangle_{ab} = \frac{1}{\sqrt{p_0}}\left\{[a_0\cos\frac{\theta}{2} + a_1 e^{i(\mu-\eta)}\sin\frac{\theta}{2}]|00\rangle + e^{-i\eta}\sin\frac{\theta}{2}[a_2|01\rangle + a_3|10\rangle + a_4|11\rangle]\right\}_{ab} \tag{7}$$

$$|\Omega_1\rangle_{ab} = \frac{1}{\sqrt{p_1}}\left\{[a_0\sin\frac{\theta}{2} - a_1 e^{i(\mu-\eta)}\cos\frac{\theta}{2}]|00\rangle - e^{-i\eta}\cos\frac{\theta}{2}[a_2|01\rangle + a_3|10\rangle + a_4|11\rangle]\right\}_{ab} \tag{8}$$

If the result of Charlie's measurement is 0, the composite system of Alice and Bob is $|\Omega_0\rangle_{ab}$ with probability p_0 and $|\Omega_1\rangle_{ab}$ with probability p_1 for result 1. To ensure that the particle c entangles with the whole system, we assume that $a_0 > 0$ and a_2, a_3, a_4 are not equal to 0 at the same time. This is equivalent to $p_0 > 0$ and $p_1 > 0$ at the same time.

Note that Step 1 is actually similar to that of controlled teleportation in [36]. We arrange it here to keep the integrity of the paper. More detailed calculation can be found in [36].

Step 2: Without loss of generality, we assume that the result of Charlie's measurement is 0. Then the composite system of Alice and Bob is $|\Omega_0\rangle_{ab}$. Using Schmidt-Decomposition of two-qubit system, there exists bases $\{|0'\rangle, |1'\rangle\}_a$ and $\{|0'\rangle, |1'\rangle\}_b$ for particle a and b respectively, such that $|\Omega_0\rangle_{ab}$ can be expressed as

$$|\Omega_0\rangle_{ab} = (\sqrt{\lambda_{00}}|0'0'\rangle + \sqrt{\lambda_{01}}|1'1'\rangle)_{ab} \tag{9}$$

where $\lambda_{00} = (1-\sqrt{1-C_0^2})/2$, $\lambda_{01} = (1+\sqrt{1-C_1^2})/2$ in [36]. On receiving the result of Charlie's measurement, the sender Alice prepares a projective measurement utilizing the classical information of $|\varphi\rangle$ in the following form:

$$\begin{pmatrix} |\mu_0\rangle \\ |\mu_1\rangle \end{pmatrix}_a = \begin{pmatrix} \alpha & \beta \\ \beta^* & -\alpha^* \end{pmatrix}\begin{pmatrix} |0'\rangle \\ |1'\rangle \end{pmatrix}_a \tag{10}$$

Then $|\Omega_0\rangle_{ab}$ could be reexpressed as

$$|\Omega_0\rangle_{ab} = |\mu_0\rangle_a(\sqrt{\lambda_{00}}\alpha^*|0'\rangle + \sqrt{\lambda_{01}}\beta^*|1'\rangle)_b + |\mu_1\rangle_a(\sqrt{\lambda_{00}}\beta|0'\rangle - \sqrt{\lambda_{01}}\alpha|1'\rangle)_b \tag{11}$$

Next we first discuss the case for real coefficients, *i.e.*, α, β are real. Then Equation (11) will be

$$|\Omega_0\rangle_{ab} = |\mu_0\rangle_a(\sqrt{\lambda_{00}}\alpha|0'\rangle + \sqrt{\lambda_{01}}\beta|1'\rangle)_b + |\mu_1\rangle_a(\sqrt{\lambda_{00}}\beta|0'\rangle - \sqrt{\lambda_{01}}\alpha|1'\rangle)_b \tag{12}$$

Alice measures her qubit under base $\{|\mu_0\rangle, |\mu_1\rangle\}_a$ and gets the outcome 0 and 1 with probability $\lambda_{00}\alpha^2 + \lambda_{01}\beta^2$ and $\lambda_{00}\beta^2 + \lambda_{01}\alpha^2$ respectively. And Alice sends her measurement result to Bob by 1 classical bit. The receiver Bob's system will collapse to

$$|\xi_0\rangle_b = \frac{1}{\sqrt{\lambda_{00}\alpha^2 + \lambda_{01}\beta^2}}(\sqrt{\lambda_{00}}\alpha|0'\rangle + \sqrt{\lambda_{01}}\beta|1'\rangle)_b \tag{13}$$

and

$$|\xi_1\rangle_b = \frac{1}{\sqrt{\lambda_{00}\beta^2 + \lambda_{01}\alpha^2}}(\sqrt{\lambda_{00}}\beta|0'\rangle - \sqrt{\lambda_{01}}\alpha|1'\rangle)_b \tag{14}$$

respectively.

Step 3: We assume that Alice's measurement result is 0. Now according to Charlie and Alice's results, Bob wants to recovery the state $|\varphi\rangle$ on his side. Bob needs to introduce an auxiliary particle in initial state $|0\rangle_{b'}$, then he makes a unitary operation $U^0_{bb'}$ on his particle b and the auxiliary particle b', and his state changes to $|\omega^0\rangle_{bb'}$, where

$$U^0_{bb'} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{\lambda_{00}}{\lambda_{01}}} & \sqrt{1-\frac{\lambda_{00}}{\lambda_{01}}} \\ 0 & 0 & -\sqrt{1-\frac{\lambda_{00}}{\lambda_{01}}} & \sqrt{\frac{\lambda_{00}}{\lambda_{01}}} \end{pmatrix} \tag{15}$$

$$\begin{aligned} |\omega^0\rangle_{bb'} &= U^0_{bb'}|\xi_0\rangle_b|0\rangle_{b'} \\ &= \frac{1}{\sqrt{\lambda_{00}\alpha^2+\lambda_{01}\beta^2}}\left[\sqrt{\lambda_{00}}(\alpha|0'0\rangle+\beta|1'0\rangle)+\sqrt{\lambda_{01}-\lambda_{00}}\beta|1'1\rangle\right]_{bb'} \end{aligned} \tag{16}$$

After the unitary operation, Bob makes a measurement on his auxiliary particle b' under the base $\{|0\rangle,|1\rangle\}_{b'}$. The probability for Bob to get measurement result 0 is $\lambda_{00}/(\lambda_{00}\alpha^2+\lambda_{01}\beta^2)$, and he can recovery state $|\varphi\rangle$ successfully. But if the result is 1, the scheme fails.

Similarly, if Alice's measurement result is 1, Bob also introduces an auxiliary particle in initial state $|0\rangle_{b'}$. But the unitary operation is $U^1_{bb'}$, and the system after the unitary operation is $|\omega^1\rangle_{bb'}$, where

$$U^1_{bb'} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sqrt{\frac{\lambda_{00}}{\lambda_{01}}} & 0 & 0 & \sqrt{1-\frac{\lambda_{00}}{\lambda_{01}}} \\ \sqrt{1-\frac{\lambda_{00}}{\lambda_{01}}} & 0 & 0 & \sqrt{\frac{\lambda_{00}}{\lambda_{01}}} \end{pmatrix}$$

$$\begin{aligned} |\omega^1\rangle_{bb'} &= U^1_{bb'}|\xi_1\rangle_b|0\rangle_{b'} \\ &= \frac{1}{\sqrt{\lambda_{00}\beta^2+\lambda_{01}\alpha^2}}\left[\sqrt{\lambda_{00}}(\alpha|0'0\rangle+\beta|1'0\rangle)-\sqrt{\lambda_{01}-\lambda_{00}}\alpha|1'1\rangle\right]_{bb'} \end{aligned}$$

The probability for Bob to successfully reconstruct the state $|\varphi\rangle$ is $\lambda_{00}/(\lambda_{00}\beta^2+\lambda_{01}\alpha^2)$.

Combining the process of Step 1 and Step 2, when the controller Charlie's measurement result is 0, the receiver Bob can reconstruct the qubit $|\varphi\rangle$ with probability

$$p_0(\lambda_{00}\alpha^2+\lambda_{01}\beta^2)\frac{\lambda_{00}}{\lambda_{00}\alpha^2+\lambda_{01}\beta^2}+p_0(\lambda_{00}\beta^2+\lambda_{01}\alpha^2)\frac{\lambda_{00}}{\lambda_{00}\beta^2+\lambda_{01}\alpha^2}=2p_0\lambda_{00} \tag{17}$$

Similarly, if Charlie's measurement result is 1 with probability p_1, the whole system collapses to $|\Omega_1\rangle_{ab}$. And there are bases $\{|\bar{0}\rangle,|\bar{1}\rangle\}_a$ and $\{|\bar{0}\rangle,|\bar{1}\rangle\}_b$ for Alice and Bob's systems (Ref. [36]), so that the Schmidt-Decomposition for $|\Omega_1\rangle_{ab}$ is

$$|\Omega_1\rangle_{ab} = (\sqrt{\lambda_{10}}|\bar{0}\,\bar{0}\rangle+\sqrt{\lambda_{11}}|\bar{1}\,\bar{1}\rangle)_{ab} \tag{18}$$

Then continuing to use the last 2 steps as those in which Charlie's measurement result is 0, we can get that the successful probability for Bob to produce the desired state is $2p_1\lambda_{10}$.

As a result, for the real case, Alice can prepare the qubit $|\varphi\rangle$ at Bob's position under the control of Charlie with probability $2(p_0\lambda_{00} + p_1\lambda_{10})$, which is the same as that of controlled teleportation in [36]. But the consumption of classical bits in CRSP is reduced to 2 cbits for the whole process, and there is no need to prepare the qubit beforehand, which could save much quantum resource.

Next we discuss the case for complex coefficients. Step 1 is the same as that of real case. In Step 2, if Alice's measurement result is 0, referring to Equation (11), the remote state preparation fails. When Alice gets the result 1 with probability $\lambda_{00}|\beta|^2 + \lambda_{01}|\alpha|^2$, the whole system collapses to

$$\frac{1}{\sqrt{\lambda_{00}|\beta|^2 + \lambda_{01}|\alpha|^2}}(\sqrt{\lambda_{00}}\beta|0'\rangle - \sqrt{\lambda_{01}}\alpha|1'\rangle) \tag{19}$$

Then Step 3 is the same as that of the real case. The whole successful probability is

$$p_0(\lambda_{00}|\beta|^2 + \lambda_{01}|\alpha|^2)\frac{\lambda_{00}}{\lambda_{00}|\beta|^2 + \lambda_{01}|\alpha|^2} + p_1(\lambda_{10}|\beta|^2 + \lambda_{11}|\alpha|^2)\frac{\lambda_{10}}{\lambda_{10}|\beta|^2 + \lambda_{11}|\alpha|^2} = p_0\lambda_{00} + p_1\lambda_{10} \tag{20}$$

which is half of the real case.

According to the discussion of [36], the maximally probability for controlled teleportation will reach unit if and only if the shared channel is

$$a_0|000\rangle + a_1|100\rangle + \frac{1}{\sqrt{2}}|111\rangle, \quad a_0 > 0, \quad a_1 \geq 0, \quad a_0^2 + a_1^2 = \frac{1}{2} \tag{21}$$

As for the controlled remote state preparation for a qubit using the above channel, the successful probability can also reach one for the real case, and $1/2$ for the complex case.

3. CRSP for a Two-Qubit State

In the CRSP for a two-qubit state, there are also three parties Alice, Bob and Charlie. They share a quantum channel which is the composite of $|\Phi\rangle_{cab}$ and the Bell state, the distribution of particles in the shared quantum channel is displayed in Figure 2, the meaning of symbols is the same as in Figure 1.

$$|\Phi\rangle_{cab}|\phi^+\rangle_{a'b'} = (a_0|000\rangle + a_1e^{i\mu}|100\rangle + a_2|101\rangle + a_3|110\rangle + a_4|111\rangle)_{cab}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{a'b'} \tag{22}$$

the particle c belongs to Charlie, a, a' to Alice and b, b' to Bob. Now the sender Alice possesses the classical information of a general two-qubit state $|\varphi\rangle$,

$$|\varphi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle, \quad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1 \tag{23}$$

she wants to prepare the state at the position of a distant receiver Bob with the help of a controller Charlie. Like CRSP in Section 2, there are three steps to complete this task.

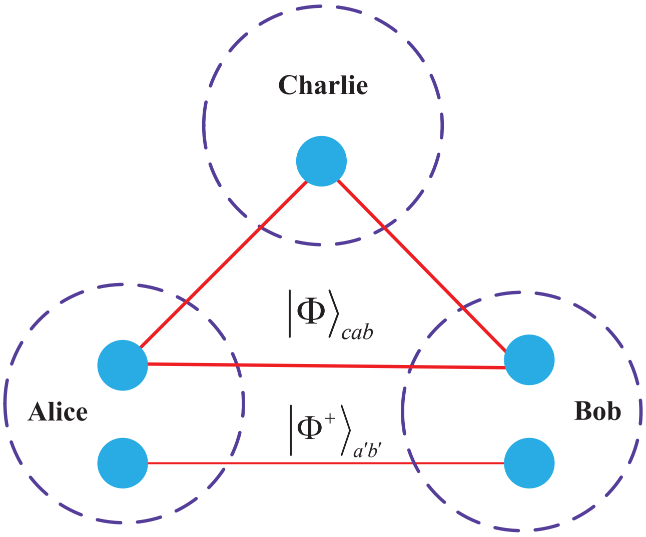

Figure 2. Particle distribution in two-qubit CRSP.

Step 1: This step is the same as that of Step 1 in Section 2. Charlie makes a projective measurement $\{|\varepsilon_c^0\rangle, |\varepsilon_c^1\rangle\}$ on his particle c, and gets the measurement result 0 and 1 with probability p_0 and p_1, respectively. The corresponding composite system of Alice and Bob is $|\Omega_0\rangle_{ab}|\phi^+\rangle_{a'b'}$ and $|\Omega_1\rangle_{ab}|\phi^+\rangle_{a'b'}$, respectively. He broadcasts his measurement result using 1 cbit.

Step 2: We assume Charlie's measurement result is 0 in Step 1. Then the system state after his measurement is $|\Omega_0\rangle_{ab}|\phi^+\rangle_{a'b'}$. Utilizing Schmidt-Decomposition of a two component quantum system, there exists bases $\{|0'\rangle, |1'\rangle\}_a$ and $\{|0'\rangle, |1'\rangle\}_b$ such that

$$\begin{aligned} |\Omega_0\rangle_{ab}|\phi^+\rangle_{a'b'} &= \frac{1}{\sqrt{2}}(\sqrt{\lambda_{00}}|0'0'\rangle + \sqrt{\lambda_{01}}|1'1'\rangle)_{ab}(|00\rangle + |11\rangle)_{a'b'} \\ &= \frac{1}{\sqrt{2}}\left[\sqrt{\lambda_{00}}|0'00'0\rangle + \sqrt{\lambda_{00}}|0'10'1\rangle + \sqrt{\lambda_{01}}|1'01'0\rangle + \sqrt{\lambda_{01}}|1'11'1\rangle\right]_{aa'bb'} \end{aligned} \tag{24}$$

Next we first discuss the case in which all the coefficients are real. According to her knowledge of the two-qubit state $|\varphi\rangle$, Alice constructs the measurement basis $\{|\mu_0\rangle, |\mu_1\rangle, |\mu_2\rangle, |\mu_3\rangle\}_{aa'}$,

$$\begin{pmatrix} |\mu_0\rangle \\ |\mu_1\rangle \\ |\mu_2\rangle \\ |\mu_3\rangle \end{pmatrix}_{aa'} = \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \beta & -\alpha & -\delta & \gamma \\ \gamma & \delta & -\alpha & -\beta \\ \delta & -\gamma & \beta & -\alpha \end{pmatrix} \begin{pmatrix} |0'0\rangle \\ |0'1\rangle \\ |1'0\rangle \\ |1'1\rangle \end{pmatrix}_{aa'} \tag{25}$$

Then the system for Alice and Bob can be rewritten as

$$\begin{aligned} & |\Omega_0\rangle_{ab}|\phi^+\rangle_{a'b'} \\ = \; & \frac{1}{\sqrt{2}}\Big\{|\mu_0\rangle[\sqrt{\lambda_{00}}(\alpha|0'0\rangle + \beta|0'1\rangle) + \sqrt{\lambda_{01}}(\gamma|1'0\rangle + \delta|1'1\rangle)] \\ & +|\mu_1\rangle[\sqrt{\lambda_{00}}(\beta|0'0\rangle - \alpha|0'1\rangle) - \sqrt{\lambda_{01}}(\delta|1'0\rangle - \gamma|1'1\rangle)] \\ & +|\mu_2\rangle[\sqrt{\lambda_{00}}(\gamma|0'0\rangle + \delta|0'1\rangle) - \sqrt{\lambda_{01}}(\alpha|1'0\rangle + \beta|1'1\rangle)] \\ & +|\mu_3\rangle[\sqrt{\lambda_{00}}(\delta|0'0\rangle - \gamma|0'1\rangle) + \sqrt{\lambda_{01}}(\beta|1'0\rangle - \alpha|1'1\rangle)]\Big\}_{aa'bb'} \end{aligned} \tag{26}$$

Thus Alice can get result 0 and 1 with probability $[\lambda_{00}(\alpha^2+\beta^2)+\lambda_{01}(\gamma^2+\delta^2)]/2$, respectively, and result 2 or 3 with probability $[\lambda_{00}(\gamma^2+\delta^2)+\lambda_{01}(\alpha^2+\beta^2))]/2$. The system state of Bob after Alice's measurement is

$$|\xi_0\rangle_{bb'} = \frac{\sqrt{\lambda_{00}}(\alpha|0'0\rangle+\beta|0'1\rangle)+\sqrt{\lambda_{01}}(\gamma|1'0\rangle+\delta|1'1\rangle)}{\sqrt{\lambda_{00}(\alpha^2+\beta^2)+\lambda_{01}(\gamma^2+\delta^2)}} \tag{27}$$

$$|\xi_1\rangle_{bb'} = \frac{\sqrt{\lambda_{00}}(\beta|0'0\rangle-\alpha|0'1\rangle)-\sqrt{\lambda_{01}}(\delta|1'0\rangle-\gamma|1'1\rangle)}{\sqrt{\lambda_{00}(\alpha^2+\beta^2)+\lambda_{01}(\gamma^2+\delta^2)}} \tag{28}$$

$$|\xi_2\rangle_{bb'} = \frac{\sqrt{\lambda_{00}}(\gamma|0'0\rangle+\delta|0'1\rangle)-\sqrt{\lambda_{01}}(\alpha|1'0\rangle+\beta|1'1\rangle)}{\sqrt{\lambda_{00}(\gamma^2+\delta^2)+\lambda_{01}(\alpha^2+\beta^2)}} \tag{29}$$

$$|\xi_3\rangle_{bb'} = \frac{\sqrt{\lambda_{00}}(\delta|0'0\rangle-\gamma|0'1\rangle)+\sqrt{\lambda_{01}}(\beta|1'0\rangle+\alpha|1'1\rangle)}{\sqrt{\lambda_{00}(\gamma^2+\delta^2)+\lambda_{01}(\alpha^2+\beta^2)}} \tag{30}$$

with respective to the result $0, 1, 2, 3$. Alice then broadcasts her measurement result to Bob using 2 cbits.

Step 3: Assume that the measurement result of Alice is 0 in Step 2. Then according to the result, Bob introduces an auxiliary particle b_a in the initial state $|0\rangle_{b_a}$, and makes unitary operation $U^0_{bb'b_a}$ on his particles, where

$$U^0_{bb'b_a} = \begin{pmatrix} I_4 & 0 \\ 0 & U_0 \end{pmatrix} \tag{31}$$

Here I_4 is the 4×4 identity matrix and

$$U_0 = \begin{pmatrix} \sqrt{\frac{\lambda_{00}}{\lambda_{01}}} & 0 & 0 & \sqrt{1-\frac{\lambda_{00}}{\lambda_{01}}} \\ 0 & -\sqrt{\frac{\lambda_{00}}{\lambda_{01}}} & \sqrt{1-\frac{\lambda_{00}}{\lambda_{01}}} & 0 \\ 0 & \sqrt{1-\frac{\lambda_{00}}{\lambda_{01}}} & \sqrt{\frac{\lambda_{00}}{\lambda_{01}}} & 0 \\ \sqrt{1-\frac{\lambda_{00}}{\lambda_{01}}} & 0 & 0 & -\sqrt{\frac{\lambda_{00}}{\lambda_{01}}} \end{pmatrix} \tag{32}$$

The state after Bob performing the unitary operation is

$$\begin{aligned} &U^0_{bb'b_a}|\xi_0\rangle_{bb'}|0\rangle_{b_a} \\ &= \frac{\sqrt{\lambda_{00}}(\alpha|0'0\rangle+\beta|0'1\rangle+\gamma|1'0\rangle+\delta|1'1\rangle)_{bb'}|0\rangle_{b_a}+\sqrt{\lambda_{01}-\lambda_{00}}(\gamma|1'0\rangle+\delta|1'1\rangle)_{bb'}|1\rangle_{b_a}}{\sqrt{\lambda_{00}(\alpha^2+\beta^2)+\lambda_{01}(\gamma^2+\delta^2)}} \end{aligned} \tag{33}$$

Thereafter, Bob makes a projective measurement on his auxiliary particles under basis $\{|0\rangle,|1\rangle\}_{b_a}$. He can get result 0 with probabilities $\lambda_{00}/(\lambda_{00}(\alpha^2+\beta^2)+\lambda_{01}(\gamma^2+\delta^2))$. As for the other three cases, Bob can successfully reconstruct the desired two-qubit state with probabilities $\lambda_{00}/(\lambda_{00}(\alpha^2+\beta^2)+\lambda_{01}(\gamma^2+\delta^2))$, $\lambda_{00}/(\lambda_{00}(\gamma^2+\delta^2)+\lambda_{01}(\alpha^2+\beta^2))$, and $\lambda_{00}/(\lambda_{00}(\gamma^2+\delta^2)+\lambda_{01}(\alpha^2+\beta^2))$.

Similarly, in the real case, if Charlie's measurement result is 1 with probability p_1, then the system state after his measurement is $|\Omega_1\rangle_{ab}|\phi^+\rangle_{a'b'}$. Using Schmidt-Decomposition of a two component quantum system, we get

$$|\Omega_1\rangle_{ab}|\phi^+\rangle_{a'b'} = \frac{1}{\sqrt{2}}(\sqrt{\lambda_{10}}|\bar{0}\,\bar{0}\rangle+\sqrt{\lambda_{11}}|\bar{1}\,\bar{1}\rangle)_{ab}(|00\rangle+|11\rangle)_{a'b'} \tag{34}$$

where λ_{10} and λ_{11} are the same as those in Section 2. Bob can also reconstruct the two-qubit state using similar method in the above three steps. As a result, for the real case, the total successful probability for the sender Alice to prepare the two-qubit state at the position of Bob under the control of controller Charlie is

$$\begin{aligned}
&2\times[p_0\frac{\lambda_{00}(\alpha^2+\beta^2)+\lambda_{01}(\gamma^2+\delta^2)}{2}\frac{\lambda_{00}}{\lambda_{00}(\alpha^2+\beta^2)+\lambda_{01}(\gamma^2+\delta^2)}\\
&+p_0\frac{\lambda_{00}(\gamma^2+\delta^2)+\lambda_{01}(\alpha^2+\beta^2)}{2}\frac{\lambda_{00}}{\lambda_{00}(\gamma^2+\delta^2)+\lambda_{01}(\alpha^2+\beta^2)}\\
&+p_1\frac{\lambda_{10}(\alpha^2+\beta^2)+\lambda_{11}(\gamma^2+\delta^2)}{2}\frac{\lambda_{10}}{\lambda_{10}(\alpha^2+\beta^2)+\lambda_{11}(\gamma^2+\delta^2)}\\
&+p_1\frac{\lambda_{10}(\gamma^2+\delta^2)+\lambda_{11}(\alpha^2+\beta^2)}{2}\frac{\lambda_{10}}{\lambda_{10}(\gamma^2+\delta^2)+\lambda_{11}(\alpha^2+\beta^2)}]\\
&=2(p_0\lambda_{00}+p_1\lambda_{10})
\end{aligned}$$

It is the same as that of the controlled teleportation for the real case of a qubit in [36], but notice that here we complete the task of reconstructing a two-qubit state, moreover we do not need to prepare the state beforehand, which may save lots of quantum resource. In the whole process the consumption of classical resource is 3 cbits.

For the case in which there is at least one complex coefficient, in Step 2, Alice constructs measurement basis$\{|\nu_0\rangle,|\nu_1\rangle,|\nu_2\rangle,|\nu_3\rangle\}_{aa'}$ in the following form,

$$\begin{pmatrix}|\nu_0\rangle\\|\nu_1\rangle\\|\nu_2\rangle\\|\nu_3\rangle\end{pmatrix}_{aa'}=\begin{pmatrix}\alpha^* & -\beta^* & \gamma^* & -\delta^*\\ \zeta\alpha^* & -\zeta\beta^* & -\zeta^{-1}\gamma^* & \zeta^{-1}\delta^*\\ -\beta & -\alpha & -\delta & -\gamma\\ -\zeta\beta & -\zeta\alpha & \zeta^{-1}\delta & \zeta^{-1}\gamma\end{pmatrix}\begin{pmatrix}|0'0\rangle\\|0'1\rangle\\|1'0\rangle\\|1'1\rangle\end{pmatrix}_{aa'} \tag{35}$$

where $\zeta=\sqrt{(|\gamma|^2+|\delta|^2)/(|\alpha|^2+|\beta|^2)}$, here we can assume that $|\alpha|^2+|\beta|^2\neq 0$. Because if $|\alpha|^2+|\beta|^2=0$, the number of coefficients decreases to two, which is actually the same as the single-qubit case. The system for Alice and Bob can be reexpressed as

$$\begin{aligned}
&|\Omega_0\rangle_{ab}|\phi^+\rangle_{a'b'}\\
=\;&\frac{1}{\sqrt{2}}\Big\{|\nu_0\rangle[\sqrt{\lambda_{00}}(\alpha|0'0\rangle-\beta|0'1\rangle)+\sqrt{\lambda_{01}}(\gamma|1'0\rangle-\delta|1'1\rangle)]\\
&+|\nu_1\rangle[\sqrt{\lambda_{00}}\zeta(\alpha|0'0\rangle-\beta|0'1\rangle)-\sqrt{\lambda_{01}}\zeta^{-1}(\gamma|1'0\rangle-\delta|1'1\rangle)]\\
&-|\nu_2\rangle[\sqrt{\lambda_{00}}(\beta^*|0'0\rangle+\alpha^*|0'1\rangle)+\sqrt{\lambda_{01}}(\delta^*|1'0\rangle+\gamma^*|1'1\rangle)]\\
&+|\nu_3\rangle[\sqrt{\lambda_{00}}\zeta(-\beta^*|0'0\rangle-\alpha^*|0'1\rangle)+\sqrt{\lambda_{01}}\zeta^{-1}(\delta^*|1'0\rangle+\gamma^*|1'1\rangle)]\Big\}_{aa'bb'}
\end{aligned} \tag{36}$$

Thus Alice can get result 0 and 1 with probability $[\lambda_{00}(|\alpha|^2+|\beta|^2)+\lambda_{01}(|\gamma|^2+|\delta|^2)]/2$ and $[\lambda_{00}\zeta^2(|\alpha|^2+|\beta|^2)+\lambda_{01}\zeta^{-2}(|\gamma|^2+|\delta|^2)]/2$, respectively. The states after Alice's measurement with respect to the result 0 and 1 are

$$|\vartheta_0\rangle=\frac{\sqrt{\lambda_{00}}(\alpha|0'0\rangle-\beta|0'1\rangle)+\sqrt{\lambda_{01}}(\gamma|1'0\rangle-\delta|1'1\rangle)}{\sqrt{\lambda_{00}(|\alpha|^2+|\beta|^2)+\lambda_{01}(|\gamma|^2+|\delta|^2)}} \tag{37}$$

and

$$|\vartheta_1\rangle = \frac{\sqrt{\lambda_{00}}\zeta(\alpha|0'0\rangle - \beta|0'1\rangle) + \sqrt{\lambda_{01}}\zeta^{-1}(-\gamma|1'0\rangle + \delta|1'1\rangle)}{\sqrt{\lambda_{00}\zeta^2(|\alpha|^2+|\beta|^2) + \lambda_{01}\zeta^{-2}(|\gamma|^2+|\delta|^2)}} \tag{38}$$

We divide it into two cases according to the value of ζ.

(i) $\zeta = 1$, *i.e.*, $|\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2$. In this case, using similar methods as in the real cases above, Bob can recover the desired two-qubit state both from states in Equations (37) and (38). And the probabilities are both $\lambda_{00}/(\lambda_{00}(|\alpha|^2+|\beta|^2)+\lambda_{01}(|\gamma|^2+|\delta|^2))$. Similar scheme applies to the case that Charlie's measurement result is 1. Thus the total successful probability for Alice remotely to prepare the two-qubit state $|\varphi\rangle$ at Bob's position under the control of Charlie is

$$\begin{aligned} & 2 \times \left\{ p_0 [\frac{\lambda_{00}(|\alpha|^2+|\beta|^2)+\lambda_{01}(|\gamma|^2+|\delta|^2)}{2}] \frac{\lambda_{00}}{\lambda_{00}(|\alpha|^2+|\beta|^2)+\lambda_{01}(|\gamma|^2+|\delta|^2)} \right. \\ & \left. + p_1 [\frac{\lambda_{10}(|\alpha|^2+|\beta|^2)+\lambda_{11}(|\gamma|^2+|\delta|^2)}{2}] \frac{\lambda_{10}}{\lambda_{10}(|\alpha|^2+|\beta|^2)+\lambda_{11}(|\gamma|^2+|\delta|^2)} \right\} \\ & = p_0\lambda_{00} + p_1\lambda_{10} \end{aligned} \tag{39}$$

which is half of the case that all the coefficients are real. As for the result 2 and 3, the CRSP protocol fails.

(ii) $\zeta \neq 1$. For this case, as Bob does not know the classical information of $|\varphi\rangle$, only when Alice's measurement result is 0, Bob can reconstruct the two-qubit state $|\varphi\rangle$. Thus the successful probability reduces to half of (i) as $(p_0\lambda_{00} + p_1\lambda_{10})/2$.

4. Conclusions

In this paper, protocols for controlled remote state preparation are presented both for a single qubit and two-qubit state. We utilize the general pure three-qubit states as the shared quantum channels, which are not LOCC equivalent to the GHz state. We discuss protocols for both states with real and complex coefficients, and find that the general pure three-qubit states can help to complete CRSP probabilistically. More than that, in some special cases, the CRSP can be achieved with unit probability, which are deterministic CRSP protocols. This overcomes the limitation that most of the existing quantum communication protocols are completed with GHz-, W- or Bell states, or the composition of these states. Moreover, due to the involvement of controller and multi-partities, this work may have potential application in controlled quantum communication, quantum network communication and distributed computation.

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Author Contributions

Yuebo Zha: Algorithm developing and writing of the paper; Zhihua Zhang: Algorithm developing and writing of the paper; Yulin Huang: Writing of the paper; Jianyu Yang: Algorithm developing. All authors have read and approved the final manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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