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A Filter Structure for Arbitrary Re-Sampling Ratio Conversion of a Discrete Signal

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Abstract: In this report, we studied the sampling synchronization of a discrete signal in the receiver of a communication system and found that the frequency of the received signal usually exhibits some unpredictable deviations. We observed many harmonics caused by the frequency deviations of the discrete received signal. These findings indicate that signal sampling synchronization is an important research technique when using discrete Fourier transforms (DFT) to analyze the harmonics of discrete signals. We investigated the influence of these harmonics on the performance of signal sampling and studied the frequency estimation of the received signal. Based on the frequency estimation of the received signal, the sampling rate of the discrete signal was converted using a modified Farrow filter to achieve sampling synchronization for the received signal. The algorithm discussed here can be applied to sampling synchronization for monitoring and control systems. Finally, simulations and experimental results are presented.

Keywords: sample rate conversion; frequency estimation; Farrow filter

1. Introduction

The sampling theorem is one of the most basic and fascinating topics in engineering sciences [1]. The best-known form uses a Fourier transform to analyze the uniform sampling rate, based on earlier work by Nyquist [2] and Shannon [3]. In Shannon's uniform-sampling theorem, the sampling rate depends on the signal frequency; however, the frequency of the received signal usually exhibits some deviations in the receiver for a communications system, which raises an important question about the sampling of the received signal: how can we estimate the frequency deviation of the received signal and also achieve a sampling rate conversion (SRC) for a discrete signal?

To answer this and related questions, we must first understand the frequency estimation of the discrete signal in the receiver and its properties. Several fast and accurate frequency estimators have been previously proposed [4–7]. Without loss of generality, these frequency estimation algorithms mainly use discrete Fourier transform (DFT), maximum likelihood (ML), and phase estimation [4]. Each of these methods has an optimal characteristic. For example, Viterbi [8] proposed a frequency estimation algorithm that was an approximation of the ML estimator. Additionally, there are several methods for using autocorrelation theory to estimate the frequency of the received signal. For example, Fitz et al. [9] calculated the autocorrelation function of a signal to estimate the signal frequency. They proposed a new frequency estimator for a single complex sinusoid in complex white Gaussian noise. To estimate the angle frequency, they estimate the angle difference primarily by using the DFT, which can be applied to accurately estimate the change of the signal phase. Using the signal phase variation, the received signal frequency can be obtained. In [10–12], they also used the

autocorrelation function of signal to study the signal frequency estimation problem. They noted that the signal frequency can be efficiently determined by the autocorrelation function. In particular, the autocorrelation function can effectively suppress Gaussian white noise. Brown et al. [13] proposed an algorithm for estimating the frequency of a complex sinusoid in noise. The estimator consists of multiple applications of low-pass filtering and decimation, frequency estimation by linear prediction, and digital heterodyning. Note that the autocorrelation function provides advantages that include both high estimation precision and simple calculation. Gedalyahu et al. [14] proposed a multichannel architecture for sampling pulse streams with arbitrary shape. Their approach was based on modulating the input signal with a set of properly chosen waveforms, followed by a bank of integrators. They noted that a stable pulse stream can be recovered from the proposed minimal-rate samples taken from the

spectral estimation using standard tools. On the other hand, the SRC operation needs to be performed, after which the frequency of the received signal can be estimated. Generally, we can use the Farrow filter [15] to perform the SRC operation. The Farrow structure is a standard for implementing polynomial-based interpolation filters. It provides a means for efficient, real-time implementation of a polynomial-based filter [16]. The classic approach to SRC is presented in [17]. There are two basic types of SRC as follows: fractional-factor SRC and integer-factor SRC. In this paper, we discuss fractional-factor SRC for the received signal of the communication system. A few earlier works have attempted to study SRC [18–21]. For example, Blok et al. [18] presented a sampling rate conversion with a continuously changing re-sampling ratio. The proposed implementation was based on a variable fractional delay filter implemented using a Farrow structure. Wang et al. [19] proposed a reconfigurable integer factor sampling rate converter for software defined radio (SDR) receivers. A cascaded integrate comb (CIC) filter with a factor-16 down sampler was used to realize large-factor and multiplier-free decimation in the front stage, while a multistage decimator is implemented together with the CIC filter to provide a finer integral downsampling rate. Similarly, Horridge et al. [20] discussed the design of re-sampling algorithms suitable for use in over-the-horizon radar (OTHR) (and because the techniques are general, in other radar systems). Similarly, Blok et al. [21] proposed a novel classification of fractional delay filter design methods, dividing them into the following three general categories: optimal fractional filter design, offset window method and polyphase decomposition. The proposed classification was based on the differences in the properties of the sample rate conversion algorithm based on fractional delay filters. In [22], to improve the measurement accuracy for distorted signals, Andria et al. examined suitable windows and interpolation algorithms to reduce undesirable effects when the sampling process is not synchronized.

Based on the above analysis, estimating the actual signal frequency and consequently interpolating in order to limit harmonic distortion are well-known in literature. However, the sampling rate conversion of a discrete signal with an arbitrary ratio between the original discrete signal and the converted sampling rate signal is a challenging problem, although estimating the actual signal frequency has been solved. This is because the frequency deviation of a received signal is random and unpredictable, and sampling rate deviation occurs in order to cause harmonic disturbance. Currently, existing techniques for the sampling rate conversion have studied the methods in the specific factors as [19–21]. Note that the SRC algorithm must compute the values of signal samples at new time instants located between the original samples [23,24]. This means that we can treat each output sample of the SRC algorithm as an input sample delayed or advanced by a fraction of the input sampling period.

In this paper, we use the angular frequency to estimate the frequency of the received signal. We present a new method using fine frequency resolution (FFR) to estimate the frequency of the received signal. The FFR is presented in [25], where it is applied to the phase difference between two nearby segments of the discrete signal to calculate the frequency deviation. Based on the frequency estimation of the received signal, the discrete signal sampling rate is converted using the modified Farrow filter to achieve the received signal sampling synchronization. The algorithm can

be applied to a variety of power distribution system devices during front-end processing of signals by synchronous sampling.

The sampling rate conversion of discrete signal is studied in detail. Our analysis reveals that harmonics become more complicated as the frequency deviation of the signal becomes larger. Given this characterization, the frequency estimation of the received signal can be derived. We extend the results by assuming the received signal is bandlimited as follows: we introduce a filter on the SRC of the signal. These theoretical results are verified through simulation experiments. Beyond the fundamental interest of precisely characterizing the SRC of the signal, the results are useful in obtaining the best sampling rate synchronization for the received signal. Our framework builds upon [19–21] and extends it in several aspects, such as sampling rate conversion for arbitrary ratio.

Our proposed scheme provides three contributions to the sampling rate conversion of the discrete signal:

- The modified Farrow filter is simple to use for sampling rate conversion of a discrete signal in an arbitrary ratio between the original discrete signal and the converted sampling rate signal. In the conversion algorithm, because the modified Farrow structure depends only on the conversion ratio, a minimal amount of calculation information is needed. Thus, the modified Farrow filter can greatly simplify the calculation complexity of sampling rate conversion theory.
- For a specific order number of the modified Farrow structure, its sampling points must be increased to improve the sampling rate accuracy. To the existing theory, our approach is a simplification of a complex signal to the modified Farrow filter.
- Our proposed method is more accurate than the existing methods in the literature. For a complex signal, such as an existing harmonic distortion, the determined sampling rate accuracy is higher when using the FFR to perform the sampling rate problem of a discrete signal.

The remainder of this paper is organized as follows. A modified frequency estimation method is presented in Section 2. In Section 3, an SRC algorithm is presented. The theory presented in this paper is then verified using simulations and real measurements in Section 4. Conclusions are drawn in Section 5.

2. Frequency Deviation Estimation

2.1. Problem Statement

We begin by briefly reviewing the problem. In a communication system, due to different clock rates between the transmitter and receiver, the signal frequency in the receiver will deviate from the signal frequency in the transmitter [19,26]. Additionally, because of superimposed noise, non-linear static characteristics and slow response [22], the frequency deviation of the received signal is increased, as shown in Figure 1. Let the frequency of the signal x(t) be f_x in the transmitter. Furthermore, the frequency of the signal y(t) is f_y in the receiver, where $f_y = f_x + \Delta f$. Without loss of generality, assume Δf is unknown. The sampling rate of the received signal y(t) is usually given by $f_s = f_x \cdot N$, where N is the number of samples in a periodic signal, as shown in Figure 2. The received signal y(t) can be defined as

$$y(t) = \sum_{i=1}^{K} A_i \cos(2\pi f_i t + \varphi_i),$$
(1)

where *K* is the number of harmonics and is related to the interference for the channel, f_i is the frequency of the i^{th} harmonic, A_i is the amplitude and φ_i is the phase.

By using the sampling rate $f_s = f_x \cdot N$ to sample the signal y(t), its components can be represented as follows:

$$y_i(nT_s) = A_i \cos[2\pi f_i nT_s + \varphi_i], \qquad (2)$$

where n = 0, 1, 2, ..., N - 1 and $T_s = 1/f_s$. Then, the relationship between f_s and the frequency of the received signal y(t) is not an integer multiple. The reason for this can be understood by considering

that there are some harmonics for the received signal that are caused by the interference of the signal for $y_i(nT_s)$. More precisely, as depicted in Figure 2, the sampling period of the received signal is T_y . However, if the received signal is sampled using T_s , the spectral leakage from the signal will occur due to the sampling period deviation, i.e., $\Delta T = |T_y - T_s|$ shown in Figure 2, where $\Delta T = 1/\Delta f$ and $T_y = 1/f_y$. Therefore, we must first estimate Δf . Subsequently, we must convert the sampling rate of the discrete received signal $y_i(nT_s)$ to eliminate the harmonic disturbances caused by sampling rate deviation. To address this problem, we analyze the frequency deviation using the method proposed in the next sections.



Figure 1. Transmission of a signal with frequency deviation.



Figure 2. Diagram of the frequency deviation for a periodic signal.

2.2. FFR Estimation

In this sub-section, we analyze how the frequency deviation is estimated. Assuming that the received signal is sampled at the sampling rate $f_s = f_x \cdot N$, we select two segments of the discrete received signal, each with length N_s . Additionally, to eliminate the leakage of the analyzed signal segment, N_s (i.e., the length of the analyzed signal segment (DFT size)) must be equal to an integer multiple of the signal's period. The two segments are represented as $x_m(n)$ and $x_{m+1}(n)$, where $n = 0, 1, 2, ..., N_s - 1$. The DFT of the discrete received signal for one section is

$$X(k) = \sum_{n=0}^{N_s - 1} x(n) W_{N_s}{}^{nk}, k = 0, 1, 2, ..., N_s - 1,$$
(3)

where $W_{N_s} = e^{-j\frac{2\pi}{N_s}}$. The DFTs of $x_m(n)$ and $x_{m+1}(n)$ are $X_m(k)$ and $X_{m+1}(k)$, respectively, where $k = 0, 1, 2, ..., N_s - 1$.

The FFR of the received signal can be determined using the relationship between the signal phases. More precisely, the highest frequency consists of some pieces of signal from $X_m(k)$ at time m, where k is the frequency component of the input signal. The initial phase $\theta_m(k)$ of the received signal can be calculated using the DFT of the received signal as follows:

$$\theta_m(k) = \tan^{-1} \left\{ \frac{\operatorname{Im}[X_m(k)]}{\operatorname{Re}[X_m(k)]} \right\},\tag{4}$$

where Im is the imaginary part of $X_m(k)$, and Re is the real component of $X_m(k)$.

Generally, the frequency deviation of the signal is very small, so we can assume that the length of the component $X_{m+1}(k)$ of the signal is 1.0 ms in a short period of time. The interval time is from *m* to m + 1. This length is equivalent to its maximum phase component. Then, the initial phase angle of a received signal at time m + 1 can be calculated as

$$\theta_{m+1}(k) = \tan^{-1} \left\{ \frac{\text{Im}[X_{m+1}(k)]}{\text{Re}[X_{m+1}(k)]} \right\}.$$
(5)

Using the two phase angles, $\theta_m(k)$ and $\theta_{m+1}(k)$, the fine frequency can be calculated as

$$f_{\Delta} = \frac{\theta_{m+1}(k) - \theta_m(k)}{2\pi N_s}.$$
(6)

The fine frequency calculated by Equation (6) can be used to obtain highly accurate results for the frequency deviation. To ensure that the frequency is not changed, the phase difference $\Delta \theta = \theta_{m+1}(k) - \theta_m(k)$ must be smaller than 2π . Furthermore, when the phase difference is 2π , the bandwidth of the non-fuzzy portion is $1/N_s$, where N_s is the delay time between two groups of continuous signal [15].

2.3. Signal Frequency Correction

If the frequency of the transmitted signal is *f*, then the frequency of the corrected received signal can be written as

$$f_{new} = f + f_{\Delta}.\tag{7}$$

From (7), we can obtain a frequency of the received signal that is closer to the frequency of the transmitted signal. Therefore, the new sampling rate of the received signal is

$$\stackrel{\wedge}{f_s} = f_{new} \cdot N_s. \tag{8}$$

Finally, we can process the two pieces of the received signal again to obtain high precision in the frequency estimation. The received signal is sampled using the new sampling rate, f_s . Using (6)–(8), the sampling rate f_s can be updated. The above expressions provide a method of re-estimating for a different sampling rate f_s . It should be observed that the sampling rate of the received signal can be approximated by the sampling rate of the transmitted signal to achieve high sampling synchronization.

2.4. Performance Analysis of the Frequency Deviation Estimation

Without loss of generality, the frequency deviation estimation is considered as a frequency estimation of the signal. This problem has been studied using many methods, such as the autocorrelation function and maximum likelihood estimation [9–13]. For our method, the frequency deviation can be estimated using (6). Firstly, the frequency deviation can be accurately estimated using the phase of the signal spectrum. Secondly, we intercept an integer multiple of the signal to perform the frequency estimation, so that the frequency of the signal can be tracked at any moment in

time. The concept of Cramer–Rao bound (CRB) can evaluate the performance of frequency estimation. The CRB of the frequency estimate, given by [27] is

$$\sigma_f^2 = \frac{6f_s^2}{(2\pi)^2 \rho N \left(N^2 - 1\right)},\tag{9}$$

where ρ is the signal-to-noise ratio (SNR). Based on (6) and (8), the CRB of the frequency estimate obtained by our proposed method can be modified as

$$\sigma_f^2 = \frac{6\hat{f}_s^2}{(2\pi)^2 \rho N_s \left(N_s^2 - 1\right)}.$$
(10)

Note the difference between (9) and (10) with respect to the sampling rate f_s . In (9), the sampling rate is not converted, and the sampling rate has been converted in (10). Therefore, we try to use (10) to analyze the performance of the frequency estimation.

3. Sampling Rate Conversion Algorithm

The above discussion should provide a reasonably clear idea of what a re-estimation of sampling rate is. After obtaining a new sampling rate for the discrete signal, the original sampling rate of the discrete signal from the received signal must be converted to obtain the SRC. Here, a modified Farrow filter is used to perform the SRC of the signal. This procedure is described below.

3.1. The Modified Farrow Filter

The Farrow filter, as proposed by Farrow [16], is a multi-rate filter structure that offers the option of a continuously adjustable re-sampling ratio [15]. Here, we use the Farrow filter and modify its coefficient selection method to optimize the performance and computational complexity of re-sampling. More precisely, the coefficients of the filter are saved to a text file. Then, we use the stored coefficients to perform the re-sampling, thus reducing the computational complexity. Without loss of generality, in the transmitter, assume the signal is represented as $x(nT_s)$, where T_s is the sampling interval. The sampling rate for the transmitted signal is $f_s = 1/T_s$. In the receiver, if the signal correction frequency is $f_y = f_x + \Delta f$, then the sampling rate for the received signal is $\hat{f_s}$. Therefore, the sampling rate of the discrete received signal must be converted to the sampling rate, $\hat{f_s}$, when its original sampling rate is f_s . The expression for the SRC can be written as

$$y(mT_y) = \sum_{n=-\infty}^{\infty} x(nT_s)h(mT_y - nT_s).$$
(11)

For an arbitrary scaling factor of sampling rate conversion, we assume that $mT_y = (k_m + \Delta)T_s$, $0 \le \Delta < 1$, where k_m is an integer. (11) can be re-written as

$$y(mT_y) = \sum_{n=-\infty}^{\infty} x(nT_s) h((k_m + \Delta - n) T_s).$$
(12)

Furthermore, (12) can be simplified to

$$y(m) = \sum_{n=-\infty}^{\infty} x(n)h(k_m + \Delta - n).$$
(13)

If $k = k_m - n$, (13) can be written as follows:

$$y(m) = \sum_{n=-\infty}^{\infty} x(k_m - n) h(k + \Delta).$$
(14)

We can see that the SRC operation mainly depends on the sampling interval Δ and the filter coefficient $h(\cdot)$. In this paper, $h(\cdot)$ is a modified Farrow filter [15]. It is described in the following subsection.

3.2. The Interpolation Arithmetic of the Modified Farrow Filter

In this sub-section, we use a *P* order polynomial such as the Taylor expansion or Lagrange polynomial to approximate $h(k + \Delta)$. The order of the modified Farrow filter is determined by the sampling rate conversion ratio. The expression for $h(k + \Delta)$ can be approximatively given by

$$h(k+\Delta) \approx \sum_{l=0}^{P} c_l(k) \Delta^l,$$
(15)

where $c_l(k)$ is the expanded form of the Taylor expansion or Lagrange polynomial.

Generally, the *P* order polynomial of $h(\cdot)$ (i.e., the prototype impulse response of the Farrow filter) can be represented by K_c coefficients. In addition, allow $K_c > P + 1$. The DFT of these K_c coefficients is

$$H(k) = DFT[h(n)] = \sum_{n=0}^{K_c - 1} h(n) W_{K_c}^{kn},$$
(16)

where $k = 0, 1, 2, ..., K_c - 1$ and $W_{K_c} = e^{-j\frac{2\pi}{K_c}}$. To obtain an arbitrary sampling rate for the discrete signal, H(k) is approximated by interpolation of the continuous signal. The DFT operation is described as

$$h(n) = \frac{1}{K_c} \sum_{k=0}^{K_c - 1} H(k) W_{K_c}^{-kn}.$$
(17)

Here, n could be arbitrary. Then, (15) is deployed as

$$h(n) = \frac{1}{K_c} \sum_{k=0}^{K_c-1} H(k) W_{K_c}^{-kn} = \frac{1}{K_c} \{ H(0) + H(1)e^{j\frac{2\pi}{K_c}n} + H(2)e^{j\frac{2\pi}{K_c}n\cdot 2} + \dots + H(K_c - 1)e^{j\frac{2\pi}{K_c}n\cdot (K_c - 1)}.$$
(18)

To improve the sampling precision for the sampling rate conversion of a discrete signal, the number of sampling points of the Farrow filter h(n) must be increased. Based on (18), the number of sampling points of the Farrow filter h(n) can be increased by the interpolation of h(n) and can be denoted N_1, N_2, \cdots . After the interpolation operation, we must analyze the interpolation error of h(n). The expression for the interpolation error is

$$h_{error}(n) = h_{N_1}(n) - h_{N_2}(n), \tag{19}$$

where N_1 represents the similar continuous signal for the needed SRC signal, and N_2 is the minimum interpolation for the corresponding discrete signal. Here, the number of interpolation points of the interpolation operation of H(k) ranges from 500 to 10,000. We find that when the number of interpolation points is 2000, the conversion from discrete signal to continuous signal produces the required effect. As shown in Figure 3, the errors in the interpolation for numbers of interpolation points between 2000 and 10,000 are very small. Therefore, using 2000 points to perform the interpolation operation of H(k) can achieve an optimal signal conversion. In this report, the 2000 point parameters

of H(k) are saved in a file, so the interpolation reconstruction for signal conversion can be performed by a look-up table.



Figure 3. Subtraction results of h(n) interpolation between 2000 points and 10,000 points.

3.3. SRC of Discrete Signal

After the interpolation coefficients of h(n) are obtained, the sampling rate of the discrete sampling signals x(n) is converted by the following expression:

$$x_{new}(n) = x(n) * h(n), \tag{20}$$

where * represents the convolution operation.

Using (20), the sampling rate of $x_{new}(n)$ is different from that of x(n). Therefore, sampling rate conversion is performed.

4. Experimental Results

Now, experimental results are presented for sampling rate conversion with arbitrary re-sampling ratio of a discrete signal. These results are then compared with results obtained by other methods.

4.1. Results of the Frequency Estimation

To evaluate the effectiveness of our proposed method using a discrete signal sequence, the sampling rate of the discrete signal is normalized to $f_s = 1$. The number of sampling points is N = 128, the sampling point interval is $T_s = 1/f_s$, the normalization factor is $M = 128 \times 50$, the fundamental frequency is $f_0 = 50/M$ Hz, and its maximum variation range is 0.2 Hz. Therefore, the signal frequency for simulation is $f_1 = 50.2/M$. The experiments include the following three cases for the frequency estimation of the signal.

- a signal without harmonic components and noise
- a signal consisting of three times, five times, and seven times harmonics
- a signal consisting of a harmonic and 20 dB of Gaussian white noise. The results obtained using the FFR estimation method are shown in Table 1.

One can observe in Table 1 that the magnitude of the frequency estimation accuracy reaches 0.001 in the first case, when the discrete signal does not contain harmonics or noise. The frequency of the discrete signal is estimated three times for the iteration estimation, and the frequency completely converges to 50.2 Hz. When the discrete signal consists of harmonics and noise, the frequency also converges after three iterative estimations, as shown in Table 1. In Table 1, note that the frequency estimation improves significantly after the third iteration, but subsequent improvements become

less significant with additional iterations. From the above analysis, it is clear that the influence of harmonics and noise on frequency estimation is very small when using our proposed method.

Loop Number	Frequency Estimation	Containing Harmonic	Containing Harmonic and Noise
1	50.200796	50.205414	50.329538
2	50.20000	50.200000	50.200333
3	50.20000	50.200000	50.200000
4	50.20000	50.200000	50.200000
5	50.20000	50.200000	50.200000

Table 1. Simulation results of fine frequency. (Units: Hz.)

4.2. Design of the Modified Farrow Filter

The above example should provide a reasonably clear idea of what frequency estimation is and how it can be performed by the FFR estimation. The effectiveness of our proposed method can be verified. After estimating the frequency of the discrete signal, we can use the correction frequency to correct the sampling rate of the discrete signal. Then, we use a modified Farrow filter to perform the SRC. The design of this modified Farrow filter is an important factor. Here, we design a modified Farrow filter h(n) using the Fdatool in MATLAB R2010a which is presented by MathWorks of America (Natick, MA, USA). We show an example where the cut-off frequency is $F_c = 6400$ Hz, the sampling rate of h(n) is $F_s = 6400 \times 4$ Hz, and the number of orders is seven. In addition, we use the Bartlett–Hanning window to perform this operation. To improve the effectiveness of SRC using the look-up table, the coefficient of h(n) is extended by (18) to 7 × 300 coefficients. According to the symmetry of DFT, let P = 3.5. The original h(n) is shown in Figure 4b, and the interpolation coefficients of h(n) are shown in Figure 4a.



Figure 4. Diagram of 7×300 points. The top (**a**) graph illustrates the interpolation coefficients of h(n), while the bottom (**b**) illustrates the original coefficients of h(n).

After that, the interpolation coefficients of h(n) are applied to perform the sampling rate conversion from the discrete signal to the continuous signal. Figure 5a shows the original discrete signal, and Figure 5b shows the resampled signal after its sampling rate has been converted. To evaluate the conversion performance, the spectra of the original discrete signal and the converted signal are presented in Figure 6. It can be observed that the spectral leakage is reduced for the converted signal, compared to the original discrete signal. Therefore, the sampling of the signal has been synchronized

because the sampling rate is an integer multiple of the frequency of the signal. This result demonstrates that the modified Farrow filter can be applied to convert the sampling rate of a discrete signal.



Figure 5. Diagram of the re-sampling signal. The top (**a**) graph illustrates the original discrete signal, while the bottom (**b**) graph illustrates the sampling rate of the converted discrete signal.



Figure 6. The spectra of the original discrete signal and the converted discrete signal for the corresponding signal in Figure 5.

4.3. Simulation Results for SRC

In this subsection, we present the SRC of a simulated signal. In general, the received signal consists of the transmitted signal plus some harmonic waves. Without loss of generality, the expression of the received signal can be given as

$$x(t) = \sin(2\pi f_1 t) + \frac{1}{3}\sin(3 \times 2\pi f_1 t) + \frac{1}{5}\sin(5 \times 2\pi f_1 t) + \frac{1}{7}\sin(7 \times 2\pi f_1 t).$$
(21)

Here, let $f_1 = 350$ Hz. The signal is sampled using the sampling rate, $f_s = 128 \times 50$ Hz. The sampling results are shown in Figure 7a. The figure shows only the result from 0 to 500 points. To clearly show the harmonic of the signal and signal synchronization, we select the cycle length of the discrete signal in (21) to perform its FFT. Note that there are 128 points per cycle length of the discrete signal according to the sampling rate f_s . The FFT results are shown in Figure 7b. It can be observed that there are many harmonics in the spectrum of the original signal; however, after the SRC operation, no harmonics remain in the spectrum of the re-sampling signal. This demonstrates that our proposed method can be used to perform the SRC, improving its effectiveness.



Figure 7. (**a**) illustrates the difference between the original signal and the resampled signal for sampling rate synchronization; (**b**) illustrates the fast Fourier transform (FFT) of the resampled signal for comparison to the original signal.

The signal x(t) is sampled using the sampling rate f_s' . Note that f_s' is an integer multiple of f_1 . For the frequency estimation, because spectral leak occurs in the re-sampling process, the frequency estimate converges to 50.20066477092068 Hz, when the iterative estimation is run five times. Table 2 shows the results of the FFT for x(n). It is corrected by the SRC before and after each cycle of x(n).

The Results of the Sampling Signal for before the Sampling Rate Conversion (SRC)	The Results for after the SRC
0	0.00485602103592
0	0.00473329018826
0.0000000000001	0.00435523550826
0.0000000000001	0.00377333106918
0.0000000000001	0.00243896125486
0.0000000000001	0.00038573119211
0.0000000000001	0.00019852719739
0	0.00013465731865
62.82496056620225	62.56533184922048
20.86262463928692	20.77828756114954
12.42215235579411	12.37393682470713
8.76973611504727	8.73749834116767
0	0.00070606042704
0.0000000000001	0.00026219741259
0	0.00016016198162

Table 2. Fast Fourier transform (FFT) comparison of the signals with the theoretical integer sampling rate and the synchronous sampling rate.

Table 2 shows the extent of the signal spectrum before and after SRC. Before the SRC, many of the sampled values are zero because f_s is an f integer number. It should be obvious that the signal spectrum still contains a small amount of leakage. This behavior is understandable because the process of sampling rate conversion inevitably has some frequency spectrum leakages. Furthermore, the sampling rate is not strictly equal to the integer time frequency of the signal. However, the base and the maximum amplitude of the harmonic wave amplitude ratio is only 7.761520465715398 × e^{-5} , showing that frequency spectrum leakage is very small. This level of accuracy could completely satisfy the face harmonic analysis of a signal and could accurately identify harmonics.

4.4. Comparison Results

4.4.1. Frequency Deviation Estimation

To enable the comparison of the accuracy of this method with the accuracies of alternative solutions, the method of iterative frequency estimation by interpolation on Fourier coefficients (IIFC) [28] is applied to perform the experimental evaluation. The authors proposed and analyzed two new frequency estimators that interpolate on the Fourier coefficients of the received signal samples. The estimators were shown to achieve identical asymptotic performances. The frequency of the signal was derived as

$$f_{new} = \frac{\hat{m}_N + \delta_N}{N} f_s,\tag{22}$$

where \hat{m}_N is the index of the bin with the largest magnitude, and δ_N is a residual in the interval [-0.5, 0.5].



Figure 8. Diagram of the channel request block (CRB) results obtained by our frequency estimation method and by the interpolation on Fourier coefficients (IIFC).

In this experiment, the signal's amplitude is 1, and the length of the signal is 256. After using (7) and (22) to estimate the signal frequency, the estimated performances of the two methods are evaluated using the concept of channel request block (CRB). We further use the proposed method and the IIFC to perform the frequency estimation, and then the CRBs of the two methods are analyzed for different signal noise ratios (SNR). For each method, the frequency estimation of the signal is performed 1000 times. We present experimental results on the CRB shown in Figure 8. It can be observed that the CRB decreases as the SNR increases for the two frequency evaluated methods. Obviously, the CRB obtained by our method is better than that obtained by the IIFC. This is because

our method intercepts the signal to estimate the signal phase deviation that reflects the variation of the signal frequency. Based on the above analysis, the results of this experiment also indicate that the proposed method outperforms the IIFC for frequency deviation estimation.

4.4.2. Sampling Rate Conversion

To enable accuract comparisons for our proposed method, we applied the batch Farrow filter resampling (BFFR) method [20] to perform the experimental evaluation. In [20], the authors presented a design for an arbitrary sample-rate resampling method using the OTHR for radar applications. The signal presented in (19) was selected to perform the SRC using our proposed method and BFFR.

The results obtained by our proposed method are depicted in Figure 7, while Figure 9 depicts the corresponding converted results obtained using BFFR. Figure 9b shows that serious harmonic wave distortion exists, but, due to the sampling rate, there is no synchronization with the frequency of the signal. However, Figure 7b shows no harmonic wave. From the above analysis, it is clear that our proposed method can be applied to perform the SRC, with better performance than BFFR.



Figure 9. (a) illustrates the sampling rate conversion after using batch Farrow filter resampling (BFFR); (b) illustrates the spectrum of sampling rate conversion after using BFFR.

5. Conclusions

In this paper, we propose a simple but effective signal frequency correction and sampling rate conversion method based on DFT. We have demonstrated the performance of the proposed method experimentally using a discrete signal with a frequency of $f_1 = 50$ Hz. Our method can be easily adapted to any other discrete signal for different frequencies. Sampling rate synchronization is critical for the resampling of any discrete signal for any application due to serious distortions of the discrete signal. The FFR estimation and the modified Farrow filter of our proposed method can effectively solve the sampling rate conversion for any discrete signal.

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