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Target Assignment Algorithm for Joint Air Defense Operation Based on Spatial Crowdsourcing Mode

Sheng He , Shaohua Yue, Gang Wang, Siyuan Wang , Jiayi Liu, Wei Liu and Xiangke Guo *

Air Defense and Antimissile School, Air Force Engineering University, Changle East Road, Xi'an 710051, China; 161180045@smail.nju.edu.cn (S.H.); stoneho32@163.com (S.Y.); sharesunny123@163.com (G.W.); dahonghuaer@163.com (S.W.); sixandone1@163.com (J.L.); long_waver@163.com (W.L.)
* Correspondence: guosyanyu@163.com

Abstract: Spatial crowdsourcing is a mode that uses distributed artificial computing power to solve specific function sets through Internet outsourcing. It has broad application value in the networked command and control of current joint air defense operations. In this paper, we introduce the spatial crowdsourcing theory into the field of target allocation for joint air defense operations and establish a weapon-target assignment model based on spatial crowdsourcing mode, which is more appropriate to the real situation and highlights the system cooperation capability of joint air defense operations. To solve the model, we propose a heuristic variable weight nonlinear learning factor particle swarm optimization (VWNF-PSO). This algorithm can significantly improve the efficiency and adaptability to weapon-target assignment problems under large-scale extreme conditions. Finally, we establish two kinds of joint air defense operation scenarios to verify the proposed model, then compare the proposed algorithm with variable weight PSO (VWPSO) and adaptive learning factor PSO (AFPSO), to validate the effectiveness and efficiency of the VWNF-PSO algorithm proposed in this paper.

Keywords: regional air defense operations; weapon-target distribution; variable weight nonlinear learning factor particle swarm optimization algorithm

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1. Introduction

With the development of ground attack aircraft, unmanned aerial vehicles, ballistic missiles and cruise missiles, air attack has become a very common means of operation in modern war. As the current air defense combat situation presents the characteristics of multiple targets and large scale, the existing combat mode based on equipment autonomous engagement has been difficult to adapt to the battlefield environment. The rational allocation of operational resources to improve interception efficiency and the reduction of operational cost have become urgent problems to be solved in air defense operations.

Spatial crowdsourcing [1] is a mode that uses distributed artificial computing power to solve specific function sets through Internet outsourcing. It has attracted wide attention since being proposed in 2006, and is now widely used in image and video marking classification, road condition detection, text recognition, Uber, takeout food delivery and other applications [2]. The efficient assignment of tasks to workers represents a research hotspot in the field of spatial crowdsourcing. The existing research objectives include maximizing the number of one-time task matching, realizing the optimal task scheduling scheme for a single worker, pursuing the quality and diversity of task completion, and iterating task allocation and task scheduling to maximize the number of global task allocation [3].

Regional air defense operation refers to intercepting enemy air attack targets to protect assets. The most important intercepting targets are cruise missiles, ballistic missiles, rockets and various types of enemy aircraft, etc. Assets mainly include cities, airports, ports and other important facilities. The main objective of defense is to maximize the destruction of enemy targets or to maximize the self-protection of assets. The multiple resources and high timeliness requirements of regional air defense operation control are in line with the characteristics of collaborative intelligence and efficient processing of spatial crowdsourcing mode. At the same time, the relationship between combat units and incoming targets is highly similar to the "worker-task" relationship in spatial crowdsourcing mode. As a problem-solving framework, spatial crowdsourcing has a broad application prospect in the target allocation of regional cooperative air defense operations.

Since Manne (1957) put forward the WTA issue [4], the study on missile assignment has been continued up to now. Matlin provided literature on missile assignment before 1970 [5]. Soland studied the weapon target assignment of the anti-ballistic missile in area defense and point defense [6]. Wacholder analyzed the WTA problem in ICBM defense, minimizing the residual threat to the incoming target as the target, and considering the upper limit of interceptor weapons owned by each combat unit and the upper limit of interceptor weapons assigned to a target [7]. Liu studied the WTA problem of maximizing the interception of enemy weapons based on different kill zones with multiple target channels of the ground-to-air missile defense system, taking into account time constraints, space constraints and resource constraints [8]. Meng analyzed the WTA problem of multi-layer ballistic missile defense systems [9]. Wang mainly studied target assignment in air defense command and control system of the tactical unit of surface-to-air missile and proposed the concept of shooting superiority of intercepting target of firepower unit [10]. Wang studied the WTA problem of general ballistic missile defense [11]. Li studied the WTA problem of interception of the multi-layer incoming ballistic missiles. Multi-layer mainly refers to the high-altitude defense layer and low-altitude defense layer. The defense side launches different interceptors according to different defense layers and considers the time window when assigning interceptors [12]. Xu studied the multi-target WTA problem under uncertain conditions based on the static perspective, to obtain the maximum interception efficiency and minimum interception loss [13]. Li studied the WTA problem of static multi-target ground air defense to maximize the protection of assets while minimizing interceptor consumption [14]. Jang focused on the WTA problem of intercepting enemy missiles by interceptors with high hit probability [15]. Guo mainly studied the problem of multi-target missile interception with fixed and adaptive grouping constraints, and the grouping strategy mainly considers the limitation of the number of weapons allocated to each target [16]. Zhang proposed a dynamic sensor/heterogeneous weapon-target integration assignment problem by extracting key factors of typical ground-air anti-penetration scenarios, that is, multiple types of near, medium, and far defense weapons and sensors are deployed to intercept the target during the penetration process of the target from far to near [17]. The current research directions of the WTA problem mainly focus on simplifying models and assumptions that more closely resemble actual operations, multi-weapon cooperation, interception cost-effectiveness optimization, and timeliness.

Since WTA has its characteristics under different combat patterns, our research mainly studies WTA in-ground joint air defense operations. Compared with the current research on WTA issues, our research is more in-depth. In terms of problem size, our study includes large, medium, and small scales. From the perspective of complexity, our research considers the cooperation between combat units and the cooperation between combat units and sensors. In addition, the model construction takes full account of the requirements of actual application scenarios, and the complexity is further improved. From the perspective of timeliness, as it involves intercepting tactical ballistic missiles, timeliness is required to a certain extent, but in general, optimality is more important than timeliness.

In spatial crowdsourcing mode, workers usually have a relatively fixed work scope (such as a circle with their location as the center and a radius of the distance affected by many personal preferences) and can complete dynamic tasks that are influenced by personal preference. In regional air defense operations, combat units, as "workers", are usually unable to move after completion. As a "working range of workers", the kill zone of the combat unit has great differences in intercepting different types of incoming targets. The incoming target is a moving individual with a specific threat level as a "worker assignment".

To adapt to the target assignment of joint air defense operations, the traditional spatial crowdsourcing model needs to be improved to adapt to the target assignment of regional air defense operations.

The target assignment problem of air defense operation is an integer nonlinear multi-dimensional combinatorial optimization problem, which belongs to the non-deterministic problem (NP) [18]. The Hungarian algorithm [19], particle swarm algorithm [20–22], ant colony algorithm [23], artificial fish swarm algorithm [24], simulated annealing algorithm [25,26], cuckoo algorithm [27] and so on are widely used in solving this kind of problem. Compared with other evolutionary algorithms, particle swarm optimization bears the advantages of fewer control parameters, better convergence, and easier implementation. Because of these advantages, PSO has attracted extensive attention in the field of evolutionary computing since its introduction. As with other swarm-based stochastic optimization algorithms, PSO is initialized with a population of random solutions (position of each particle) in the search space, and subsequently begins to enter a loop to continue searching for optimal solutions by updating the particle's velocities and positions until some termination conditions are satisfied [28]. Thus, the methodology of generating high-quality initial particles represents a worthy research direction in the PSO field. Moreover, the proper selection of control parameters, such as inertia weight and learning factor, can significantly influence the convergence of PSO. Therefore, the current research on PSO is mainly aimed at improving the above two aspects.

For swarm initialization, Tian used two kinds of chaotic maps to improve the quality of the initial swarm for PSO with promising results [29]. Gao offered a similar chaotic opposition-based swarm initialization [30]. Li applied two kinds of chaotic maps to initialize the swarm in which the logistic map was for positions while the cubic map was for velocities of the particles [31].

For parameter selection, Clerc introduced a constriction factor into the standard PSO that was a function of learning factors c_1 and c_2 to insure the convergence of particle swarm optimization [32]. Ratnaweera put forward a self-organizing hierarchical particle swarm optimizer (HPSO) with time-varying acceleration coefficients (TVAC) to control the local search and convergence to the global optimum solution. Conducted experiments revealed that the performance of HPSO with TVAC was markedly better than that of HPSO with fixed learning factor [33]. Tang brought forward a modified particle swarm optimization by exploiting the exponential time-varying acceleration coefficients [34].

Compared with the existing spatial crowdsourcing model applied to ride-hailing and takeout delivery services, the main difficulties to be solved in this problem are as follows:

A single incoming target may be intercepted by too many combat units, which will seriously occupy combat resources.

The performance parameters of different types of combat units are quite different, combat units with better performance will be used under heavy load in conventional assignment algorithms. This would leave more economical combat units idle. Combat unit targets are not evenly distributed.

Aiming at the above difficulties, we introduce the spatial crowdsourcing theory into the weapon-target assignment problem of joint air defense operations. To adapt to the real scene and highlight the system coordination capability of joint air defense operations, according to the characteristics of the WTA problem, we establish a target allocation model of joint air defense operations based on spatial crowdsourcing mode. Firstly, we preprocess the data, and calculate the potential intercept task matrix, combat unit synergy relation diagram, and intercept mission reward matrix by combat unit data and incoming target data. Then we use the tree decomposition algorithm to segment the combat units set according to the cooperative relationship between combat units, to reduce the size of the solution space and improve the efficiency of the optimization algorithm. To solve the model, we propose a heuristic variable weight nonlinear learning factor particle swarm optimization (VWNF-PSO). We use the potential interception capability matrix to constrain the generation of the primary particles to further compress the solution space in the initialization stage, and use it to constrain the particle movement velocity to avoid the particles moving out of the solution space in the optimization stage; then we add the adaptive mutation method, calculate the mutation probability according to the particle individual fitness and population average fitness to improve the ability to jump out of local optimal; finally, we improve the algorithm inertia weight and learning factor value strategy, adjust the inertia weight timely according to the individual fitness and the average fitness of the population, adjust the learning factor by using the nonlinear cloud selection method according to the number of iterations, and improve the ability of the particles to move to the optimal solution at different stages of the search. Through the above improvements, the efficiency of the algorithm is effectively improved, and the adaptability is stronger in the face of large-scale extreme conditions.

2. Problem Definition

This paper defines regional air defense operations as a model similar to the typical spatial crowdsourcing model, including workers (combat units), requestors (incoming targets), space missions (interception missions), and platforms (command and control systems). Different combat units are equipped with interceptors of different performance. They communicate with the command and control (C2) system and other combat units through communication links, and dynamically inform the C2 system of their position, channel status, and the number of interceptors. Different combat units have different reward indicators for different incoming targets. The incoming target issues an intercept mission with a clear message. The C2 system receives all the information from the combat unit and the incoming target, and assigns the incoming target to the combat unit through the designed target assignment mechanism.

The main symbols in this paper are shown in Table 1.

Table 1. Summary of notations.

Symbol	Definition	Symbol	Definition
w	Combat unit	Ft	Latest completion time of target interception mission
$w.tp$	Type of combat unit	Tr	A set of incoming targets
$w.l$	Location of the combat unit	Tw	An interception task set of w
rt	Average response time of combat units	$ETS(w)$	An effective interception task set of w
A	Kill zone parameters of combat units	$EETS(w)$	Extremely effective interception task set of w
tr	Incoming target	$t(w,tr)$	The time when w completes the interception task of tr
T	The trajectory of the incoming target	P	Interception capability matrix
ch	Characteristic parameters of the incoming target	S	Interception task assignment scheme

Definition 1. *Combat unit.*

In this paper, combat units generally refer to ground air defense units that carry out joint air defense operations in the region and are capable of intercepting incoming targets in the air. A single combat unit is defined by a five-dimensional cell array such as $w_i = \langle w.tp_i, w.l_i, A_i, rt_i, c_i, a_i \rangle$. $w.tp_i$ is the type of combat unit. $w.l_i = \langle lon_i, lat_i, h_i \rangle$ is the position of the combat unit, it includes longitude, latitude and altitude data of the combat unit.$A_i = (P_i, h\max_i, h\min_i, d\max_i, d\min_i, \gamma\max_i, \varepsilon\max_i, \varepsilon\min_i, \alpha_i, \beta_i, K)$ is the kill zone parameter of the combat unit.Pi is the combat unit radar normal direction.$h\max_i$ is the high limit of the kill zone, $h\min_i$ is the low limit of the kill zone, $d\max_i$ is the far limit of the kill zone, $d\min_i$ is the near limit of the kill zone, $\gamma\max_i$ is the maximum route angle, $\varepsilon\max i$ is the maximum high and low angle. $\varepsilon\min_i$ is the minimum high and low angle, αi is the azimuth of the radar sector of the combat unit. βi is the pitching angle of the radar sector, K is the radar power coefficient, rt_i is the average response time of combat units, c_i is the number of combat unit fire channels, and a_i is the number of combat unit interceptors.

Definition 2. *Incoming target.*

The incoming target is defined by a three-dimensional cell array such as $tr_j = \langle T_j, ch_j, Ft_j \rangle$. T_j is the trajectory of the incoming target, consisting of multiple sets of longitude, latitude, and altitude data. $ch_j = \langle rcs_j, tr.tp_j \rangle$ is the characteristic parameter of the incoming target and is a two-dimensional array composed of the RCS of the incoming target and the target type. Ft_j is the latest completion time of target interception task set by the command and control system according to the characteristics of the incoming target. After receiving the target intercept order, the combat unit needs to complete the intercept before the latest completion time.

Definition 3. *Task sequences.*

T_w is the set of incoming targets assigned to the combat unit w. $Tr(T_w)$ is a sequential sequence of T_w, represents the time sequence in which the combat unit intercepts the incoming target. $t(w, tr)$ is the time when the combat unit w finishes intercepting the incoming target tr, and must meet the condition: $t(w, tr) \leq Ft(tr)$.

Definition 4. *Effective interception task set (ETS).*

The interception task set T_w is called the effective intercept task set of combat unit w if and only if the following conditions hold:

$$\forall tr_j \in T_w, t(w, tr_j) \leq Ft_j; a \geq 0. \tag{1}$$

Definition 5. *Extremely effective interception task set (EETS).*

If any superset of the effective blocking task set T_{w} is not the effective intercepting task set, it is the extremely effective task set.

Definition 6. *Interception task assignment.*

Given the combat unit set W and the incoming target set Tr, the intercept task assignment is $S = \{\langle w_1, ETS(w_1)\rangle, \langle w_2, ETS(w_2)\rangle, \ldots, \langle w_n, ETS(w_n)\rangle\}$. $S.Tr = \cup_{w \in W} T_w$ is defined as the set of interception missions assigned to all operational units.

Definition 7. *Intercept mission reward metrics.*

The intercept mission reward parameter refers to the interception effect parameter calculated by various factors when the combat unit completes an interception mission. According to the characteristics of combat units, the factors that need to be considered in the judgment of reward parameters are compared and screened, and the important and relatively independent factors are extracted as the basic elements for the judgment of reward parameters. In this paper, the direct assessment method is used to judge the threat mainly by the target type and supplemented by other factors.

1. Incoming target property parameter

   (1) Intercept arc length (time parameter)

The longer the target passes through the kill zone of the combat unit, the more likely the combat unit will capture and intercept it successfully, and the higher the success rate of interception. Normalized, the duration of the target passing through the kill zone to obtain the intercept arc length parameters of the incoming target:

$$\mu_1(t) = \begin{cases} \frac{t_{ij}-t_i\min}{t_i\max-t_i\min} & t > 0 \\ 0 & t = 0 \end{cases}. \tag{2}$$

In this formula, t_{ij} is the time for the incoming target tr_j to pass through the kill zone of combat unit w_i. $t_i\min$ and $t_i\max$ are the minimum and maximum arc lengths of intercepting all incoming targets by combat unit w_i, respectively.

(2) The target type

For different types of targets, their flight speed and attack capability are different, and therefore their threat level is different. Usually, according to the threat degree, the order is anti-radiation missile, typical target, slow target, cruise missile target, and jamming target. The target type parameter function is:

$$\mu_2(l) = \begin{cases} 1 & \text{anti-radiation missile} \\ 0.7 & \text{typical target} \\ 0.6 & \text{cruise missile target} \\ 0.5 & \text{slow target} \\ 0.2 & \text{jamming target} \end{cases}. \tag{3}$$

(3) Electronic jamming

When the enemy carries out an air raid, they usually add electronic jamming equipment to the air raid weapon to improve its penetration ability. The purpose of jamming:

Disturbing the search radar, affects the ability of radar to detect the target, so that it cannot receive the correct information about the target;

Jamming the tracking guidance radar in the surface-to-air missile weapon system makes it unable to track the target effectively;

Interferes with the on-board electronic equipment of surface-to-air missile to make the missile lose control and reduce the hit probability;

Destroying ground air defense weapons directly to render it incapable of anti-aircraft operations.

The parameter function of electronic interference capability is:

$$\mu_3(e) = \begin{cases} 0.8 & \text{very strong} \\ 0.6 & \text{strong} \\ 0.4 & \text{normal} \\ 0.2 & \text{weak} \\ 0 & \text{none} \end{cases}. \tag{4}$$

According to the calculation of the above function, the comprehensive attribute parameter of target j is given by the above formula:

$$td_j = \sum_{k=1}^{3} \omega_k \mu_{kj}. \tag{5}$$

The weight vector composed of each weight value is:

$$\vec{\omega} = [\omega_1, \omega_2, \omega_3], \sum_{k=1}^{3} \omega_k = 1. \tag{6}$$

According to the influence of each attribute factor on the threat degree, the comprehensive target attribute parameters can be obtained by integrating the weights set by experts.

2. Combat unit adaptability parameters

(1) Combat unit effectiveness indicators

Since the effectiveness of combat units in intercepting different incoming targets is different, when setting the reward mechanism, it is necessary to introduce the effectiveness index e_{ij} of combat units based on the evaluation of incoming target attribute parameters, which is given by experts according to combat unit type $w.tp_i$ and incoming target type $tr.tp_j$.

(2) Interceptor cost penalty

To avoid excessive consumption of interceptors and maximize the benefits of air defense operations, it is necessary to set the penalty parameter pu_i for the consumption of interceptors. If the combat unit fires n interceptors, its total penalty parameter is: $Pu_i = n \times pu_i$.

The combat unit adaptability parameter is: $ad_{ij} = e_{ij} - Pu_i$.

Finally, the elements of the intercept mission reward matrix **R** are:

$$r_{ij} = td_j + ad_{ij}. \tag{7}$$

Problem definition:

The problem is to find the task allocation problem in the crowd-sourcing mode based on the reward mechanism under the limitation of the latest completion time of the target interception task and the number of interceptors of the combat unit, under the condition that the combat unit set W and the incoming target set Tr are determined. The objective is to find a globally optimal task assignment scheme S_{opt} to achieve $\forall S_i \in S, S.r \leq S_{opt}.r$. S represents all target assignment schemes, and $S.r$ represents the total value of reward parameters for this target assignment scheme.

3. Algorithm

The global optimal task assignment scheme in spatial crowdsourcing mode is a typical NP problem. Greedy algorithms can be used to find the most efficient set of interceptor tasks for this unit, and then the assigned interceptor tasks can be weighted. However, for the incoming targets that can be intercepted by multiple combat units, if the target is randomly assigned to any combat unit for the interception, other incoming targets may not be intercepted. To solve this problem, firstly, using the dynamic programming method to find all extremely effective interception task sets for each combat unit, and the optimal assignment scheme is some combination of maximum effective task sets for each combat unit. Secondly, using the tree decomposition technology to divide unrelated combat units into different combat unit sets and then employing the tree index to search combat unit sets to improve the search efficiency of the extremely effective interception task sets. Finally, introducing the reward mechanism, and searching the search tree established in the previous step by the search algorithm combined with the success rate of interception.

3.1. Calculate the Effective Interception Task Set

3.1.1. Finding Potential Interception Missions

Due to the limitation of deployment position and equipment performance of combat units, each combat unit can only form interception conditions for part of the incoming targets. Therefore, it is necessary to find the incoming target set with potential interception conditions for each combat unit under the condition of satisfying constraints. The set of incoming targets with potential interception conditions of combat unit w is $Tr.T_w$, and combat unit A shall meet the following conditions:

$$\forall tr \in Tr.T_w, t(w, tr) \leq Ft(tr). \tag{8}$$

$$\forall tr \in Tr.T_w, T(tr) \cap D(w) \neq \varnothing. \tag{9}$$

$$\forall tr \in Tr.T_w, T(tr) \cap A(w) \neq \varnothing. \tag{10}$$

$$\forall tr \in Tr, t(T(tr) \cap A(w)) > rt(w). \tag{11}$$

$T(tr) \cap D(w)$ refers to the overlap between the trajectory of the incoming target and the radar detection area of the combat unit. $T(tr) \cap A(w)$ refers to the overlap between the trajectory of the incoming target and the combat unit's kill zone. $t(T(tr) \cap A(w))$ is the time when the incoming target crosses the combat unit's kill zone. The above four conditions ensure that a combat unit can effectively fire on an incoming target passing through its kill zone before the interception mission expires.

3.1.2. Finding Extremely Effective Interception Task Set

To avoid traversing and searching all incoming targets, it is necessary to assign multiple interceptable incoming targets to the combat unit at one time under the limitation of the number of fire channels and interceptors. In this paper, a dynamic programming algorithm is used to solve the extremely effective interception task set of the combat unit. The algorithm expands the effective interception task set of combat units by gradually increasing the size of the effective interception task set, and finds all EETS under the set in each iteration. Given a combat unit w and an interception task set $Q \subseteq Tr.T_w$, $opt(Q, tr)$ is the maximum number of interception tasks that can be completed when intercepting incoming target \$tr\$ after intercepting an incoming target in Q. R is the target assignment sequence of Q. tr_i represents the target before tr_j in sequence R. $opt(Q, s_j)$ can be obtained by the following formula:

$$opt(Q, tr_j) = \begin{cases} 1 & \text{when}|Q|=1 \\ \max\limits_{tt_i \in Q, tr_i \neq tr_j} opt(Q - \{tr_j\}, tr_i) + \sigma_{ij} & \text{others} \end{cases}. \quad (12)$$

$$\sigma_{ij} = \begin{cases} 1 & \text{when } t(w, tr_j) \le Ft(tr_j), T(tr_j) \cap D(w) \neq \varnothing, T(tr_j) \cap A(w) \neq \varnothing, t(T(tr_j) \cap A(w)) > rt(w) \\ 0 & \text{others} \end{cases}. \quad (13)$$

$\sigma_{ij} = 1$ indicates that adding the incoming target tr_j to the end of the sequence R, and target tr_j can still be intercepted, meeting the limitation of the number of combat unit channels and interceptors at the same time. $opt(tr_i, tr_i) = 1$ when Q contains only one incoming target tr_i. Q needs to be searched to obtain the full effective interception task set, and tr_i is found to achieve the maximum $opt(Q, tr_j)$, when $|Q| > 1$.

Meanwhile, constructing the potential interception task matrix $\mathbf{M}$, and its element m_{ij} is:

$$m_{ij} = \begin{cases} 1 & w_i \text{ have the potential interception ability to intercept } tr_j \\ 0 & \text{others} \end{cases}. \quad (14)$$

3.2. Split the Set of Combat Units

With the increase in the number of combat units and incoming targets, the solution space of optimal target assignment will increase rapidly. To meet the high real-time requirements of air defense operations, the search algorithm needs to be optimized to improve search efficiency.

Definition 8. *Combat unit synergy.*

Given two combat units w_i, w_j, and their interceptable mission sets $Tr.Tw_i$, $Tr.T_{w_j}$, and if $Tr.T_{w_i} \cap Tr.T_{w_j} = \varnothing$, they are independent of each other. Otherwise, there is a synergistic relationship between the two combat units.

Definition 9. *Combat unit synergy relation diagram.*

According to the known combat unit set W and incoming target set Tr, the synergy relation diagram $(WTrG)G(N, E)$ of combat units is constructed, hereinafter referred to as the synergy relation diagram. Each node n in the diagram represents a weapon system

$w_n \in W$. If two combat units w_m and w_n have a synergistic relationship, there is an edge $e(m, n) \in E$ between them.

In the mathematical model, synergy relation diagram $(WTrG)G(N, E)$ is represented as an upper triangular matrix $\mathbf{P}$, where each element p_{ij} is:

$$p_{ij} = \begin{cases} 1 & \text{combat unit } w_i \text{ and } w_j \text{ have cooperation relationship} \\ 0 & \text{combat unit } w_i \text{ and } w_j \text{ don't have cooperation relationship} \end{cases}. \tag{15}$$

In the decomposition of the coordination diagram of combat units, the combat units without synergy relationship can be divided into different sets to divide the synergy relation diagram in a balanced way [35,36]. This not only improves search efficiency but also further optimizes interception tasks and avoids wasting interceptor resources. The algorithm flow is shown in Figure 1.

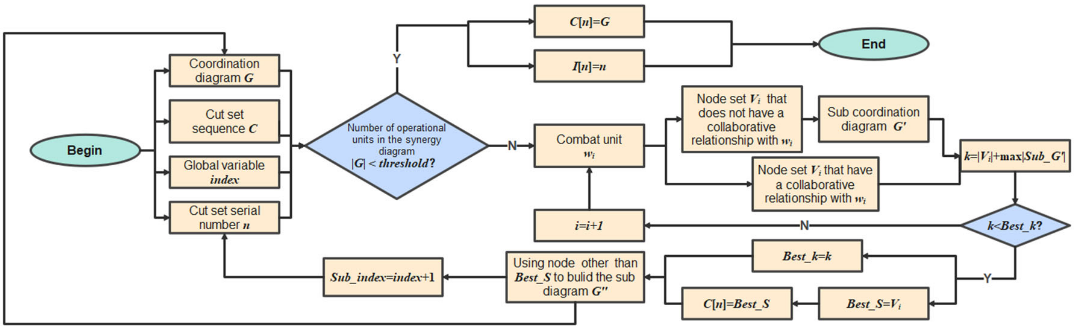

Figure 1. Cooperative diagram tree decomposition algorithm.

In each step of decomposition, a node and the node associated with the node are taken as the cut set, which is used to cut the whole synergy relation diagram. Then, the size of the cut set (including the number of combat units in the cut set) is recorded and the sum of the number of combat units in the largest subgraph after the graph is divided. The cut set that minimizes the sum above is selected to segment the original graph again, and the algorithm is recursively called for the subgraph after segmentation until the number of nodes in the sub-synergy relation diagram is less than the threshold set in initialization [37,38].

3.3. Search

After the operation unit synergy diagram is transformed into a tree structure, the search algorithm is used to solve the optimal task assignment scheme.

Given the combat unit set W and the incoming target set Tr. For each combat unit w_i firstly calculates its maximum interception task set Q_{w_i}, T_{w_i} is the interception task set, and constructs the corresponding synergy diagram G. For each subgraph in the synergy diagram $g \in G$, the tree decomposition algorithm is used to divide the combat units into different sets of combat units to reduce the search times, and the search tree structure is established according to the sequence of combat unit sets. Finally, the variable weight nonlinear factor particle swarm optimization algorithm is used to search the search tree constructed in the previous step to find the optimal target assignment scheme. Since the different subgraphs of G are not correlated with each other, the final target allocation scheme is a collection of assignment schemes of different subgraphs. The detailed optimization structure is shown in Figure 2.

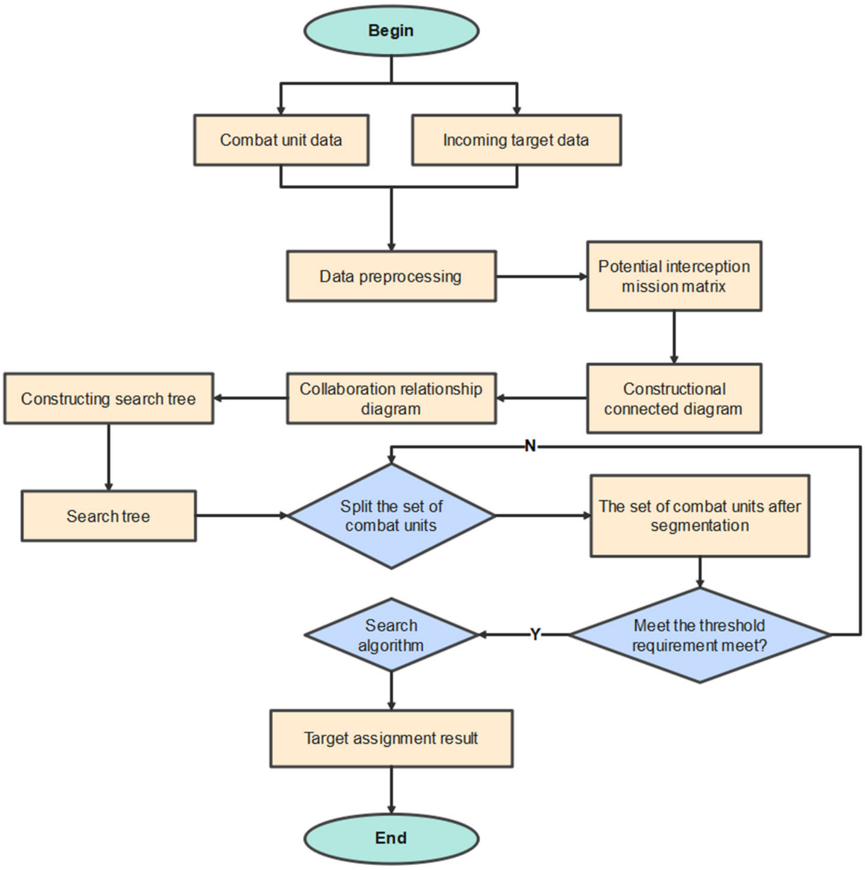

Figure 2. Optimal structure.

3.3.1. Particle Swarm Optimization

Particle swarm optimization (PSO) was first proposed by Eberhart and Kennedy in 1995 based on the foraging behavior of birds [39]. It is a method to find the optimal solution by iteratively calculating the fitness based on the random solution.

In the PSO, each particle has two attributes: position X and speed V. Position X represents the candidate solution of the problem, and velocity V represents the position change between successive iterations. The fitness of each particle in the swarm is calculated, then compared with the fitness of specific particle P_{gbest} (whose fitness is the global best fitness) and the current population best particle P_{ibest} to evaluate the merits and disadvantages of the current particle. The result drives the current particle to its position at a speed determined by the distance between the current particle and P_{gbest}, the distance between the current particle and P_{ibest}, while maintaining inertia to prevent falling into local optimization. The state transition is shown in the following formula:

$$V_i(t+1) = w \times V_i(t) + c_1 r_1 \Big(P_{gbest}(t) - X_i(t) \Big) + c_2 r_2 (P_{ibest}\ (t) - X_i(t)). \tag{16}$$

$$X_i(t+1) = X_i(t) + V_i(t+1). \tag{17}$$

In this formula w is the inertia factor of velocity, adjusting its value can balance global and local optimization. c_1 and c_2 are the learning factors of speed, and adjusting their values can realize faster convergence and prevent falling into the local optimum. r_1 and r_2 are two random numbers evenly distributed between 0 and 1. Set the boundary conditions to ensure:

$$X_{\min} \leq X_i(t+1) \leq X_{\max}. \tag{18}$$

$$V_{\min} \leq V_i(t+1) \leq V_{\max}. \tag{19}$$

3.3.2. Variable Weight Nonlinear Factor PSO

Because of the disadvantages of traditional PSO algorithms with fixed inertia weight and learning factors, such as prematurity, and low efficiency in late iteration, they can easily fall into local optimization. In this paper, a variable weight nonlinear factor PSO (VWNF-PSO) is proposed to solve the WTA problem. Using the VWNF-PSO can quickly find the optimal solution to achieve the optimal allocation between the combat unit and the incoming target. The algorithm improvements are as follows:

By introducing adaptive variation, when the current particle fitness is lower than the swarm average fitness, the position of particles can be adjusted by mutation operation, which can avoid prematurity and effectively jump out of local optimum.

The inertia weight w in PSO is used to balance the global and local search capabilities. Many researchers have advocated that the value of w should be large in the exploration state and small in the exploitation state. However, it is not necessarily correct to decrease w purely with time [40]. The learning factor c_1 shares some characteristics with the inertia weight w in that c_1 is also relatively large during the exploration state and becomes relatively small in the convergence state. Hence, it would be beneficial to allow w to follow the evolutionary states using the variable weight method. When the particle fitness is less than the average fitness, it means that the particle is relatively far from the maximum value, and the search scope needs to be expanded to find the maximum value. When the particle fitness is greater than the average fitness, it means that the particle is close to the maximum value, and the search scope needs to be narrowed for local accurate search [41]. Using the variable weight method, according to the particle fitness, express the following formula to adjust the inertia weight:

$$w_i^d = \begin{cases} w_{\min} + (w_{\max} - w_{\min}) \frac{f_{\max^d} - f(x_i^d)}{f_{\max^d} - f_{\text{average}}^d} & f\left(x_i^d\right) \geq f_{\text{average}}^d \\ w_{\max} & f\left(x_i^d\right) < f_{\text{average}}^d \end{cases} \tag{20}$$

Presetting the maximum and minimum inertia weights: $w\text{min} = 0.4, w\text{max} = 0.9$; f_{average}^d is the average fitness of all particles in the d iteration; $f_{\max}^d = \max\left\{f\left(x_1^d\right), f\left(x_2^d\right), \cdots, f\left(x_n^d\right)\right\}$ is the maximum fitness of all particles in the d iteration. Through the above improvements, the algorithm can adjust the inertia weight in real time according to the current fitness during optimization, so that the particles have strong detection ability and avoid falling into local optimal.

The learning factor c_1 represents the acceleration weight of the particle moving towards the global optimal particle, and c_2 represents the acceleration weight of the particle moving towards the current swarm optimal particle. In the initial search process, because of the wide distribution of particles, we should be more inclined to search for the global optimum. In the later search process, if still using similar learning factors, the particle position will change too much and jump out of the global optimal group, reducing the efficiency of the later search [42]. Adopting the strategy of nonlinear cloud learning factor to make the learning factor c_1 nonlinearly decrease with the number of iterations, and c_2 nonlinearly increase with the number of iterations, which can effectively prevent the particles from clustering into the local optimal early, and quickly gather in the late iteration, and improve the accuracy and speed of search.

$$c_1 = c_{1e} + (c_{1i} - c_{1e}) \cdot \exp\left[-(4t/T)^2\right] + rand(-0.1, +0.1). \tag{21}$$

$$c_2 = c_{2e} + (c_{2i} - c_{2e}) \cdot \exp\left[-(4t/T)^2\right] + rand(-0.1, +0.1). \tag{22}$$

where, c_{1i} and c_{2i} are the left bounds of their respective value ranges, and c_{1e} and c_{2e} are the right bounds of their value ranges. The reference range of learning factor is set to $c_1 \in (0.25, 0.85); c_2 \in (0.3, 0.9)$, and its value is shown in the Figure 3.

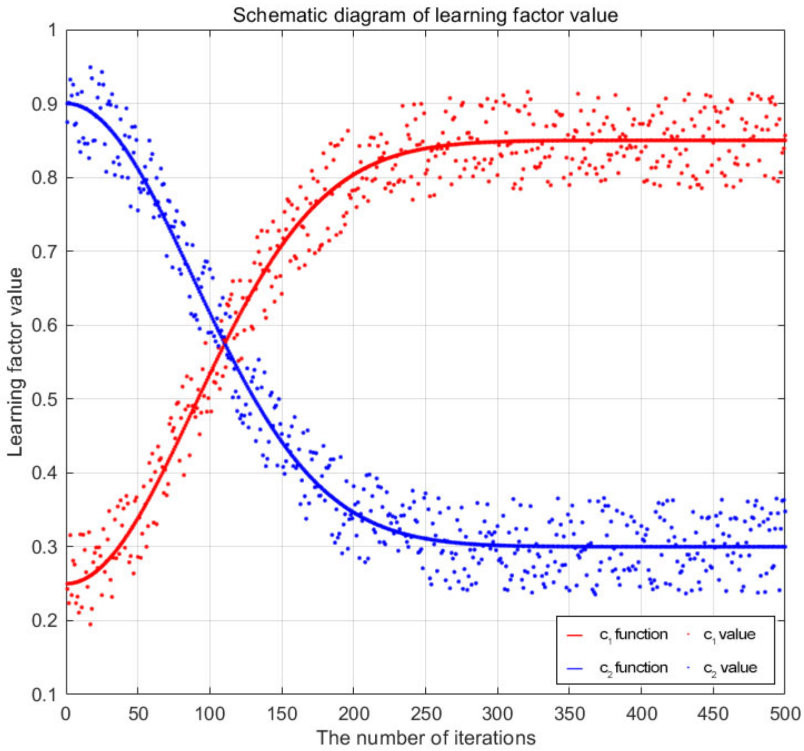

Figure 3. Learning factor value diagram.

For regional joint air defense operations, because combat units usually cannot move after deployment, a single combat unit does not have the interception capability (i.e., the interception arc length is 0) for part of the incoming targets. In this case, if using the traditional PSO algorithm's particle coding mode, a large number of invalid solutions outside the boundary conditions will be generated, which will occupy a large number of computing resources. To solve this problem, a particle coding strategy based on the interception task is proposed with the proposed model: randomly generating the primary particles are based on the potential interception mission matrix **M**. For the combat unit w_i, if its potential interception mission matrix $\mathbf{M}_{w_i-tr_j} = [\underbrace{0,1,0,1,\cdots,1}_{j}]$, its particle value space is:

$$X_{w_i-tr_j} = \begin{cases} rand(0,1) & \mathbf{M}_{w_i-tr_j} = 1 \\ 0 & \mathbf{M}_{w_i-tr_j} = 0 \end{cases} \tag{23}$$

The generation of primary particles by this strategy can significantly reduce the search space, thus improving the search efficiency and avoiding the effects of the algorithm due to the large search space.

3.3.3. Solution Steps Based on VWNF-PSO

Using VWNF-PSO, the specific steps of the optimization process of the proposed model are as follows:

Step 1: According to the input combat unit and incoming target data, preprocess the parameters involved in each combat unit and incoming target. Encoding the target allocation matrix and randomly setting the initial value to obtain the initial particle swarm, which is processed according to the particle coding strategy based on the potential interception task.

Step 2: Calculate the fitness and update the global extremum. According to the objective function and constraint conditions of the model, evaluating the fitness of each particle, and calculating the individual extreme value p_{ibest}. Comparing the fitness of each particle with the global best fitness and updating p_{gbest}, w_i, c_1, c_2.

Step 3: Update particle status.

Step 4: Carry out adaptive mutation operation. To judge whether the particle fitness is lower than the population average fitness, performing the adaptive mutation operation for the particle fitness is lower than the population average fitness, and performing boundary absorption processing for the data beyond the boundary.

Step 5: When the algorithm reaches the stop condition, break the search, output the results, and obtain the optimal target allocation scheme. Otherwise, return to Step 3 to continue the loop.

4. Experimental Analysis

All experiments were carried out on a Windows 11 experimental platform configured with Core I5-10210U, CPU 1.6 GHz, 8G RAM. The simulation calculates in Matlab 2017B experimental environment. Finally, the VWNF-PSO proposed in this paper is analyzed and compared with variable weight PSO (VWPSO) and adaptive learning factor PSO (AFPSO).

4.1. Small-Scale Experimental

In order to verify the effectiveness and performance of the algorithm in this paper in solving WTA problems, constructing a regional air defense operation scenario: C2 center obtains the current combat unit sequence $W = (w_1, w_2, w_3, \cdots, w_7)$(Type A weapon systems A1-A3, Type B weapon systems B1-B4), to intercept 20 targets $Tr = (tr_1, tr_2, tr_3, \cdots, tr_{20})$ (missile1–missile20) that are attacking secure positions (AreaTarget1–AreaTarget4).

Set type A weapon system combat unit effectiveness at 0.68 and interceptor consumption penalty at 0.5; the Type B weapon system has an operational unit effectiveness indicator of 0.7 and an interceptor consumption penalty of 0.2. The incoming targets 1–10 are cruise missiles, 11–20 are anti-radiation missiles, and their electronic jamming capabilities are of a general level. Set interception arc length, target type and electronic jamming capability weights as $[\omega_1, \omega_2, \omega_3] = [0.4, 0.3, 0.3]$. All experiments were carried out on a Windows 11 experimental platform configured with Core I5-10210U, CPU 1.6 GHz, 8G RAM. The simulation calculates in Matlab 2017B experimental environment. The battle settings are shown in Tables 2–5. The trajectory of incoming targets and deployment of combat units are shown in Figures 4 and 5.

Table 2. Combat unit and incoming target information.

Combat Units	Incoming Target	Maximum Number of Interceptors Available to a Target
Type A: 3 Type B: 4	tr1-tr10: Cruise missile tr11-tr20: Anti-radiation missile	6

Table 3. Combat unit performance parameters.

Combat Unit	Distance of Kill Zone (km) [1]	Maximum Route Angle	Kill Zone Pitching Angle	Radar Sector Azimuth	Radar Sector Pitching Angle	Radar Power Factor	Kill Probability
A	200/70/150/45	45°	30°~70°	±50°	30°~70°	1000	0.8
B	30/2/25/2	50°	−2°~70°	±45°	0°~75°	200	0.7

[1] Farthest/Nearest/Highest/Lowest.

Table 4. Combat unit deployment location.

Combat Unit	Serial Number	Deployment Location [1]	Radar Normal Direction (0° Due East)
A	w_1	(32.43, 113.14)	0°
A	w_2	(31.44, 113.55)	0°
A	w_3	(30.58, 113.55)	0°
B	w_4	(30.27, 114.02)	15°
B	w_5	(31.41, 114.41)	15°
B	w_6	(31.71, 113.05)	0°
B	w_7	(32.75, 112.77)	0°

[1] (Longitude, Latitude, Altitude).

Table 5. Incoming target parameters.

Incoming Target	Serial Number	Launching Point Coordinates 1	Placement Coordinate 1
Cruise missile	tr1	(35.59, 129.20)	(30.27, 114.02)
Cruise missile	tr2	(34.48, 132.38)	(32.75, 112.77)
Cruise missile	tr3	(34.48, 132.38)	(30.27, 114.02)
Cruise missile	tr4	(35.59, 129.20)	(32.75, 112.77)
Cruise missile	tr5	(26.58, 134.46)	(31.41, 114.41)
Cruise missile	tr6	(33.57, 130.77)	(30.27, 114.02)
Cruise missile	tr7	(35.59, 129.20)	(31.71, 113.05)
Cruise missile	tr8	(34.48, 132.38)	(31.71, 113.05)
Cruise missile	tr9	(28.32, 133.42)	(30.27, 114.02)
Cruise missile	tr10	(28.32, 133.42)	(31.41, 114.41)
Anti-radiation missile	tr11	(28.32, 133.42)	(31.71,113.05)
Anti-radiation missile	tr12	(33.57, 130.77)	(31.41,114.41)
Anti-radiation missile	tr13	(28.32, 133.42)	(32.75,112.77)
Anti-radiation missile	tr14	(28.32, 133.42)	(31.41,114.41)
Anti-radiation missile	tr15	(35.59, 129.20)	(31.41,114.41)
Anti-radiation missile	tr16	(26.58, 134.46)	(31.41,114.41)
Anti-radiation missile	tr17	(26.58, 134.46)	(31.71,113.05)
Anti-radiation missile	tr18	(26.58, 134.46)	(30.27,114.02)
Anti-radiation missile	tr19	(26.58, 134.46)	(32.75,112.77)
Anti-radiation missile	tr20	(33.57, 130.77)	(31.71,113.05)

[1] (Longitude, Latitude, Altitude).

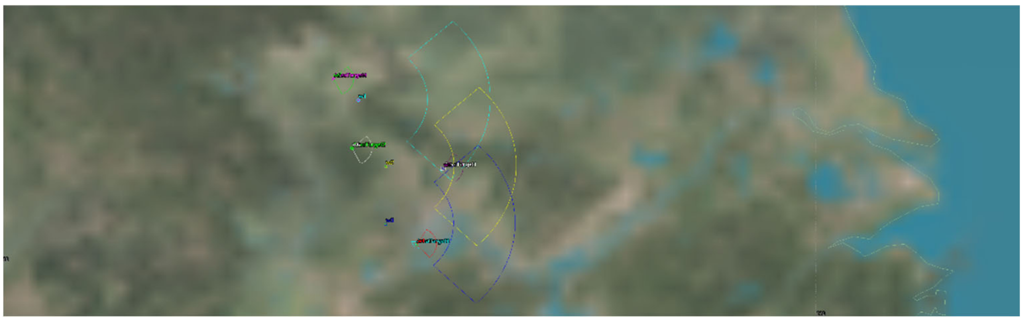

Figure 4. 2D schematic diagram of operational scenarios.

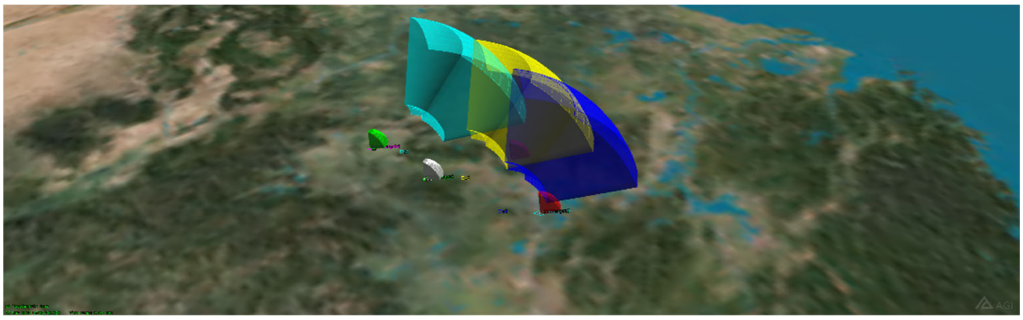

Figure 5. 3D schematic diagram of operational scenarios.

Firstly, preprocess the data to obtain the intercept sequence of the combat unit to the incoming target, the result is shown in Figure 6.

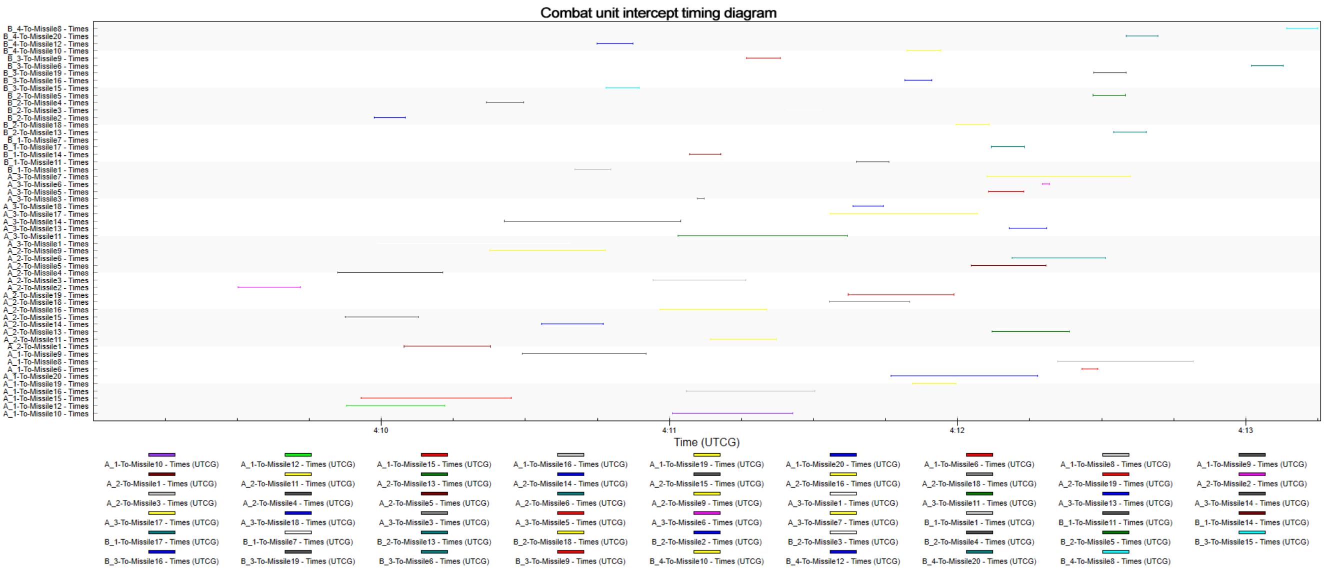

Figure 6. Sequence diagram of incoming target interception.

Construct intercept arc length matrix:

$$\mathbf{T}=\begin{bmatrix} 0&0&0&0&0&4&0&31&34&25&0&20&0&0&32&27&0&0&9&30\\ 18&13&19&22&16&19&0&0&24&0&14&0&16&13&15&22&0&16&22&0\\ 40&0&2&0&7&2&29&0&0&0&36&0&8&37&0&0&31&6&0&0\\ 7&0&0&0&0&0&6&0&0&0&7&0&0&6&0&0&7&0&0&0\\ 0&7&7&8&6&0&0&0&0&0&0&0&7&0&0&0&0&7&0&0\\ 0&0&0&0&0&6&0&0&7&0&0&0&0&0&7&5&0&0&7&0\\ 0&0&0&0&0&0&0&7&0&7&0&8&0&0&0&0&0&0&0&6 \end{bmatrix} \tag{24}$$

According to potential interception capability matrix **M**, synergistic relationship matrix **P** can be obtained as follows:

$$\mathbf{M}=\begin{bmatrix} 0&0&0&0&0&0&0&1&1&1&0&1&0&0&1&1&0&0&0&1\\ 1&1&1&1&1&1&0&0&1&0&1&0&1&1&1&1&0&1&1&0\\ 1&0&0&0&0&0&1&0&0&0&1&0&0&1&0&0&1&0&0&0\\ 1&0&0&0&0&0&1&0&0&0&1&0&0&1&0&0&1&0&0&0\\ 0&1&1&1&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0&1&0&0&0&0&0&1&1&0&0&1&0\\ 0&0&0&0&0&0&0&1&0&1&0&1&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{25}$$

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ & 1 & 1 & 1 & 1 & 1 & 0 \\ & & 1 & 1 & 1 & 1 & 0 \\ & & & 1 & 0 & 0 & 0 \\ & & & & 1 & 0 & 0 \\ & & & & & 1 & 0 \\ & & & & & & 1 \end{bmatrix} \tag{26}$$

Using the tree decomposition method, as shown in the figure below, to divide the combat unit into three sub-sets $\{C_1, C_2, C_3\}$, the result is shown in Figure 7.

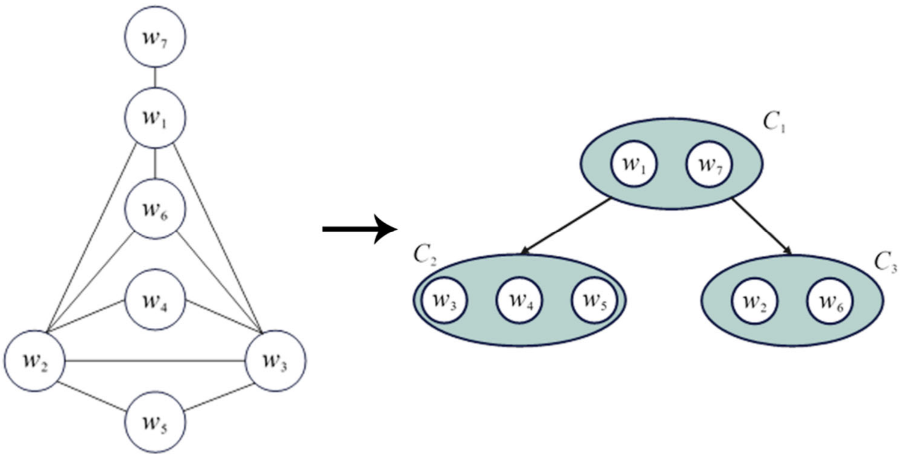

Figure 7. Combat unit tree segmentation results.

Use VWNF-PSO, AFPSO and VWPSO to optimize the set of three combat units sun-sets $\{C_1, C_2, C_3\}$ respectively. Set the inertia weight of AFPSO as $w = 0.8$, and the learning factor of VWPSO as $c_1 = 0.5, c_2 = 0.5$, obtain the target allocation matrix:

$$\mathbf{S}_{\text{VWNF-PSO}} = \begin{bmatrix} 0&0&0&0&0&0&0&1&0&0&0&1&0&0&1&1&0&0&0&1 \\ 0&1&0&1&1&0&0&0&1&0&1&0&0&1&1&1&0&1&1&0 \\ 1&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0 \\ 0&0&1&1&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&1&0&1&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{27}$$

$$\mathbf{S}_{\text{AFPSO}} = \begin{bmatrix} 0&0&0&0&0&0&0&1&1&1&0&1&0&0&0&0&0&0&0&1 \\ 1&0&1&1&0&1&0&0&0&0&1&0&1&0&0&1&0&1&0&0 \\ 1&0&0&0&0&0&1&0&0&0&1&0&0&1&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&1&0&0&1&0&0&1&0&0&0 \\ 0&1&1&1&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&1&1&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&1&0&1&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{28}$$

$$\mathbf{S}_{\text{VWPSO}} = \begin{bmatrix} 0&0&0&0&0&0&0&1&0&1&0&0&0&0&1&1&0&0&0&0 \\ 1&1&1&1&1&1&0&0&1&0&1&0&0&1&0&0&0&0&1&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0 \\ 1&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&1&0&0&0 \\ 0&1&1&1&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{29}$$

The target allocation and algorithm contrast are shown in the Tables 6–8 and Figures 8 and 9.

Table 6. Target assignment result.

Incoming Target	Combat Unit	Incoming Target	Combat Unit
tr_1	w_3, w_4	tr_{11}	w_2, w_4
tr_2	w_2	tr_{12}	w_1, w_7
tr_3	w_5	tr_{13}	w_5
tr_4	w_2, w_5	tr_{14}	w_2, w_3
tr_5	w_2, w_5	tr_{15}	w_1, w_2
tr_6	w_6	tr_{16}	w_1, w_2, w_6
tr_7	w_3	tr_{17}	w_3, w_4
tr_8	w_1, w_7	tr_{18}	w_2, w_5
tr_9	w_2, w_6	tr_{19}	w_2, w_6
tr_{10}	w_7	tr_{20}	w_1, w_7

Table 7. Algorithm contrast.

Algorithm	Test Times	Average Generations	Average Results	Optimal Results
VWPSO	50	23.4	16.578	18.563
AFPSO	50	25.1	19.188	19.989
VWAF-PSO	50	20.3	20.225	21.570

Table 8. Interceptor consumption.

Combat Unit	VWNF-PSO	AFPSO	VWPSO
w_1	5	9	5
w_2	13	12	12
w_3	6	8	3
w_4	5	5	6
w_5	6	8	8
w_6	4	6	1
w_7	8	5	5

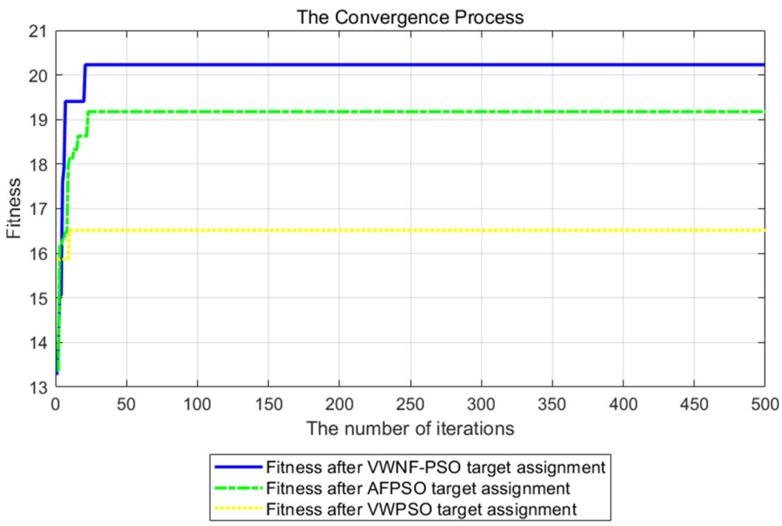

Figure 8. Algorithm fitness comparison.

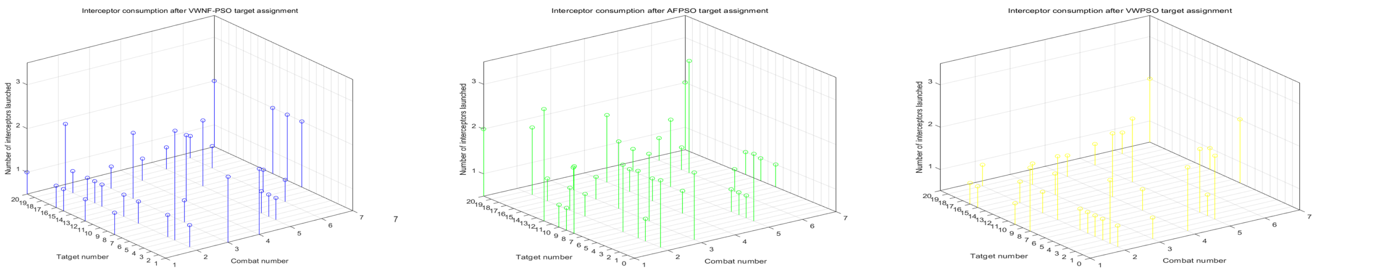

Figure 9. Number of interceptors consumed.

4.2. Large-Scale Experimental

To verify the adaptability of the model algorithm in dealing with large-scale scenarios, we set 60 future targets (missile1–missile60), under the condition that the number, type, and deployment position of combat units are not changed. Among them, incoming targets 1–20 are anti-radiation missiles, 31–50 are cruise missiles, and 21–30 and 51–60 are jamming targets, the electronic jamming ability of cruise missiles is strong, the electronic jamming ability of anti-radiation missiles is very strong, and the electronic jamming ability of jamming target is normal. Set interception arc length, target type and electronic jamming capability weights as $[\omega_1, \omega_2, \omega_3] = [0.4, 0.3, 0.3]$. The battle settings of the large-scale scenario are shown in Table 9.

Table 9. Incoming target parameters.

Serial Number	Launching Point Coordinates [1]	Placement Coordinate [1]
tr_1, tr_{21}, tr_{41}	(35.59, 129.20)	(30.27, 114.02)
tr_2, tr_{22}, tr_{42}	(34.48, 132.38)	(32.75, 112.77)
tr_3, tr_{23}, tr_{43}	(34.48, 132.38)	(30.27, 114.02)
tr_4, tr_{24}, tr_{44}	(35.59, 129.20)	(32.75, 112.77)
tr_5, tr_{25}, tr_{45}	(26.58, 134.46)	(31.41, 114.41)
tr_6, tr_{26}, tr_{46}	(33.57, 130.77)	(30.27, 114.02)
tr_7, tr_{27}, tr_{47}	(35.59, 129.20)	(31.71, 113.05)
tr_8, tr_{28}, tr_{48}	(34.48, 132.38)	(31.71, 113.05)
tr_9, tr_{29}, tr_{49}	(28.32, 133.42)	(30.27, 114.02)
tr_{10}, tr_{30}, tr_{50}	(28.32, 133.42)	(31.41, 114.41)
tr_{11}, tr_{31}, tr_{51}	(28.32, 133.42)	(31.71, 113.05)
tr_{12}, tr_{32}, tr_{52}	(33.57, 130.77)	(31.41, 114.41)
tr_{13}, tr_{33}, tr_{53}	(28.32, 133.42)	(32.75, 112.77)
tr_{14}, tr_{34}, tr_{54}	(28.32, 133.42)	(31.41, 114.41)
tr_{15}, tr_{35}, tr_{55}	(35.59, 129.20)	(31.41, 114.41)
tr_{16}, tr_{36}, tr_{56}	(26.58, 134.46)	(31.41, 114.41)
tr_{17}, tr_{37}, tr_{57}	(26.58, 134.46)	(31.71, 113.05)
tr_{18}, tr_{38}, tr_{58}	(26.58, 134.46)	(30.27, 114.02)
tr_{19}, tr_{39}, tr_{59}	(26.58, 134.46)	(32.75, 112.77)
tr_{20}, tr_{40}, tr_{60}	(33.57, 130.77)	(31.71, 113.05)

[1] (Longitude, Latitude, Altitude).

Due to the large scale of the data, the intended calculation process is omitted. After target assignment of the incoming target by three algorithms, the results are shown in Tables 10–12 and Figures 10 and 11.

Table 10. Target assignment result.

Combat Unit	Incoming Target
w_1	tr_8, tr_9, tr_{10}, tr_{12}, tr_{16}, tr_{20}, tr_{30}, tr_{32}, tr_{35}, tr_{36}, tr_{40}, tr_{48}, tr_{50}, tr_{52}, tr_{60}
w_2	tr_1, tr_2, tr_5, tr_{13}, tr_{15}, tr_{16}, tr_{18}, tr_{21}, tr_{22}, tr_{23}, tr_{24}, tr_{26}, tr_{33}, tr_{38}, tr_{39}, tr_{43}, tr_{44}, tr_{46}, tr_{49}
w_3	tr_1, tr_7, tr_{11}, tr_{14}, tr_{17}, tr_{21}, tr_{27}, tr_{31}, tr_{34}, tr_{37}, tr_{41}, tr_{47}, tr_{51}
w_4	tr_1, tr_7, tr_{14}, tr_{17}, tr_{21}, tr_{34}, tr_{47}, tr_{57}
w_5	tr_3, tr_4, tr_{13}, tr_{18}, tr_{19}, tr_{22}, tr_{33}, tr_{42}, tr_{44}, tr_{45}, tr_{58}
w_6	tr_6, tr_9, tr_{16}, tr_{19}, tr_{35}, tr_{39}, tr_{46}, tr_{55}
w_7	tr_8, tr_{10}, tr_{12}, tr_{20}, tr_{32}, tr_{40}, tr_{48}

Table 11. Algorithm contrast.

Algorithm	Test Times	Average Generations	Average Results	Optimal Results
VWPSO	50	298.4	39.88	48.329
AFPSO	50	228.3	15.34	52.391
VWNF-PSO	50	29.7	65.47	69.239

Table 12. Interceptor consumption.

Combat Unit	VWNF-PSO	AFPSO	VWPSO
w_1	24	14	19
w_2	24	24	23
w_3	24	8	9
w_4	12	8	8
w_5	12	7	9
w_6	12	5	6
w_7	12	3	3

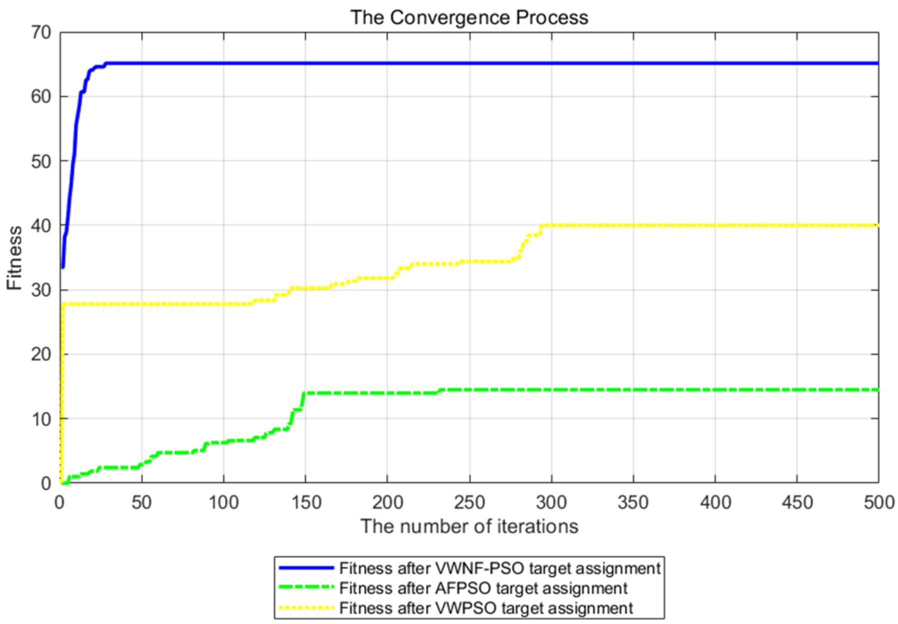

Figure 10. Algorithm fitness comparison.

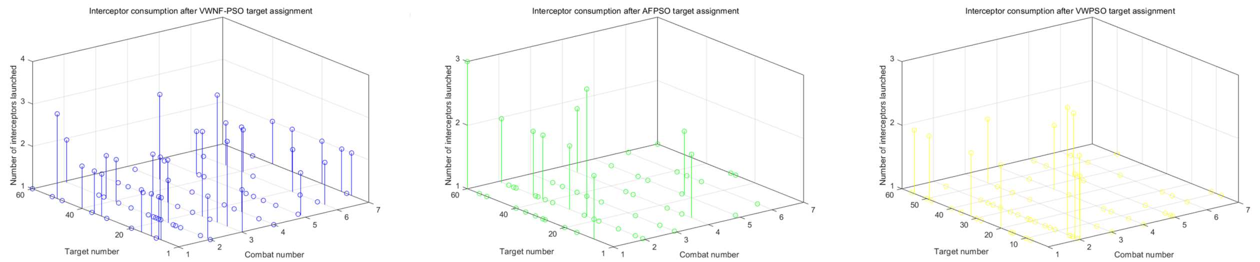

Figure 11. Number of interceptors consumed.

4.3. Data Analysis

From the perspective of algorithm performance. Under the condition of small-scale experimental, the VWPSO algorithm and the AFPSO algorithm have a small gap with the VWNF-PSO algorithm in the calculation of fitness, the output plans of VWPSO, and AFPSO can also be used as a feasible solution for target assignment in practical application. However, the abilities of the VWPSO and the AFPSO to jump out of local optimum are poor, and they can easily fall into local optimum. In addition, they need more algebra to converge. When scaled up to the limits of the combat units' interception capabilities, the disadvantages of the VWPSO algorithm and AFPSO algorithm tendency to fall into local optimal are more obvious. There is a big gap between the obtained scheme and the actual optimal solution, and the plan is no longer usable as a viable solution. VWNF-PSO algorithm has better global search ability and faster search speed, which can adapt well to the requirements of solving the model proposed in this paper.

From the perspective of target allocation results. In terms of interception probability, the interception probability of the three algorithms can all meet the requirements under small-scale scenarios, but the interception resource consumption of the AFPSO algorithm is too large. Large-scale scenarios under the condition of incoming target scale have reached the limit units of intercept capability of the combat units (60 incoming targets, combat units have 120 interceptors). VWPSO and AFPSO are often trapped in the local optimal solution, and always miss the targets, which cannot be tolerated in the practical application of target assignment. In the cooperation of combat units, the VWPSO algorithm and AFPSO algorithm tend to select type A combat units with higher performance in small-scale scenarios, resulting in the idle situation of type B weapons. In a large-scale experiment, this problem of the VWPSO algorithm and AFPSO algorithm remains very prominent. VWNF-PSO shows excellent stability under two scenarios, and its outstanding advantage is that it can fully use interceptors to solve the problem of target assignment under large-scale extreme conditions.

5. Results

This paper studies the regional joint air defense operation unit coordinated target assignment problem, builds a multi-type of ground air defense weapon system coordination target assignment model of air defense combat under the spatial crowdsourcing model, and solves the problem of target assignment plan for dealing with multi-type, multi-target, and multi-directional threat scenarios. Using the VWAF-PSO to solve the model, the following conclusions were obtained through simulation:

Based on the requirement of optimal interception quality, this method can generate a cooperative target assignment plan for a multi-type ground air defense weapon system to manage complex multi-target threats. Moreover, it provides a new idea for the design and configuration of regional air defense coordination plans in the future.

By adjusting the inertia weight and learning factor of the basic binary particle swarm optimization algorithm, there are improvements in the global search ability and the convergence speed of the algorithm.

The problem of dynamic and real-time target allocation in the cooperative operation of multi-type ground air defense weapon systems and the anti-saturation attack efficiency of regional joint air defense will be the focus of future research.

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