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Multiple Signal TDOA/FDOA Joint Estimation with Coherent Integration

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Abstract: Passive localization relies significantly on the estimation of the Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) to accurately determine the location of a target. The precision of TDOA and FDOA estimation is affected by signal parameters of time and frequency distribution. In case of multiple signals arising at different frequency bands and intercepted simultaneously by spatially separate sensors covering a wide frequency band, the traditional method is first to separate the signals from the mixed wideband signal through digital down conversion (DDC), which brings multiple narrowband signals, and then the estimation of TDOA and FDOA of each narrowband signal can be performed using cross ambiguity function (CAF). The paper introduces a novel approach for estimating TDOA and FDOA of multiple signals simultaneously, which employs a coherent integration method. First, the cross ambiguity function for each signal is realized with the narrowband signal as the same as the traditional method. Next, the phase relation of each CAF is analyzed, then the joint CAF can be obtained with phase compensation, from which multiple signal TDOA and FDOA estimations will be implemented simultaneously. Numerical simulations are performed to compare the two methods, and the results demonstrate the superiority of the proposed algorithm.

Keywords: TDOA estimation; FDOA estimation; passive localization; multiple signal; coherent integration

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1. Introduction

Over the past few years, Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) localization have garnered significant interest in the field of passive localization [1–12]. TDOA and FDOA localization systems need three or more stations to locate the emitters; the location procedure includes two steps, first, to get the TDOA and FDOA estimations between the receiver stations, and second, to solve the location equations with the TDOA and FDOA measurements. So, the emitter localization performance depends on how well the estimation of TDOA and FDOA is executed.

The precision of TDOA estimation primarily hinges on the signal bandwidth [1]. To enhance the accuracy of estimation and localization, previous studies have explored the potential of multi-carrier signals due to their wider bandwidth [13–15]. At the same time, the performance of FDOA estimation depends mainly on the duration of signals [1]. Algorithms for joint TDOA and FDOA estimation have attracted much attention in recent years. Most of them are cross-correlation-based methods, and the cross-ambiguity function is most commonly used; the TDOA and FDOA estimation need to search for the peak within a two-dimensional grid of TDOA and FDOA values. Higher-order statistics or lower-order statistics methods are also used for special cases to get better estimation performance [16,17]. When advantage knowledge of signal and noise power spectra is obtained, the Generalized

Cross Correlation methods have been analyzed by Knapp and Carter [18]. These methods mainly focused on single signal localization or single TDOA and FDOA estimation; for the case of multiple signal localization or multiple TDOA and FDOA estimation, considerable efforts have also been devoted [19–22]. Wax derived a Maximum Likelihood (ML) algorithm for the differential delay, Doppler, and phase estimation [19]. Then Ianniello applied the ML algorithm to multipath time delay estimation in the case of multipath signals intercepted at a single observation station [20] while in the presence of power spectrum knowledge for signal and noise. ML estimation requires a multidimensional grid search or complex iterative calculations. Belanger proposed expectation maximization (EM) estimation for multipath signals as an iterative calculation method [21]. Owing to the ability of super-resolution, Multiple Signal Classification (MUSIC) based high-resolution spectral analysis tools are also applied to the problem of multipath TDOA and FDOA estimation [22].

Multipath signals are emitted from the same target at the same frequency with different TDOAs and FDOAs; multi-signals with different frequencies from the same or different emitters are also used in passive radar to improve performance [13–15]. Frequency-Hopping (FH) is a special multi-frequency signal, and a coherent integration technique is employed to improve TDOA and FDOA estimation performance [23]. In the case of the reference signal and target signal occupying different frequency bands, the joint target TDOA and reference signal TDOA estimation method with coherent integration can improve the target TDOA estimation performance [24].

The problems above focus on the localization or parameter estimation of a single target. In scenarios where multiple signals from different targets with varying frequencies are present, we discuss the coherent integration TDOA and FDOA estimation method to get high-accuracy passive localization. From the expressions of the cross-correlation function in the time and frequency domain, the cross-ambiguity function of different target signals is derived. Meanwhile, the phase change of CAF is specially analyzed during the processing of digital down conversion. Then the phase difference and relation of each signal CAF are analyzed, and the joint CAF can be obtained with phase compensation. Lastly, the multiple signals merged CAFs could be obtained with coherent integration, and multiple signal TDOA and FDOA estimations will be implemented simultaneously by searching the peak of the merged CAF. A better TDOA estimation performance can be obtained with the proposed method, which is shown in the simulations.

The paper is structured as follows. In Section 2, we provide the expression of intercepted signals for multiple target signals with different TDOA and FDOA, analyze the CAFs separately, and reveal the CAF phase relations. Section 3 first shows the traditional separate method for multiple signals TDOA and FDOA estimation, then the merged CAF for multiple signals joint estimation method with phase difference compensation is derived. In Section 4, numerical simulations are offered to demonstrate performance improvement. Finally, in Sections 5 and 6, the discussion and conclusions are presented, respectively.

2. Signal Model and Cross Ambiguity Function Analysis

For convenience, the case of two target signals is analyzed as an example. Figure 1 depicts a typical scenario for two target signals intercepted by two sensors simultaneously. The figure shows that the two sensors locate at different positions; the expression of received signals mixed with noise at the two sensors can be modeled as

$$\begin{cases} x_1(t) = \gamma_1 e^{j\varphi_1}[s_a(t) + s_b(t)] + n_1(t) \\ x_2(t) = \gamma_2 e^{j\varphi_2}[s_a(t-\tau_a)e^{j2\pi f_{da}t} + s_b(t-\tau_b)e^{j2\pi f_{db}t}] + n_2(t) \end{cases} \tag{1}$$

where $s_a(t)$ and $s_b(t)$ are the two target signals, γ_1 and γ_2 are the attenuation factors at the two sensors, φ_1 and φ_2 represent the random phase brought by the sensors [25], τ_a and τ_b denote the TDOA between the two receivers of the two target signals, and f_{da} and f_{db} are the corresponding FDOA of the two signals. The additive noise terms, $n_1(t)$ and $n_2(t)$ stand for independent white Gaussian noise, and both of them are uncorrelated with $s_a(t)$ and $s_b(t)$.

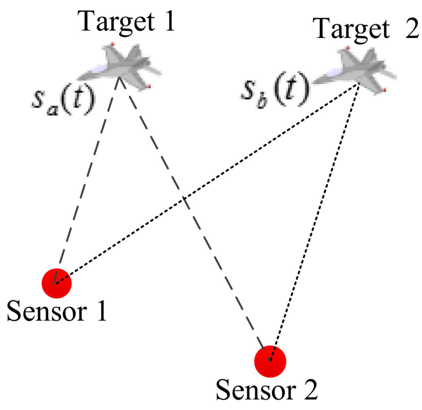

Figure 1. Interception of multiple signals with two sensors.

Assuming that the frequency centers of the two target signals are f_a and f_b, and the bandwidth of them are B_a and B_b, the intercepted signal expression in the frequency domain can then be transformed to

$$\begin{cases} X_1(f) = \gamma_1 e^{j\varphi_1}[S_a(f) + S_b(f)] + N_1(f) \\ X_2(f) = \gamma_2 e^{j\varphi_2}[S_a(f + f_{da})e^{-j2\pi f \tau_a} + S_b(f + f_{db})e^{-j2\pi f \tau_b}] + N_2(f) \end{cases} \tag{2}$$

The two target signals can be separated through suited filters since they are located at different frequency bands. Then the cross-ambiguity function for each signal can be obtained separately since the cross-ambiguity function can be regarded as the product of cross-correlation in the time dimension and cross-correlation in the frequency dimension. Hence, the cross-correlation in each dimension is analyzed first.

The cross-correlation in the time dimension for the signal $s_a(t)$ can be described as

$$\begin{aligned} R_a(\Delta\tau) &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} \int_{f_a-\frac{B_a}{2}}^{f_a+\frac{B_a}{2}} S_a(f)S_a^*(f)e^{-j2\pi f\tau_a} e^{j2\pi f\Delta\tau} df \\ &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} \int_{f_a-\frac{B_a}{2}}^{f_a+\frac{B_a}{2}} \Big|S_a(f)\Big|_2 e^{j2\pi f(\Delta\tau-\tau_a)} df \\ &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_a \mathrm{sinc}(B_a(\Delta\tau - \tau_a))e^{j2\pi f_a(\Delta\tau-\tau_a)} \end{aligned} \tag{3}$$

where E_a is the energy of the signal $s_a(t)$. The cross-correlation in the frequency dimension is

$$\begin{aligned} R_a(\Delta f) &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} \int_{f_a-\frac{B_a}{2}}^{f_a+\frac{B_a}{2}} S_a(f)S_a^*(f + f_d - \Delta f)df \\ &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} \int_{T_{ca}-\frac{T_a}{2}}^{T_{ca}+\frac{T_a}{2}} \Big|s_a(t)\Big|_2 e^{-j2\pi f_{da} t} e^{j2\pi\Delta f t} dt \\ &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_a \mathrm{sinc}((\Delta f - f_{da})T_a)e^{j2\pi(\Delta f - f_{da})T_{ca}} \end{aligned} \tag{4}$$

where T_a is the duration of the signal $s_a(t)$, and T_{ca} is the time center of the signal $s_a(t)$. So, we have

$$\begin{aligned} &\frac{R_a(\Delta\tau)R_a(\Delta f)}{\gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_a} \\ &= \frac{\gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} \int_{f_a-\frac{B_a}{2}}^{f_a+\frac{B_a}{2}} S_a(f)S_a^*(f)e^{-j2\pi f\tau_a} e^{j2\pi f\Delta\tau} df \cdot \int_{f_a-\frac{B_a}{2}}^{f_a+\frac{B_a}{2}} S_a(f)S_a^*(f + f_d - \Delta f)df}{E_a} \\ &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} \frac{\int_{f_a-\frac{B_a}{2}}^{f_a+\frac{B_a}{2}} \int_{f_a-\frac{B_a}{2}}^{f_a+\frac{B_a}{2}} S_a(f)S_a{}^*(f)e^{j2\pi f(\Delta\tau-\tau_a)} S_a(f)S_a{}^*(f + f_{da} - \Delta f)dfdf}{E_a} \\ &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_a \,\mathrm{sinc}(B_a(\Delta\tau - \tau_a))e^{j2\pi f_a(\Delta\tau-\tau_a)} \,\mathrm{sinc}((\Delta f - f_{da})T_a)e^{j2\pi(\Delta f - f_{da})T_{ca}} \end{aligned} \tag{5}$$

Then the CAF for the signal $s_a(t)$ can be expressed as

$$\begin{aligned} CAF_a(\Delta\tau, \Delta f) &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_a \mathrm{sinc}(B_a(\Delta\tau-\tau_a)) e^{j2\pi f_a(\Delta\tau-\tau_a)} \\ &\cdot \mathrm{sinc}((\Delta f - f_{da})T_a) e^{j2\pi(\Delta f - f_{da})T_{ca}} + W_a \end{aligned} \tag{6}$$

where W_a is produced by the noise. Ignoring the influence of the noise, the peak values will be at $\Delta\tau = \tau_a, \Delta f = f_{da}$, and the phases of $CAF_a(\Delta\tau, \Delta f)$ here are zero.

Correspondingly, the CAF for the signal $s_b(t)$ is

$$\begin{aligned} CAF_b(\Delta\tau, \Delta f) &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_b \mathrm{sinc}(B_b(\Delta\tau-\tau_b)) e^{j2\pi f_b(\Delta\tau-\tau_b)} \\ &\cdot \mathrm{sinc}((\Delta f - f_{db})T_b) e^{j2\pi(\Delta f - f_{db})T_{cb}} + W_b \end{aligned} \tag{7}$$

Multiple signals detected in the wideband spectrum usually undergo digital down conversion to convert to narrow baseband signals in particular applications. After conversion to narrow baseband signals, the frequency centers and time centers of the signals will be changed to zero, then the CAF for the signal $s_a(t)$ and $s_b(t)$ could be

$$\begin{aligned} CAF_a(\Delta\tau, \Delta f) &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_a \mathrm{sinc}(B(\Delta\tau-\tau_a)) e^{-j2\pi f_a\tau_a} \\ &\cdot \mathrm{sinc}((\Delta f - f_{da})T_a) e^{-j2\pi f_{da}T_{ca}} + W_a \end{aligned} \tag{8}$$

$$\begin{aligned} CAF_b(\Delta\tau, \Delta f) &= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_b \mathrm{sinc}(B(\Delta\tau-\tau_b)) e^{-j2\pi f_b\tau_b} \\ &\cdot \mathrm{sinc}((\Delta f - f_{db})T_b) e^{-j2\pi f_{db}T_{cb}} + W_b \end{aligned} \tag{9}$$

It can be found that the phases of $CAF_a(\Delta\tau, \Delta f)$ and $CAF_b(\Delta\tau, \Delta f)$ changed to $-2\pi(f_a\tau_a + f_{da}T_{ca})$ and $-2\pi(f_b\tau_b + f_{db}T_{cb})$, so the phases of them at the true estimation $\Delta\tau = \tau_a, \Delta f = f_{da}$ and $\Delta\tau = \tau_b, \Delta f = f_{db}$ will no longer be zero.

3. Multiple Signal TDOA and FDOA Joint Estimation Method

3.1. Traditional Separate Estimation Method

When multiple signals were separated through DDC from a mixed wideband signal, the CAFs of each signal can be obtained with the separated narrow baseband signal. Then, the TDOAs and FDOAs estimation can be separately completed by searching the CAF correlation peak using each narrow baseband signal.

$$\begin{cases} (TDOA_a, FDOA_a) = \underset{\Delta\tau,\Delta f}{\mathrm{argmax}} |CAF_a(\Delta\tau, \Delta f)| \\ (TDOA_b, FDOA_b) = \underset{\Delta\tau,\Delta f}{\mathrm{argmax}} |CAF_b(\Delta\tau, \Delta f)| \end{cases} \tag{10}$$

3.2. Joint Estimation with Coherent Integration Method

The cross-ambiguity function serves as a foundation for TDOA and FDOA joint estimation, and the estimation accuracy is mainly influenced by the parameters of frequency and time distribution, respectively. Stein has also pointed out that the correlation lobe width is of the order 1/B in the time dimension and 1/T in the frequency, where B and T are the signal bandwidths and duration. So, the estimation performance of TDOA will be affected by the estimation result of FDOA since there will be no correlation peak at a wrong FDOA estimation position far away from the true FDOA value. In most applications, the TDOA and FDOA joint estimation aims for TDOA localization; FDOA estimation is only used to compensate for the cross-correlation function in order to get more accurate TDOA estimation results. Ignoring the influence of FDOA, multiple signals hold wider frequency bands, so the estimation accuracy of TDOA could potentially benefit from multiple signals' coherent integration processing.

The analysis ahead has shown that random phases brought by the sensors for multiple signals are identical; the CAF phase difference is caused by the different carrier frequencies, time center, TDOA, and FDOA. At their true TDOA and FDOA position, the CAF phases are $-2\pi(f_a\tau_a + f_{da}T_{ca})$ and $-2\pi(f_b\tau_b + f_{db}T_{cb})$, respectively. The phase difference must be compensated to realize coherent integration, then the phases of multiple CAFs at their true position could be the same to get a coherent integration peak.

The true TDOAs and FDOAs of multiple signals are unknown, and the phase difference cannot be compensated directly. Estimation results need searching the peak of CAF, so the phase compensation could be completed by multiplying $e^{j2\pi(f_a\Delta\tau+\Delta f T_{ca})}$ and $e^{j2\pi(f_b\Delta\tau+\Delta f T_{cb})}$, respectively, during searching. When the searched position is $\Delta\tau = \tau_a, \Delta f = f_{da}$ and $\Delta\tau = \tau_b, \Delta f = f_{db}$, the phases of CAFs would be the same as zero, and then the coherent integration can be realized. So, the merged CAF could be displayed as

$$\begin{aligned}
&CAF(\Delta\tau_1, \Delta\tau_2, \Delta f_1, \Delta f_2) \\
&= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_a \,\mathrm{sinc}(B_a(\Delta\tau_1 - \tau_a))\,\mathrm{sinc}((\Delta f_1 - f_{da})T_a) e^{-j2\pi(f_a\tau_a + f_{da}T_{ca})} e^{j2\pi(f_a\Delta\tau_1 + \Delta f_1 T_{ca})} + W_a \\
&+\gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} E_b \,\mathrm{sinc}(B_b(\Delta\tau_2 - \tau_b))\,\mathrm{sinc}((\Delta f_2 - f_{db})T_b) e^{-j2\pi(f_b\tau_b + f_{db}T_{cb})} e^{j2\pi(f_b\Delta\tau_2 + \Delta f_2 T_{cb})} + W_b \\
&= \gamma_1\gamma_2^* e^{j(\varphi_1-\varphi_2)} [E_a \,\mathrm{sinc}(B_a(\Delta\tau_1 - \tau_a))\,\mathrm{sinc}((\Delta f_1 - f_{da})T_a) e^{j2\pi(f_a\Delta\tau_1 + \Delta f_1 T_{ca} - f_b\tau_b - f_{db}T_{cb})} \\
&+E_b \,\mathrm{sinc}(B_b(\Delta\tau_2 - \tau_b))\,\mathrm{sinc}((\Delta f_2 - f_{db})T_b) e^{j2\pi(f_b\Delta\tau_2 + \Delta f_2 T_{cb} - f_b\tau_b - f_{db}T_{cb})}] + W_a + W_b
\end{aligned} \tag{11}$$

So multiple signal TDOA and FDOA joint estimation results are

$$(TDOA_a, TDOA_b, FDOA_a, FDOA_b) = \underset{\Delta\tau_1,\Delta\tau_2,\Delta f_1,\Delta f_2}{\mathrm{argmax}} \left| CAF(\Delta\tau_1, \Delta\tau_2, \Delta f_1, \Delta f_2) \right| \tag{12}$$

From Equation (11), it can be found that when

$$f_a\Delta\tau_1 + \Delta f_1 T_{ca} - f_b\tau_b - f_{db}T_{cb} = f_b\Delta\tau_2 + \Delta f_2 T_{cb} - f_b\tau_b - f_{db}T_{cb} \tag{13}$$

the coherent integration $CAF(\Delta\tau_1, \Delta\tau_2, \Delta f_1, \Delta f_2)$ has period peaks besides at the position of true value $(\tau_a, \tau_b, f_{da}, f_{db})$. In the presence of a low signal-to-noise ratio (SNR), the accuracy of TDOA and FDOA estimation could be significantly degraded by the occurrence of periodic peaks.

It can also be found that the merged CAF for the proposed joint estimation method needs to search for the peak within a four-dimensional grid of two TDOA and two FDOA values, while that of the separated method only needs to search for the peak within a two-dimensional grid for each signal. Assuming that each grid needs N points to search, then the number of computation points for the proposed joint estimation method is N^4, while that of the separated method is $2N^2$.

4. Numerical Results

This section presents Monte Carlo simulations to evaluate the performance of the proposed joint estimation method for multiple signals TDOA and FDOA, referred to as CI. The performance of CI is compared to that of a separated estimation method, and the Cramer-Rao bounds (CRB) are also calculated for the separated estimation method using the results from [1]. Table 1 provides the simulation parameters used in the study.

The SNRs of the signal in the simulation vary from 10 dB to 20 dB. To evaluate the performance, 100 Monte Carlo experiments were performed for each SNR. The results obtained using the separated estimation method and CRB are denoted as SE and SE CRB, respectively. Figures 2 and 3 show the simulation results for TDOA and FDOA estimation.

Table 1. Parameters for Numerical Simulation.

Parameters	Value
Prior Sample Rate	1 MHz
Sample Rate after DDC	100 kHz
Target 1 Signal Frequency Center	400 kHz
Target 2 Signal Frequency Center	100 kHz
Target 1 Signal Bandwidth	50 kHz
Target 2 Signal Bandwidth	50 kHz
Sample Time	0.1 s
Target 1 Signal TDOA	2 μs
Target 2 Signal TDOA	1.5 μs
Target 1 Signal FDOA	5.5 Hz
Target 2 Signal FDOA	10.2 Hz

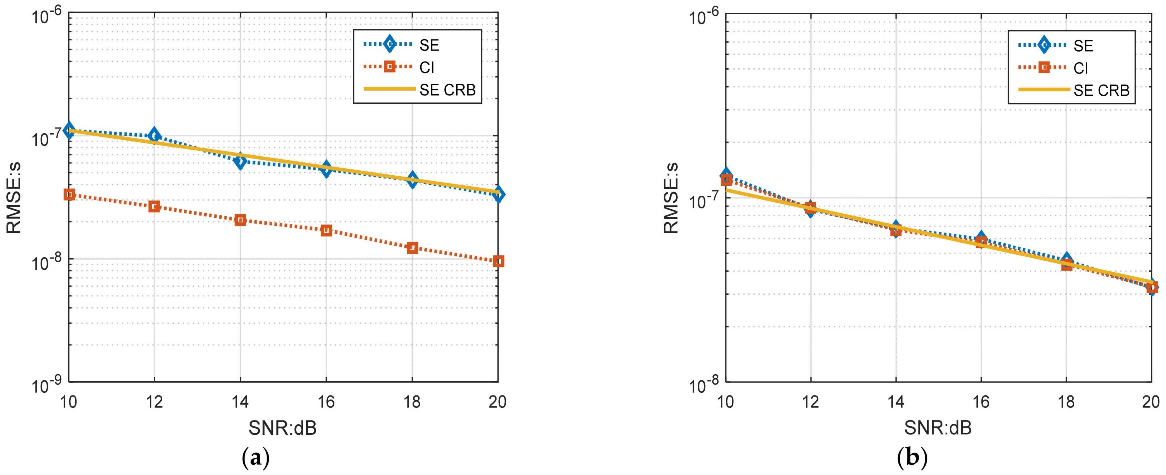

Figure 2. Comparison of TDOA estimation performance between the two methods and CRB. (**a**) Target 1 signal results; (**b**) Target 2 signal results.

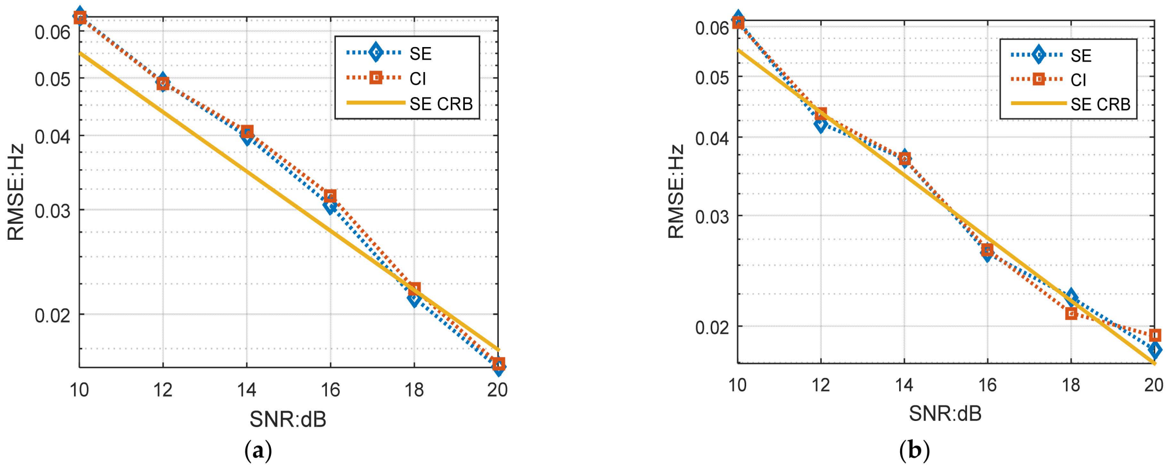

Figure 3. Comparison of FDOA estimation performance between the two methods and CRB. (**a**) Target 1 signal results; (**b**) Target 2 signal results.

The root mean square error (RMSE) is usually used to reflect the performance of estimation results, and the RMSE of TDOA estimation can be expressed as

$$RMSE = \sqrt{\sum_{i=1}^{N} (\tau_i - TDOA)^2 / N} \tag{14}$$

where τ_i is the estimation result, *TDOA* is the true value, and *N* represents the number of estimation results. The RMSE results of the two methods for TDOA estimation are compared with the corresponding Cramer-Rao bounds (CRB) in Figure 2. From the simulation results of Figure 2a, it can be found that the TDOA estimation performance of the Target 1 signal obtained evident improvement via the proposed joint estimation method, while the performance of the separated method is near to the CRB. At SNR = 20 dB, the RMSE of TDOA estimation results for CI is 9.5 ns, and that of SE is 33 ns; the performance has been improved 3.47 times, while the average improvement is 3.35 times. The TDOA estimation performance of the Target 2 signal is almost the same as that of the separated method, and both of them are near CRB. It has been found that the wider the frequency distribution is, the better the TDOA estimation performs [1]. Since the joint estimation method used both the two target signals' frequency information, the better TDOA estimation performance of the Target 1 signal is logical, but the reason that the TDOA estimation performance of the Target 2 signal got little improvement still needs to be found.

Figure 3 provides the comparison of CRB and the estimation RMSE results of the two methods for FDOA estimation. The experimental results indicated that the estimation performances of the joint estimation method and the separated method are nearly the same for both the Target 1 signal and Target 2 signal, and all the simulation results are near the CRB. The theory results in [1] have revealed that the FDOA estimation performance depends mainly on the time distribution. Target 1 signal and Target 2 signal appeared simultaneously in the simulation condition, so the estimation performance obtained no improvement for the reason that there is no improvement in time distribution via the joint estimation method. Since the FDOA estimation accuracy can be improved when time distribution expands, then, in the case that multiple signals appear at different times and hold wider time distribution than each signal, the proposed joint estimation method will get better FDOA estimation.

5. Discussion

When sensors work with a wide range in the frequency domain, it is common that multiple signals at different frequencies are intercepted simultaneously. Based on the hypothesis that the random phase brought by the sensor is the same for signals received simultaneously, this paper put forward a multiple signal joint method for TDOA and FDOA estimation. The simulation results demonstrate an obvious improvement in TDOA estimation performance for the Target 1 signal, while that of the Target 2 signal performs no better. The reason for this fact and what is the theoretical performance of the proposed joint estimation method still need to be clarified. In many applications, the response time is very important; if the time spent on computation is too long, the results are worthless. So, the computational complexity also needs optimization since the searching dimensions are according to the number of multiple signal TDOAs and FDOAs.

6. Conclusions

In striving for precise emitter localization results, the high estimation accuracy is the identical goal of various TDOA and FDOA parameter estimation methods or algorithms. Focusing on the problem of multiple signal TDOA and FDOA estimation, this paper proposed a joint estimation method with multiple signal CAFs coherent integration, in which the multiple signal TDOAs and FDOAs are estimated simultaneously. First, the separated CAF of each signal is deduced, and the phase difference and relation are specially analyzed. After that, the compensation of different phases for coherent integration is accomplished, then the merged CAF is obtained, and the parameters estimation can be realized through peak searching. Numerical simulations were conducted to compare the proposed method with the traditional separated method. The results demonstrate the better TDOA estimation accuracy of the proposed method. The results show that the TDOA estimation performance for Target 1 signal can be improved 3.35 times on average.

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