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Coordinated Multicast Precoding for Multi-Cell Massive MIMO Transmission Exploiting Statistical Channel State Information

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Abstract: This paper considers coordinated multi-cell multicast precoding for massive multiple-input-multiple-output transmission where only statistical channel state information of all user terminals (UTs) in the coordinated network is known at the base stations (BSs). We adopt the sum of the achievable ergodic multicast rate as the design objective. We first show the optimal closed-form multicast signalling directions of each BS, which simplifies the coordinated multicast precoding problem into a coordinated beam domain power allocation problem. Via invoking the minorization-maximization framework, we then propose an iterative power allocation algorithm with guaranteed convergence to a stationary point. In addition, we derive the deterministic equivalent of the design objective to further reduce the optimization complexity via invoking the large-dimensional random matrix theory. Numerical results demonstrate the performance gain of the proposed coordinated approach over the conventional uncoordinated approach, especially for cell-edge UTs.

Keywords: coordinated multi-cell multicast transmission; statistical channel state information; beam domain; Massive MIMO; cell-edge user terminal

1. Introduction

In massive multiple-input-multiple-output (MIMO) systems, large numbers of antennas are equipped at the base stations (BSs) to enable simultaneous transmission for different user terminals (UTs) in the same time and frequency transmission resources. Massive MIMO is a promising means of improving the spectral efficiency, energy efficiency and reliability of wireless systems, and has attracted extensive research interest from both academia and industry [1–7].

Due to the explosive demand for group-oriented wireless data services such as satellite communications and video streaming, physical layer wireless multicasting has received significant research attention [8–13]. Physical layer wireless multicasting has been incorporated in some 3GPP LTE standards as evolved Multimedia Broadcast Multicast Service (eMBMS) for efficient multicast transmission [14]. Compared with small scale MIMO systems, massive MIMO can shape the transmission signals efficiently and further improve the transmission quality of service. Thus, massive MIMO multicasting is promising for physical layer multicasting in future wireless networks [15,16].

For multi-cell MIMO multicast transmission, coordinated precoding between neighbouring cells is a powerful approach for inter-cell interference management and has been extensively investigated in the literature [17–22]. The performance of coordinated precoding in multi-cell MIMO transmission relies on the quality of channel state information (CSI) available at the BSs. Most of the existing works on coordinated multicast precoding for multi-cell MIMO transmission assume the availability of instantaneous CSI at the BSs. However, exchangement of instantaneous CSI across the BSs in

coordinated multi-cell MIMO transmission will lead to a significant signalling overhead. In addition, for massive MIMO transmission with large numbers of BS antennas, acquisition of instantaneous CSI at the BS is not an easy task due to some practical limitations such as hardware imperfections and feedback overhead [23–26]. Thus, it is of more practical interest to exploit statistical CSI for coordinated multi-cell massive MIMO transmission since statistical CSI varies much more slowly than instantaneous CSI.

Motivated by the above practical considerations, we investigate coordinated multicast precoding for multi-cell massive MIMO transmission exploiting statistical CSI in this paper. In our work, each BS only utilizes statistical CSI of UTs in the corresponding coordinated network to perform multicast transmission. In addition, the massive MIMO channel structures are exploited to simplify multicast transmission design. The major contributions of this work are summarized as follows:

- With the design goal to maximize the sum of the achievable ergodic multicast rate in coordinated multi-cell massive MIMO multicast precoding, we show that it is optimal for each BS to transmit the multicast signals in the beam domain.
- We propose an iterative beam domain power allocation algorithm for coordinated multicast transmission via invoking the minorization-maximization (MM) framework. Our proposed algorithm is guaranteed to converge to a stationary point.
- We employ the large-dimensional random matrix theory to derive the deterministic equivalent (DE) of the optimization objective to further reduce the computational complexity.

The rest of this paper is organized as follows. We present the system model in Section 2. Coordinated multicast precoding for multi-cell massive MIMO transmission is investigated in Section 3. Numerical results are presented in Section 4 and the paper is concluded in Section 5.

The following notations are used throughout the paper. Upper and lower case boldface letters denote matrices and column vectors, respectively. We adopt $\mathbb{C}^{M \times N}$ and $\mathbb{R}^{M \times N}$ to denote the $M \times N$ dimensional complex-valued and real-valued vector space, respectively. We adopt \mathbf{I}_M to denote the $M \times M$ dimensional identity matrix. Notations $(\cdot)^H$, $(\cdot)^*$, and $(\cdot)^T$ denote conjugate-transpose, conjugate, and transpose operations, respectively. We utilize \odot to denote the Hadamard product. The operators tr $\{\cdot\}$ and det (\cdot) denote the matrix trace and determinant operations, respectively. We adopt \mathcal{CN} ($\mathbf{0}, \mathbf{A}$) to denote the circular symmetric complex-valued Gaussian distribution with mean $\mathbf{0}$ and covariance \mathbf{A} . We adopt $[\mathbf{A}]_{i,j}$, $[\mathbf{A}]_{i,i}$, and $[\mathbf{A}]_{:,j}$ to denote the (i, j)th element, the *i*th row, and the *j*th column of the matrix \mathbf{A} , respectively. We employ $\mathbf{A} \succeq \mathbf{0}$ to denote that \mathbf{A} is positive semidefinite. We adopt $|\mathcal{K}|$ to denote the cardinality of set \mathcal{K} . We adopt diag $\{\mathbf{x}\}$ to denote the diagonal matrix with \mathbf{x} along its main diagonal.

2. System Model

We consider coordinated multicast precoding for multi-cell massive MIMO transmission, as shown in Figure 1. The number of cells in the considered coordinated multi-cell massive MIMO transmission is *U* and each cell includes several multiple antenna UTs and a BS, which is equipped with *M* antennas. We focus on the case where there is only one multicast group in each cell.

Denote by (k, u) the *k*th UT in cell *u*, equipped with $N_{k,u}$ antennas, and \mathcal{K}_u the UT set in cell *u*. Let $\mathbf{x}_u \in \mathbb{C}^{M \times 1}$ be the multicast transmitted signal for all the UTs in cell *u* by the BS in cell *u*, then the received signal of UT (k, u) can be written as

$$\mathbf{y}_{k,u} = \mathbf{H}_{k,u,u} \mathbf{x}_u + \underbrace{\sum_{\ell \neq u} \mathbf{H}_{k,u,\ell} \mathbf{x}_{\ell}}_{\text{inter-cell interference}} + \mathbf{z}_{k,u} \in \mathbb{C}^{N_{k,u} \times 1}, \tag{1}$$

where $\mathbf{H}_{k,u,\ell} \in \mathbb{C}^{N_{k,u} \times M}$ is the downlink channel matrix between the BS in the ℓ th cell and UT (k, u), and $\mathbf{z}_{k,u} \in \mathbb{C}^{N_{k,u} \times 1}$ represents the additive complex-valued white Gaussian noise distributed as

 $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_{k,u}})$. Assume that the multicast signal \mathbf{x}_{ℓ} sent by the BS in the ℓ th cell satisfies $\mathsf{E}\{\mathbf{x}_{\ell}\} = \mathbf{0}$ and $\mathsf{E}\{\mathbf{x}_{\ell}\mathbf{x}_{\ell}^{H}\} = \mathbf{Q}_{\ell} \in \mathbb{C}^{M \times M}$ where \mathbf{Q}_{ℓ} is the multicast transmit covariance matrix.



Figure 1. Coordinated multicast transmission for multi-cell massive MIMO.

In this paper, we adopt the jointly correlated Rayleigh fading MIMO channel model [27] (also referred to as Weichselberger's channel model [28]). Different from the conventional independent and identically distributed MIMO channel model, the adopted jointly correlated MIMO channel model can capture the joint correlation properties of the channels between the transmitter and the receiver. Specifically, the downlink channel matrix from the BS in the ℓ th cell to UT (k, u) is given by

$$\mathbf{H}_{k,\mu,\ell} = \mathbf{U}_{k,\mu,\ell} \mathbf{G}_{k,\mu,\ell} \mathbf{V}_{k,\mu,\ell}^{H}, \tag{2}$$

where $\mathbf{U}_{k,u,\ell} \in \mathbb{C}^{N_{k,u} \times N_{k,u}}$ and $\mathbf{V}_{k,u,\ell} \in \mathbb{C}^{M \times M}$ are deterministic unitary matrices, and $\mathbf{G}_{k,u,\ell} \in \mathbb{C}^{N_{k,u} \times M}$ is a random matrix with independently distributed zero-mean elements. Note that $\mathbf{G}_{k,u,\ell}$ is referred to as the beam domain channel matrix [29]. The statistics of the beam domain channel matrix $\mathbf{G}_{k,u,\ell}$ can be described by the following matrix

$$\mathbf{\Omega}_{k,u,\ell} = \mathbb{E}\left\{\mathbf{G}_{k,u,\ell} \odot \mathbf{G}_{k,u,\ell}^*\right\} \in \mathbb{R}^{N_{k,u} \times M},\tag{3}$$

where the elements of $\Omega_{k,u,\ell}$ represent the average power of the corresponding beam domain channel elements. Statistical CSI $\Omega_{k,u,\ell}$ varies much more slowly than instantaneous CSI $G_{k,u,\ell}$. In addition, the channel statistics have been shown to stay constant over a wide frequency interval [30,31]. Therefore, statistical CSI can be obtained via averaging over time and frequency in a wideband wireless transmission system with guaranteed accuracy and can be adopted to facilitate practical wideband transmission.

In massive MIMO systems, when the number of BS antennas *M* goes to infinity, the eigenvector matrices of the BS correlation matrices for different links tend to be equal, which only depends on the array topology adopted at the BS [23]. For example, it has been shown in [23] that for the case where the BS is equipped with the uniform linear array (ULA), the eigenvector matrices of the BS correlation matrices will tend to be a discrete Fourier transform (DFT) matrix. Then, the downlink channel matrix $\mathbf{H}_{k,u,\ell}$ can be well approximated by

$$\mathbf{H}_{k,u,\ell} \stackrel{M \to \infty}{=} \mathbf{U}_{k,u,\ell} \mathbf{G}_{k,u,\ell} \mathbf{V}^H, \tag{4}$$

where **V** is a deterministic matrix which depends on the array structure equipped at the BS. It is worth noting that the approximated channel model in (4) has been shown to be with very high accuracy in typical transmission scenarios and widely adopted in previous works [23,31,32]. In the following, we will adopt the massive MIMO channel model in (4).

3. Coordinated Multicast Transmission Design

Assume that each BS only has access to statistical CSI of all UTs in the coordinated network, and each UT has knowledge of its instantaneous CSI in the associated cell. For each UT (k, u), the aggregate interference-plus-noise $\mathbf{z}'_{k,u} = \sum_{\ell \neq u} \mathbf{H}_{k,u,\ell} \mathbf{x}_{\ell} + \mathbf{z}_{k,u}$ is treated as the worst-case Gaussian noise [33] in detection with covariance given by

$$\mathbf{K}_{k,u} = \mathbf{I}_{N_{k,u}} + \sum_{\ell \neq u} \mathbb{E} \left\{ \mathbf{H}_{k,u,\ell} \mathbf{Q}_{\ell} \mathbf{H}_{k,u,\ell}^{H} \right\} \in \mathbb{C}^{N_{k,u} \times N_{k,u}}.$$
(5)

For multicast transmission, the transmitted multicast signals have to be decodable by all UTs in the corresponding multicast UT group. Then, the achievable ergodic multicast rate in cell u can be defined as a minimum of the UT rates across the cell [34], which can be explicitly written as

$$\mathcal{R}_{u} = \min_{k \in \mathcal{K}_{u}} \left\{ \mathbb{E} \left\{ \operatorname{logdet} \left(\mathbf{K}_{k,u} + \mathbf{H}_{k,u,u} \mathbf{Q}_{u} \mathbf{H}_{k,u,u}^{H} \right) \right\} - \operatorname{logdet} \left(\mathbf{K}_{k,u} \right) \right\}$$

$$\stackrel{(a)}{=} \min_{k \in \mathcal{K}_{u}} \left\{ \mathbb{E} \left\{ \operatorname{logdet} \left(\overline{\mathbf{K}}_{k,u} + \mathbf{G}_{k,u,u} \mathbf{V}^{H} \mathbf{Q}_{u} \mathbf{V} \mathbf{G}_{k,u,u}^{H} \right) \right\} - \operatorname{logdet} \left(\overline{\mathbf{K}}_{k,u} \right) \right\},$$
(6)

where (a) follows from the massive MIMO channel model in (4), and the following definition

$$\overline{\mathbf{K}}_{k,u} \triangleq \mathbf{U}_{k,u,\ell}^{H} \mathbf{K}_{k,u} \mathbf{U}_{k,u,\ell}$$
$$= \mathbf{I}_{N_{k,u}} + \sum_{\ell \neq u} \mathbb{E} \left\{ \mathbf{G}_{k,u,\ell} \mathbf{V}^{H} \mathbf{Q}_{\ell} \mathbf{V} \mathbf{G}_{k,u,\ell}^{H} \right\} \in \mathbb{C}^{N_{k,u} \times N_{k,u}}.$$
(7)

For notational convenience, we define a matrix-valued function as follows

$$\mathbf{A}_{k,u,\ell}\left(\mathbf{X}\right) \triangleq \mathbb{E}\left\{\mathbf{G}_{k,u,\ell}\mathbf{X}\mathbf{G}_{k,u,\ell}^{H}\right\}.$$
(8)

Using the beam domain massive MIMO channel properties, it is not difficult to verify that $\mathbf{A}_{k,u,\ell}(\mathbf{X})$ defined in (8) is a diagonal matrix-valued function with the elements given by

$$\left[\mathbf{A}_{k,u,\ell}\left(\mathbf{X}\right)\right]_{i,j} = \begin{cases} \operatorname{tr} \left\{ \operatorname{diag} \left\{ \left(\left[\mathbf{\Omega}_{k,u,\ell}\right]_{i,:}\right)^T \right\} \mathbf{X} \right\}, & i = j, \\ 0, & i \neq j. \end{cases}$$
(9)

Then, $\overline{\mathbf{K}}_{k,u}$ defined in (7) can be rewritten as

$$\overline{\mathbf{K}}_{k,u} = \mathbf{I}_{N_{k,u}} + \sum_{\ell \neq u} \mathbf{A}_{k,u,\ell} \left(\mathbf{V}^H \mathbf{Q}_{\ell} \mathbf{V} \right).$$
(10)

We investigate coordinated multicast precoding design for multi-cell massive MIMO transmission in the following. Our design objective is to identify the optimal transmit covariance matrices $\{\mathbf{Q}_1, \dots, \mathbf{Q}_U\}$ of all BSs in the coordinated network that can maximize the sum of the achievable ergodic multicast rate under individual power constraints, which can be formulated as the following problem:

$$\left\{ \mathbf{Q}_{1}^{\mathrm{op}}, \cdots, \mathbf{Q}_{U}^{\mathrm{op}} \right\} = \underset{\mathbf{Q}_{1}, \cdots, \mathbf{Q}_{U}}{\operatorname{arg\,max}} \quad \sum_{u=1}^{U} \mathcal{R}_{u}$$
subject to $\operatorname{tr} (\mathbf{Q}_{u}) \leq P_{u},$
 $\mathbf{Q}_{u} \succeq \mathbf{0}, \ u = 1, \cdots, U,$

$$(11)$$

where \mathcal{R}_u is the ergodic multicast rate of cell *u* defined in (6), and P_u is the transmit power constraint of the BS in cell *u*.

Denote by $\mathbf{Q}_u = \mathbf{\Phi}_u \mathbf{\Lambda}_u \mathbf{\Phi}_u^H$ the eigenvalue decomposition of the multicast transmit covariance matrix in cell *u*, where the columns of $\mathbf{\Phi}_u$ are the eigenvectors of \mathbf{Q}_u and $\mathbf{\Lambda}_u$ is a diagonal matrix composed of the eigenvalues of \mathbf{Q}_u . In practice, $\mathbf{\Phi}_u$ represents the directions in which signals are transmitted and $\mathbf{\Lambda}_u$ represents the transmit power allocated onto each direction, respectively. We first identify the optimal multicast signalling directions in the following theorem.

Theorem 1. The eigenvectors of the optimal transmit covariance matrix $\mathbf{Q}_{u}^{\text{op}}$ for each cell *u* to problem (11) are given by the columns of the eigenvector matrices of the BS correlation matrix **V**, *i.e.*,

$$\mathbf{Q}_{u}^{\mathrm{op}} = \mathbf{V} \boldsymbol{\Lambda}_{u} \mathbf{V}^{H}, \quad \forall u.$$
 (12)

Proof. Please refer to the Appendix A. \Box

Theorem 1 shows that the optimal multicast transmit directions should align with the eigenvectors of the transmit correlation matrix at the BS for each cell. This implies that optimal coordinated multicast transmission should be performed in the beam domain to maximize the achievable ergodic sum multicast rate.

Then, we focus on the beam domain coordinated multicast transmission. In particular, based on Theorem 1, the matrix-valued coordinated multicast precoding optimization problem in (11) can be simplified to the following beam domain power allocation problem

$$\begin{array}{ll} \underset{\Lambda = \{\Lambda_{1}, \cdots, \Lambda_{U}\}}{\operatorname{subject to}} & \sum_{u=1}^{U} \mathcal{R}_{u} \left(\Lambda \right) \\ \underset{\Lambda_{u} \succeq \mathbf{0}, \ \Lambda_{u} \text{ diagonal, } u = 1, \cdots, U, \end{array}$$
(13)

where

$$\mathcal{R}_{u}\left(\boldsymbol{\Lambda}\right) \triangleq \min_{k \in \mathcal{K}_{u}} \left\{ \mathcal{R}_{k,u,1}\left(\boldsymbol{\Lambda}\right) - \mathcal{R}_{k,u,2}\left(\boldsymbol{\Lambda}\right) \right\},\tag{14}$$

$$\mathcal{R}_{k,u,1}\left(\mathbf{\Lambda}\right) \triangleq \mathbb{E}\left\{ \text{logdet}\left(\overline{\mathbf{K}}_{k,u}\left(\mathbf{\Lambda}\right) + \mathbf{G}_{k,u,u}\mathbf{\Lambda}_{u}\mathbf{G}_{k,u,u}^{H}\right) \right\},\tag{15}$$

$$\mathcal{R}_{k,u,2}\left(\mathbf{\Lambda}\right) \triangleq \operatorname{logdet}\left(\overline{\mathbf{K}}_{k,u}\left(\mathbf{\Lambda}\right)\right),$$
(16)

$$\overline{\mathbf{K}}_{k,u}\left(\mathbf{\Lambda}\right) \triangleq \mathbf{I}_{N_{k,u}} + \sum_{\ell \neq u} \mathbf{A}_{k,u,\ell}\left(\mathbf{\Lambda}_{\ell}\right).$$
(17)

Although the number of variables needs to be optimized in the beam domain power allocation problem in (13) is significantly reduced compared with the original precoding optimization problem in (11), it is still challenging to solve (13) due to the complexity of objective function. We adopt the MM algorithmic framework [35] to address this problem. The MM algorithmic framework is a sequential optimization approach, and the idea is to handle a difficult maximization problem via solving a sequence of maximization problems that are easy to handle. The key step of MM framework is to construct a surrogate lower-bound function of the objective so that the maximization problems are

easy to handle. For the considered coordinated power allocation problem in (13), we construct the surrogate lower-bound function in each iteration by replacing $\mathcal{R}_{k,u,2}(\Lambda)$ for $\forall k, u$ with their first-order Taylor expansions and then solve it, which further yields the next iteration. Specifically, the problem in (13) is handled via iteratively solving the following sequence of optimization problems

$$\left\{ \boldsymbol{\Lambda}^{(i+1)} \right\} = \arg \max_{\boldsymbol{\Lambda}} \qquad \sum_{u=1}^{U} \min_{k \in \mathcal{K}_{u}} \left\{ \mathcal{R}_{k,u,1} \left(\boldsymbol{\Lambda} \right) - \mathcal{R}_{k,u,2} \left(\boldsymbol{\Lambda}^{(i)} \right) - \sum_{\ell \neq u} \operatorname{tr} \left\{ \left(\frac{\partial}{\partial \boldsymbol{\Lambda}_{\ell}} \mathcal{R}_{k,u,2} \left(\boldsymbol{\Lambda}^{(i)} \right) \right)^{T} \left(\boldsymbol{\Lambda}_{\ell} - \boldsymbol{\Lambda}_{\ell}^{(i)} \right) \right\} \right\},$$
subject to $\operatorname{tr} \left(\boldsymbol{\Lambda}_{u} \right) \leq P_{u},$
 $\boldsymbol{\Lambda}_{u} \succeq \mathbf{0}, \ \boldsymbol{\Lambda}_{u} \text{ diagonal}, \ u = 1, \cdots, U,$

$$(18)$$

where the superscript *i* denotes the iteration index, $\Lambda^{(i)} = \{\Lambda_1^{(i)}, \dots, \Lambda_U^{(i)}\}$, and the gradient of $\mathcal{R}_{k,u,2}(\Lambda)$ with respect to Λ_ℓ for $\ell \neq u$ is a diagonal matrix, whose diagonal elements can be explicitly calculated as follows

$$\left[\frac{\partial}{\partial \mathbf{\Lambda}_{\ell}} \mathcal{R}_{k,u,2}\left(\mathbf{\Lambda}^{(i)}\right)\right]_{t,t} = \sum_{n=1}^{N_{k,u}} \frac{[\mathbf{\Omega}_{k,u,\ell}]_{n,t}}{1 + \sum_{v \neq u} \sum_{m=1}^{M} [\mathbf{\Omega}_{k,u,v}]_{n,m} \left[\mathbf{\Lambda}_{v}^{(i)}\right]_{m,m}}.$$
(19)

We can observe that each subproblem in (18) is a convex program and can be more efficiently solved compared with the optimization problem in (13). In addition, from [35], it is not difficult to verify that the sequence $\{\Lambda^{(i)}\}_{i=0}^{\infty}$ generated by the proposed MM optimization approach for coordinated beam domain multi-cell massive MIMO multicast transmission in (18) monotonically converges to a stationary point of the problem in (13).

The computational complexity of $\mathcal{R}_{k,u,1}(\Lambda)$ in (18) is still high due to the involved expectation operation. To further reduce the optimization complexity, we utilize the large dimensional random matrix theory [36] to calculate the deterministic equivalent (DE) of $\mathcal{R}_{k,u,1}(\Lambda)$ as follows

$$\overline{\mathcal{R}}_{k,u,1}\left(\mathbf{\Lambda}\right) = \operatorname{logdet}\left(\mathbf{I}_{M} + \mathbf{\Gamma}_{k,u}\mathbf{\Lambda}_{u}\right) + \operatorname{logdet}\left(\widetilde{\mathbf{\Gamma}}_{k,u} + \overline{\mathbf{K}}_{k,u}\left(\mathbf{\Lambda}\right)\right) - \operatorname{tr}\left\{\mathbf{I}_{N_{k,u}} - \left(\widetilde{\mathbf{\Phi}}_{k,u}\right)^{-1}\right\}, \quad (20)$$

where $\Gamma_{k,u}$, $\widetilde{\Gamma}_{k,u}$, and $\widetilde{\Phi}_{k,u}$ can be obtained by solving the following fixed-point equations

$$\mathbf{\Gamma}_{k,u} = \mathbf{\Pi}_{k,u} \left(\left(\overline{\mathbf{K}}_{k,u} \left(\mathbf{\Lambda} \right) \widetilde{\mathbf{\Phi}}_{k,u} \right)^{-1} \right) \in \mathbb{C}^{M \times M},$$
(21a)

$$\widetilde{\mathbf{\Gamma}}_{k,u} = \mathbf{\Xi}_{k,u} \left(\left(\mathbf{I}_M + \mathbf{\Gamma}_{k,u} \mathbf{\Lambda}_u \right)^{-1} \mathbf{\Lambda}_u \right) \in \mathbb{C}^{N_{k,u} \times N_{k,u}},\tag{21b}$$

$$\widetilde{\mathbf{\Phi}}_{k,u} = \mathbf{I}_{N_{k,u}} + \widetilde{\mathbf{\Gamma}}_{k,u} \left(\overline{\mathbf{K}}_{k,u} \left(\mathbf{\Lambda} \right) \right)^{-1} \in \mathbb{C}^{N_{k,u} \times N_{k,u}},$$
(21c)

with $\Xi_{k,u}(\mathbf{X}) \triangleq \mathbb{E}\left\{\mathbf{G}_{k,u,u}\mathbf{X}\mathbf{G}_{k,u,u}^{H}\right\} \in \mathbb{C}^{N_{k,u} \times N_{k,u}}$ and $\mathbf{\Pi}_{k,u}(\mathbf{Y}) \triangleq \mathbb{E}\left\{\mathbf{G}_{k,u,u}^{H}\mathbf{Y}\mathbf{G}_{k,u,u}\right\} \in \mathbb{C}^{M \times M}$ both being diagonal matrix-valued functions with the diagonal elements given by

$$\left[\Xi_{k,u}\left(\mathbf{X}\right)\right]_{n,n} = \operatorname{tr}\left\{\operatorname{diag}\left\{\left(\left[\mathbf{\Omega}_{k,u,u}\right]_{n,:}\right)^{T}\right\}\mathbf{X}\right\},\tag{22}$$

$$\left[\boldsymbol{\Pi}_{k,u}\left(\mathbf{Y}\right)\right]_{m,m} = \operatorname{tr}\left\{\operatorname{diag}\left\{\left[\boldsymbol{\Omega}_{k,u,u}\right]_{:,m}\right\}\mathbf{Y}\right\},\tag{23}$$

respectively. Using (21), the DE results can usually be calculated with a high accuracy in a few iterations [36], and the computational complexity can be significantly reduced compared with the

Monte-Carlo approach. By replacing $\mathcal{R}_{k,u,1}(\Lambda)$ with its DE in (21) in each iteration, the sequence of optimization problems in (18) can be rewritten as the following problem

$$\left\{ \mathbf{\Lambda}^{(i+1)} \right\} = \arg \max_{\mathbf{\Lambda}} \qquad \sum_{u=1}^{U} \min_{k \in \mathcal{K}_{u}} \left\{ \overline{\mathcal{R}}_{k,u,1} \left(\mathbf{\Lambda} \right) - \mathcal{R}_{k,u,2} \left(\mathbf{\Lambda}^{(i)} \right) - \sum_{\ell \neq u} \operatorname{tr} \left\{ \left(\frac{\partial}{\partial \mathbf{\Lambda}_{\ell}} \mathcal{R}_{k,u,2} \left(\mathbf{\Lambda}^{(i)} \right) \right)^{T} \left(\mathbf{\Lambda}_{\ell} - \mathbf{\Lambda}_{\ell}^{(i)} \right) \right\} \right\},$$
subject to $\operatorname{tr} \left(\mathbf{\Lambda}_{u} \right) \leq P_{u},$
 $\mathbf{\Lambda}_{u} \succeq \mathbf{0}, \mathbf{\Lambda}_{u} \text{ diagonal}, u = 1, \cdots, U.$

$$(24)$$

For the sequence of optimization problems in (24), the DE result $\overline{\mathcal{R}}_{k,u,1}(\Lambda)$ is concave with respect to Λ . In addition, the DE result $\overline{\mathcal{R}}_{k,u,1}(\Lambda)$ is a quite good approximation of $\mathcal{R}_{k,u,1}(\Lambda)$ for massive MIMO transmission [36]. Thus, the sequence of optimization problems in (24) can be efficiently solved. Based on the above formulation, we propose the coordinated beam domain power allocation algorithm for multi-cell massive MIMO multicast transmission in Algorithm 1.

Algorithm 1 Beam Domain Coordinated Multicast Power Allocation Algorithm

Require: An initialization power allocation $\Lambda^{(0)}$, the beam domain channel statistics $\Omega_{k,u,\ell}$, the preset

threshold ϵ

Ensure: Beam domain power allocation pattern Λ

- 1: Initialize $\overline{\mathcal{R}}^{(-1)} = 0$, and iteration index i = 0
- 2: Calculate $\overline{\mathcal{R}}^{(i)} \triangleq \sum_{u=1}^{U} \min_{k \in \mathcal{K}_{u}} \left\{ \overline{\mathcal{R}}_{k,u,1} \left(\mathbf{\Lambda}^{(i)} \right) \mathcal{R}_{k,u,2} \left(\mathbf{\Lambda}^{(i)} \right) \right\}$ 3: while $\overline{\mathcal{R}}^{(i)} \overline{\mathcal{R}}^{(i-1)} \ge \epsilon$ do
- Update $i \leftarrow i + 1$ 4:
- Calculate $\Lambda^{(i)}$ via solving (24) 5:
- Calculate $\overline{\mathcal{R}}^{(i)}$ 6:
- 7: end while
- 8: Return $\Lambda = \Lambda^{(i)}$

4. Simulation Results

Numerical results are presented to evaluate the performance of our proposed coordinated multicast precoding for multi-cell massive MIMO transmission. We adopt the WINNER II channel model in the suburban macro-cell propagation environment in the simulation [37]. We adopt the 120° tri-sector cellular model. We focus on a coordinated network which is composed U = 3 neighbouring sectors and assume that the user terminals are uniformly distributed in each sector. The number of multicast UTs in each sector is $|\mathcal{K}_u| = 4$. The BSs and UTs are all equipped with the ULAs with half wavelength antenna spacing. The numbers of antennas at the BSs and UTs are set to be M = 128 and $N_{k,u} = 4 (\forall k, u)$, respectively. The power budgets in all cells are set to be equal as $P_u = P (\forall u)$, and the signal-to-noise ratio (SNR) is defined as P. The major simulation setup parameters are listed in Table 1.

Parameter	Value
Channel model	WINNER II
Scenario	Suburban macro-cell
Array topology	ULA with half wavelength antenna spacing
Number of Sectors	U = 3
Number of BS antennas	M = 128
Number of UTs in each sector	$ \mathcal{K}_u = 4 \; (orall u)$
Number of UT antennas	$N_{k,u} = 4 \; (\forall k, u)$

Table 1. Simulation Setup Parameters.

We first evaluate the convergence performance of the proposed iterative coordinated power allocation procedure in Algorithm 1. We can observe from Figure 2 that the proposed algorithm exhibits very fast convergence performance under different values of SNRs.



Figure 2. Convergence performance of Algorithm 1 for different values of SNRs.

We then compare the performance of the proposed coordinated multi-cell multicast precoding approach with that of the conventional uncoordinated approach, where the BSs do not coordinate and the inter-cell interference is not taken into account in precoding optimization. We consider two specific UT distribution cases. In Case A, all UTs are randomly distributed within the associated cell, as shown in Figure 3, and in Case B, all UTs are randomly distributed within a circle with a radius of 0.2 times the cell radius from the corresponding BS, as shown in Figure 4. We can observe from Figure 5 that the proposed coordinated approach outperforms the conventional uncoordinated approach, especially in the high SNR regime where the inter-cell interference dominates. In addition, the performance gain of the proposed coordinated approach over the conventional uncoordinated approach in Case A is larger than that in Case B, which shows that our proposed coordinated multicast approach is especially beneficial for cell-edge UTs. The accuracy of the DE results compared with the Monte-Carlo results in a wide range of SNRs is also verified in Figure 5.



Figure 3. Illustrative schematic diagram of the UT distributions considered in Case A where UTs are randomly distributed within each cell (unit: meter).



Figure 4. Illustrative schematic diagram of the UT. distributions considered in Case B where UTs are randomly distributed within a circle with a distance of 0.2 times the cell radius from the corresponding BS (unit: meter).



Figure 5. Achievable ergodic sum multicast rate performance comparison between the proposed coordinated approach and the conventional uncoordinated approach for Cases A and B. Both the DE and Monte-Carlo results are depicted.

We evaluate the performance of the proposed coordinated approach for different numbers of BS antennas in Figure 6. We can observe from Figure 6 that the coordinated multicast sum rate performance increases as the number of BS antennas increases. In addition, the DE results exhibit similar performance as the Monte-Carlo results with typical numbers of BS antennas in massive MIMO.

Finally, we compare the performance of our proposed statistical CSI approach with that of the full CSI approach where the BSs can know the instantaneous CSI of all the UTs in the coordinated network. We can observe from Figure 7 that there exits a performance gap between our proposed approach and the full CSI approach. However, with a 3/7 pilot overhead taken into account [1], our proposed approach outperforms the full CSI approach in terms of the net sum rate. For the larger pilot overhead cases such as the high mobility, high frequency cases, our proposed approach will exhibit larger performance gains.



Figure 6. Achievable ergodic sum multicast rate performance of the proposed coordinated approach with different numbers of BS antennas. Both the DE and Monte-Carlo results are depicted.



Figure 7. Comparison of the achievable ergodic sum multicast rate performance of the proposed coordinated approach with statistical CSI, the full CSI approach, and the full CSI approach with a 3/7 pilot overhead taken into account.

5. Conclusions

In this paper, we have investigated coordinated multicast precoding for multi-cell massive MIMO transmission where only the statistical CSI of all UTs in the coordinated network is known at the BSs. Our design goal was to maximize the sum of the achievable ergodic multicast rate. Based on a beam domain massive MIMO channel model, we first showed the closed-form optimal transmit directions of each BS, which simplified the coordinated precoding optimization problem into a beam domain power allocation problem. We then proposed an iterative power allocation algorithm with guaranteed convergence to a stationary point based on the MM framework and the DE. Numerical results showed that our proposed coordinated approach exhibits performance gain over the conventional uncoordinated approach, especially for cell-edge UTs.

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Appendix A. Proof of Theorem 1

Firstly, we can observe from (9) and (10) that the off-diagonal elements of $\mathbf{V}^H \mathbf{Q}_u \mathbf{V}$ do not affect the values of the elements of $\overline{\mathbf{K}}_{k,u}$ for all *u* and *k*. In addition, using a proof technique similar to [38], we can show that in order to maximize R_u defined in (6), $\mathbf{V}^H \mathbf{Q}_u \mathbf{V}$ should be a diagonal matrix for all *u*. Moreover, the multicast transmit power in each cell, namely tr { \mathbf{Q}_u } for all *u*, is only related to the diagonal elements of $\mathbf{V}^H \mathbf{Q}_u \mathbf{V}$. Thus, we can conclude that $\mathbf{V}^H \mathbf{Q}_u \mathbf{V}$ for $\forall u$ should be diagonal to maximize the objective in (6). This concludes the proof.

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