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Effect of the Cosmological Constant on Light Deflection: Time Transfer Function Approach

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Academic Editor: Lorenzo Iorio
Received: 9 February 2016; Accepted: 8 March 2016; Published: 14 March 2015

Abstract: We revisit the role of the cosmological constant Λ in the deflection of light by means of the Schwarzschild–de Sitter/Kottler metric. In order to obtain the total deflection angle α, the time transfer function approach is adopted, instead of the commonly used approach of solving the geodesic equation of photon. We show that the cosmological constant does appear in expression of the deflection angle, and it diminishes light bending due to the mass of the central body M. However, in contrast to previous results, for instance, that by Rindler and Ishak (Phys. Rev. D. 2007), the leading order effect due to the cosmological constant does not couple with the mass of the central body M.

Keywords: light deflection; cosmological constant; time transfer function; relativity

PACS: 95.30.Sf; 98.62.Sb; 98.80.Es

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1. Introduction

The cosmological constant problem is the old one concerning closely the general theory of relativity (See reviews by, e.g., [1,2]). After establishing the general theory of relativity by Einstein in 1915–1916, he introduced the cosmological constant Λ to describe the static universe since the original Einstein equation cannot represent the picture of static universe. Though the discovery of the cosmic expansion by Hubble made a denial of Einstein's first purpose, the cosmological constant is realized again because of the find of accelerating expansion of the Universe [3–5], and it is popularly considered that the cosmological constant Λ or dark energy generally has the highest potential for explaining the observed accelerating expansion of the Universe. However, its details are still far from clear; therefore, this hypothesis must be verified through not only cosmological observations but also other astronomical/astrophysical measurements.

Among such efforts, the most straightforward approach is to investigate the role of Λ in the classical tests of general relativity, such as the perihelion advance of planetary orbits and the bending of light rays. Thus far, it was found that the cosmological constant Λ contributes to the perihelion shift in principle even though this contribution is presently difficult to detect because of its very small effect (See [6–8] and the references therein, and corresponding topic to perihelion advance [9–12]).

While in the case of bending of light under the Schwarzschild–de Sitter/Kottler spacetime (see Equation (12)), contrary to the expectation, the second-order geodesic equation of a photon does not contain Λ,

$$\frac{d^2u}{d\phi^2} = -u + \frac{3}{2}r_g u^2, \quad r_g = \frac{2GM}{c^2}, \quad u = \frac{1}{r} \tag{1}$$

then, as a consequence, it is considered that the deflection angle in the Schwarzschild–de Sitter or Kottler metric coincides with that of Schwarzschild case. However, recently, Rindler and Ishak [13] reported that Λ does affect the bending of light by means of the Schwarzschild–de Sitter or Kottler metric and the invariant formula for the cosine. Subsequently, many authors have argued its appearance in many different ways and the generality of these arguments advocated the appearance of Λ in the deflection angle α. Nevertheless, presently, it seems that a conclusion has not yet been reached, for instance, on whether the leading order effect due to Λ is coupled with the mass of the central body M or not. See [14] for a review and also [15–25]. In addition, for the cosmological constant and cosmological lensing equation, see, e.g., [17,18,26,27].

As we assess the circumstances, the origin of confusion, e.g., the appearance/disappearance of Λ or the coupling/uncoupling with the mass of the central body M, is essentially attributable to the use of the standard geodesic equation of a photon to obtain light deflection due to Λ, because Λ does not appear. Therefore, it is worthy to revisit this problem using another theoretical approach.

In this paper, we will revisit the role of the cosmological constant Λ in terms of the time transfer function recently proposed in [28,29], which is originally related to Synge's world function $\Omega(x_A, x_B)$ and which enables us to circumvent the integration of the null geodesic equation. In Section 2, we will briefly summarize the time transfer function method. In Section 3, the effect of Λ on light deflection will be re-investigated. Section 4 is devoted to a short summary of this paper.

2. Outline of the Time Transfer Function

Before calculating the light deflection due to the cosmological constant Λ, let us briefly summarize the time transfer function method presented in [28,29].

Synge's world function is defined by [30]

$$\Omega(x_A, x_B) \equiv \frac{1}{2}(\lambda_B - \lambda_A) \int_{\lambda_A}^{\lambda_B} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda \tag{2}$$

where $g_{\mu\nu}$ is a metric tensor of spacetime; $x_A = (x_A^0 = ct_A, x_A^i = \vec{x}_A)$ and $x_B = (x_B^0 = ct_B, x_B^i = \vec{x}_B)$ are the coordinates of the two end-points A and B, respectively, on the geodesic world-line; and λ is the affine parameter. Then, the world function $\Omega(x_A, x_B)$ is defined as the half length of the world-line between A and B.

It is generally difficult to acquire the form of the world function concretely. Nonetheless, in the case of the Minkowskian flat spacetime, the world function is easily obtained using the parameter equation $x(\lambda) = (x_B - x_A)\lambda + x_A$ and by setting $\lambda_A = 0$ and $\lambda_B = 1$ [28,30],

$$\Omega^{(0)}(x_A, x_B) = \frac{1}{2}\eta_{\mu\nu}(x_B^\mu - x_A^\mu)(x_B^\nu - x_A^\nu) \tag{3}$$

where x^μ ($\mu = 0, 1, 2, 3$) are the Minkowskian coordinates with respect to the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$.

For the null geodesic, the world function $\Omega(x_A, x_B)$ satisfies the condition

$$\Omega(x_A, x_B) = 0 \tag{4}$$

because $ds^2 = 0$. Hence, from Equations (3) and (4), the travel time between A and B, namely $t_B - t_A$, in the Minkowskian flat spacetime becomes

$$c^2(t_B - t_A)^2 = \delta_{ij}(x_B^i - x_A^i)(x_B^j - x_A^j) = R_{AB}^2 \tag{5}$$

where δ_{ij} is Kronecker's delta, and c is the speed of light in vacuum. The time transfer function starts from Equation (5), and the weak-field approximation is developed recursively with respect to the gravitational constant G.

If the metric has the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1 \tag{6}$$

where $h_{\mu\nu}$ is a perturbation to $\eta_{\mu\nu}$, the time transfer functions that yield the travel time of the light ray are formally expressed as follows:

$$t_B - t_A = \mathcal{T}_e(t_A, \vec{x}_A, \vec{x}_B) = \frac{1}{c}\left[R_{AB} + \Delta_e(t_A, \vec{x}_A, \vec{x}_B)\right] \tag{7}$$

$$= \mathcal{T}_r(\vec{x}_A, t_B, \vec{x}_B) = \frac{1}{c}\left[R_{AB} + \Delta_r(\vec{x}_A, t_B, \vec{x}_B)\right] \tag{8}$$

where $\mathcal{T}_e(t_A, \vec{x}_A, \vec{x}_B)$ is the emission time transfer function for the spatial coordinates $\vec{x}_A, \vec{x}_B$ and signal emission time t_A; $\mathcal{T}_r(\vec{x}_A, t_B, \vec{x}_B)$ is the reception time transfer function for the spatial coordinates $\vec{x}_A, \vec{x}_B$ and signal reception time t_B; $R_{AB} = |\vec{x}_B - \vec{x}_A|$; and Δ_e and Δ_r are called the emission time delay function and reception time delay function, respectively. Δ_e and Δ_r characterize the gravitational time delay. R_{AB} in Equations (7) and (8) comes from Equation (5). Henceforth, A corresponds to the emission and B corresponds to the reception.

In general, the time transfer function depends on either the emission time t_A or reception time t_B, and this dependence feature is applied to obtain the gravitational time delay in the McVittie spacetime [31]. However, if the spacetime is static, the first order formulae reduce to

$$\Delta^{(1)}(\vec{x}_A, \vec{x}_B) = -\frac{R_{AB}}{2}\int_0^1 \left[g_{(1)}^{00} - 2N_{AB}^i g_{(1)}^{0i} + N_{AB}^i N_{AB}^j g_{(1)}^{ij}\right] d\mu \tag{9}$$

where $\vec{N}_{AB} = N_{AB}^i = (x_B^i - x_A^i)/R_{AB}$. The above equation is integrated along the parameter equation $\vec{x}(\mu) = \vec{x}_A + \mu(\vec{x}_B - \vec{x}_A)$ on the Minkowskian spacetime. From Equation (9), the time delay is calculated with the remaining form of the metric $g_{\mu\nu}$, though the weak-field approximation is presumed.

Once the time transfer function $\mathcal{T}$ is determined, the direction of the light ray can be obtained by

$$(k_0)_A = -1, \quad (k_i)_A = -c\frac{\partial \mathcal{T}}{\partial x_A^i} \tag{10}$$

$$(k_0)_B = -1, \quad (k_i)_B = c\frac{\partial \mathcal{T}}{\partial x_B^i} \tag{11}$$

Equations (10) and (11) enable us to calculate light deflection directly from the time transfer function $\mathcal{T}$.

We note that Equations (9)–(11) have an opposite sign with respect to the corresponding equations given in [28,29], as we now adopt the signature of Minkowski metric as $(-,+,+,+)$ and because of which, the time transfer function should essentially be a positive value, $\mathcal{T} > 0$.

3. Effect of the Cosmological Constant on Light Deflection

Now, let us revisit the contribution of Λ to the light deflection with consideration of the time transfer function $\mathcal{T}$. To this end, we adopt the Schwarzschild–de Sitter or Kottler metric [32];

$$\begin{aligned} ds^2 &= -\left(1 - \frac{r_g}{r} - \frac{\Lambda}{3}r^2\right)c^2dt^2 + \left(1 - \frac{r_g}{r} - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 + r^2 d\Omega^2 \\ &= -\left(1 - \frac{r_g}{r} - \frac{\Lambda}{3}r^2\right)c^2dt^2 \\ &\quad + \left(1 + \frac{r_g}{r} + \frac{\Lambda}{3}r^2 + \mathcal{O}(r_g^2, \Lambda^2)\right) dr^2 + r^2 d\Omega^2 \end{aligned} \tag{12}$$

where $r_g = 2GM/c^2$ is the Schwarzschild radius, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and the dr^2 component is linearized from the first line to the second.

Here, we consider the validity and limitation of weak-field approximation supposed in Equation (12), The bending of light due to the point mass M is characterized by r_g/r; on the other hand, the bending due to the cosmological constant Λ is derived from $\Lambda r^2/3$ term. Therefore, it may be suitable to estimate the validity of approximation by

$$\frac{\Lambda}{3}r^2 < \frac{r_g}{b}, \quad \frac{r_g}{b} \ll 1 \tag{13}$$

where b is the impact parameter. Then, r should range $b < r < d$ and d is estimated from the relation $r_g/b \sim \Lambda d^2/3$. As an example of a deflector or lens object, let us choose the Sun ($M \approx M_\odot = 2.0 \times 10^{30}$ [kg], $b \approx R_\odot = 7.0 \times 10^8$ [m]) and the galaxy ($M \approx 10^{12} M_\odot$, $b \approx R_{\rm galaxy} \approx 10^5$ [ly]); it is found that $d \sim 10^{23}$ [m] ~ 10 [Mpc] in both cases (we assumed $\Lambda \approx 10^{-52}$ [m^{-2}]). This value is comparable with the distance from our galaxy to the Virgo Cluster but one or two orders of magnitude smaller than the distance from our galaxy to quasars, the typical range of which is from 100 [Mpc] to 1000 [Mpc].

It is beneficial to transform the spherical coordinates into rectangular ones since it is easy to set up the rectilinear line as the first approximation of the light path (straight line in flat spacetime). However, it is difficult to transform the standard Schwarzschild–de Sitter/Kottler metric into the isotropic form; hence, employing the approach used in [33], we recast Equation (12) in rectangular form. By the coordinate transformation,

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta \tag{14}$$

Equation (12) is rewritten as

$$\begin{aligned} ds^2 &= -\left(1 - \frac{r_g}{r} - \frac{\Lambda}{3}r^2\right) c^2 dt^2 \\ &\quad + \left[\delta_{ij} + \left(\frac{r_g}{r} + \frac{\Lambda}{3}r^2\right)\frac{x^i x^j}{r^2} + \mathcal{O}(r_g^2, \Lambda^2)\right] dx^i dx^j \end{aligned} \tag{15}$$

in which indices i, j run from 1 to 3 (spatial coordinates). We presume that the light travels in x-y plane; that is, $\vec{x}_A = (x_A, y_A), \vec{x}_B = (x_B, y_B)$, and from Equations (9) and (15), the time transfer function $\mathcal{T}(\vec{x}_A, \vec{x}_B)$ can be obtained as

$$\mathcal{T} = \frac{1}{c}(R_{AB} + \Delta\mathcal{T}) \tag{16}$$

$$\begin{aligned} \Delta\mathcal{T} &= \frac{r_g}{2}\ln\frac{R_B + \vec{x}_B\cdot\vec{N}_{AB}}{R_A + \vec{x}_A\cdot\vec{N}_{AB}} \\ &\quad + \frac{1}{2}\left[(N^x_{AB})^2(x_B - x_A)^2 + 2N^x_{AB}N^y_{AB}(x_B - x_A)(y_B - y_A) + (N^y_{AB})^2(y_B - y_A)^2\right] \\ &\quad \times \left\{ r_g \left[\frac{1}{R^2_{AB}}\ln\frac{R_B + \vec{x}_B\cdot\vec{N}_{AB}}{R_A + \vec{x}_A\cdot\vec{N}_{AB}} \right.\right. \\ &\quad \left.\left. - \frac{(\vec{x}_A\cdot\vec{N}_{AB})[2\vec{x}_A\cdot\vec{x}_B - R_A(R_A + R_B)] - R_{AB}R_A^2}{R_B\{[\vec{x}_A\cdot(\vec{x}_B - \vec{x}_A)]^2 - R^2_{AB}R^2_A\}}\right] + \frac{\Lambda}{9}\right\} \\ &\quad + \left[(N^x_{AB})^2 x_A(x_B - x_A) + 2N^x_{AB}N^y_{AB}(x_B y_A + x_A y_B - 2x_A y_A) + (N^y_{AB})^2 y_A(y_B - y_A)\right] \\ &\quad \times R_{AB}\left[r_g\frac{\vec{x}_A\cdot(\vec{x}_B - \vec{x}_A)R_A(R_B - R_A)}{R_B\left\{[\vec{x}_A\cdot(\vec{x}_B - \vec{x}_A)]^2 - R^2_{AB}R^2_A\right\}} + \frac{\Lambda}{6}\right] \\ &\quad + \left[(N^x_{AB})^2 x_A^2 + 2N^x_{AB}N^y_{AB}x_A y_A + (N^y_{AB})^2 y_A^2\right] \\ &\quad \times \frac{R_{AB}}{2}\left[r_g\frac{(R_B - R_A)[\vec{x}_A\cdot(\vec{x}_B - \vec{x}_A)] - R^2_{AB}R_A}{R_A R_B\left\{[\vec{x}_A\cdot(\vec{x}_B - \vec{x}_A)]^2 - R^2_{AB}R^2_A\right\}} + \frac{\Lambda}{3}\right] \end{aligned} \tag{17}$$

in which $R_A = |\vec{x}_A|$, $R_B = |\vec{x}_B|$, $\vec{N}_{AB} = (N^x_{AB}, N^y_{AB})$. The slightly complicated expression in Equation (17) originates from $g^{ij}_{(1)}$ in Equation (15). We are interested in how Λ modulates the total deflection angle in the Schwarzschild case, $\alpha_{\rm GR} = 4GM/(c^2 b)$, where b is the impact parameter. Then, in order to extract the influence of Λ on the bending angle of light rays, let us re-define the coordinate system in such a way that the emission point A and the reception point B have the same value of the y coordinate, namely, $y_A = y_B = b$. Further, let us assume that the source and the observer are at rest with respect to the lens (deflector), the light is emitted at x_A and received at x_B and that $x_A < x_B$ holds, then $N^x_{AB} = 1$, $|\vec{x}_B - \vec{x}_A| = x_B - x_A > 0$, and so on. Then, Equation (17) reduces to a simple form,

$$\mathcal{T} = \frac{1}{c}\left(R_{AB} + \Delta\mathcal{T}_{\rm GR} + \Delta\mathcal{T}_\Lambda\right) \tag{18}$$

$$\Delta\mathcal{T}_{\rm GR} = \frac{GM}{c^2}\left[2\ln\frac{x_B + \sqrt{x_B^2 + b^2}}{x_A + \sqrt{x_A^2 + b^2}} - \left(\frac{x_B}{\sqrt{x_B^2 + b^2}} - \frac{x_A}{\sqrt{x_A^2 + b^2}}\right)\right] \tag{19}$$

$$\Delta\mathcal{T}_\Lambda = \frac{\Lambda}{18}\left[2(x_B^3 - x_A^3) + 3b^2(x_B - x_A)\right] \tag{20}$$

From Equations (18)–(20), the direction of light at the emission point A and reception point B are computed using Equations (10) and (11).

Since we are now choosing the emission point A and reception point B as being located upon the line $y = b$ and $x_A < x_B$, then $R_{AB} = |\vec{x}_B - \vec{x}_A| = x_B - x_A$, $\vec{N}_{AB} = ((x_B - x_A)/R_{AB}, 0) = (1, 0)$; the photon may travel along this straight-line with the minimum value of the coordinate $r = b$ (the impact parameter) if the light ray were un-deflected in the absence of central mass M and cosmological constant Λ. Thus, let us define the angle θ_A as that between $\vec{N}_{AB}$ and $\vec{k}_A$ and angle θ_B as that between $\vec{N}_{AB}$ and $\vec{k}_B$. Further, we suppose that θ_A and θ_B have a small value, $\theta_A \ll 1$, $\theta_B \ll 1$, then the direction vectors $\vec{k}_A$ and $\vec{k}_B$ can be expressed by the following form

$$\vec{k}_A = \begin{pmatrix}\cos\theta_A \\ \sin\theta_A\end{pmatrix} \simeq \begin{pmatrix}1 \\ \theta_A\end{pmatrix} = \vec{N}_{AB} + \delta\vec{k}_A = \begin{pmatrix}1 \\ 0\end{pmatrix} + \begin{pmatrix}\delta k_{xA} \\ \delta k_{yA}\end{pmatrix} \tag{21}$$

$$\vec{k}_B = \begin{pmatrix}\cos\theta_B \\ \sin\theta_B\end{pmatrix} \simeq \begin{pmatrix}1 \\ \theta_B\end{pmatrix} = \vec{N}_{AB} + \delta\vec{k}_B = \begin{pmatrix}1 \\ 0\end{pmatrix} + \begin{pmatrix}\delta k_{xB} \\ \delta k_{yB}\end{pmatrix} \tag{22}$$

Hence, we may obtain θ_A and θ_B from the y components of $\delta\vec{k}_A$ and $\delta\vec{k}_B$, namely, δk_{yA} and δk_{yB}, respectively.

Let us take the deflection angle α in such a way that $\alpha > 0$. As a consequence, the deflection angle α is given by

$$\alpha \equiv \theta_A - \theta_B = \alpha_{\rm GR} + \alpha_\Lambda + \mathcal{O}(r_g^2, \Lambda^2) \tag{23}$$

$$\alpha_{\rm GR} = \frac{GM}{c^2}b\left(\frac{2}{x_A\sqrt{x_A^2 + b^2} + x_A^2 + b^2} - \frac{2}{x_B\sqrt{x_B^2 + b^2} + x_B^2 + b^2} + \frac{x_A}{\sqrt{x_A^2 + b^2}^3} - \frac{x_B}{\sqrt{x_B^2 + b^2}^3}\right) \tag{24}$$

$$\alpha_\Lambda = -\frac{2\Lambda}{3}b(x_B - x_A) \tag{25}$$

Again, we note $x_B > x_A$ in our case. The Equation (25) is similar and comparable with previous results, that is, the third term of Equation (13) in [20], and the fourth term of Equation (25) in [22].

The reason why $\mathcal{O}(m\Lambda)$ term disappear in our result comes from the fact that present calculation is first (linear) order with respect to $\epsilon \sim r_g \sim \Lambda$, see Equations (9) and (15). If we extend to second order $\mathcal{O}(\epsilon^2)$, $\mathcal{O}(m\Lambda)$ terms appear. See, e.g., Equation (41) in [29].

In the case of Schwarzschild spacetime, the total deflection angle $\alpha_{\rm GR}$ is obtained for the limit $r \to \infty$; however, in the case of Schwarzschild–de Sitter/Kottler spacetime, we cannot impose this limit since the term $(1 - r_g/r - \Lambda r^2/3)^{-1} dr^2$ in Equation (12) diverges at $r = \sqrt{3/\Lambda}$ and the coordinate value r does not range $r > \sqrt{3/\Lambda}$ (here we assume $r_g/r \ll 1$). Then we shall define the total deflection angle due to Λ, α_Λ, in such a way that $x_A = -\sqrt{3/\Lambda}$ and $x_B = \sqrt{3/\Lambda}$. Hence, inserting these values into Equation (25), we have,

$$\alpha_\Lambda = -\frac{4\sqrt{3\Lambda}}{3} b \tag{26}$$

We note that the transformation from coordinate distance into angular distance is discussed, e.g., in [34].

It is worthwhile to show that Equation (24) can result in $\alpha_{\rm GR} = 4GM/(c^2 b)$ when $\Lambda = 0$. Equation (24) is rewritten as

$$\alpha_{\rm GR} = \frac{GM}{c^2 b}\left[2(\cos\phi_B - \cos\phi_A) + \sin^2\phi_B \cos\phi_B - \sin^2\phi_A \cos\phi_A\right] \tag{27}$$

where we introduced

$$\sin\phi_A = \frac{b}{\sqrt{x_A^2 + b^2}}, \quad \cos\phi_A = \frac{x_A}{\sqrt{x_A^2 + b^2}} \tag{28}$$

$$\sin\phi_B = \frac{b}{\sqrt{x_B^2 + b^2}}, \quad \cos\phi_B = \frac{x_B}{\sqrt{x_B^2 + b^2}} \tag{29}$$

For $\phi_A \to \pi$ (the emission point A is located at $-\infty$) and $\phi_B \to 0$ (the reception point B is located at $+\infty$), Equation (29) gives

$$\alpha_{\rm GR} = \frac{4GM}{c^2 b} \tag{30}$$

thus replicating the light deflection in the Schwarzschild case.

It should be mentioned that the time transfer function Equation (9), which is used to determine the defection, is justified as long as the zeroth-order straight line that joins $\vec{x}_A$ and $\vec{x}_B$ does not intersect an event horizon such as the Schwarzschild horizon. This implies that $|\phi_B - \phi_A| < \pi$ is a necessary condition to apply the method. However, this condition may be violated if $\vec{x}_A$ and/or $\vec{x}_B$ are sufficiently far from the mass center. To avoid this difficulty, it may be a much more satisfactory procedure to introduce the periapsis $\vec{x}_P$ of the light ray, calculate the defection angle between $\vec{x}_A$ and $\vec{x}_P$ as well as that between $\vec{x}_P$ and $\vec{x}_B$, and finally add these two contributions.

4. Summary

We revisited the effect of the cosmological constant Λ on light deflection by means of the Schwarzschild–de Sitter or Kottler metric. To obtain the deflection angle α, we adopted the time transfer function approach, instead of solving the geodesic equation of photon. We showed that the cosmological constant appears in the deflection angle α, and it diminishes the light bending due to the mass of the central body M.

We list in Table 1 the expressions of bending angle due to the cosmological constant previously obtained [14–25], and estimate the numerical value using $c = 3.0 \times 10^8$ [m/s], $G = 6.674 \times 10^{-11}$ [$\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$], Mass of galaxy $M \approx 10^{12} M_\odot = 2.0 \times 10^{42}$ [kg], $\Lambda \approx 10^{-52}$ [m^{-2}],

$b, R, r_0, B \sim 10^5$ [ly] $\sim 10^{21}$ [m] (typical radius of galaxy), $r_s, r_o, d_{\rm OL}, d_{\rm LS}, r_S, r_{obs}, x_B, -x_A \sim 10$ [Mpc] $\sim 10^{23}$ [m]. R_1 in [24] is calculated $1/R_1 = 2GM/c^2B^2 + 15\pi(GM)^2/8c^4B^3$. The underline indicates the leading order term.

Table 1. Comparison with previous results.. We estimate the numerical value using $c = 3.0 \times 10^8$ [m/s], $G = 6.674 \times 10^{-11}$ [$\rm m^3 \cdot kg^{-1} \cdot s^{-2}$], Mass of galaxy $M \approx 10^{12} M_\odot = 2.0 \times 10^{42}$ [kg], $\Lambda \approx 10^{-52}$ [$\rm m^{-2}$], $b, R, r_0, B \sim 10^5$ [ly] $\sim 10^{21}$ [m] (typical radius of galaxy), $r_s, r_o, d_{\rm OL}, d_{\rm LS}, r_S, r_{obs}, x_B, -x_A \sim 10$ [Mpc] $\sim 10^{23}$ [m]. R_1 in [24] is calculated $1/R_1 = 2GM/c^2B^2 + 15\pi(GM)^2/8c^4B^3$. The underline indicates the leading order term.

Authors	Deflection Due to Λ	Numerical Value [rad]
Rindle & Ishak [13,14]	$-\frac{c^2\Lambda R^3}{6GM}$	-1.1×10^{-5}
Park [15]	Not contribute	-
Khriplovich & Pomeransky [16]	Not contribute	-
Sereno [17,18]	$+\frac{2GMb\Lambda}{3c^2} + \underline{\frac{b^3\Lambda}{6}\left(\frac{1}{r_s}+\frac{1}{r_o}\right)}$	$+3.3 \times 10^{-13}$
Simpson *et al.* [19]	Not contribute	-
Bhadra *et al.* [20]	$\frac{2GM\Lambda b}{3c^2} - \underline{\frac{\Lambda b}{6}(d_{\rm OL}+d_{\rm LS})} + \frac{\Lambda b^3}{6}\left(\frac{1}{d_{\rm OL}}+\frac{1}{d_{LS}}\right)$	-3.3×10^{-9}
Miraghaei *et al.* [21]	$-\sqrt{\frac{2\Lambda}{3}}R$	-8.2×10^{-6}
Biressa *et al.* [22]	$+\frac{2GMb\Lambda}{3c^2} - \underline{\frac{b\Lambda}{6}(r_S+r_{obs})}$ $-\frac{b^3\Lambda}{12}\left(\frac{1}{r_S}+\frac{1}{r_{obs}}\right) + \frac{GMb^3\Lambda}{6c^2}\left(\frac{1}{r_S^2}+\frac{1}{r_{obs}^2}\right)$	-3.3×10^{-9}
Arakida & Kasai [23]	Not contribute	-
Hammad [24]	$-\frac{\sqrt{2}}{3}\Lambda\sqrt{\frac{GMR_1^3}{c^2}}$	-1.1×10^{-5}
Batic *et al.* [25]	$-\underline{\frac{2}{\sqrt{3}}r_0\sqrt{\Lambda}} - \frac{2\sqrt{\Lambda}}{\sqrt{3}}\frac{2GM}{c^2} - \frac{\sqrt{3\Lambda}}{4}\frac{(2GM)^2}{c^4 r_0} - \frac{5\sqrt{\Lambda}}{8\sqrt{3}}\frac{(2GM)^3}{c^6 r_0^2}$	-1.1×10^{-5}
Present Paper	$-\frac{2\Lambda}{3}b(x_B - x_A)$	-1.3×10^{-8}

Our result seems to be similar and comparable with the third term of Equation (13) in [20], and the fourth term of Equation (25) in [22]. Also, as [13,14,20–22,24,25], the cosmological constant leads to diminishing the bending angle due to the mass of the central body M.

However, contrary to previous results such as [13,14,17,18,20,22,24,25], in our case the bending angle due to Λ does not couple with the mass of the central body M. As mentioned in Section 3, it comes from the fact that our calculation is first (linear) order with respect to $\epsilon \sim r_g \sim \Lambda$, (see Equations (9) and (15)), then if we extend to second order $\mathcal{O}(\epsilon^2)$, the coupling term $\mathcal{O}(m\Lambda)$ appears.

Acknowledgments: We would like to acknowledge anonymous referees for reading our manuscript carefully and for giving fruitful comments and suggestions, which significantly improved the quality of the manuscript. This work was supported by JSPS KAKENHI Grant Number 15K05089.

Conflicts of Interest: The authors declare no conflict of interest.

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