universe

MDPI

Article

Rotating Quark Stars in General Relativity

Enping Zhou [1,2,*,†], Antonios Tsokaros [2,4], Luciano Rezzolla [2,3], Renxin Xu [1,5] and Kōji Uryū [6]

1 State Key Laboratory of Nuclear Science and Technology and School of Physics, Peking University, Beijing 100871, China; r.x.xu@pku.edu.cn
2 Institute for Theoretical Physics, Frankfurt am Main 60438, Germany; tsokaros@illinois.edu (A.T.); rezzolla@th.physik.uni-frankfurt.de (L.R.)
3 Frankfurt Institute of Advanced Studies, 60438 Frankfurt am Main, Germany
4 Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA
5 Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, China
6 Department of Physics, University of the Ryukyus, Senbaru, Nishihara, Okinawa 903-0213, Japan; uryu@sci.u-ryukyu.ac.jp
* Correspondence: zhouenpingz715@sina.com
† Current address: State Key Laboratory of Nuclear Science and Technology and School of Physics, Peking University, Beijing 100871, China

Received: 4 January 2018; Accepted: 26 February 2018; Published: 5 March 2018

Abstract: We have built quasi-equilibrium models for uniformly rotating quark stars in general relativity. The conformal flatness approximation is employed and the Compact Object CALculator (COCAL) code is extended to treat rotating stars with surface density discontinuity. In addition to the widely used MIT bag model, we have considered a strangeon star equation of state (EoS), suggested by Lai and Xu, that is based on quark clustering and results in a stiff EoS. We have investigated the maximum mass of uniformly rotating axisymmetric quark stars. We have also built triaxially deformed solutions for extremely fast rotating quark stars and studied the possible gravitational wave emission from such configurations.

Keywords: pulsars; quark stars; general relativity

---

1. Introduction

The gravitational-wave (GW) event GW170817 and the associated electromagnetic emission observations [1,2] from a binary neutron star (BNS) merger has announced the birth of a multi-messenger observation era. Apart from enriching our knowledge on origins of short gamma-ray bursts [3] and heavy elements in the universe [4,5], it provides an effective way for us to constrain the equation of state (EoS) of neutron stars (NSs). In addition to BNS systems , rapidly rotating compact stars are also important candidates of GW sources [6], which could be detected by ground-based GW observatories [7–11]. Further, the properties of both uniformly and differentially rotating stars is tightly related to the evolution of the post-merger product during a BNS merger, for example, whether or not there will be a prompt collapse to a black hole. Hence, studying the properties of rotating compact stars has long been important and of great interest.

Following the first study on the equilibrium models of uniformly rotating, incompressible fluid stars in a Newtonian gravity scheme [12], various works have been done with more realistic EoSs and general relativity [13,14]. Among those studies, quasi-universal relationship has been found for both uniformly rotating and differentially rotating NSs [15–20]. Quasi-equilibrium figures of triaxially rotating NSs have also been created and studied in full general relativity [21,22].

However, it is worth noting that the EoS of compact stars is still a matter of lively debate as it originates from complicated problems in non-perturbative quantum choromodynamics (QCD). In addition to the conventional NS model, strange quark stars (QSs) are also suggested [23,24], after it was conjectured that strange quark matter (SQM) consists of up, down, and strange quarks that could be absolutely stable [25,26]. Additionally, the small tidal deformability of QSs passes the test of GW170817 [27], which requires that a dimensionless tidal deformability of a 1.4 solar mass star is smaller than 800. A more detailed analysis based on the probability distribution of each star in the binary system also indicates that the strangeon star model is consistent with the observation GW170817 together with other EoSs such as APR4. Possible models are also suggested to explain the electromagnetic counterparts (c.f. [27,28]).

Following this possibility, we here use the Compact Object CALculator code, COCAL, to build general-relativistic rotating QS solution sequences using different EoS models. COCAL is a code to calculate general-relativistic equilibrium and quasi-equilibrium solutions for binary compact stars (black hole and NSs) as well as rotating NSs [21,22,29–32]. The EoS part of COCAL is modified to treat quark stars that have a surface density discontinuity. With the modified code, we have built a uniformly rotating axisymmetric and triaxial sequence for quark stars.

2. Results

2.1. Maximum Mass of Axisymmetric Rotating Quark Stars

The maximum mass of a static spherical compact star and an axisymmetric rotating compact star depends on the EoS and is also closely related to the post-merger phase of a BNS merger. The total mass of the binary system could be obtained according to the GW observation. By comparing the total mass of the system with the maximum mass of a rotating star, it can be interpreted whether the post-merger product is a long-lived supramassive NS or short-lived hypermassive NS.

Various nuclear EoS models have been applied to build both uniform and differentially rotating NSs. It has been found that the maximum mass of uniformly rotating NSs, compared with the TOV maximum mass, depends very weakly on EoSs [15]. More specifically, regardless of the EoS model, the star could support approximately 20% more mass by uniform rotation. Another universal relationship is also discussed by [16] for differentially rotating NSs. Such relations have been invoked to interpret the observation of GW170817, and constraints on the maximum mass of NSs have been set accordingly [33]. In order to see whether this relationship still holds for rotating QSs, we have built axisymmetric rotating QS sequences for both the MIT bag model and the strangeon star model [34]. We first build a TOV solution sequence for both EoSs with 24 successive central densities. From each of those TOV solutions, we construct a rotating QS solution sequence by fixing the central density and decrease the axis ratio R_z/R_x. The axis ratio parameter determines the rotation of the star and preserves axisymmetry at the same time. Those rotating QS solutions terminate at the mass shedding limit. In this way, we manage to explore the parameter space for rotating QSs for various central densities and angular velocities.

Once we have all the solutions ready, we can obtain the TOV maximum mass ($M_{\rm TOV}$) and angular momentum at the mass shedding limit ($J_{\rm kep}$) as well as the maximum mass for a certain angular momentum ($M_{\rm crit}$). The relationship between normalized mass ($M_{\rm crit}/M_{\rm TOV}$) and angular momentum ($J/J_{\rm kep}$) can therefore be re-investigated for rotating QSs. The result is shown in Figure 1. As can be seen, the universal relationship for NSs no longer holds for QSs. Moreover, even for rotating QSs with different EoSs, the relation is quite different.

Although we couldn't extend the universal relationship or find a new one for rotating QSs, it does provide a potential to distinguish between NS and QS models from a BNS merger event. In particular, quark stars could be more massive when supported by uniform rotation compared with neutron stars (40% compared with 20%). Consequently, a post-merger phase might be longer before collapsing to a black hole if a QS is formed during the merger.

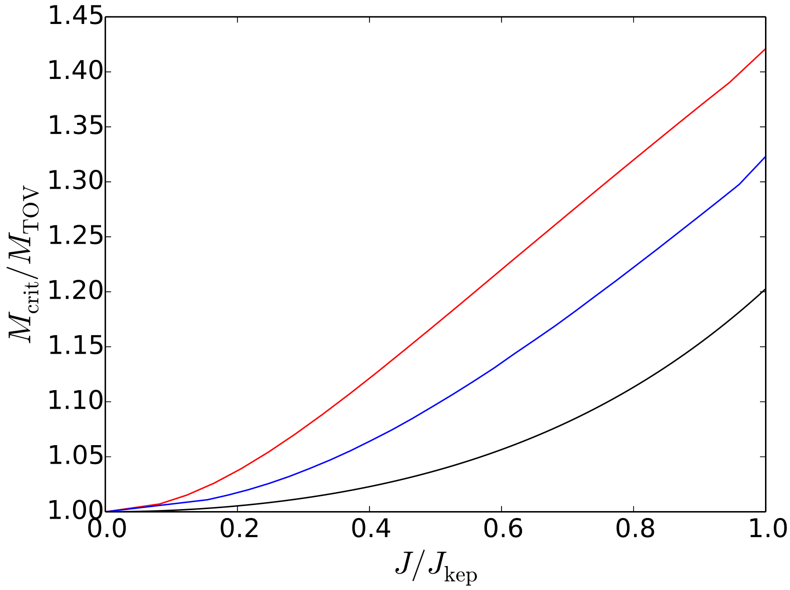

Figure 1. Relationship between normalized critical mass and the normalized angular momentum for rotating eutron stars (NSs) and quark stars (QSs). Bottom black line is the quasi-universal relationship found by [15]. Blue curve in the middle is the relationship for rotating QSs with strangeon star model and the top red line represents the relationship for the MIT bag model. The universal relationship cannot be extended for QSs easily.

2.2. Triaxial Rotating Quark Stars

Rotating QSs in a triaxial Jacobian sequence are another interesting type of QS and occur when the kinetic energy to potential ratio ($T/|W|$) is sufficient large. On the one hand, the post merger product in a BNS merger or a newly born compact star from a core collapse supernova possesses quite a large angular momentum, which might lead to a sufficiently large $T/|W|$ ratio for the bifurcation for a triaxial Jacobian sequence to occur [35–39]. On the other hand, triaxially rotating compact stars is an effective GW radiator itself [40].

Unlike NSs, which are bound by gravity, QSs are self-bound by strong interaction. Consequently, rotating QSs can reach a much larger $T/|W|$ ratio compared with NSs due to the finite surface density. Therefore, the triaxial instability can play a more important role [41–43] for QSs. The triaxial bar mode (Jacobi-like) instability for the MIT bag-model EoS has been investigated in a general relativistic framework [44].

Here, we build quasi-equilibrium constant rest mass sequences (axisymmetric and triaxial) for both the MIT bag-model EoS and the strangeon star model EoS. The surface fit coordinates used in COCAL allows us to treat the surface density discontinuity properly. We begin with the axisymmetric sequence in which we calculate solution sequences with varying parameters, i.e., the central rest-mass density (ρ_c) and the axis ratio (R_z/R_x). We first impose axisymmetry as a separate condition and manage to reach the mass shedding limit for all the sequences. In order to access the triaxial solutions, we recompute the above sequence of solutions but this time without imposing axisymmetry. As the rotation rate increases (R_z/R_x decreases), the triaxial deformation ($R_y/R_x < 1$) is *spontaneously* triggered, since at a large rotation rate the triaxial configuration possesses a lower total energy and is therefore favored over the axisymmetric solution.

Overall, three main properties of triaxially rotating QSs are found according to our calculations. Firstly, QSs generally have triaxial sequences of solutions that are longer than those of NSs. In another words, QSs can see larger triaxial deformations before the sequence is terminated at the mass-shedding limit (c.f. Figures 2 and 3 for an example for the comparison with the $n = 0.3$ NS model), due to the much higher $T/|W|$ ratio attained by rotating QSs. Secondly, when considering similar triaxial configurations, QSs are (slightly) more efficient GW sources because of the finite surface rest-mass density and hence larger mass quadrupole for QSs (c.f. Figure 4). Thirdly, triaxial supramassive solutions can be found for QSs.

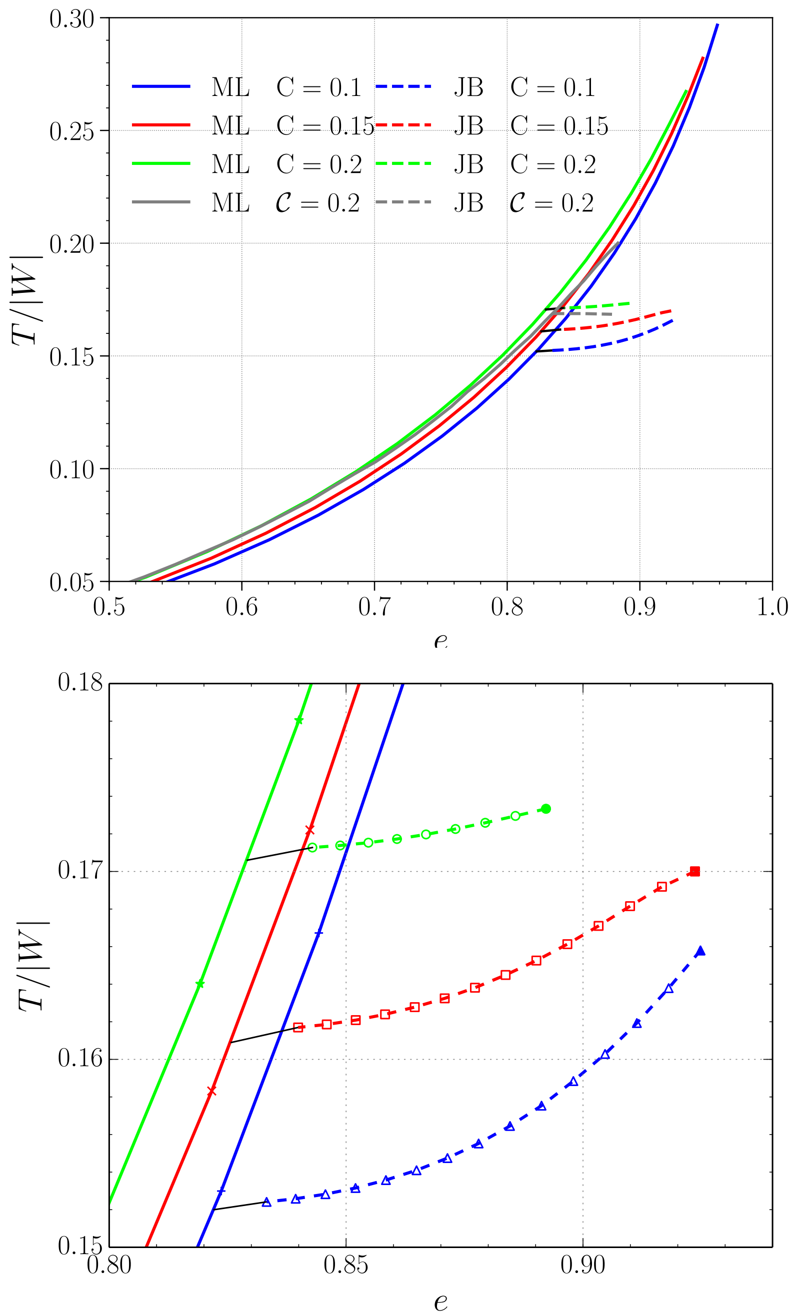

Figure 2. *Upper panel:* $T/|W|$ versus eccentricity $e := \sqrt{1-(z/x)^2}$ (in proper length) for the MIT bag-model Equation of State (EoS) sequences as well as for NSs with $n = 0.3$ EoS reported in [21] (labeled with gray curves). Solid curves are axisymmetric solution sequences, and dashed curves are triaxial solution sequences, which correspond to $\mathcal{C} = M/R = 0.2$ (green curves), 0.15 (red curves), and 0.1 (blue curves), respectively. Note that M is the spherical ADM mass. *Lower panel:* Magnification of the region near the onset of the triaxial solutions marked with empty symbols. Filled symbols mark the models at the mass-shedding limit. Solutions labeled with "ML" are axisymmetric solutions (Maclaurin spheroids), while those labeled "JB" are triaxial solutions (Jacobi ellipsoids).

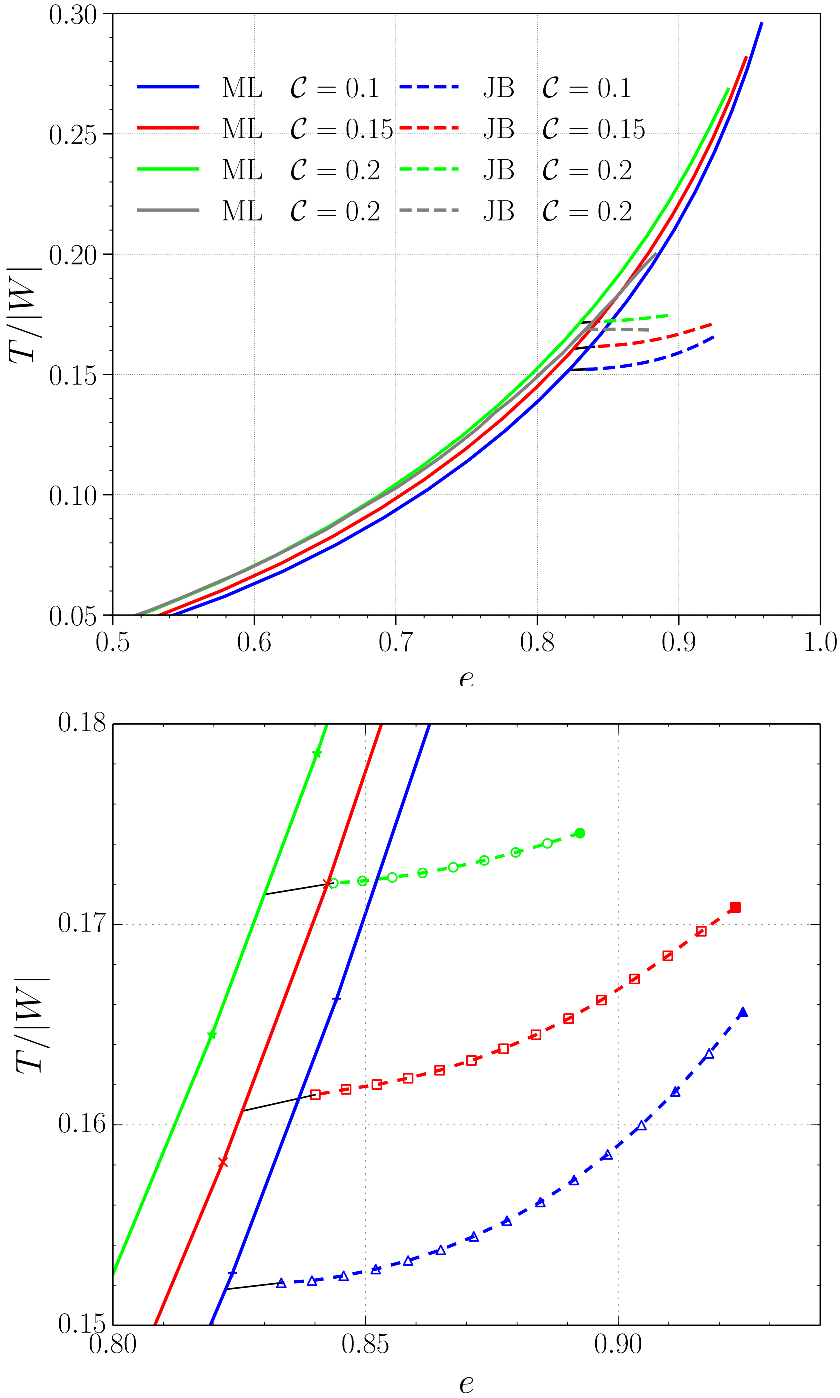

Figure 3. The same as Figure 2 but for the LX EoS sequences.

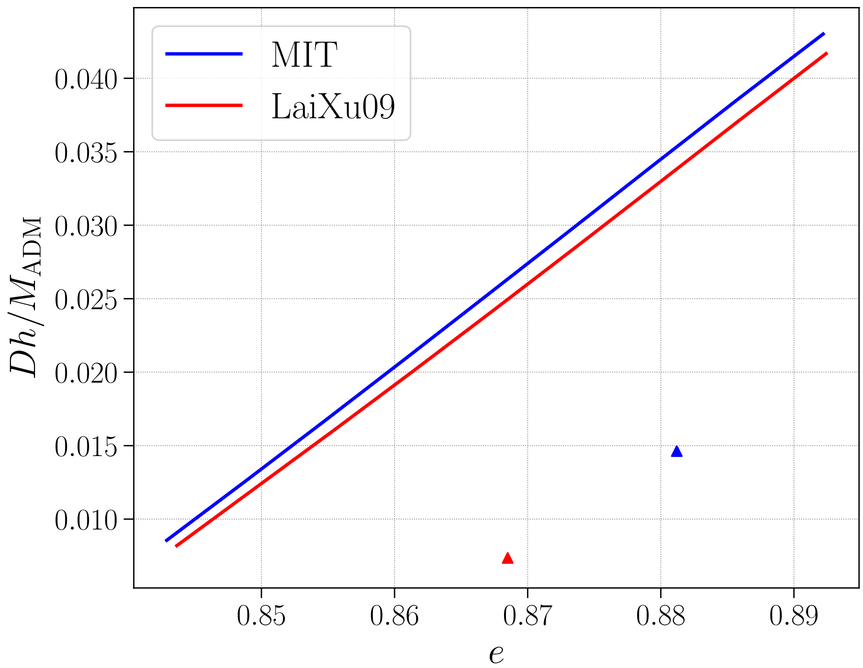

Figure 4. Estimates of the GW strain amplitude according to the quadrupole formula for the $\mathcal{C} = 0.2$ triaxial sequence for both the MIT bag-model EoS (blue solid curve) and the LX EoS (red dashed curve). Shown is the GW strain for the $\ell = m = 2$ mode normalized by the distance and the ADM mass of the source. Triangle markers in blue and red stand for the triaxially rotating NS cases G4C010 and G4C025 models in [40], which indicates that triaxially deformed QSs are more effective graviational-wave (GW) sources compared with NSs.

3. Discussion

We have built both axisymmetric and triaxial solutions for uniformly rotating QSs. For axisymmetric rotating QSs, we have investigated the critical mass-angular momentum relation and find that it deviates from the universal relationship for rotating NSs. Especially, uniformly rotating QSs can be more massive compared with $M_{\rm TOV}$, indicating a quite different post-merger phase in binary merger events. For triaxially rotating QSs, we have identified the bifurcation point from axisymmetric solutions. Triaxial solutions have been constructed from the bifurcation point to the mass shedding limit. The GW emission from such a triaxial rotating QS is estimated with a quadrupole formula and is found to be more effective than that of an NS, which can also be tested with future GW observations.

Additionally, since the spin period of a triaxially rotating star increases as the angular momentum increases, the spin frequency at the bifurcation point somehow represents a maximum spin frequency that can be attained by a pulsar when spun-up by accretion. Particularly, a solid QS model is suggested for the strangeon star model [45], which means that r-mode instability could be totally suppressed for such a star and a strangeon star might be spun up to the bifurcation frequency. With the construction of more power radio telescopes such as SKA and FAST, this limit could be tested by searching for faster spinning pulsars and might provide an important clue on the properties of the dense matter in compact stars.

Acknowledgments: It's a pleasure to thank the China Scholarship Council for supporting E. Z. on the joint PhD training in Frankfurt. This work was supported in part by the ERC synergy grant "BlackHoleCam: Imaging the Event Horizon of Black Holes" (Grant No. 610058), by "NewCompStar", COST Action MP1304, by the LOEWE-Program in the Helmholtz International Center (HIC) for FAIR, and by the European Union's Horizon 2020 Research and Innovation Programme (Grant 671698) (call FETHPC-1-2014, project ExaHyPE). R.X. was supported by National Key R&D Program (No.2017YFA0402600) and NNSF (11673002,U1531243). A.T. was supported by NSF Grants PHY-1662211 and PHY-1602536, and NASA Grant 80NSSC17K0070. K.U. was supported by JSPS Grant-in-Aid for Scientific Research(C) 15K05085.

Author Contributions: E.Z. modified the COCAL code to include QS models. E.Z. and A.T. performed the simulations of triaxially rotating QSs and the convergence tests. L. R. has designed those studies. R.X. helped with the EoS part of the code, and K.U. modified the COCAL code, which substantially improved the accuracy and convergence performance for QSs.

Conflicts of Interest: We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted. We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work. The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results .

References

1. Abbott, B.P.; Abbott, R.; Acernese, F.; Ackley, F.; Adams, C.; Adamsa, T.; Addesso, P.; Adhikari, R.X.; Adya, V.B.; et al. GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral. *Phys. Rev. Lett.* **2017**, *119*, 161101.
2. Abbott, B.P.; Abbott, R.; Acernese, F.; Ackley, F.; Adams, C.; Adamsa, T.; Addesso, P.; Adhikari, R.X.; Adya, V.B.; et al. Multi-messenger Observations of a Binary Neutron Star Merger. *Astrophys. J. Lett.* **2017**, *848*, L12.
3. Abbott, B.P.; Abbott, R.; Acernese, F.; Ackley, F.; Adams, C.; Adamsa, T.; Addesso, P.; Adhikari, R.X.; Adya, V.B.; et al. Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A. *Astrophys. J. Lett.* **2017**, *848*, L13.
4. Abbott, B.P.; Abbott, R.; Acernese, F.; Ackley, F.; Adams, C.; Adamsa, T.; Addesso, P.; Adhikari, R.X.; Adya, V.B.; et al. Estimating the Contribution of Dynamical Ejecta in the Kilonova Associated with GW170817. *Astrophys. J. Lett.* **2017**, *850*, L39.
5. Baiotti, L.; Rezzolla, L. Binary neutron-star mergers: A review of Einstein's richest laboratory. *Rep. Prog. Phys.* **2017**, *80*, 096901.
6. Andersson, N.; Ferrari, V.; Jones, D.I.; Kokkotas, K.D.; Krishnan, B.; Read, J.S.; Rezzolla, L.; Zink, B. Gravitational waves from neutron stars: Promises and challenges. *Gen. Relativ. Gravit.* **2011**, *43*, 409–436.
7. Abramovici, A.A.; Althouse, W.; Drever, R.P.; Gursel, Y.; Kawamura, S.; Raab, F.; Shoemaker, D.; Sievers, L.; Spero, R.; Thorne, K.S.; et al. LIGO: The Laser Interferometer Gravitational-Wave Observatory. *Science* **1992**, *256*, 325–333.
8. Punturo, M.; Abernathy, M.; Acernese, F.; Allen, B.; Andersson, N.; Arun, K.; Barone, F.; Barr, B.; Barsuglia, M.; Beker, M.; et al. The third generation of gravitational wave observatories and their science reach. *Class. Quantum Gravity* **2010**, *27*, 084007.
9. Accadia, T.; Acernese, F.; Antonucci, F.; Astone, P.; Gallardin, G.; Barone, F.; Barsuglia, M.; Basti, A.; Bauer, T.S.; Bebronne, M.; et al. Status of the Virgo project. *Class. Quantum Gravity* **2011**, *28*, 114002.
10. Kuroda, K. Status of LCGT. *Class. Quantum Gravity* **2010**, *27*, 084004.
11. Aso, Y.; Michimura, Y.; Somiya, K.; Ando, M.; Miyakawa, O.; Sekiguchi, T.; Tatsumi, D.; Yamamoto, H. Interferometer design of the KAGRA gravitational wave detector. *Phys. Rev. D* **2013**, *88*, 043007.
12. Chandrasekhar, S. *Ellipsoidal Figures of Equilibrium*; Yale Univ. Press: New Haven, CT, USA, 1969.
13. Meinel, R.; Ansorg, M.; Kleinwächter, A.; Neugebauer, G.; Petroff, D. *Relativistic Figures of Equilibrium*; Cambridge University Press: Cambridge, UK, 2008.
14. Friedman, J.L.; Stergioulas, N. *Rotating Relativistic Stars*; Cambridge University Press: Cambridge, UK, 2013.
15. Breu, C.; Rezzolla, L. Maximum mass, moment of inertia and compactness of relativistic stars. *Mon. Not. R. Astron. Soc.* **2016**, *459*, 646–656.
16. Weih, L.R.; Most, E.R.; Rezzolla, L. On the stability and maximum mass of differentially rotating relativistic stars. *Mon. Not. R. Astron. Soc.* **2017**, *473*, L126–L130.
17. Bozzola, G.; Stergioulas, N.; Bauswein, A. Universal relations for differentially rotating relativistic stars at the threshold to collapse. *Mon. Notices Royal Astron. Soc.* **2017**, *474*, 3557–3564.
18. Yagi, K.; Yunes, N. I-Love-Q. *Science* **2013**, *341*, 365.
19. Yagi, K.; Yunes, N. I-Love-Q relations in neutron stars and their applications to astrophysics, gravitational waves, and fundamental physics. *Phys. Rev. D* **2013**, *88*, 023009.

20. Yagi, K.; Yunes, N. Approximate universal relations for neutron stars and quark stars. *Phys. Rep.* **2017**, *681*, 1–72.
21. Huang, X.; Markakis, C.; Sugiyama, N.; Uryū, K. Quasi-equilibrium models for triaxially deformed rotating compact stars. *Phys. Rev. D* **2008**, *78*, 124023.
22. Uryū, K.; Tsokaros, A.; Galeazzi, F.; Hotta, H.; Sugimura, M.; Taniguchi, K.; Yoshida, S. New code for equilibriums and quasiequilibrium initial data of compact objects. III. Axisymmetric and triaxial rotating stars. *Phys. Rev. D* **2016**, *93*, 044056.
23. Itoh, N. Hydrostatic Equilibrium of Hypothetical Quark Stars. *Prog. Theor. Phys.* **1970**, *44*, 291–292.
24. Alcock, C.; Farhi, E.; Olinto, A. Strange stars. *Astrophys. J.* **1986**, *310*, 261–272.
25. Bodmer, A.R. Collapsed Nuclei. *Phys. Rev. D* **1971**, *4*, 1601–1606.
26. Witten, E. Cosmic separation of phases. *Phys. Rev. D* **1984**, *30*, 272–285.
27. Lai, X.Y.; Yu, Y.W.; Zhou, E.P.; Li, Y.Y.; Xu, R.X. Merging Strangeon Stars. *Res. Astron. Astrophys.* **2018**, *18*, 024.
28. Li, A.; Zhang, B.; Zhang, N.B.; Gao, H.; Qi, B.; Liu, T. Internal x-ray plateau in short GRBs: Signature of supramassive fast-rotating quark stars? *Phys. Rev. D* **2016**, *94*, 083010.
29. Uryū, K.; Tsokaros, A. New code for equilibriums and quasiequilibrium initial data of compact objects. *Phys. Rev. D* **2012**, *85*, 064014.
30. Uryū, K.; Tsokaros, A.; Grandclement, P. New code for equilibriums and quasiequilibrium initial data of compact objects. II. Convergence tests and comparisons of binary black hole initial data. *Phys. Rev. D* **2012**, *86*, 104001.
31. Tsokaros, A.; Uryū, K. Binary black hole circular orbits computed with cocal. *J. Eng. Math.* **2012**, *82*, 133–141.
32. Tsokaros, A.; Uryū, K.; Rezzolla, L. New code for quasiequilibrium initial data of binary neutron stars: Corotating, irrotational, and slowly spinning systems. *Phys. Rev. D* **2015**, *91*, 104030.
33. Rezzolla, L.; Most, E.R.; Weih, L.R. Using Gravitational-wave Observations and Quasi-universal Relations to Constrain the Maximum Mass of Neutron Stars. *Astrophys. J. Lett.* **2018**, *852*, L25.
34. Lai, X.Y.; Xu, R.X. Strangeon and Strangeon Star. *J. Phys. Conf. Ser.* **2017**, *861*, 012027.
35. Lai, D.; Shapiro, S.L. Gravitational radiation from rapidly rotating nascent neutron stars. *Astrophys. J.* **1995**, *442*, 259–272.
36. Bildsten, L. Gravitational Radiation and Rotation of Accreting Neutron Stars. *Astrophys. J. Lett.* **1998**, *501*, L89–L93.
37. Woosley, S.; Janka, T. The physics of core-collapse supernovae. *Nat. Phys.* **2005**, *1*, 147–154.
38. Watts, A.L.; Krishnan, B.; Bildsten, L.; Schutz, B.F. Detecting gravitational wave emission from the known accreting neutron stars. *Mon. Not. R. Astron. Soc.* **2008**, *389*, 839–868.
39. Piro, A.L.; Thrane, E. Gravitational Waves from Fallback Accretion onto Neutron Stars. *Astrophys. J.* **2012**, *761*, 63.
40. Tsokaros, A.; Ruiz, M.; Paschalidis, V.; Shapiro, S.L.; Baiotti, L.; Uryō, K. Gravitational wave content and stability of uniformly, rotating, triaxial neutron stars in general relativity. *Phys. Rev. D* **2017**, *95*, 124057.
41. Gondek-Rosinska, D.; Haensel, P.; Zdunik, J.L.; Gourgoulhon, E. Rapidly rotating strange stars. In *IAU Colloq. 177: Pulsar Astronomy-2000 and Beyond*; Astronomical Society of the Pacific Conference Series; Kramer, M., Wex, N., Wielebinski, R., Eds.; Astronomical Society of the Pacific: San Francisco, CA, USA, 2000; Volume 202, p. 661.
42. Gondek-Rosińska, D.; Bulik, T.; Zdunik, L.; Gourgoulhon, E.; Ray, S.; Dey, J.; Dey, M. Rapidly rotating compact strange stars. *Astron. Astrophys.* **2000**, *363*, 1005–1012.
43. Gondek-Rosińska, D.; Stergioulas, N.; Bulik, T.; Kluźniak, W.; Gourgoulhon, E. Lower limits on the maximum orbital frequency around rotating strange stars. *Astron. Astrophys.* **2001**, *380*, 190–197.
44. Gondek-Rosińska, D.; Gourgoulhon, E.; Haensel, P. Are rotating strange quark stars good sources of gravitational waves? *Astron. Astrophys.* **2003**, *412*, 777–790.
45. Xu, R.X. Solid Quark Stars? *Astrophys. J. Lett.* **2003**, *596*, L59–L62.