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Quantum Measurements in a Finite Space–Time Domain †

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Abstract: In this paper, we discuss the quantum Unruh–DeWitt detector, which couples to the field bath for a finite amount of its proper time. It is demonstrated that due to the renormalization procedure, a new dimensionful parameter appears, having the meaning of a detector's recovery proper time. It plays no role in the leading order of the perturbation theory, but can be important non-perturbatively. We also analyze the structure of finite time corrections in two cases—perturbative switching on, and switching off when the detector is thermalized.

Keywords: Unruh–DeWitt detector; time-dependent coupling

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1. Introduction

Any result in physics refers to a corresponding measurement procedure. In general, people have in mind two physical systems (which, in some cases, can be considered as two distinct parts of the same system) interacting with each other, with one system denoted as "the system" being measured, while the other system is "the detector". The main difference between these systems is the following: The detector's dynamics is under full control of the experimenter, and the detector's subsystem state at some chosen moment in time is supposed to be accurately known. Then, when reading the detector state ("the measurement result") some moments later, one can come to conclusions about properties of the system the detector has been interacting with. In real measurements, of course, many other factors come into play—the still imperfect knowledge of the detector's state, interaction, or dynamics, and external forces, such as noise.

In most cases in particle physics, we assume that dynamics of interest at a microscopic scale are uncorrelated with dynamics of the detector device. Indeed, the typical scale of the former is given by a strong interaction distance of $\sim 10^{-15}$ m and an even smaller one for weak interactions, while detectors are macroscopic objects. The use of all the standard perturbative quantum field theory machinery is implicitly based on this assumption. One usually tries to disentangle the theoretical or analytical part of the problem (symmetries, diagrams, amplitudes, etc.) from the computational detector part (concrete material models of the detector, GEANT4 simulations, etc.). On the other hand, they are tightly glued together by the very nature of the measurement problem and, in many cases, like the celebrated Unruh effect, the above distinction is by far the most conceptually nontrivial. In general, how a detector does influence a system is always a quantitative, physical question.

There are important constraints which any measurement in quantum theory is to obey. The most well-known is, of course, the Heisenberg uncertainty principle. Slightly less well-known are their relatives, like the Robertson–Schrödinger uncertainty relations, the entropic uncertainty principle, the Margolus–Levitin bound, Gabor limit, Landauer limit, etc. These constraints are not dynamical (in the sense that they are independent on the concrete theory's dynamics) and hence are very fundamental. The same Planck constant $\hbar$ controls magnitudes of, for example, zero fluctuations of currents in conductors and oscillations of neutral B-mesons, despite drastic differences in the scale and physical mechanisms of these phenomena.

One of the main physical outcomes of the Heisenberg uncertainty relations is the impossibility to measure the particle's momentum better than $\hbar/L$ if this particle is confined in the box with the linear size L. One can say that the idea of a free quantum particle being at absolute rest in a box of finite size is unphysical, even at zero temperature. However, if one fixes the particle's trajectory "by hand", its spatial dynamics are fully determined by this pre-defined trajectory and it is no longer a quantum, in the Heisenberg uncertainty sense. On the other hand, if the particle has some internal degrees of freedom, their "quantumness" remains intact. This is exactly the situation realized in the well-known model of the Unruh–DeWitt point-like detector (UDW detector) [1–3] coupled to a field bath. The space–time detector's trajectory is supposed to be predefined, while one is interested in the dynamics of internal degrees of freedom.

In this contribution, we present the main results of the formalism [4] for systematic studies of the UDW detector interacting with the field bath for a finite proper time. Various aspects of this problem were discussed in the literature [5–11], and our work [4] is based on these findings.

2. Unruh–DeWitt Detector with Time-Dependent Coupling

The detector-field system state is described as a vector from the space $|n, \Phi\rangle$, where the index n encodes the discrete state of the detector, while Φ stays for the state of the field subsystem. For a two-level UDW detector interacting with the heat bath, the index n takes values 0 or 1, and Φ_β stays for the thermal state of a free massless scalar field with an inverse temperature $\beta = (k_B T)^{-1}$. The evolution operator we focused on read:

$$U_\chi = \mathrm{T}\exp\left[ig\int d\tau\,\chi(\tau)\mu(\tau)\phi(x(\tau))\right], \tag{1}$$

where $x(\tau)$ parameterizes the detector's world-line, τ is a proper time along it, and $\mu(\tau)$ is a monopole transition operator for the detector, where the latter is assumed to have evolved with Hamiltonian H_d having a discrete spectrum $\{E_n\}$. The field-detector coupling constant is g, and the field $\phi(x(\tau))$ is assumed to be an elementary massless scalar field. The window function $\chi(\tau)$ takes nonnegative real values not exceeding 1, and parameterizes the measurement procedure. We will consider smooth functions $\chi(t)$ with smooth derivatives $\chi'(t)$ in what follows, and discuss sharp boundaries $\chi_{12}(\tau) = \theta(\tau-\tau_1) - \theta(\tau-\tau_2)$ only as a limiting case, where we take $\tau_2 > \tau_1$ and the standard θ-function is 1 for $x > 1$ and 0 for $x < 0$. Operators in Equation (1) are ordered along the world-line by the standard T-ordering recipe. If the detector is at rest, the proper time can be chosen to coincide with the usual time, $x(\tau) = (\tau, 0, 0, 0)$.

Fundamental couplings, like the gauge couplings of the Standard Model, are usually considered as universal space–time constants, despite attempts to find their time-dependence continue since the famous Dirac paper [12]. In a model like that in Equation (1), the time-dependent coupling need not be thought of as some fundamental one. It can instead be understood as, for example, an effective parametrization of the detector's finite lifetime, caused by some dynamics, irrelevant for its coupling to the field ϕ. Since the internal parameter is nothing other than the detector's energy, we could expect that the energy–time uncertainty relation should manifest itself either in this or in that way.

It is convenient to consider the evolution for the semi-infinite interval:

$$U_\chi(t) = \mathrm{T}\exp\left[ig\int\limits_{-\infty}^{t} d\tau\,\chi(\tau)\mu(\tau)\phi(x(\tau))\right]. \tag{2}$$

Then, an amplitude for the detector's transition to the state $|n\rangle$ from another state $|m\rangle$ is given by the following expression:

$$\mathcal{A}_{m\to n} = \langle \Phi', n|U_\chi(t)|m, \Phi\rangle, \tag{3}$$

where the state $|\Phi'\rangle$ represents the final state of the field subsystem, while initially it is supposed to be in the initial state $|\Phi\rangle$. The transition probability is given by:

$$\mathcal{P}_{0\to 0}(t) = \langle 0, \Phi|U_\chi^\dagger(t)|0\rangle\langle 0|U_\chi(t)|0, \Phi\rangle, \tag{4}$$

with the normalization condition $\mathcal{P}_{0\to 0}(t) + \mathcal{P}_{0\to 1}(t) = 1$.

In a Markovian approximation, the probability evolution with time can be approximated by the following equation (see Reference [4] for details):

$$\frac{d\mathcal{P}_{0\to 0}(t)}{dt} = -(C_+(t) + C_-(t))\cdot \mathcal{P}_{0\to 0}(t) + C_+(t), \tag{5}$$

where $C_+(t)$ is coefficient function $C(t, \Omega)$:

$$C_+(t) \equiv C(t, \Omega) = \bar{g}\int\limits_{-\infty}^{t} d\tau \chi(t)\chi(\tau)\left(G^+(t,\tau)e^{i\Omega(t-\tau)} + G^+(\tau, t)e^{-i\Omega(t-\tau)}\right), \tag{6}$$

where we denote $\bar{g} = g^2|\langle 0|\mu(0)|1\rangle|^2$ and $G^+(t,\tau)$ is the Wightmann function, given for free massles scalar field at finite temperature (in regularized form) by the expression:

$$G^+_\beta(\tau, \tau') = -\frac{1}{4\pi^2}\sum_{l=-\infty}^{\infty}\frac{1}{(\tau - \tau' - i\epsilon\zeta(\tau,\tau') + i\beta l)^2}, \tag{7}$$

with the infinitesimal regulator parameter $\epsilon > 0$ physically representing the finite size (divided by c) of any realistic detector (see discussion in Reference [13]). The function $\zeta(\tau, \tau')$ can be taken as unity in the simplest case of the detector at rest.

3. Results

Before we come to dynamics of the UDW detector with time-dependent coupling, let us first consider the $\chi(\tau) \equiv 1$ case. Notice that in the physical condition for renormalization, $C_-(t) = C(t, -\Omega) \equiv 0$ for $T = 0$, which means that the detector at rest located in the vacuum at zero temperature cannot be excited and $\mathcal{P}_{0\to 1}(t) \equiv 0$. It is easy to see that this condition is satisfied by the chosen regularization. One can check that in the infinite measurement case, $C_\pm(t)$ are t-independent factors given by:

$$\frac{1}{\bar{g}}C_-(t) = \int\limits_0^\infty ds\left(G^+_\beta(s)e^{-i\Omega s} + G^+_\beta(-s)e^{i\Omega s}\right) = F_\beta(\Omega) = \frac{\Omega}{2\pi}\frac{e^{\epsilon\Omega}}{e^{\beta\Omega} - 1}, \tag{8}$$

so that $p(t) = (1 + e^{\beta\Omega})^{-1}$, which is nothing but the Boltzmann distribution, as it should be.

In the nonstationary case the result is given by (see details in Reference [4]):

$$\begin{aligned} C(t,\Omega) = \bar{g}\chi^2(t)F_\beta(-\Omega) + \bar{g}\chi(t)\chi'(t)\zeta + \bar{g}\chi(t)\int\limits_0^\infty ds\,(\chi(t-s) - \chi(t))(\tilde{G}^+_\beta(s) - \tilde{G}^+(s)(s))\cos\Omega s \\ + \bar{g}\chi(t)\int\limits_0^\infty ds\,(\chi(t-s) - \chi(t) + s\chi'(t))\tilde{G}^+(s)\cos\Omega s \end{aligned}, \tag{9}$$

where:

$$\tilde{G}^+_\beta(s) = -\frac{1}{4\beta^2}\left(\frac{1}{\sinh^2\left(\frac{\pi(s-i\epsilon)}{\beta}\right)} + \frac{1}{\sinh^2\left(\frac{\pi(s+i\epsilon)}{\beta}\right)}\right), \tag{10}$$

and the zero temperature function (where we omit the β index) is:

$$\tilde{G}^{+}(s) = -\frac{1}{4\pi^2}\left(\frac{1}{(s-i\epsilon)^2} + \frac{1}{(s+i\epsilon)^2}\right), \tag{11}$$

and the renormalization procedure corresponds to the replacement:

$$\zeta = -\int_0^\infty ds\, s\, \tilde{G}^{+}(s) \cos\Omega s \to \zeta^{(r)} = -\frac{\mathrm{Ci}(|\Omega|\tau_s)}{2\pi^2}, \tag{12}$$

for renormalized coefficients $C_{\pm}^{(r)}(t)$. This guarantees that the physical condition for normalization is satisfied: $C_{-}^{(r)}(t) \to 0$ if $\Omega \to \infty$ provided τ_s—dead time of the detector—is kept finite.

The appearance of a new dimensionful parameter—τ_s—is remarkable. It is worth mentioning the Unruh effect for circular motion in this respect ([14], see also References [11,15,16] for more recent exposition of the subject). Linear motion with constant acceleration is characterized by the only parameter—acceleration itself, which corresponds to the temperature in the standard Unruh effect. To parameterize the planar circular motion, on the other hand, one needs two independent parameters—the radius of the circle R and angular velocity ω. It occurs that the corresponding rotating detector's levels distribution is nonthermal as soon as R is finite. Due to the global nature of the Unruh effect, the excitation probability depends on the geometry of the detector's world-line as a whole and not on just instantaneous acceleration. This is to be compared with the nonstationary case we are interested in here—the dynamics start "to feel" microstructure of the detector, which is encoded in the new parameter τ_s.

The structure of Equation (9) reflects different physical mechanisms behind the detector's evolution. The first term in the rhs is the only one surviving in the stationary limit and it corresponds to ordinary thermal excitation. In what follows, we illustrate the role of other terms in this expression, responsible for nonstationary effects.

Now, let us look at finite time corrections to the excitation probability at the leading order in g. Naively, at this order, the excitation probability is $p = \bar{g}\tau_m F_\beta(\Omega)$ where $\tau_m = \int_{-\infty}^{\infty} (\chi(\tau))^2 d\tau$ corresponds to the total interaction time. Of course, this expression is valid only if $p \ll 1$. More precisely, one can define the initial state forgetting time via the difference:

$$\mathcal{P}_{0\to 0}(t) - \mathcal{P}_{1\to 0}(t) = e^{-\int_{-\infty}^{t} d\tau (C_{+}(\tau) + C_{-}(\tau))}. \tag{13}$$

It is worth stressing that Equation (13) is nonperturbative, i.e., it is valid for any value of the coupling constant. In $t \to \infty$, the limit expression in the exponent takes the form:

$$\int_{-\infty}^{\infty} d\tau (C_{+}(\tau) + C_{-}(\tau)) = \bar{g}\tau_m \int_{-\infty}^{\infty} ds\, D_\chi(s)(G^{+}(s) + G^{+}(-s))e^{i\Omega s}, \tag{14}$$

where the function $D_\chi(s)$ is given by:

$$D_\chi(s) = \frac{1}{\tau_m}\int_{-\infty}^{\infty} d\tau\, \chi(\tau + s/2)\, \chi(\tau - s/2)\,. \tag{15}$$

A similar separation of variables was proposed in Reference [5]. Replacing in Equation (15) $s \to -i(\partial/\partial\Omega)$, Equation (13) can be written in terms of the operator $D_\chi(-\partial^2/\partial\Omega^2)$:

$$\mathcal{P}_{0\to 0} - \mathcal{P}_{1\to 0} = e^{-\bar{g}\tau_m (D_\chi F_\beta(\Omega) + D_\chi F_\beta(-\Omega))}. \tag{16}$$

This defines critical time τ_m for transition of the problem from weak to strong coupling regimes.

It is remarkable that all the finite time physics can be factorized in terms of the operators D_χ. However, it is *not* the interaction time τ_m only which governs expressions like $D_\chi F_\beta(\pm\Omega)$. Instead, one can expand (at large times τ_m) D_χ as:

$$D_\chi(-\partial^2/\partial\Omega^2) = 1 + \frac{1}{2\tau_{eff}^2}\frac{\partial^2}{\partial\Omega^2} + ..., \tag{17}$$

where:

$$\frac{1}{\tau_{eff}^2} = \frac{\int d\tau\,(\chi'(\tau))^2}{\int d\tau\,(\chi(\tau))^2}. \tag{18}$$

There is no reason why τ_{eff} should coincide in a parametric sense with τ_m in the general case. The universal character of $1/\tau_{eff}^2$ asymptotic Equation (17) could well mean actual $1/\tau_m$ (and not naively expected $1/\tau_m^2$) dependence for the leading finite time correction; see concrete examples in Reference [4]. This means that correction to the leading perturbative answer for the transition probability in time τ_m could be constant (not decreasing with rise of τ_m).

Let us consider another situation, when the detector is already thermalized and is being perturbatively switched off. To see what happens in this case, it is instructive to proceed with some concrete example of the switching profile, which we take here as $\chi_-(\tau) = (1-\tanh(\lambda\tau))/2$. One can compute the difference $p_f - p_i$ in the adiabatic limit $\delta\tau = 1/\lambda \to \infty$, where $\delta\tau$ is a typical switching time:

$$\frac{p_f - p_i}{\bar{g}\,(1/2 - p_i)} = \frac{1}{2\pi^2}\mathrm{Ci}(|\Omega|\tau_s) + \frac{1}{2\beta^2\Omega^2}\int\limits_0^\infty ds\, s\cos s\left[\left(\frac{\beta\Omega}{\pi s}\right)^2 - \mathrm{csch}^2\frac{\pi s}{\beta\Omega}\right]. \tag{19}$$

The remarkable property of this expression is that it is nonvanishing. It seems, naively, that adiabatically slow switching should keep the initial thermal state intact. Indeed, the probability of transition in unit time decays as $1/\delta\tau$ in a large $\delta\tau$ limit. However, since the probability is an integral over time, it gives a compensating $\delta\tau$ factor. The second term in the rhs of (19) represents thermal contribution and vanishes in small temperatures or large gap limits $\beta\Omega \to \infty$, while the term, proportional to integral cosine, is the temperature-independent vacuum contribution (also vanishing in a large gap limit). It corresponds to the second term in the rhs of Equation (9) and nontrivially reflects the logarithmic divergency in the original problem. It would be very interesting to observe this kind of oscillatory behavior in real experiments.

4. Conclusions

The time-dependent dynamics of the UDW detector have many peculiar features that are absent in stationary cases. There are a few relevant time parameters which encode these features in an integral form. The most obvious one is the total interaction time $\tau_m = \int\limits_{-\infty}^{\infty}(\chi(\tau))^2 d\tau$. However, other parameters come into play if subleading or nonperturbative effects are considered. One is the effective time τ_{eff}, defined by (18). Another one is a recovery time τ_s, which characterizes the detector's intrinsic dynamics. It is interesting that any τ_s-dependent effects are proportional (in a Markov approximation) to $\int_{\tau_1}^{\tau_2} d\tau\chi(\tau)\chi'(\tau) = \left(\chi^2(\tau_2) - \chi^2(\tau_1)\right)/2$. It explains why they are not seen at the leading order in perturbation theory, since in this case, $\tau_1 \to -\infty$, $\tau_2 \to \infty$, and the function $\chi(\tau)$ vanishes at infinity. In the case of switching off, on the other hand, $\chi(\tau) \neq 0$ at the initial time; hence, the result becomes τ_s-dependent.

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