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Review

Searching for Wormholes Beyond Horndeski Theories†

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Abstract: We discuss whether it is possible to construct a stable, static, spherically symmetric Lorentzian wormhole in beyond Horndeski theory. The deep analogy between the cosmological bounce and wormhole scenarios is described in detail. We show explicitly that going beyond Horndeski enables one to evade the no-go theorem formulated for the wormholes in the general Horndeski case.

Keywords: beyond Horndeski; Lorentzian wormholes; stability; cosmological bounce; ghosts; gradient instabilities

1. Introduction and Summary

Despite being purely hypothetical space objects, wormholes are quite peculiar for numerous reasons [1–3]. For instance, unlike other gravitational structures in General Relativity (GR), a wormhole requires quite an exotic matter to prevent its throat from collapsing. In particular, this matter has to violate the Null Energy Condition (NEC) to make the wormhole traversable. One of the possible approaches to modelling this exotic matter is to make use of Horndeski theories [4]. Horndeski (or, equivalently, generalized Galileon) theories are scalar field theories whose Lagrangian includes second order derivatives, but the corresponding equations of motion remain second order. The central feature of Horndeski theories is their ability to violate NEC without giving rise to Ostrogradsky instabilities. Horndeski theories have a generalization, which is usually referred to as beyond Horndeski theories [5]. The equations of motion in beyond Horndeski case include third order derivatives, nonetheless, there are the same number of degrees of freedom in beyond Horndeski theories as compared to Horndeski case. Recently, an even more general class of theories has been put forward and dubbed DHOST theories (see e.g., Ref. [6]). In this review we consider Horndeski and beyond Horndeski subclasses only.

Apart from other applications, (beyond) Horndeski theories are widely used for constructing the early Universe scenarios, which require NEC violation, e.g., cosmological bounce or Genesis scenario (there exist many sound specific models, we name just a few [7–11]). The central issue for all cosmological solutions constructed in Horndeski theory so far has been their stability. It was shown that any cosmological solution in the general Horndeski theory becomes plagued with gradient instabilities, provided one considers entire evolution [12,13]. In other words, the general Horndeski theories are not suitable for constructing complete cosmological scenarios. However, it has been found that going beyond Horndeski enables one to evade the no-go and construct a complete cosmological solution, which is free of any kind of pathologies at all times (see, e.g., Refs. [14–18]).

To some extent wormhole is similar to the bouncing scenario. Indeed, the evolution of the bouncing scale factor $a(t)$ and the profile of the wormhole $R(r)$ in terms of radial coordinate r are quite alike (Figure 1): the narrowest region of the throat corresponds to the moment of bounce and flaring out sleeves of the wormhole are analogous to the contracting and expanding epochs of the bouncing scenario.

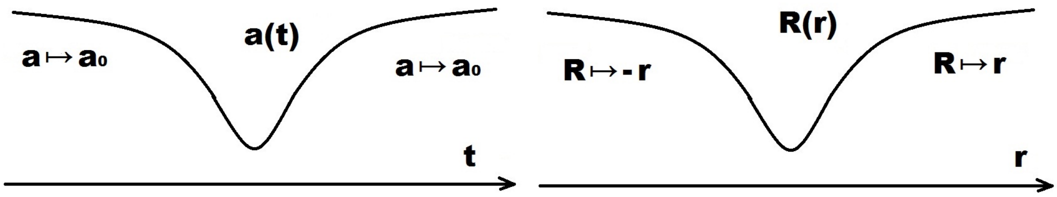

Figure 1. Time dependence of the scale factor of a bouncing solution (**a**) and a wormhole profile (**b**).

Hence, one might assume that the wormhole studies in Horndeski theory should give the results which are analogous to those for the bouncing solution. And indeed Refs. [19–21] showed that in the cubic Horndeski theory ghost instabilities inevitably develop somewhere in space where a Lorentzian wormhole is present. A general form of stability conditions for the spherically symmetric, static solution in the general Horndeski theory has been derived in Refs. [22,23]. These conditions were used to formulate a full-blown no-go theorem for the everywhere-stable wormholes in the general Horndeski theory [24]. Thus, the analogy with the case of a cosmological bounce seems to be valid so far. The next natural step is to try going beyond Horndeski. In this note we review the latest results of Refs. [25,26][1] , which demonstrated that beyond Horndeski terms in the Lagrangian indeed enable one to evade the no-go theorem for wormholes. Similar results were obtained within the EFT formalism in Ref. [27]. What is more important, Ref. [26] contains an explicit example of the beyond Horndeski Lagrangian, which describes the theory admitting an everywhere stable wormhole solution[2]. The suggested solution is not entirely flawless: although it describes an asymptotically flat space, gravity is still modified as compared to GR even far away from the throat. Despite having quite specific features, this wormhole solution is a promising sign in the context of the analogy with a bouncing case. Indeed, after having constructed a complete, stable bouncing solution in beyond Horndeski theory with non-trivial asymptotical behaviour in Ref. [16], another completely healthy bounce was suggested in Ref. [18], whose asymptotical past and future are described by GR with a conventional massless scalar field. Consequently, one might expect that the construction of a stable wormhole with a conventional form of asymptotical regions is possible as well.

This review aims to highlight the similarities between the cosmological bounce and static, spherically symmetric wormhole settings within beyond Horndeski theories. We mainly focus on a healthy behaviour of beyond Horndeski theory at the linearized level in both settings, comparing the corresponding stability conditions. It occurs that the central stability requirements coincide in both cases as well as the mechanism of evading the no-go theorems is identical. This makes the analogy between the bounce and wormhole even deeper. However, we consider a specific subset of stability conditions for the wormhole solution, namely, we analyse the constraints following from the no-ghost requirement only.

This note is organized as follows. In Section 2 we introduce the Lagrangian for Horndeski theory, specifying which terms belong to beyond Horndeski case. We collect the existing results for the

[1] In Ref. [25] there occurred a calculational mistake, which resulted in erroneously driven conclusions about the fine-tuning issue. The error was corrected in Ref. [26].

[2] Let us point out, that Ref. [26] considers only a part of the whole set of stability conditions, namely, the solution is not checked against the stability conditions, eliminating angular gradient instabilities in the parity even sector.

cosmological bounce in beyond Horndeski theory in Section 3. In Section 4 we discuss the linearized theory for a static, spherically symmetric wormhole setting in beyond Horndeski theory and give a subset of stability conditions in this case. We show explicitly the way to circumvent the no-go theorem in the case of a wormhole solution.

2. General Horndeski Theory and Beyond

Horndeski theories are the most general scalar field theories with modified gravity, which are free of Ostrogradsky instabilities despite the presence of the second derivatives in the Lagrangian:

$$\begin{aligned}
&\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}},\\
&\mathcal{L}_2 = F(\pi, X),\\
&\mathcal{L}_3 = K(\pi, X)\Box\pi,\\
&\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],\\
&\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \tfrac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}^{\ \ \nu}\right],\\
&\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} + F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'},
\end{aligned} \tag{1}$$

where π is the scalar (Galileon) field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{4X} = \partial G_4/\partial X$, etc. The terms $\mathcal{L}_2 - \mathcal{L}_5$ make up the general Horndeski theory, while $\mathcal{L}_{\mathcal{BH}}$ appears beyond Horndeski.

As it is mentioned above, the difference between the general Horndeski theory and beyond Horndeski theory reveals itself in the order of field equations: in beyond Horndeski theory the equations contain terms with the third order derivatives. However, the Hamiltonian analysis shows that no extra degrees of freedom appear in the theory, meaning that beyond Horndeski is still free of Ostrogradsky instabilities [5].

Throughout this note we keep $K(\pi, X) = 0$, $G_5(\pi, X) = 0$ and $F_5(\pi, X) = 0$, since the rest of functions in the Lagrangian (1) give rise to all the structures in the linearized theory, which are essential for the argument.

3. Stability Conditions in a Homogeneous Case

One of the main concerns regarding the cosmological solutions in (beyond) Horndeski theory is the absence of any instabilities in the linearized theory throughout entire evolution. To study the linearized theory for a homogeneous background solution we consider flat FLRW metric and parametrize metric perturbations as follows:

$$ds^2 = (1 + 2\alpha)dt^2 - \partial_i\beta\, dtdx^i - a^2(1 + 2\zeta\delta_{ij} + 2\partial_i\partial_j E + h^T_{ij})dx^i dx^j, \tag{2}$$

where α, β, ζ and E are scalar perturbations and h^T_{ij} denote transverse, traceless tensor modes (here and in what follows vector perturbations are omitted). Let us fix the gauge by removing $\partial_i\partial_j E$ in (2) and set the perturbations of the Galileon field π to zero (unitary gauge approach). Due to a specific structure of the Lagrangian (1) (see e.g., Refs. [28,29] for details) α and β are non-dynamical degrees of freedom, so they give two constraint equations. Thus, there are one scalar (ζ) and two tensor degrees of freedom left. The general form of quadratic action for the Lagrangian (1) reads:

$$S = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\frac{\mathcal{G_T}}{8}\left(\dot{h}^T_{ij}\right)^2 - \frac{\mathcal{F_T}}{8a^2}\left(\partial_k h^T_{ij}\right)^2 + \mathcal{G_S}\dot{\zeta}^2 - \mathcal{F_S}\frac{(\nabla\zeta)^2}{a^2}\right], \tag{3}$$

where the coefficients are

$$\mathcal{G}_{\mathcal{S}} = \frac{\Sigma \mathcal{G}_{\mathcal{T}}{}^2}{\Theta^2} + 3\mathcal{G}_{\mathcal{T}}, \tag{4a}$$

$$\mathcal{F}_{\mathcal{S}} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_{\mathcal{T}}, \tag{4b}$$

$$\xi = \frac{a\left(\mathcal{G}_{\mathcal{T}} + \mathcal{D}\dot{\pi}\right)\mathcal{G}_{\mathcal{T}}}{\Theta}, \tag{4c}$$

and

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4G_{4X}X - \dot{\pi}(2F_4X\dot{\pi}), \tag{5a}$$

$$\mathcal{D} = 2F_4X\dot{\pi}, \tag{5b}$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4, \tag{5c}$$

$$\Theta = 2G_4H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi} - 10HF_4X^2 - 4HF_{4X}X^3 \tag{5d}$$

$$\begin{aligned}\Sigma = {} & F_XX + 2F_{XX}X^2 - 6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3 \\ & - 6HG_{4\pi}\dot{\pi} - 30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi} + 90H^2F_4X^2 + 78H^2F_{4X}X^3 + 12H^2F_{4XX}X^4.\end{aligned} \tag{5e}$$

An overdot stands for the time derivative. The coefficients (5) include both general Horndeski and beyond Horndeski terms. According to (3) it is sufficient to impose the following constraints to avoid the appearance of ghost and gradient instabilities

$$\mathcal{G}_{\mathcal{T}} > \mathcal{F}_{\mathcal{T}} > 0, \quad \mathcal{G}_{\mathcal{S}} > \mathcal{F}_{\mathcal{S}} > 0. \tag{6}$$

The conditions above also ensure that there are only subluminal (or at most luminal) modes in the theory, i.e.,

$$c_{\mathcal{T}}^2 = \frac{\mathcal{F}_{\mathcal{T}}}{\mathcal{G}_{\mathcal{T}}} < 1, \qquad c_{\mathcal{S}}^2 = \frac{\mathcal{F}_{\mathcal{S}}}{\mathcal{G}_{\mathcal{S}}} < 1, \tag{7}$$

where $c_{\mathcal{T}}$ and $c_{\mathcal{S}}$ are the sound speed for tensor and scalar perturbations respectively.

Let us revisit the no-go theorem formulated for stable cosmological solutions in Horndeski theory [13]. The no-go argument is based on Equation (4b) and the no-gradient instabilities requirement in the scalar sector, i.e., $\mathcal{F}_S > 0$. These assumptions basically state that for any non-singular cosmological scenario

$$\dot{\xi} > a\mathcal{F}_{\mathcal{T}} > 0. \tag{8}$$

The latter relation suggests that ξ is a monotonously growing function of time and, therefore, it has to cross zero at some point. But according to definition (4c) zero-crossing of ξ is impossible in the general Horndeski theory. Indeed, in Horndeski theory $F_4(\pi, X) = 0$ so $\mathcal{D} = 0$ and definition (4c) reads:

$$\xi = \frac{a\mathcal{G}_T{}^2}{\Theta}, \tag{9}$$

where $\mathcal{G}_T$ still has to be strictly positive to avoid ghosts in the tensor sector. However, there is an option to allow $\mathcal{F}_T$ in Equation (8) to hit zero value, but, at least naively, the theory becomes strongly coupled in this case, which is considered to be undesirable. One might also consider discontinuous ξ by allowing Θ to cross zero in Equation (9), so that ξ could cross zero in the asymptotic past and/or future. But this scenario still does not meet the condition (8) unless one allows $\mathcal{F}_{\mathcal{T}} \to 0$ and again risks to run into a strong coupling regime (see e.g., Ref. [30] for a detailed discussion of the discontinuous ξ issue). Therefore, it is impossible to construct a non-singular cosmological solution in the general Horndeski theory, which is free of any kind of instabilities during entire evolution.

The situation changes in beyond Horndeski theory. The coefficient $\mathcal{D}$ in Equation (4c) is non-zero and, while $\mathcal{G}_T > 0$, allows one to arrange a model so that ξ crosses zero at the moment when

$\mathcal{G}_T + \mathcal{D}\dot{\pi} = 0$. Hence, in beyond Horndeski theory the extra coefficient $\mathcal{D}$ enables one to construct a complete, stable cosmological solution. In the following section we show that the linearized theory has similar structure and features in the case of a static, spherically symmetric background.

4. Stability Conditions for Wormholes in Horndeski Theory and Beyond

In this section we consider static, spherically symmetric solutions in the same class of theories with Lagrangian (1) and analyze their stability. Let us choose the background metric in the following form:

$$ds^2 = A(r)dt^2 - \frac{dr^2}{B(r)} - R(r)^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{10}$$

with

$$A > 0, \quad B > 0, \quad R > 0, \tag{11}$$

where the radial coordinate r ranges from $-\infty$ to $+\infty$. With a specific choice of the functions A, B and R, the metric (10) describes a spherically symmetric wormhole. We study the behaviour of both metric and scalar field perturbations. The linearized theory in the case of the general Horndeski theory has been studied in Refs. [22,23], where the authors adopted the Regge-Wheeler approach to parametrization of perturbations about a spherically symmetric background [31]. In this note we generalize the results of Refs. [22,23] to the case of beyond Horndeski theories and show how the stability conditions get modified. Let us note that we do not consider lower multipoles: both monopole ($\ell = 0$) and dipole ($\ell = 1$) modes should be addressed separately, since the resultant quadratic action for dynamic degrees of freedom, if exists, has significantly different form, and, hence, the stability conditions should be obtained from the scratch (see Refs. [22,23,32] for details). However, in the case of Horndeski theory these conditions for $\ell = 0, 1$ modes do not give any new restrictions as compared to $\ell > 1$. Explicit calculations have shown that this is also the case in beyond Horndeski theory, i.e., the stability conditions for monopole and dipole modes do not impose any new restrictions, see Ref. [27] for computations within the EFT approach[3] .

In the spherically symmetric case perturbations can be divided into parity odd and parity even sectors according to the their transformation properties with respect to reflection [31]. There is only one dynamical degree of freedom Q in the parity odd sector with the quadratic action of the following form

$$\mathcal{S}_{odd} = \int dtdr \frac{\ell(\ell+1)}{2(\ell-1)(\ell+2)} \sqrt{\frac{B}{A}} R^2 \left[\frac{\mathcal{H}^2}{A\mathcal{G}} \dot{Q}^2 - \frac{B\mathcal{H}^2}{\mathcal{F}} (Q')^2 - \frac{\ell(\ell+1)}{R^2} \cdot \mathcal{H} Q^2 - V(r) Q^2 \right], \tag{12}$$

where prime stands for the derivative with respect to radial coordinate r and ℓ is the angular momentum. We do not give the "potential" $V(r)$ explicitly here, since it is irrelevant for the argument. The squared speed of propagation in the radial and angular directions reads, respectively,

$$c_r^2 = \frac{\mathcal{G}}{\mathcal{F}}, \quad c_\theta^2 = \frac{\mathcal{G}}{\mathcal{H}}. \tag{13}$$

The stability conditions immediately follow from action (12):

$$\mathcal{H} > \mathcal{G} > 0, \quad \mathcal{F} > \mathcal{G} > 0, \tag{14}$$

[3] Similar calculations were carried out in a covariant case discussed in Ref. [26], but have not been published yet.

where

$$\mathcal{F} = 2G_4,$$
$$\mathcal{H} = \mathcal{G} = 2G_4 + 2G_{4X}B\pi'^2 + 2F_4B^2\pi'^4. \tag{15}$$

These conditions ensure the absence of ghost and gradient instabilities in both radial and angular directions as well as subluminal propagation of the odd-parity modes.

The quadratic action for the even-type perturbations contains two dynamical degrees of freedom v^i $(i = 1, 2)$:

$$\mathcal{S}_{even} = \int dtdr\sqrt{\frac{A}{B}}R^2 \cdot \left[\frac{1}{2}\mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \frac{1}{2}\mathcal{P}_{ij}v^{i\prime}v^{j\prime} - \mathcal{Q}_{ij}v^i v^{j\prime} - \frac{1}{2}\mathcal{M}_{ij}v^i v^j\right], \tag{16}$$

where the coefficients $\mathcal{K}_{ij}$, $\mathcal{P}_{ij}$, $\mathcal{Q}_{ij}$ and $\mathcal{M}_{ij}$ are some expressions of Lagrangian functions evaluated on a spherically symmetric background. While in the parity odd sector both ghost and gradient instabilities are avoided by imposing requirements (14), in the parity even case we concentrate on ghosts only. To have a ghost-free solution one has to require

$$det(\mathcal{K}) > 0, \qquad \mathcal{K}_{11} > 0, \tag{17}$$

to make the quadratic form $\mathcal{K}_{ij}$ positive definite. Modulo the overall positive factors the conditions (17) reduce to the following constraints

$$det(\mathcal{K}) \sim \mathcal{F}(2\xi_w' - \mathcal{F}) > 0, \tag{18a}$$
$$\mathcal{K}_{11} \sim \ell(\ell+1)\xi_w' - \mathcal{F} > 0, \tag{18b}$$

where

$$\mathcal{D}_w = 2F_4B^2\pi'^4. \tag{18c}$$

Note that it is sufficient to satisfy the condition (18a) only, since for $\ell > 1$ the inequality (18b) is valid automatically. Both constraints in Equation (18) involve ξ_w, which is the analogue of ξ in Equation (4c) and reads

$$\xi_w = \frac{R^2\mathcal{H}\,(\mathcal{H} - \mathcal{D}_w)}{\Theta_w}, \tag{19}$$

with

$$\Theta_w = 2\mathcal{H}RR' + \pi'\Big(-2G_{4\pi}R^2 - 4G_{4X}B\,RR'\pi' + 2G_{4\pi X}BR^2(\pi')^2 + 4G_{4XX}B^2\,RR'(\pi')^3$$
$$-16F_4B^2\,RR'(\pi')^3 + 4F_{4X}B^3\,RR'(\pi')^5\Big).$$

According to Equation (18) the situation appears to be identical to the homogeneous one: Equation (18a) implies that $\xi_w' > \mathcal{F}/2 > 0$ and, hence, ξ_w has to be a monotonously growing function, which means that it must cross zero somewhere.

Now ξ_w and ξ have basically the same structure (see Equations (19) and (4c)) and, in full analogy with the homogeneous case, the definition (19) explicitly shows that it is necessary to go beyond Horndeski to make ξ_w cross zero. Indeed, according to Equation (18c) $\mathcal{D}_w$ vanishes in the general Horndeski case and leaves $\mathcal{H}^2$ in the numerator, while letting Θ_w cross zero in the denominator still does not help out (see the comment in Section 3). The fact that ξ_w cannot cross zero in the general Horndeski case in a healthy way has been formulated in the form of a no-go theorem in Ref. [24]. Hence, the stability conditions with respect to ghost degrees of freedom can be met everywhere only in beyond Horndeski theory.

Thus, we have explicitly showed that in full analogy with a cosmological setting the no-go theorem, forbidding ghost-free wormholes, can be evaded by going beyond Horndeski. The central relation for the stability analysis (19) in the spherically symmetric case of beyond Horndeski theory is identical to the one in the cosmological setup. In this note we have not considered the possibility of tachyonic instabilities in the parity odd sector, while the corresponding stability analysis has been carried out in Refs. [27,32]. We also have not discussed gradient instabilities in parity even sector, which are partly addressed in Ref. [26]. What is more, the stability analysis for the even-parity perturbations should be further developed to rule out angular gradient instabilities and tachyonic degrees of freedom, which are governed by matrices $\mathcal{Q}_{ij}$ and $\mathcal{M}_{ij}$ in the quadratic action (16) (both matrices are quite cumbersome, therefore the issue is left for the future studies). Although off hand it is unlikely that these types of pathologies occur in the spherically symmetric case, there might be peculiar results as well.

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