econometrics

MDPI

Article

Modeling Real Exchange Rate Persistence in Chile

Leonardo Salazar

Department of Economics and Finance, Universtiy of Bío-Bío, Avenida Collao 1202, Concepción, Chile; lsalazar@ubiobio.cl; Tel.: +56-41-3111481

Academic Editor: Katarina Juselius
Received: 28 February 2017; Accepted: 30 June 2017; Published: 7 July 2017

Abstract: The long and persistent swings in the real exchange rate have for a long time puzzled economists. Recent models built on imperfect knowledge economics seem to provide a theoretical explanation for this persistence. Empirical results, based on a cointegrated vector autoregressive (CVAR) model, provide evidence of error-increasing behavior in prices and interest rates, which is consistent with the persistence observed in the data. The movements in the real exchange rate are compensated by movements in the interest rate spread, which restores the equilibrium in the product market when the real exchange rate moves away from its long-run benchmark value. Fluctuations in the copper price also explain the deviations of the real exchange rate from its long-run equilibrium value.

Keywords: exchange rate; long swings; I(2) analysis

JEL Classification: C32; F31; F41; G15

1. Introduction

The purchasing power parity (PPP) theory establishes that identical goods will have the same price in different economies when prices are expressed in the same currency (Krugman et al. 2011). In other words, the aggregate relative prices between two countries should be equal to the nominal exchange rate between them (Taylor and Taylor 2004).[1]

The PPP has been broadly used in economics, in both theoretical models and empirical applications. For instance, a number of general equilibrium models use the PPP as an equilibrium condition; that is, the PPP is assumed to hold over time, and the main results in these models rely on the PPP assumption (Duncan and Calderón 2003). In addition, estimates of PPP exchange rates are used to compare national income levels, determining the degree of misalignment of the nominal exchange rate around relative prices and the appropriate policy response (Sarno and Taylor 2002).

However, empirical evidence shows that over time, the nominal exchange rate exhibits long and persistent swings around relative prices. Specifically, while the ratio of domestic to foreign good prices changes slowly over time, the nominal exchange rate exhibits long and persistent swings away from its benchmark value. Consequently, these persistent swings are observed in the real exchange rate. See Figure 1 for the Chilean case.

Long and persistent fluctuations in the real exchange rate (RER) may generate allocative effects on the economy. Indeed, the competitiveness of a country might be negatively affected by a prolonged real appreciation (Mark 2001). Furthermore, these fluctuations might affect domestic real interest rates, wages, unemployment, and output, generating structural slumps in economies (Phelps 1994).

[1] This concept is known as absolute PPP.

Frydman and Goldberg (2007) developed a monetary model based on imperfect knowledge economics (IKE), know as IKE-based model, that was proposed as a solution to the puzzle of the long swings in exchange rates. Its empirical validity has been tested by Johansen et al. (2010), Juselius (2017a), and Juselius and Assenmacher (2017). For instance, using a cointegrated vector autoregressive (CVAR) scenario,[2] Juselius (2017a) argues, based on German-US data, that the IKE-based scenario is empirically supported by every testable hypothesis that describes the underlying assumptions of this model.

Departures from PPP have also been related to theories were the markup over costs of firms operating on imperfectly competitive markets is negatively affected by the inflation rate. For instance, Bacchiocchi and Fanelli (2005) found that persistent deviations from PPP in France, the United Kingdom and Germany, versus the United States, might be attributed to the presence of $I(2)$ stochastic trends in prices which can be associated with inflation rates that reduces the markup of profit-maximizing firms acting on imperfectly competitive markets.

The evidence on PPP is generally mixed and the results depend on the covered period, the variables included in the analysis, and the econometric methodology used to test the PPP hypothesis.[3]

In the case of Chile, the evidence is also mixed, and the results depend primarily on the methodology used to test the PPP hypothesis. On the one hand, when augmented Dickey-Fuller (ADF) test is used in a single equation that includes the nominal exchange rate, domestic price, and foreign price, the PPP hypothesis seems to hold. That is, RER is found to be a stationary process (Délano and Valdés 1998; Duncan and Calderón 2003). On the other hand, if multivariate cointegration techniques are used, the results show that RER behaves as a nonstationary $I(1)$ process. However, it cointegrates with other $I(1)$ variables to a stationary process. Indeed, there is evidence of cointegration between RER, productivity, net foreign assets, government expenditures, and terms of trade (Céspedes and De Gregorio 1999) and between RER and black exchange rates (parallel market) (Diamandis 2003). It also seems that the stationarity of RER depends on the analyzed period; for instance, Délano and Valdés (1998) shows that RER behaves as an $I(0)$ process when the period 1830–1995 is considered but as an $I(1)$ process in the period 1918–1995.

The Chilean economy, similar to other economies in South America, depends strongly on its commodities prices. Copper is the main export commodity in Chile; it accounted for 54% of Chile's exports, 14% of fiscal revenue, and 13% of nominal GDP in 2012 (Wu 2013). Chile has become increasingly important in the world copper market because its share of global production has increased to somewhat more than a third since the late 1960s (De Gregorio and Labbé 2011).

A number of studies have analyzed how copper prices affect the Chilean economy through its effects on nominal exchange rates, terms of trade, and business cycles. The results suggest that a positive shock to the copper price leads to appreciation in nominal and real exchange rates, output expansion, and an increased inflation rate (Cowan et al. 2007; Medina and Soto 2007).

In the long run, copper prices appear to explain most of the fluctuations in the Chilean peso, but in the short run, other factors, including interest rate spread, global financial risk, and local pension funds foreign exchange derivative position, may explain these fluctuations (Wu 2013). The fact that RER has acted as a shock absorber due to the flexible exchange rate regime, a rule-based fiscal policy, and a flexible inflation targeting system might explain why the Chilean economy has become increasingly resilient to copper price shocks in the last 25 years (De Gregorio and Labbé 2011).

This paper finds, based on the estimation of a CVAR model, that the long and persistent swings in the real exchange rate are compensated by movements in the interest rate spread, which restores the equilibrium in the product market when the real exchange rate moves away from its long-run benchmark value. Fluctuations in the copper price also explain the deviations of the real exchange

[2] A CVAR scenario tests the empirical consistency of the basic underlying assumptions of a model rather than imposing them on the data from the outset (Juselius 2017a).

[3] Duncan and Calderón (2003), and Froot and Rogoff (1995) present a thorough review of the literature on PPP testing.

rate from its long-run equilibrium value. The latter is consistent with the finding that in commodity exporters economies, variations in exchange rates are not random, but tightly linked to movements in commodity prices (Kohlscheen et al. 2017). Additionally, the results indicate error-increasing behavior in prices and interest rates, which is consistent with the persistence in the data.

The paper is organized as follows. Section 2 presents a theoretical framework based on IKE for exchange rate determination. Section 3 introduces the cointegrated vector of autoregressive model for variables that are integrated of order 2, $I(2)$. Section 4 presents stylized facts about Chilean data. Section 5 shows an empirical analysis of the data and presents a long-run structure. Section 6 concludes.

2. Theoretical Framework

2.1. Parity Conditions

This subsection introduces one of the most important parity conditions of open-economy macroeconomic models: the purchasing power parity (PPP) condition. This parity condition states that once converted to a common currency, via nominal exchange rate, national price levels should equalize (Bacchiocchi and Fanelli 2005). The absolute form (or strong form) of the PPP condition is expressed as:

$$P_{d,t} = S_t P_{f,t} \tag{1}$$

where $P_{d,t}$ is the domestic price level, $P_{f,t}$ is the foreign price level, S_t is the nominal exchange rate defined as the domestic-currency price in a unit of foreign currency, and t stands for time.

If p_d, p_f and s are, respectively, the natural logarithm of P_d, P_f, and S, Equation (1) can be rewritten as:

$$p_{d,t} = p_{f,t} + s_t \tag{2}$$

and the long-run PPP condition is expressed as:

$$p_{d,t} - p_{f,t} - s_t = \mu + ppp_t \tag{3}$$

where μ is a constant that reflects differences both in units of measure and in base-year normalization of price indices (Mark 2001), and ppp_t is a stationary error term that represents the deviations from PPP.[4] If the PPP condition holds in the goods market, then by definition, the log of the real exchange rate,[5] q_t, behaves as a stationary process, that is:

$$q_t = s_t + p_{f,t} - p_{d,t} \sim I(0). \tag{4}$$

Moreover, deviations from the uncovered interest parity (UIP) condition, that is, the excess returns on foreign exchange, er_t, would be stationary,[6] so that:

$$er_t = \left(i_d - i_f\right) - \left(s^e_{t+1} - s_t\right) \sim I(0) \tag{5}$$

where i_d and i_f are, respectively, the domestic and foreign interest rated and the superscript e denotes an expected value.

Empirical evidence finds, however, that the real exchange rates and excess returns behave as nonstationary processes, suggesting that the assumptions behind Equations (4) and (5) are

[4] In empirical testing, the PPP condition is normally replaced by $s_t = \mu + \gamma_1 p_{d,t} + \gamma_2 p_{f,t} + ppp_t$, where $\gamma_1 = -\gamma_2 = 1$ is expected.

[5] The real exchange rate is defined as $Q_t = S_t \frac{P_{f,t}}{P_{d,t}}$. It corresponds to the ratio of the foreign price level and the domestic price level, once the foreign price has been converted to the domestic currency through the nominal exchange rate.

[6] If deviations from PPP are assumed to be near $I(1)$, the deviations from UIP also behave as nonstationary, near-$I(1)$ processes.

untenable when using real data (see Juselius 2010, 2017a, 2017b; Juselius and Assenmacher 2017; Johansen et al. 2010; and Frydman and Goldberg 2007).

2.2. Persistence in the Data

This subsection[7] presents a theoretical framework, developed in Juselius (2017a) and based on IKE, that is consistent with the long and persistent swings in the real exchange rate. The model assumes that the nominal exchange rate is mainly driven by relative prices, that is:

$$s_t = B_0 + B_{1,t}\left(p_d - p_f\right)_t + \nu_t \tag{6}$$

where ν_t is a standard i.d.d. Gaussian error term that captures changes in interest rates and income. B_0 is a constant term, and $B_{1,t}$ is a time-varying coefficient that represents the weight to relative prices in financial actors' forecasts. Generally, the weight depends on how far the nominal exchange is from its long-run benchmark value. Based on (6), changes in the nominal exchange can be expressed as:

$$\triangle s_t = B_{1,t}\triangle\left(p_d - p_f\right)_t + \triangle B_{1,t}\left(p_d - p_f\right)_t + \triangle\nu_t. \tag{7}$$

One can assume, as in Frydman and Goldberg (2007), that $\left|B_{1,t}\triangle\left(p_d - p_f\right)_t\right| \gg \left|\triangle B_{1,t}\left(p_d - p_f\right)_t\right|$,[8] so that:

$$\triangle s_t \simeq B_{1,t}\triangle\left(p_d - p_f\right)_t + \triangle\nu_t. \tag{8}$$

Before estimating the above model using the CVAR, the issue of time-varying parameters must be addressed. Tabor (2014) simulates data for the process $y_t = \beta_t' x_t + \varepsilon_t$ where x_t is nonstationary $I(1)$, ε_t is an i.i.d. Gaussian error term and $\beta_t = \beta + Z_t$ where $Z_t = \varrho Z_{t-1} + \varepsilon_{Z,t}$ and $\varrho < 1$. Tabor (2014) showed that when a CVAR model is applied to the simulated data, the estimated cointegrated coefficient corresponds to $E\left(\beta_t\right)$. Hence, based on this result, one can argue that the CVAR model may be used to estimate average long-run relationships when the underlying data-generating process involves bounded-parameter instability.

Then, the change in the real exchange rate should behave as a near $I(1)$ process provided that $B_{1,t} = B + \rho B_{1,t-1} + \varepsilon_{B_1,t}$ with $\rho < 1$, but close to one. Juselius (2014) argues that the latter behavior can be used to approximate the the change in the real exchange rate through the following process:

$$\triangle q_t = a_t + \nu_{q,t} \tag{9}$$

where $\nu_{q,t}$ is an i.i.d. Gaussian error term and the time-varying drift term, a_t, measures the appreciation or depreciation of the real exchange rate due to changes in individual forecasting strategies.[9] This drift is assumed to follow a mean zero stationary autoregressive process, so that:

$$a_t = \rho_t a_{t-1} + \nu_{a,t} \tag{10}$$

---

7 This subsection is based mainly on Juselius (2017a), Juselius and Assenmacher (2017), and Frydman and Goldberg (2007, 2011).

8 This assumption is based on simulations that show that $\triangle B_{1,t}$ has to be extremely large for $\triangle B_{1,t}\left(p_d - p_f\right)_t$ to have a marked effect on $\triangle s_t$. Frydman and Goldberg (2007) use this assumption ("conservative revision") in their IKE-based monetary model to illustrate the fact that forecasting behavior is led by new realizations of the causal variables, $\triangle\left(p_d - p_f\right)_t$, rather than revision of forecasting strategies, $\triangle B_{1,t}$.

9 This is consistent with the FG IKE-based model developed by Frydman and Goldberg (2007), which assumes that individuals recognize their imperfect knowledge about the underlying processes that drive outcomes. Thus, they use a multitude of forecasting strategies that are revised over time in a way that cannot be fully prespecified. Indeed, given the diversity of forecasting strategies, this model assumes two kinds of individuals in the foreign currency market: bulls, who speculate on the belief that the asset price will rise, and bears, who speculate on its fall.

where $\nu_{a,t}$ is an i.i.d. Gaussian error term and ρ_t is a time-varying coefficient that is close to one when the real exchange rate is in the vicinity of its long-run benchmark value, and otherwise $\rho_t \ll 1$.[10] The average of this coefficient, $\bar{\rho}$, is generally close to one whenever the sample period is sufficiently long (Juselius 2017a). Then, a_t describes a persistent near $I(1)$ process, and modeling the real exchange rate as a near $I(2)$ process is consistent with swings of shorter and longer duration, implying that the length of these swings is not predictable (Frydman and Goldberg 2007).

Since excess return on the foreign exchange rate is often found to behave like a nonstationary process—the excess return puzzle—it has been argued that volatility in the foreign currency market should be taken into account. Specifically, a risk premium, rp, might be added to (5) to obtain a stationary relationship. However, it is unlikely that a risk premium, assumed to be stationary, accounts for the persistent swings in the real interest rate spread. Frydman and Goldberg (2007), in their FG IKE-based model, proposed to replace the uncovered interest rate parity, UIP, condition—the market clearing mechanism between the expected change in the nominal exchange rate and the nominal interest rate spread—by an uncertainty adjusted uncovered interest rate parity (UA-UIP) condition, that is defined as:

$$\left(i_d - i_f\right)_t = \left(s^e_{t+1} - s_t\right) + rp_t + up_t \tag{11}$$

where up_t stands for an uncertainty nonstationary premium, a measure of agents' loss averseness.[11] The interest rate spread corrected for the uncertainty premium is a minimum return that agents require to speculate in the foreign exchange market. This premium starts increasing when the nominal exchange rate moves away from its long-run benchmark value and decreases when the nominal exchange rate moves toward equilibrium. In the foreign exchange market, the uncertainty premium is related to the PPP gap (Frydman and Goldberg 2007). Then, the UA-UIP is formulated as:

$$\left(i_d - i_f\right)_t = \left(s^e_{t+1} - s_t\right) + rp_t + f\left(p_d - p_f - s_t\right). \tag{12}$$

This equation suggests that in a world of imperfect knowledge, the expected change in the nominal exchange rate may not be directly related to the interest rate spread, but to the spread corrected by the PPP gap and the risk premium. The latter might be associated with short-term changes in interest rates, inflation rates and nominal exchange rates (Juselius 2017a).

2.3. Theory-Consistent CVAR Scenario Results

A consequence of the UA-UIP condition is that both domestic and foreign interest rates are affected by the uncertainty premium. Juselius (2017a) suggests the following data-generating process to describe changes in the interest rate:

$$\triangle i_{j,t} = \omega_{j,t} + \triangle rp_{j,t} + \nu_{j,t} \tag{13}$$

where $\nu_{j,t}$ is a white nose error term and $j = d, f$. The term $\omega_{j,t}$ stands for changes in the domestic uncertainty premium, $\omega_{j,t} = \triangle up_{j,t}$, and is assumed to follow a mean zero stationary autoregressive process:

$$\omega_{j,t} = \rho^{\omega}_{j,t}\omega_{j,t-1} + \nu^{\omega}_{j,t} \tag{14}$$

where $\nu^{\omega}_{j,t}$ is a stationary error term. The time-varying autoregressive coefficient, $\rho^{\omega}_{j,t}$, is assumed to be almost on the unit circle when the nominal exchange rate is in the vicinity of its long-run

10 When periods where a_t is far from its benchmark value are shorter compared with the near vicinity periods, it describes a persistent but mean-reverting process.

11 Frydman and Goldberg (2007) extend the concept of loss aversion given by Kahneman and Tversky (1979) to the concept of endogenous loss aversion, which says that the greater the potential loss, the higher the degree of loss aversion. This definition establishes that the UA-UIP equilibrium exists.

benchmark value—the relative price—otherwise the coefficient is strictly less than one. Nevertheless, $\rho_j^{\bar{\omega}} \approx 1$ provided that periods where the coefficient is close to one are much longer than otherwise. When $\rho_j^{\bar{\omega}} \approx 1$, (14) describes a near $I(1)$ domestic uncertainty premium. Consequently, under IKE, the interest rate change behaves as a persistent near $I(1)$ process, implying that nominal interest rates are near $I(2)$.

Using a CVAR scenario, Juselius (2017a) demonstrates that the following hypotheses are consistent with IKE:

$$s_t \sim \text{near } I(2) \tag{15}$$

$$\left(p_{d,t} - p_{f,t}\right) \sim \text{near } I(2) \tag{16}$$

$$\left(i_{d,t} - i_{f,t}\right) \sim \text{near } I(2) \tag{17}$$

$$\left(s_t + p_{f,t} - p_{d,t}\right) \sim \text{near } I(2) \tag{18}$$

$$\left\{\left(i_{d,t} - i_{f,t}\right) - c\left(s_t + p_{f,t} - p_{d,t}\right)\right\} \sim \text{near } I(1) \tag{19}$$

where c is a constant coefficient. These relationships show that when allowing for IKE, real exchange rate, interest rate spread, and relative price are likely to behave as near $I(2)$.

3. The CVAR Model and the $I(2)$ Representation

A VAR model in second order differences is expressed as:[12]

$$\triangle^2 \boldsymbol{x}_t = \boldsymbol{\Pi}\boldsymbol{x}_{t-1} - \boldsymbol{\Gamma}\triangle\boldsymbol{x}_{t-1} + \sum_{i=1}^{k-2} \boldsymbol{\Psi}_i \triangle^2 \boldsymbol{x}_{t-1} + \boldsymbol{\Phi}\boldsymbol{D}_t + \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 t + \boldsymbol{\varepsilon}_t \tag{20}$$

where $\boldsymbol{x}_t' = \left[x_{1,t}, x_{2,t}, \ldots, x_{p,t}\right]$ is a p-dimensional vector of stochastic variables, $\boldsymbol{D}_t$ is a matrix of deterministic terms (shift dummies, seasonal dummies, etc) with coefficient matrix $\boldsymbol{\Phi}$. $\boldsymbol{\Pi}, \boldsymbol{\Gamma}$ are $p \times p$ coefficient matrices, $\boldsymbol{\mu}_0$ is an unrestricted constant, t is an unrestricted trend with coefficient matrix $\boldsymbol{\mu}_1$, and $\boldsymbol{\varepsilon}_t$ is a multivariate white noise process, that is $\boldsymbol{\varepsilon}_t \sim \text{i.i.d.}\,\mathcal{N}_p\left(\boldsymbol{0}, \boldsymbol{\Omega}\right)$.

If $\boldsymbol{\Pi}$ has reduced rank, $0 < r < p$, it can be decomposed into $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times r$ matrices of full column rank. The orthogonal complement of matrix $\boldsymbol{z}$ is denoted as $\boldsymbol{z}_\perp$, and $\bar{\boldsymbol{z}} = \boldsymbol{z}\left(\boldsymbol{z}'\boldsymbol{z}\right)^{-1}$. Structuring the $I(2)$ representation of the CVAR model is a bit more complicated, and additional definitions must be given. The $I(2)$ model is defined by the two following reduced rank restrictions:

$$\begin{aligned} \boldsymbol{\Pi} &= \boldsymbol{\alpha}\boldsymbol{\beta}' \\ \boldsymbol{\alpha}_\perp'\boldsymbol{\Gamma}\boldsymbol{\beta}_\perp &= \boldsymbol{\xi}\boldsymbol{\eta}' \end{aligned} \tag{21}$$

where $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are $(p-r) \times s_1$ matrices, s_1 is the number of $I(1)$ trends, or unit root processes, and it is such that $p - r = s_1 + s_2$, where s_2 is the number of $I(2)$ trends, or double unit root processes, in vector $\boldsymbol{x}_t$. Whereas the first rank condition in (21) is associated with the variables in levels, the second rank condition is related to the differentiated variables.

$\boldsymbol{\beta}_\perp$ and $\boldsymbol{\alpha}_\perp$ can, respectively, be decomposed into $\boldsymbol{\beta}_\perp = [\boldsymbol{\beta}_{\perp 1}, \boldsymbol{\beta}_{\perp 2}]$ and $\boldsymbol{\alpha}_\perp = [\boldsymbol{\alpha}_{\perp 1}, \boldsymbol{\alpha}_{\perp 2}]$. Matrices $\boldsymbol{\alpha}_{\perp 1} = \overline{\boldsymbol{\alpha}_\perp}\boldsymbol{\eta}$ and $\boldsymbol{\beta}_{\perp 1} = \overline{\boldsymbol{\beta}_\perp}\boldsymbol{\eta}$ are of dimension $p \times s_1$. Matrices $\boldsymbol{\alpha}_{\perp 2} = \boldsymbol{\alpha}_\perp\boldsymbol{\xi}_\perp$ and $\boldsymbol{\beta}_{\perp 2} = \boldsymbol{\beta}_\perp\boldsymbol{\eta}_\perp$ have dimension $p \times s_2$.

[12] This section is based mainly on Doornik and Juselius (2017) and Juselius (2006).

Using the Johansen (1997) parametrization, model (20) can be written as follows:

$$\begin{aligned}\triangle^2 x_t =& \alpha\left(\rho'\tau' x_{t-1} + d'\triangle x_{t-1}\right) + \zeta'\tau'\triangle x_{t-1} + \sum_{i=1}^{k-2}\Lambda_i\triangle^2 x_{t-i} + \\ & \Phi D_t + \mu_0 + \mu_1 t + \varepsilon_t\end{aligned} \tag{22}$$

where $\rho = [\mathbf{I}, \mathbf{0}]'$, $\tau = [\beta, \beta_{\perp 1}]$, $d' = -\left(\left(\alpha'\Omega^{-1}\alpha\right)^{-1}\alpha\Omega^{-1}\Gamma\right)\tau_{\perp}\left(\tau'_{\perp}\tau_{\perp}\right)^{-1}\tau'_{\perp}$, $\zeta = [\zeta_1, \zeta_2]'$ is a matrix of medium-run adjustment.

In this model, the term in $(\cdot)$ represents the long-run equilibrium or polynomially cointegrating relationships. The term $\zeta'\tau'\triangle x_{t-1}$ can be interpreted as a medium-run equilibrium relationship, defining the $r + s_1$ relationship that needs to be differentiated to become stationary.

The moving average (MA) representation of the $I(2)$ model is expressed as:

$$\begin{aligned}x_t =& C_2\sum_{i=1}^{t}\sum_{s=1}^{i}\underbrace{(\varepsilon_s + \Phi D_s + \mu_0 + \mu_1 s)}_{\varepsilon_s} + C_1\sum_{i=1}^{t}\underbrace{(\varepsilon_i + \Phi D_i + \mu_0 + \mu_1 i)}_{\varepsilon_i} + \\ & C^*(L)\left(\varepsilon_t + \Phi D_t + \mu_0 + \mu_1 t\right) + A + Bt\end{aligned} \tag{23}$$

where $C_2 = \beta_{\perp 2}\left(\alpha'_{\perp 2}\Theta\beta_{\perp 2}\right)^{-1}\alpha'_{\perp 2}$, $\beta' C_1 = \bar{\alpha}'\Gamma C_2$, $\beta'_{\perp 1} C_1 = \overline{\alpha_{\perp 1}}'\left(\mathbf{I}_p - \Theta C_2\right)$, and $\Theta = \Gamma\bar{\beta}\bar{\alpha}'\Gamma + \left(\mathbf{I}_p - \sum_{i=1}^{k-2}\Lambda_i\right)$. A and B are functions of both the initial values and the model parameters (Johansen 1992).[13]

Matrix C_2 can be expressed as $C_2 = \breve{\beta}_{\perp 2}\alpha'_{\perp 2}$, where $\breve{\beta}_{\perp 2} = \beta_{\perp 2}\left(\alpha'_{\perp 2}\Theta\beta_{\perp 2}\right)^{-1}$, so that $\alpha'_{\perp 2}\sum_{i=1}^{t}\sum_{s=1}^{i}\varepsilon_s$ can be interpreted as the measure of the s_2 trends which load into the variables in x_t with the weights $\breve{\beta}_{\perp 2}$ (Juselius 2006).

The likelihood ratio test for the joint hypothesis of r cointegrating relationships and s_1 and s_2 trends, labeled $\mathcal{H}(r, s_1, s_2)$, versus $\mathcal{H}(p)$ is given by:

$$-2\log Q\left(\mathcal{H}(r, s_1, s_2) \mid \mathcal{H}(p)\right) = -T\log\left|\tilde{\Omega}^{-1}\hat{\Omega}\right| \tag{24}$$

where $\tilde{\Omega}$ and $\hat{\Omega}$ are, respectively, the covariance matrices estimated under $\mathcal{H}(r, s_1, s_2)$ and $\mathcal{H}(p)$.[14]

4. Stylized Facts

Figure 1a shows the evolution of the natural logarithm (log) of the nominal exchange rate, measured as Chilean pesos (CLP) per US dollar (USD) and the log of the relative prices, measured as the ratio between the Chilean consumer price index (CPI) and the US CPI. Relative prices exhibit a positive but decreasing slope, reflecting the fact that from 1986 until 1999, Chilean prices were growing faster than US prices, but after 1999 the growth in relative prices decreased. This might be associated with the partial implementation of inflation targeting in Chile in 1990, which reduced annual inflation from 26% in 1990 to 3% in 1997. In the same panel, the nominal exchange rate undergoes long and persistent swings around relative prices, suggesting that PPP may hold only as a very long-run condition.

---

13 From the MA representation (23), it follows that the unrestricted constant, μ_0, cumulates once to a linear trend and twice to a quadratic trend. In addition, the unrestricted trend, μ_1, cumulates once to a quadratic trend and twice to a cubic trend. To avoid the latter, quadratic and cubic trends have been restricted to zero in the subsequent analysis. For further information, see Chapter 17 in Juselius (2006).

14 The distribution of the this is found in Johansen (1995) provided that model (22) does not restrict deterministic components; otherwise see Rahbek et al. (1999).

Figure 1b shows the PPP gap, defined as the difference between the log of relative prices and the log of the nominal exchange rate. The deviations exhibit long persistent swings, but it seems that the upward trend in relative prices is canceled by the upward trend in nominal exchange rate.

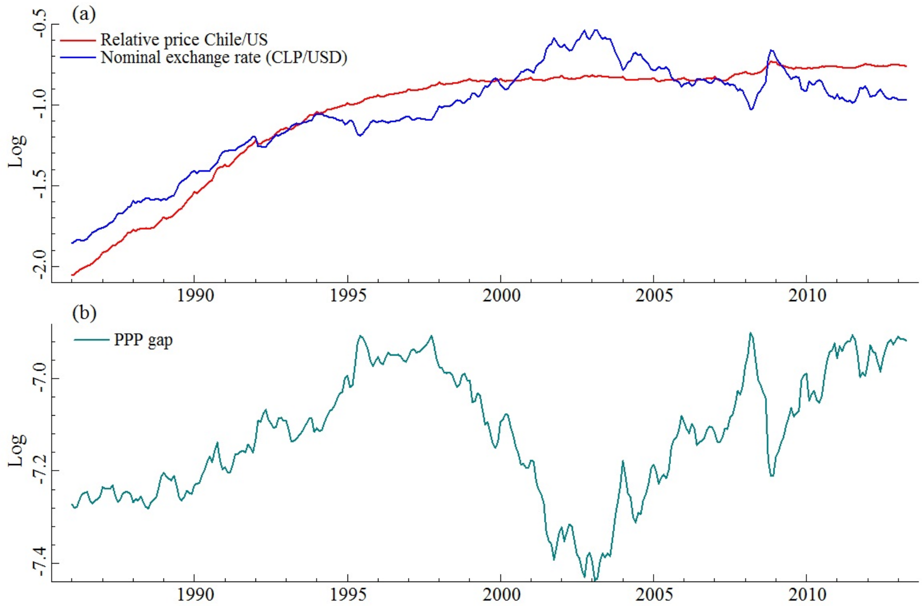


Figure 1. (**a**) Nominal exchange rate (CLP/USD) and relative prices (Chilean CPI/US CPI); (**b**) Deviations from PPP. Monthly data 1986:1–2013:04. CLP: Chilean peso, USD: U.S. dollar.

Figure 2a shows that relative inflation rates exhibit a high persistence, which is corroborated by the 12-month moving average. This persistence seems, however, to decrease steadily beginning in 1990, which may be associated with the implementation of inflation targeting in Chile in 1990. In Figure 2b shows the changes in the nominal exchange rate, which seems stationary. Nevertheless, the 12-month moving average exhibits some persistence around the mean. It also appears that appreciations and depreciations are more volatile since 2000, which might be related to the free-floating exchange rate regime that was implemented by the Central Bank of Chile in September 1999. Figure 2c, shows that changes in the PPP gap behave as a persistent but mean-reverting process. The 12-month moving average exhibits persistence around the mean that seems higher since 2000.

Figure 3a,b show, respectively, the Chilean interest rate and its first difference. The latter exhibits a large decrease in volatility since 2000. This might be associated with two major reforms that were introduced in the Chilean financial market between 2000 and 2001. While the first reform, promulgated in 2000, gave greater protection to both domestic and foreign investors, the second reform, enacted in 2001, liberalized the financial system, implying, among other things, capital account deregulation.

When the Chilean interest rate and its first difference are compared with their US counterparts, which are shown in Figure 3c,d, an important difference in levels and volatility is noticeable. The Chilean interest rate has been historically higher than the US interest rate, and this seems to have changed since 2000. The latter is clearly reflected in the interest rate spread shown in Figure 3e. The changes in the interest rate spread shown in Figure 3f seem to mimic the changes in the Chilean interest rate volatility.

Figure 4a plots the copper price and the PPP gap. Two facts are noticeable. First, it seems that both variables are positively co-moving over time, suggesting that there is a negative relationship between copper prices and real exchange rates. Second, since 2005, the copper price has been higher than in the previous years, which might be associated with an increase in world copper demand. The decrease of in the copper price observed in 2008 was mainly caused by lower copper demand due to the international financial crisis. Figure 4b shows that the copper price was more volatile at the beginning and end of the sample, and its 12-month moving average suggests some persistence around its mean.

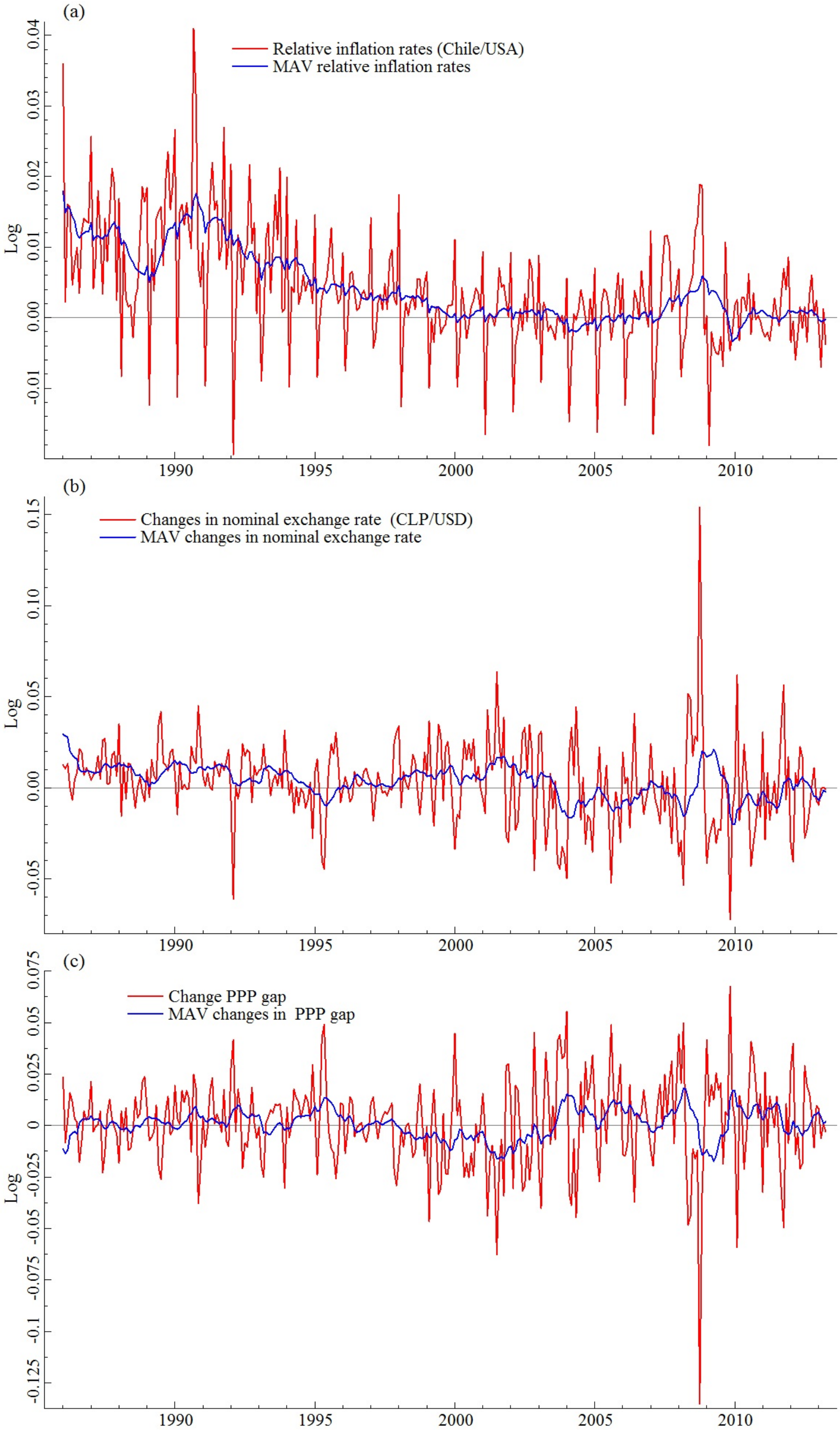


Figure 2. (**a**) Relative inflation rates (Chile/US); (**b**) Changes in nominal exchange rate (CLP/USD); (**c**) Change in the PPP gap. Monthly data 1986:1–2013:4. MAV is the 12-month moving average process.

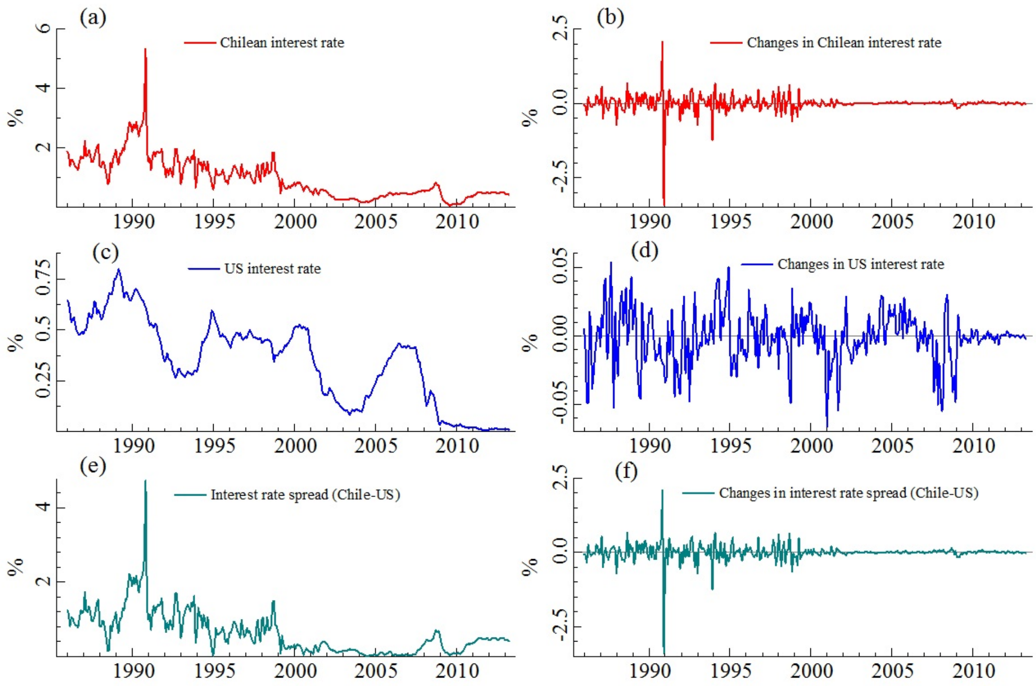


Figure 3. (**a**) Chilean interest rate; (**b**) Changes in Chilean interest rate; (**c**) US interest rate; (**d**) Changes in US interest rate; (**e**) Interest rate spread (Chile-US); (**f**) Changes in interest rate spread. Monthly data: 1986:1–2013:4.

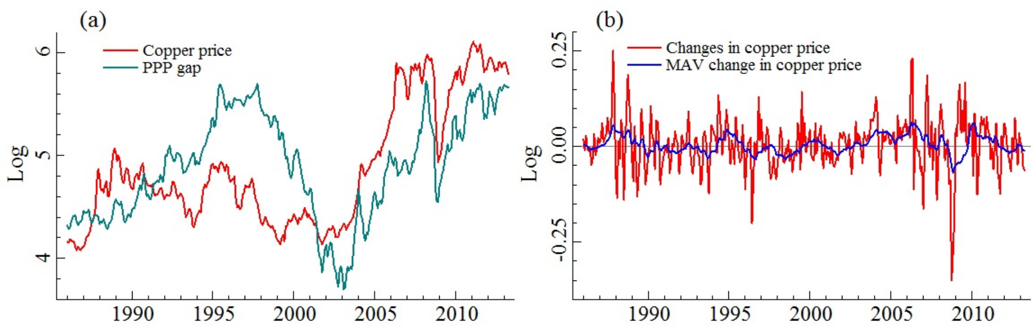


Figure 4. (**a**) Copper price and PPP gap; (**b**) Changes in the copper price. Monthly data 1986:1–2013:4. MAV is the 12-month moving average of the changes in copper price. PPP gap is calculated as the difference between the log of relative prices and the log of the nominal exchange rate.

This section discussed the pronounced persistence exhibited in the data. For instance, the graphical analysis seems to suggest that nominal exchange rate, real exchange rate, and relative prices behave as a nonstationary near $I(2)$ process. However, this persistence has to be formally tested, which is done in Section 5.

5. Empirical Model Analysis

The monthly data cover the period 1986:1–2013:4 and the baseline model, which contains three lags,[15] is expressed as:

[15] Appendix B presents the selection of the number of lags.

$$\triangle^2 x_t = \alpha \left[\tilde{\rho}' \underbrace{\begin{pmatrix} \tau \\ \tau_0 \end{pmatrix}'}_{\tilde{\tau}'} \underbrace{\begin{pmatrix} x_{t-1} \\ t-1 \end{pmatrix}}_{\tilde{x}_{t-1}} + \underbrace{\begin{pmatrix} d \\ d_0 \end{pmatrix}'}_{\tilde{d}'} \underbrace{\begin{pmatrix} \triangle x_{t-1} \\ \mathbf{1} \end{pmatrix}}_{\triangle \tilde{x}_{t-1}} \right] + \zeta' \tilde{\tau}' \triangle \tilde{x}_{t-1} + \Lambda_1 \triangle^2 x_{t-1} + \Phi_p D_{p,t} + \Phi_s D_{s,t} + \varepsilon_t \quad (25)$$

where $x_t' = \left[p_{d,t}, p_{f,t}, s_t, cp_t, i_{d,t}, i_{f,t} \right]$, $p_{d,t}$ is the Chilean CPI, $p_{f,t}$ is the US CPI, s_t is the nominal exchange rate, defined as CLP per USD, cp_t is the copper price, $i_{d,t}$ is the Chilean interest rate, and $i_{f,t}$ is the US interest rate.[16] All variables except interest rates are in natural logarithms. $\tilde{\rho} = [\rho, \mathbf{0}]$ and picks out the r cointegrating vectors, including the restricted trend, $\mathbf{1}$ is a vector of constant terms and t is a linear trend. $D_{p,t}$ is a (9×1) vector of intervention dummies,[17] and $D_{s,t}$ is a (11×1) vector of centered seasonal dummies.[18] The software CATS 3 for OxMetrics (Doornik and Juselius 2017) was used in the econometric analysis.

Table 1 reports the residual misspecification tests of model (25).[19] The upper part indicates that the hypotheses of normality and non-ARCH of orders 1 and 2 can be rejected but not the hypothesis of non-autocorrelation. The univariate tests, reported in the lower part, show that all equations exhibit residual non-normality and that only the residuals of the copper price do not show ARCH effects. It appears that the normality problem is due to excess kurtosis rather than excess skewness. Financial variables usually exhibit non-normality and ARCH problems, but adding more dummy variables is not necessarily a solution (Juselius 2010). Moreover, VAR estimates are robust for moderately excess kurtosis (Gonzalo 1994; Juselius 2006).

Table 1. Misspecification tests for CVAR model (25).

Multivariate Specification Tests				
Autocorrelation		**Normality**	**ARCH**	
Order 1: χ^2 (36)	Order 2: χ^2 (36)	χ^2 (12)	Order 1: χ^2 (441)	Order 2: χ^2 (882)
45.25 [0.14]	41.66 [0.24]	128.94 [0.00]	514.01 [0.01]	1007.42 [0.00]

Univariate Specification Tests						
Equation	$\triangle^2 p_{d,t}$	$\triangle^2 p_{f,t}$	$\triangle^2 s_t$	$\triangle^2 cp_t$	$\triangle^2 i_{d,t}$	$\triangle^2 i_{f,t}$
ARCH Order 2: χ^2 (2)	27.92 [0.00]	11.15 [0.00]	6.57 [0.04]	0.88 [0.64]	21.93 [0.00]	23.27 [0.00]
Normality χ^2 (2)	12.83 [0.00]	15.99 [0.00]	12.86 [0.00]	34.31 [0.00]	36.05 [0.00]	6.26 [0.04]
Skewness	0.23	0.07	−0.12	0.03	0.02	−0.17
Kurtosis	3.99	4.12	3.98	4.81	4.87	3.60
S.E.$\times 10^3$	4.38	1.96	16.68	52.17	1.51	0.15

$[\cdot]$ is the p-value of the test; S.E. is the residual standard error.

16 Appendix A presents the source, description, and transformation of the data. Dataset and code to replicate the results are available from the author.
17 Appendix C specifies the intervention dummies and their estimated coefficients.
18 Initially, the cointegration space considered a broken linear trend that started in September 1999, corresponding to the beginning of the floating exchange rate regime in Chile. However, this broken linear trend was revealed to be non-significant. The potential effect of the new regime on the nominal exchange rate was, possibly, offset by changes in the Chilean inflation rate and/or interest rate.
19 For a thorough description of the tests see Doornik and Juselius (2017).

5.1. Rank Determination

Table 2 reports the $I(2)$ trace test and shows the maximum likelihood test of the joint hypothesis of (r, s_1), which corresponds to the two rank restrictions in (21), together with simulated p-values of the trace test. The standard test procedure starts with the most restricted model, $(r = 0,\ s_1 = 0,\ s_2 = 6)$, which is reported in the first row with a likelihood ratio test of 1120.90; it then continues from this point to the right, and row by row, until the first joint hypothesis is not rejected. The first rejection corresponds to the case $(r = 2,\ s_1 = 2,\ s_2 = 2)$ with a p-value of 0.12. The case $(r = 1,\ s_1 = 4,\ s_2 = 1)$ is also not rejected, though with a lower p-value of 0.07.

Table 2. Simulated asymptotic distribution of the cointegration rank indices model (25).

$p - r$	r	$s_2 = 6$	$s_2 = 5$	$s_2 = 4$	$s_2 = 3$	$s_2 = 2$	$s_2 = 1$	$s_2 = 0$
6	0	1120.90 [0.00]	797.61 [0.00]	582.76 [0.00]	425.17 [0.00]	314.38 [0.00]	232.61 [0.00]	195.29 [0.00]
5	1		579.62 [0.00]	413.11 [0.00]	289.11 [0.00]	179.09 [0.00]	96.24 [0.07]	92.28 [0.01]
4	2			286.97 [0.00]	169.23 [0.00]	84.13 [0.12]	59.00 [0.39]	53.96 [0.18]
3	3				76.83 [0.17]	47.78 [0.61]	31.90 [0.77]	28.59 [0.48]
2	4					26.20 [0.86]	18.59 [0.74]	16.60 [0.37]
1	5						9.31 [0.65]	7.13 [0.28]

p is the number of variables in vector $\boldsymbol{x}$; r is the number of cointegrating relationships; s_1 and s_2 are, respectively, the number of $I(1)$ and $I(2)$ trends.

As a robustness check, Table 3 reports the seven largest characteristic roots for $r = 2$, and $r = 6$. The unrestricted model, $(r = 6,\ s_1 = 0,\ s_2 = 0)$, has six large roots: five almost on the unit circle and one large but less close to 1 (0.82). Under the assumption that $\boldsymbol{x}_t \sim I(1)$, that is $(r = 2,\ s_1 = 4,\ s_2 = 0)$, there would be two large roots (0.98 and 0.82) in the model. Under such persistence, treating the process $\boldsymbol{x}_t$ as $I(1)$ is likely to yield unreliable inference (Johansen et al. 2010).

Therefore, the reduced rank model should account for 6 unit roots. The case $(r = 2,\ s_1 = 2,\ s_2 = 2)$ implies six characteristic roots to be on the unit circle and leaves 0.56 as the largest unrestricted root. Thus, based on the above discussion, the analysis considers the case $(r = 2,\ s_1 = 2,\ s_2 = 2)$, which implies $\boldsymbol{x}_t \sim$ near $I(2)$.

Table 3. Model adequacy.

Seven Largest Characteristic Roots							
Model	**Moduli**						
$(r = 6,\ s_1 = 0,\ s_2 = 0)$	0.98	0.98	0.98	0.95	0.95	0.82	0.56
$(r = 2,\ s_1 = 4,\ s_2 = 0)$	1.00	1.00	1.00	1.00	0.98	0.82	0.56
$(r = 2,\ s_1 = 2,\ s_2 = 2)$	1.00	1.00	1.00	1.00	1.00	1.00	0.56

r is the number of cointegrating relationships; s_1 and s_2 are, respectively, the number of $I(1)$ and $I(2)$ trends.

5.2. Partial System

The copper price was found to be a strong exogenous variable based on $\chi^2\,(15) = 13.83$ with p-value of 0.05. Thus, copper price is pushing the system but not adjusting to it. Because the copper price is internationally determined, this finding is economically plausible. Therefore, a partial system can be modeled with vector $\boldsymbol{x}_t' = \left\{\boldsymbol{x}_{1,t}', \boldsymbol{x}_{2,t}'\right\}$, where $\boldsymbol{x}_{1,t}' = \left[p_{d,t}, p_{f,t}, s_t, i_{d,t}, i_{f,t}\right]$ and $\boldsymbol{x}_{2,t}' = [cp_t]$. Then, Equation (25) is reformulated as:

$$\begin{aligned}\triangle^2 \boldsymbol{x}_{1,t} =& \boldsymbol{\alpha}\left(\tilde{\boldsymbol{\rho}}'\tilde{\boldsymbol{\tau}}'\tilde{x}_{t-1} + \tilde{\boldsymbol{d}}'\triangle\tilde{x}_{t-1}\right) + \boldsymbol{\zeta}'\tilde{\boldsymbol{\tau}}'\triangle\tilde{x}_{t-1} + \\ &+ \boldsymbol{\Lambda}_1\triangle^2\boldsymbol{x}_{1,t-1} + \sum_{j=0}^{1}\boldsymbol{\pi}_j\triangle^2\boldsymbol{x}_{2,t-i} + \boldsymbol{\Phi}_p\boldsymbol{D}_{p,t} + \boldsymbol{\Phi}_s\boldsymbol{D}_{s,t} + \boldsymbol{\varepsilon}_t\end{aligned} \tag{26}$$

where the left-hand side excludes the acceleration rate of the copper price and the right-hand side adds two second-order lagged differences of the copper price.

In the full model (25) the number of $I(2)$ trends was $s_2 = 2$. In the partial model (26) the number of $I(2)$trends decreases by one because the copper price was found to be an exogenous variable. This suggests that one of the previous two $I(2)$ trends is now accounted for the exogenous copper price. Therefore, the following analysis considers the case $(r = 2, s_1 = 2, s_2 = 1)$.

5.2.1. Testing Non-Identifying Hypotheses in the $I(2)$ Model

- **Same restrictions on all $\tilde{\boldsymbol{\tau}}$**

The hypothesis of same restrictions on all $\tilde{\boldsymbol{\tau}}$ is formulated as $\mathbf{R}'\tilde{\boldsymbol{\tau}} = \mathbf{0}$, where $\mathbf{R}$ is of dimension $p_1 \times (p_1 - m)$, p_1 is the dimension of $\tilde{x}$ and m is the number of free parameters. The test is asymptotically $\chi^2((r + s_1)(p_1 - m))$ distributed (Johansen 2006).

The upper part of Table 4 reports three hypothesis restrictions on all $\tilde{\boldsymbol{\tau}}$. The null hypothesis $\mathcal{H}_1$ entails that the nominal to real transformation may be used (Kongsted 2005). That is, $\boldsymbol{x}_t$ that is near $I(2)$ can be transformed into the $I(1)$ vector $\breve{x}'_t = \left[ppp_t, \triangle p_{d,t}, \triangle p_{f,t}, i_{d,t}, i_{f,t}, cp_t\right]$ without loss of information (Johansen et al. 2010). The result of $\mathcal{H}_1$ indicates that the PPP restriction can be rejected; that is, the transformation $\left(p_{d,t} - p_{f,t} - s_t\right)$ is not statistically supported.

The null hypothesis $\mathcal{H}_2$ entails price homogeneity. That is, vector $\boldsymbol{x}_t$ can be transformed into $\breve{\breve{x}}'_t = \left[p_{d,t} - p_{f,t}, s_t, \triangle p_{d,t}, i_{d,t}, i_{f,t}, cp_t\right]$ without loss of information. The result of $\mathcal{H}_2$ indicates that price homogeneity can be rejected; that is, the transformation $\left(p_d - p_f\right)_t$ is not statistically supported. Finally, the result of hypothesis $\mathcal{H}_3$ indicates that the restricted linear trend is no long-run excludable.

- **A known vector in $\tilde{\boldsymbol{\tau}}$**

In this case, a variable or relationship can be tested to be $I(1)$ in the $I(2)$ model. The restriction is expressed as $\tilde{\boldsymbol{\tau}} = (\mathbf{b}, \mathbf{b}_{\perp}\boldsymbol{\varphi})$ where $\mathbf{b}$ is a $p_1 \times n$ known vector, n is the number of known vectors in $\tilde{\boldsymbol{\tau}}$, and $\boldsymbol{\varphi}$ is a matrix of unknown parameters. The test is asymptotically $\chi^2((p_1 - r - s_1)n)$ distributed unless $\mathbf{b}$ is also a vector in $\tilde{\boldsymbol{\beta}}$ (Johansen 1996). Thus, $\mathbf{b} \in \text{sp}(\tilde{\boldsymbol{\beta}})$ must be checked to ensure the correct distribution of the test. If the hypothesis $\tilde{\boldsymbol{\tau}} = (\mathbf{b}, \mathbf{b}_{\perp}\boldsymbol{\varphi})$ is not rejected and $\mathbf{b} \notin \text{sp}(\tilde{\boldsymbol{\beta}})$, then the analyzed variable, or relationship, can be considered $I(1)$.

The lower part of Table 4 reports the test results[20] of which hypotheses $\mathcal{H}_4$ to $\mathcal{H}_7$ and $\mathcal{H}_9$ are consistent with the CVAR scenario based on IKE under which nominal exchange rate, prices, relative prices, and nominal interest rate are likely to behave as a near $I(2)$ process. According to IKE, the real exchange rate is likely to behave as a near $I(2)$ process, but the result of $\mathcal{H}_8$ indicates that the hypothesis of the real exchange rate being $I(1)$ cannot be rejected based on a p-value of 0.11. This is, nevertheless, consistent with the high persistence observed in the real exchange rate. In addition, the result of $\mathcal{H}_{10}$ indicates that the copper price is likely to behave as near $I(2)$.

---

[20] The hypothesis $\mathbf{b}_i \in \text{sp}(\tilde{\boldsymbol{\beta}})$ was rejected in all cases, except for the Chilean interest rate based on $\chi^2(5) = 10.42$ with a p-value of 0.06 and for the interest rate spread based on $\chi^2(5) = 6.80$ with a p-value of 0.23. Thus, the hypotheses $i_{d,t} \sim I(1)$ and $\left(i_{d,t} - i_{f,t}\right) \sim I(1)$ are not presented because the distribution of the test is not necessarily χ^2.

Table 4. Restrictions on $\tilde{\boldsymbol{\tau}}$.

Hypothesis	Matrix Restriction Design	Distribution	*p*-Value
PPP restriction $\mathcal{H}_1 : \mathbf{R}_1'\tilde{\boldsymbol{\tau}} = \mathbf{0}$	$\mathbf{R}_1' = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$	$\chi^2\,(8) = 40.70$	0.00
Price homogeneity $\mathcal{H}_2 : \mathbf{R}_2'\tilde{\boldsymbol{\tau}} = \mathbf{0}$	$\mathbf{R}_2' = [1, 1, 0, 0, 0, 0, 0]$	$\chi^2\,(4) = 38.66$	0.00
Excludable trend $\mathcal{H}_3 : \mathbf{R}_3'\tilde{\boldsymbol{\tau}} = \mathbf{0}$	$\mathbf{R}_3' = [0, 0, 0, 0, 0, 0, 1]$	$\chi^2\,(4) = 39.14$	0.00
Chilean price $\mathcal{H}_4 : \tilde{\boldsymbol{\tau}} = (\mathbf{b}_1, \mathbf{b}_{1\perp}\boldsymbol{\varphi})$	$\mathbf{b}_1 = [1, 0, 0, 0, 0, 0, 0]$	$\chi^2\,(3) = 25.15$	0.00
US price $\mathcal{H}_5 : \tilde{\boldsymbol{\tau}} = (\mathbf{b}_2, \mathbf{b}_{2\perp}\boldsymbol{\varphi})$	$\mathbf{b}_2 = [0, 1, 0, 0, 0, 0, 0]$	$\chi^2\,(3) = 27.19$	0.00
Relative price $\mathcal{H}_6 : \tilde{\boldsymbol{\tau}} = (\mathbf{b}_3, \mathbf{b}_{3\perp}\boldsymbol{\varphi})$	$\mathbf{b}_3 = [1, -1, 0, 0, 0, 0, 0]$	$\chi^2\,(3) = 24.74$	0.00
Nominal exchange rate $\mathcal{H}_7 : \tilde{\boldsymbol{\tau}} = (\mathbf{b}_4, \mathbf{b}_{4\perp}\boldsymbol{\varphi})$	$\mathbf{b}_4 = [0, 0, 1, 0, 0, 0, 0]$	$\chi^2\,(3) = 14.15$	0.00
PPP gap $\mathcal{H}_8 : \tilde{\boldsymbol{\tau}} = (\mathbf{b}_5, \mathbf{b}_{5\perp}\boldsymbol{\varphi})$	$\mathbf{b}_5 = [1, -1, -1, 0, 0, 0, 0]$	$\chi^2\,(3) = 6.01$	0.11
US interest rate $\mathcal{H}_9 : \tilde{\boldsymbol{\tau}} = (\mathbf{b}_6, \mathbf{b}_{6\perp}\boldsymbol{\varphi})$	$\mathbf{b}_6 = [0, 0, 0, 0, 1, 0, 0]$	$\chi^2\,(3) = 10.07$	0.01
Copper price $\mathcal{H}_{10} : \tilde{\boldsymbol{\tau}} = (\mathbf{b}_7, \mathbf{b}_{7\perp}\boldsymbol{\varphi})$	$\mathbf{b}_7 = [0, 0, 0, 0, 0, 1, 0]$	$\chi^2\,(3) = 28.25$	0.00

5.2.2. Testing Identifying Restrictions on the Long-Run Structure

To identify plausible economic relationships among the variables, a set of restrictions, $\mathcal{H}_{\tilde{\beta}} : \tilde{\boldsymbol{\beta}} = (\mathbf{H}_1\boldsymbol{\vartheta}_1, \ldots, \mathbf{H}_r\boldsymbol{\vartheta}_r)$, must be imposed on $\tilde{\boldsymbol{\beta}} = \tilde{\boldsymbol{\tau}}\tilde{\boldsymbol{\rho}}$, where $\mathbf{H}_i$ is a $p_1 \times m_i$ restriction matrix, $\boldsymbol{\vartheta}_i$ is a $m_i \times 1$ vector of unknown parameters, and m_i is the number of free parameters in $\tilde{\boldsymbol{\beta}}_i$. The test is asymptotically χ^2 distributed with degrees of freedom equal to $\sum_{i=1}^{r}\left((p_1 - m_i) - (r-1)\right)$ (Johansen et al. 2010).

Furthermore, to understand the persistence observed in the variables in the system, it is useful to study the signs and significance of the coefficients in $\boldsymbol{\beta}$, $\boldsymbol{d}$, and $\boldsymbol{\alpha}$. Juselius and Assenmacher (2017) suggest that the different types of adjustment for the variable $x_{i,t}$, $i = 1, 2, \ldots, p$, may be illustrated using the expression $\triangle^2 x_{it} = \sum_{j=1}^{r}\alpha_{ij}\sum_{m=1}^{p}\left(\beta_{mj}x_{m,t-1} + d_{mj}\triangle x_{m,t-1}\right) + \cdots + \varepsilon_{it}$, which corresponds to the i-th equation in the baseline empirical model (25). The error correcting- and error-increasing behavior of the variables can be analyzed using the following rules:

- If $d_{ij}\beta_{ij} > 0$ (given $\alpha_{ij} \neq 0$), then $\triangle x_{i,t}$ is equilibrium error correcting to $\boldsymbol{\beta}_j'\boldsymbol{x}_{t-1}$ (medium run).
- If $\alpha_{ij}\beta_{ij} < 0$, then the acceleration rate $\triangle^2 x_{i,t}$ is equilibrium error correcting to the polynomially cointegrated relation $\left(\boldsymbol{\beta}_j'\boldsymbol{x}_{t-1} + \boldsymbol{d}_j'\triangle\boldsymbol{x}_{t-1}\right)$ (long run).

In all other cases, there is equilibrium error increasing behavior.

The selected case, $(r = 2,\ s_1 = 2,\ s_2 = 1)$, entails two stationary polynomially cointegrating relationships, $\tilde{\boldsymbol{\beta}}_i'\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{d}}_i'\triangle\tilde{\boldsymbol{x}}_t$, where $\tilde{\boldsymbol{\beta}}_i' = \tilde{\boldsymbol{\rho}}_i'\tilde{\boldsymbol{\tau}}_i'$ and $i = 1, 2$. Table 5 reports an identified long-run structure on $\tilde{\boldsymbol{\beta}}$, together with unrestricted estimates of $\tilde{\boldsymbol{d}}$ and restricted estimates of $\boldsymbol{\alpha}$, which could not be rejected based on $\chi^2\,(9) = 6.75$ with a p-value of 0.66.[21] To facilitate interpretation, a coefficient in boldface (italics) stands for equilibrium error-correcting (increasing) behavior. Table 3 showed that all eigenvalues are inside the unit circle, so that the system is stable and any error-increasing behavior is compensated by error-correcting behavior.

[21] The estimated long-run $\tilde{\boldsymbol{\beta}}$ structure is identified. That is, $r-1$ restrictions were imposed, at least, on each of the vectors. See Doornik and Juselius (2017) for further information.

Table 5. The estimated long-run $\tilde{\beta}$ structure $\left(\chi^2(9) = 6.75\ [0.66]\right)$. t-values are given in $(\cdot)$, "-" is a zero restriction. A coefficient in boldface (italics) stands for equilibrium error-correcting (increasing) behavior.

	$p_{d,t}$	$p_{f,t}$	s_t	$i_{d,t}$	$i_{f,t}$	cp_t	$t \times 10^3$	c
$\tilde{\beta}_1'$	-0.01 (-3.9)	0.01 (3.9)	0.01 (3.9)	1.00	-1.00	0.002 (2.8)	-	
$\tilde{d}_1'$	$\mathbf{-0.52}$ (-8.5)	-0.07 (-8.5)	-0.44 (-8.6)	$\mathbf{0.0006}$ (8.5)	0.0006 (8.6)	-0.03 (-1.8)		-0.06 (-4.2)
α_1'	$\mathit{-0.36}$ (-3.0)	-	-	$\mathbf{-0.26}$ (-5.9)	-			
$\tilde{\beta}_2'$	-0.03 (-14.2)	0.28 (17.4)	-	1.00	-1.00	-	-0.47 (-16.3)	
$\tilde{d}_2'$	$\mathbf{-1.21}$ (-8.4)	$\mathit{-0.17}$ (-8.5)	-1.01 (-8.4)	$\mathbf{0.001}$ (8.3)	0.001 (8.7)	-0.05 (-1.6)		-1.13 (-16.8)
α_2'	$\mathbf{0.58}$ (9.7)	$\mathit{0.05}$ (3.0)	0.37 (2.5)	$\mathit{0.06}$ (3.0)	-			

The first polynomially cointegrating relationship, $\tilde{\beta}_1'\tilde{x}_t + \tilde{d}_1'\triangle\tilde{x}_t$, is interpreted as the UA-UIP condition:[22]

$$\left(i_d - i_f\right)_t = 0.01ppp_t - 0.002cp_t + 0.52\triangle p_{d,t} - 0.0006\triangle i_{d,t} + 0.06 + \hat{v}_{1,t} \tag{27}$$

where $\hat{v}_{1,t} \sim I(0)$ is the equilibrium error. The equation shows that the interest rate spread has been positively co-moving with the PPP gap—a measure of the risk premium—and the copper price. This relationship resembles the UA-UIP condition, Equation (12), where the term $(0.51\triangle p_d - 0.0007\triangle i_d)_t$ is likely to be related to the expected change in the nominal exchange rate and to a risk premium. Moreover, Equation (27) indicates that the uncovered interest parity is stationary after being adjusted by the PPP gap—the uncertainty premium—and copper price.

Equation (27) shows that, exactly as the IKE theory predicts, movements in the interest rate spread co-move with swings in the real exchange rate. That is, the interest rate spread moves in a compensatory manner to restore the equilibrium in the product market when the nominal exchange rate has been away from its benchmark value.

The copper price also enters the relationship that describes the excess returns under IKE, though with a small coefficient. A higher copper price increases the dollar supply in Chile, generating an appreciation of the exchange rate and, consequently, a larger PPP gap. This indicates that the Chilean economy might be affected by the so called commodity super-cycle (Erten and Ocampo 2013) through the effects that fluctuations in the copper price have on the real exchange rate and, consequently, on competitiveness.

The adjustment coefficients show that the Chilean interest rate is equilibrium error correcting in the long and medium run. The domestic price is equilibrium error increasing in the long run but equilibrium error correcting in the medium run. Thus, if the domestic price is above its long-run benchmark value, in the medium run both the domestic inflation rate and changes in the domestic interest rate will tend to increase, generating an increase in the equilibrium error term $\hat{v}_{1,t}$. In the long run, however, the domestic price will tend to increase, which generates a decrease in $\hat{v}_{1,t}$. To restore the long-run equilibrium, the domestic interest rate starts increasing.

The second polynomially cointegrating relationship, $\tilde{\beta}_2'\tilde{x}_t + \tilde{d}_2'\triangle\tilde{x}_t$, can be interpreted as a long-run relationship between the interest rate spread, trend-adjusted prices, and changes in the nominal exchange rate and is expressed as:

[22] When $\alpha_{ij} = 0$, the corresponding d_{ij} is not shown in Equations (27) and (28). Furthermore, only d_{ij} coefficients with a $|\text{t-value}| \geq 2.5$ are shown.

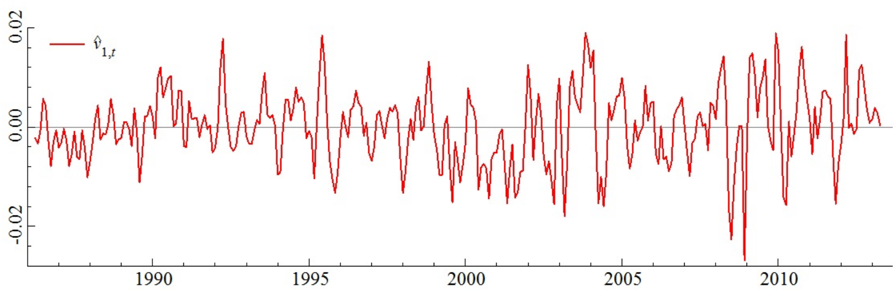
$$\left(i_{d,t} - i_{f,t}\right) = 0.03\tilde{p}_{d,t} - 0.28\tilde{p}_{f,t} + 1.21\triangle p_{d.t} + 0.17\triangle p_{f,t} + 1.01\triangle s_t - 0.001\triangle i_{d,t} + 1.13 + \hat{v}_{2,t} \quad (28)$$

where $\tilde{p}_{f,t}$ and $\tilde{p}_{d,t}$ are, respectively, the trend-adjusted prices in US and Chile and $\hat{v}_{2,t} \sim I(0)$ is the equilibrium error. The equation shows that the interest rate spread is positively co-moving with the relative trend-adjusted level of prices, domestic and foreign inflation rates, and changes in both nominal exchange rate and domestic interest rate. This relationship might describe a central bank's reaction rule.

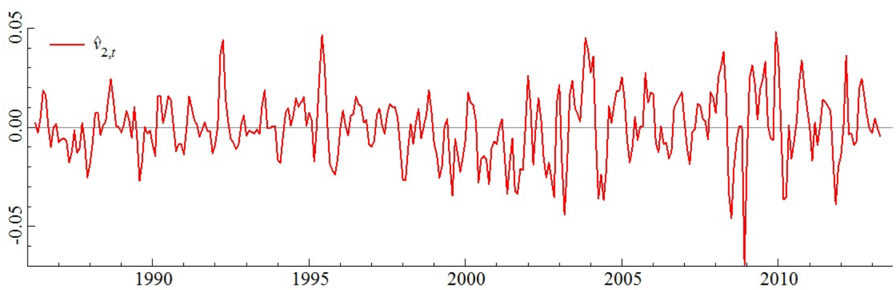
The Chilean trend-adjusted price, $\tilde{p}_{d,t}$, might tentatively be interpreted as a proxy for a long-run indicator of the inflation target. That is, given the US interest rate and US trend-adjusted price, if the domestic price is above (below) its long-run trend, the central bank may use contractionary (expansionary) monetary policy that increases (decreases) the domestic interest rate. The above argument may be used to explain the relationship between the interest rate spread and the changes in the nominal exchange rate. For example, the central bank may use contractionary monetary policy to counteract inflationary pressures due to exchange rate depreciation.

The adjustment coefficients show that when the interest rate spread has been under its long-run value, the domestic inflation rate and the domestic interest rate will tend to decrease in the medium run. Furthermore, the domestic price is equilibrium error correcting to the central bank's reaction rule in the long run, whereas the domestic interest rate is equilibrium error increasing in the long run. Then, if the interest rate spread is under its long-run equilibrium value, the domestic interest rate will tend to decrease. This generates further decreases in the equilibrium error $\hat{v}_{2,t}$. However, at the same time, the domestic price will tend to decrease, so it starts to restore the equilibrium.

Figure 5 shows the graph of the polynomial cointegration relationships and despite some signs of volatility change, they seem mean-reverting.



(**a**)



(**b**)

Figure 5. Polynomial cointegrating relationships. The graphs are corrected by short-run effects (for further details, see Juselius (2006)). (**a**) $\tilde{\boldsymbol{\beta}}_1'\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{d}}_1'\triangle\tilde{\boldsymbol{x}}_t$: UA-UIP condition; (**b**) $\tilde{\boldsymbol{\beta}}_2'\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{d}}_2'\triangle\tilde{\boldsymbol{x}}_t$: Central bank reaction's rule.

5.2.3. The Common Stochastic Trends

Table 6 reports the estimated $I(2)$ trend, $\boldsymbol{\alpha}_{\perp 2}$, and its respective estimated loading, $\breve{\boldsymbol{\beta}}_{\perp 2}$. The former may be interpreted as a relative price shock because it loads into prices and exchange rate rather than into exchange rate and interest rates. The estimate of $\boldsymbol{\alpha}_{\perp 2}$ suggests, however, that only shocks to the US price have generated the $I(2)$ trend. The coefficients in $\breve{\boldsymbol{\beta}}_{\perp 2}$ indicate that the $I(2)$ trend loads into nominal exchange rate and relative prices with coefficients of the same sign but different magnitude, which is consistent with the results of hypotheses $\mathscr{H}_4$, $\mathscr{H}_5$, and $\mathscr{H}_7$ in Table 4 that prices and exchange rate behave as a near $I(2)$ process. Equations (29) and (30) show, respectively, the $I(2)$ properties of the relative price and PPP gap.

The relative price is expressed as:

$$\left(p_{d,t} - p_{f,t}\right) = (0.25 - 0.03)\, \boldsymbol{\alpha}'_{\perp 2} \sum_{i=1}^{t} \sum_{s=1}^{i} \hat{\boldsymbol{\varepsilon}}_s. \tag{29}$$

The loading coefficients to the Chilean CPI and US CPI have the same sign but not the same size. Its difference, 0.22, has to be significant because the result of hypothesis $\mathscr{H}_6$ in Table 4 showed that the relative price is likely to behave as a near $I(2)$ process. The positive loading is consistent with the upward sloping trend in Figure 1a.

The PPP gap is expressed as:

$$\left(p_{d,t} - p_{f,t} - s_t\right) = (0.25 - 0.03 - 0.22)\, \boldsymbol{\alpha}'_{\perp 2} \sum_{i=1}^{t} \sum_{s=1}^{i} \hat{\boldsymbol{\varepsilon}}_s. \tag{30}$$

The long-run stochastic trend in relative prices and nominal exchange rate cancels out. This is consistent with both the result of hypothesis $\mathscr{H}_8$ in Table 4, which showed that deviations from PPP are likely to behave as an $I(1)$ process, and the long swings in Figure 1b.

The MA representation suggests that the Chilean economy is primarily affected by external shocks, which is natural when a small and open economy is participating in global markets. Chile has one of the most open economies in the world and also a developed financial market that is almost fully integrated into international markets.

Table 6. MA representation. $(\cdot)$ is the t-value. c_{ij} are constant terms.

$$\begin{bmatrix} p_d \\ p_f \\ s \\ i_d \\ i_f \end{bmatrix}_t = \underbrace{\begin{bmatrix} 0.25 \\ 0.03 \\ 0.22 \\ -0.00 \\ -0.00 \end{bmatrix}}_{\breve{\boldsymbol{\beta}}_{\perp 2}} \left[\boldsymbol{\alpha}'_{\perp 2} \textstyle\sum_{i=1}^{t} \sum_{s=1}^{i} \hat{\boldsymbol{\varepsilon}}_s\right] + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \\ c_{51} & c_{53} & c_{53} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}'_{\perp 2} \sum_{i=1}^{t} \hat{\boldsymbol{\varepsilon}}_i \\ \boldsymbol{\alpha}'_{\perp 1,1} \sum_{i=1}^{t} \hat{\boldsymbol{\varepsilon}}_i \\ \boldsymbol{\alpha}'_{\perp 1,2} \sum_{i=1}^{t} \hat{\boldsymbol{\varepsilon}}_i \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ b_{41} \\ b_{51} \end{bmatrix} t$$

$$\boldsymbol{\alpha}'_{\perp 2} = \begin{bmatrix} \underset{(-0.7)}{-0.07} & \mathbf{1.00} & \underset{(-0.3)}{-0.03} & \underset{(1.1)}{0.16} & \underset{(0.3)}{0.33} \end{bmatrix}$$

6. Conclusions

The long and persistent swings of the real exchange rate have for a long time puzzled economists. Recent models that build on IKE seem to provide theoretical explanations for this persistence.

This paper has analyzed the empirical regularities behind the PPP gap and the uncovered interest rate parity in Chile. The results, based on an $I(2)$ cointegrated vector autoregressive model, gave support for the theoretical exchange rate model based on imperfect knowledge, which assumes that individuals use a multitude of forecasting strategies that are revised over time in ways that cannot be fully prespecified. This is further supported by the results that showed a complex and fairly informative mix of error-increasing and error-correcting behavior.

The results showed that, exactly as the IKE theory predicts, movements in the interest rate spread co-move with swings in the real exchange rate. That is, the interest rate spread moves in a compensatory manner to restore the equilibrium in the product market when the real exchange rate has been away from its long-run value. The copper price also explain the deviations of the real exchange rate from its long-run equilibrium value. Copper is the main export commodity in Chile and accounts for a large share in total exports; its price fluctuations seems to affect the real exchange rate through its effect on the exchange market.

Altogether, the results indicate that when the interest rate spread is corrected by the uncertainty premium (the PPP gap) and by the fluctuations in the copper price one gets a stationary market-clearing mechanism.

Conflicts of Interest: The author declares no conflict of interest.

Appendix A. Data

Table A1 describes the variables used in this study, their sources, notations, and transformations.

Table A1. Data Description.

Variable	Description	Source	Transformation
$p_{d,t}$	Chilean Consumer Price Index	Central Bank of Chile	Natural logarithm
$p_{f,t}$	US Consumer Price Index	Bureau of Labor Statistics, United States	Natural logarithm
s_t	Nominal exchange rate (Chilean pesos per US dollar)	Central Bank of Chile	Natural logarithm
$i_{d,t}$	1-year Chilean average weighted rates of all transactions in the month by financial commercial banks in Chilean pesos (nominal). Nominal interest rates are annualized (base 360 days) using the conversion of simple interest.	Own elaboration based on data from the Central Bank of Chile	The original variable was divided by 1200 to make it comparable with monthly data
$i_{f,t}$	United States interest rate, Constant Maturity Yields, 1 Year, Average, USD	Own elaboration based on data from the International Monetary Fund	The original variable was divided by 1200 to make it comparable with monthly data
cp_t	Real copper price (USD cents./lb.)	Comisión Chilena del Cobre	Natural logarithm

Appendix B. Lag-Length Selection

Table A2 reports the lag-length selection and lag reduction test. The upper part suggests that $k = 2$ should be selected based on SC and H-Q criteria. However, there is evidence of autocorrelation of order 1 and 2 when $k = 2$. If $k = 3$ is selected, the hypotheses of autocorrelation of orders 1 and 3 can be rejected. The lower part of Table A2 shows that only the reduction from 4 to 3 lags cannot be rejected.

Table A2. Lag-length selection model and lag reduction test.

Lag-Length Selection				
Lag: k	**SC**	**H-Q**	**LM(1)**	**LM(k)**
4	−63.42	−65.35	0.34	0.52
3	−63.97	−65.62	0.13	0.38
2	**−64.28**	**−65.71**	0.05	0.04
1	−64.02	−65.20	0.00	0.00

Table A2. *Cont.*

Lag Reduction		
Reduction from - to	**Test**	***p*-Value**
VAR(4) - VAR(3)	χ^2 (36) $= 41.55$	0.24
VAR (3) - VAR(2)	χ^2 (36) $= 95.74$	0.00
VAR(2)-VAR(1)	χ^2 (36) $= 291.88$	0.00

SC: Schwarz Criterion, H-Q: Hannan-Quinn Criterion; LM(i) stands for a LM-test for autocorrelation of order i; a number in boldface stands for the lowest criteria value.

Appendix C. Dummy Variables

In model (25), nine dummies were incorporated. Table A3 describes the economic facts that justify the dummies, and Table A4 reports its estimated coefficients.

Table A3. Dummy justification.

Dummy	Variable	Justification
P 1990:9	$+p_d$	The Central Bank of Chile started the partial implementation of an inflation targeting system
T 1990:11	$+i_d$	INA
P 1993:12	$-i_d$	INA
P 1998:9	$+i_d$	Central Bank of Chile increased the real monetary policy interest rate from 8.5% to 14%
P 2005:9	$+p_f$	Energy costs increased sharply. Overall, the index for energy commodities (petroleum-based energy)
P 2006:04	$+cp$	The copper price increased in 30% in April triggered by the lower inventories and higher demand
P 2008:10	$-p_f$, $+s$	The energy index fell 8.6% and the transportation index fell in 5.4% in October . The nominal exchange rate depreciated 12% due to the dollar strengthening in international markets
P 2008:11	$-p_f$	The overall CPI index decreased mainly due to a decrease in energy prices, particularly gasoline .
P 2010:2	$+s$, $+p_d$	The nominal exchange rate depreciated due to changes in the forward position of the pension funds

P and T stand for a permanent dummy, $(0,\ldots,0,1,0,\ldots,0)$, and a transitory dummy, $(0,\ldots,0,1,-1,0,\ldots,0)$, respectively. The signs "$-$" and "$+$" stand for decreases and increases, respectively; INA official information regarding the variable increase or decrease is not available.

Table A4. Estimated outlier coefficients.

Dummy	$\triangle^2 p_d$	$\triangle^2 p_f$	$\triangle^2 s$	$\triangle^2 cp$	$\triangle^2 i_d$	$\triangle^2 i_f$
P 1990:9	0.01 (5.08)	*	*	*	*	*
T 1990:11	*	*	*	*	0.02 (18.44)	*
P 1993:12	*	*	*	*	-0.009 (-5.68)	*
P 1998:9	*	*	*	*	0.005 (3.32)	*
P 2005:9	*	0.01 (4.62)	*	*	*	*
P 2006:4	*	*	*	0.21 (3.98)	*	*
P 2008:10	*	-0.01 (-5.361)	0.14 (8.48)	-0.25 (-4.54)	*	*
P 2008:11	*	-0.01 (-6.73)	*	*	*	*
P 2010:2	0.01 (2.88)	*	0.07 (4.18)	*	*	*

$(\cdot)$ is the t-value. * stands for a $|\text{t-value}| \leq 2.0$; P and T stand, respectively, for a permanent and a transitory dummy.

References

Bacchiocchi, Emanuele, and Luca Fanelli. 2005. Testing the PPP through I (2) cointegration techniques. *Journal of Applied Econometrics* 20: 749–70.

Céspedes, Luis.F., and José De Gregorio. 1999. Tipo de cambio real, desalineamiento y devaluaciones: teoría y evidencia para Chile. Unpublished paper, University of Chile, Santiago, Chile, March.

Cowan, Kevin, David Rappoport, and Jorge Selaive. 2007. *High Frequency Dynamics of the Exchange Rate in Chile*. Santiago: Central Bank of Chile.

De Gregorio, José, and Felipe Labbé. 2011. *Copper, the Real Exchange Rate and Macroeconomic Fluctuations In Chile*. Santiago: Central Bank of Chile.

Délano, Valentín, and Rodrigo Valdés. 1998. Productividad y Tipo Cambio Real en Chile. *Central Bank of Chile Working Paper* 38: 1–19.

Diamandis, Panayiotis F. 2003. Market efficiency, purchasing power parity, and the official and parallel markets for foreign currency in Latin America. *International Review of Economics & Finance* 12: 89–110.

Doornik, Jurgen A., and Katarina Juselius. 2017. *Cointegration Analysis of Time Series Using CATS 3 for OxMetrics*. London: Timerlake Consultants Ltd.

Duncan, Roberto, and César Calderón. 2003. Purchasing Power Parity in an Emerging Market Economy: A Long-Span Study for Chile. *Estudios de Economía* 30: 103–132.

Erten, Bilge, and José Antonio Ocampo. 2013. Super cycles of commodity prices since the mid-nineteenth century. *World Development* 44: 14–30.

Froot, Kenneth A., and Kenneth Rogoff. 1995. Perspectives on PPP and long-run real exchange rates. In *Handbook of International Economics*. Amsterdam: Elsevier, Volume 3, pp. 1647–88.

Frydman, Roman, and Michael D. Goldberg. 2011. *Beyond Mechanical Markets: Asset Price Swings, Risk, and The Role of the State*. Princeton: Princeton University Press.

Frydman, Roman, and Michael D. Goldberg. 2007. *Imperfect Knowledge Economics: Exchange Rates and Risk*. Princeton: Princeton University Press.

Gonzalo, Jesus. 1994. Five alternative methods of estimating long-run equilibrium relationships. *Journal of Econometrics* 60: 203–33.

Johansen, Søren. 1992. A representation of vector autoregressive processes integrated of order 2. *Econometric Theory* 8: 188–202.

Johansen, Søren. 1995. A statistical analysis of cointegration for I(2) variables. *Econometric Theory* 11: 25–59.

Johansen, Søren. 1996. *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.

Johansen, Søren. 1997. Likelihood analysis of the I(2) model. *Scandinavian Journal of Statistics* 24: 433–62.

Johansen, Søren. 2006. Statistical analysis of hypotheses on the cointegrating relations in the I(2) model. *Journal of Econometrics* 132: 81–115.

Johansen, Søren, Katarina Juselius, Roman Frydman, and Michael Goldberg. 2010. Testing hypotheses in an I(2) model with piecewise linear trends. An analysis of the persistent long swings in the Dmk/$ rate. *Journal of Econometrics* 158: 117–29.

Juselius, Katarina. 2006. *The Cointegrated VAR Model: Methodology and Applications*. Oxford: Oxford University Press.

Juselius, Katarina. 2010. *Imperfect Knowledge, Asset Price Swings and Structural Slumps: A Cointegrated VAR Analysis of Their Interdependence.* Technical Report. Copenhagen: Department of Economics, University of Copenhagen.

Juselius, Katarina. 2014. Testing for Near I(2) Trends When the Signal-to-Noise Ratio Is Small. *Economics: The Open-Access, Open-Assessment E-Journal* 8: 2014-21. doi:10.5018/economicsejournal.ja.2014-21.

Juselius, Katarina. 2017a. Using a Theory-Consistent CVAR Scenario to Test an Exchange Rate Model Based on Imperfect Knowledge. Unpublished manuscript, Department of Economics, University of Copenhagen, Copenhagen, Demark.

Juselius, Katarina. 2017b. A Theory Consistent CVAR Scenario for a Standard Monetary Model Using Data-Generated Expectaions. Unpublished manuscript, Department of Economics, University of Copenhagen, Copenhagen, Denmark.

Juselius, Katarina, and Katrin Assenmacher. 2017. Real exchange rate persistence and the excess return puzzle: The case of Switzerland versus the US. *Journal of Applied Econometrics*, doi:10.1002/jae.2562.

Kahneman, Daniel, and Amos Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* 47: 263–91.

Kohlscheen, Emanuel, Fernando Avalos, and Andreas Schrimpf. 2017. When the Walk Is Not Random: Commodity Prices and Exchange Rates. *International Journal of Central Banking*, June.

Kongsted, Hans Christian. 2005. Testing the nominal-to-real transformation. *Journal of Econometrics* 124: 205–25.

Krugman, Paul R., Maurice Obstfeld, and Marc Melitz. 2011. *International Economics: Theory and Policy*, 9th ed. Boston: Addison-Wesley.

Mark, Nelson C. 2001. *International Macroeconomics and Finance: Theory and Econometric Methods*, 1st ed. Malden: Wiley-Blackwell.

Medina, Juan Pablo, and Claudio Soto. 2007. *Copper Price, Fiscal Policy and Business Cycle in Chile*. Documentos de Trabajo. Santiago: Banco Central de Chile, p. 1.

Phelps, Edmund S. 1994. *Structural Slumps: The Modern Equilibrium Theory of Unemployment, Interest, and Assets*. Harvard: Harvard University Press.

Rahbek, Anders, Hans Christian Kongsted, and Clara Jørgensen. 1999. Trend stationarity in the I(2) cointegration model. *Journal of Econometrics* 90: 265–89.

Sarno, Lucio, and Mark P. Taylor. 2002. *The Economics of Exchange Rates*. Cambridge: Cambridge University Press.

Tabor, Morten Nyboe. 2014. Essays on Imperfect Knowledge Economics, Structural Change, And Persistence in the Cointegrated VAR Model. Ph.D. thesis, University of Copenhagen, Copenhagen, Denmark.

Taylor, Alan M., and Mark P. Taylor. 2004. The Purchasing Power Parity Debate. *Journal of Economic Perspectives* 18: 135–58.

Wu, Yi. 2013. *What Explains Movements in the Peso/Dollar Exchange Rate?* IMF Working papers No. 13/171, International Monetary Fund, Washington, DC, USA.