econometrics

Editorial

Recent Developments in Copula Models

Jean-David Fermanian

ENSAE-Crest, J120, 3 Avenue Pierre-Larousse, 92245 Malakoff Cedex, France; jean-david.fermanian@ensae.fr

Academic Editor: Marc S. Paolella
Received: 14 July 2017; Accepted: 15 July 2017 ; Published: 24 July 2017

Copula models have become very popular and well studied among the scientific community. Now, most academic researchers, engineers, modelers, etc, own at least some basic copula toolkit and are able to apply it in real situations. Based on the famous Sklar's theorem (Sklar 1959), copulas allow to put in place the fruitful idea of splitting the specification of a multivariate model into two parts: the marginal distributions on one side, the dependence structure (copula) on the other part. This simple way of thinking has induced an impressive number of theoretical and applied papers during the last two decades and this trend is exponentially increasing. Indeed, all families of multivariate models and their associated statistical techniques (inference, testing, simulation, etc) potentially have to be revisited under a copula point of view for theoretical and practical reasons. This huge and necessary task has just started. On the theoretical side, a renewed focus on semiparametric techniques has been fuelled because the underlying marginal distributions are often replaced by their empirical counterparts in copula models. Moreover, two- or three-stage estimators are common under a copula point of view, inducing particular asymptotics and finite distance performances.

The earliest applications of copulas have been proposed in survival analysis (biostatistics, reliability, actuarial science), but the scientific community rapidly understood the far wider scope of dependence modelling. All applied fields are now affected by copulas. In economics and finance, this path is now clearly engaged. There, a particular difficulty is due to time-dependencies that cannot be straightforwardly managed by (sequences of) copulas: see Darsow et al. (1992) for markovian features with copulas, Cherubini et al. (2011) for a recent survey of results, etc. Nonetheless, significant advances have been observed in terms of copula modelling for univariate or even multivariate time series in the last years. The objective of this special issue is to provide new contributions into the field of copula models in general, with applications in financial econometrics.

Standard families of multivariate models for financial time series are of GARCH-type and/or stochastic volatility-type: see the surveys Asai et al. (2006) and Bauwens et al. (2006), for instance. Once the first two conditional moments of asset returns are controlled, we are faced with dependencies across the estimated (vectors of) residuals. It has been found in Chen and Fan (2006) that the maximum-likelihood estimator of the copula parameter that is associated to residuals does not depend on the first-level estimates, under some conditions of regularity. Here, Bruno Rémillard provides the limiting distribution of the sequential empirical process and the sequential empirical copula process that are associated to estimated residuals in a general multivariate GARCH framework. He proves that the limiting behavior of the latter process does not depend on the conditional mean and conditional variance estimated parameters, when the conditional correlations are constant in time. As a by-product, the limiting distribution of rank-based dependence measures computed with the residuals are the same as if the dependence measures were computed with the innovations. He applies such results to tests of structural change and specification tests of copulas.

Beside multivariate extensions of standard "historical" models in econometrics, copulas have induced a remarkable and fruitful new approach called "pair-copula constructions": combinations of bivariate copulas allow to build very flexible models through so-called "vines", that are connected trees with some particular features (Bedford and Cooke 2002). Kjersti Aas explains the main intuition and techniques behind such vine approaches: specification, inference, testing, parsimony, the "simplifying copula assumption", etc. Moreover, she provides an overview of applications in finance. The potential of vine models appears in a very convincing way in the paper of Holger Fink, Yulia Klimova, Claudia Czado and Jakob Stöber too. They propose a time-varying dependence structure based on Markov-switching copulas. Those copulas are defined through regular vines, and are related to dependencies between stock returns and volatility indices in different geographical areas. The authors identify times of "normal" and "abnormal" states, confirming the existence of joint points in time at which dependencies globally switch between different vine structures.

Beside vines, hierarchical Archimedean copula (HAC) models have emerged as realistic competitors: see McNeil (2008), Okhrin et al. (2013), e.g., They are restricted to Archimedean families, but allow complex combinations of p-dimensional copulas (not only $p = 2$ as with vines). Ostap Okhrin and Anastasija Tetereva propose to use high-frequency observations in dependence modeling. As in Fengler and Okhrin (2016), they infer (daily) realized covariances of vectors of asset returns and Hoeffding's identity provides some moment conditions that will be used to estimate the copula parameters. In order to estimate the structure of the underlying HAC, they generalize the clustering method proposed by Górecki et al. (2016) by adapting the algorithm introduced in Segers and Uyttendaele (2014). The proposed approach has been applied to predict daily VaR exceedances of portfolios of US stocks. High-dimensional realized copula models seem to outperform some benchmark models in higher dimensions, especially for low VaR levels.

In financial econometrics, latent factor models are now recognized as an efficient way of building static and/or dynamic high-dimensional models. In this special issue, two related papers adapt such techniques to dynamic factor copula models, in the vein of Murray et al. (2013), Krupskii and Joe (2013) and Oh et al. (2017). First, Eugen Ivanov, Aleksey Min and Franz Ramsauer apply the copula autoregressive (COPAR) model of Brechmann and Czado (2015). They consider two dynamic factor models and estimate them separately by Kalman filtering/smoothing. The estimated factors are then combined with a COPAR model, inducing a non-Gaussian dependence structure of simulated latent factors. In their empirical application, they consider monthly U.S. financial and macroeconomic panel data to filter driving factors later employed for a mean-variance portfolio optimization. The forecasted factors with such a non-linear dependence structure are used to assess the future variability of multivariate asset returns. The obtained optimal portfolios performs well compared to more usual techniques.

Second, Benedikt Schamberger, Lutz F. Gruber and Claudia Czado develop Bayesian inference for a recently proposed latent factor copula model Krupskii and Joe (2013), which utilizes a pair copula construction to couple the variables with the latent factor. They use adaptive rejection Metropolis sampling within Gibbs sampling for posterior simulation. Their approach is illustrated to forecast the value-at-risk and the expected shortfall of a portfolio of European bank stocks.

Finally, contagion and systemic risks provide a natural field for applications of copulas. Indeed, such questions naturally involve joint and conditional distributions. In a certain sense, contagion is a particular way of analyzing dependence. Fabrizio Durante, Enrico Foscolo and Alex Weissensteiner focus on dependence between sovereign risk (measured through credit spreads) and the failure risk of Italian banks (measured through their stock returns), during the agitated period of time 2003–2015. They found this dependence structure suffered from a structural break located around Lehman's collapse. Moreover, they try to know whether an increasing sovereign risk has transferred through contagion to the equity returns of Italian banks. By measuring potential contagions with a combination of conditional Spearman's rho, the answer seems to be negative.

I think all these papers offer an interesting and wide range of theoretical and applied contributions under the topic of copula modelling in economics and finance. Thanks are due to the Editor Marc Paolella, the assistant editors Michele Cardani, Lu Liao and Nancy Zhang, as well as the numerous referees who provided useful comments and advice.

Conflicts of Interest: The author declares no conflict of interest.

References

Asai, Manabu, Michael McAleer, and Jun Yu. 2006. Multivariate stochastic volatility: A review. *Econometric Reviews* 25: 145–75.

Bauwens, Luc, Sébastien Laurent, and Jeroen VK Rombouts. 2006. Multivariate GARCH models: A survey. *Journal of Applied Econometrics* 21: 79–109.

Bedford, Tim, and Roger M. Cooke. 2002. Vines—A new graphical model for dependent random variables. *Annals of Statistics* 30(4): 1031–68.

Brechmann, Eike Christian, and Claudia Czado. 2015. COPAR-Multivariate time series modeling using the copula autoregressive model. *Applied Stochastic Models in Business and Industry* 31: 495–514.

Chen, Xiaohong, and Yanqin Fan. 2006. Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspecification. *Journal of Econometrics* 135: 125–54.

Cherubini, Umberto, Sabrina Mulinacci, Fabio Gobbi, and Silvia Romagnoli. 2011. *Dynamic Copula Methods in Finance*. Hoboken: John Wiley & Sons.

Darsow, William F., Bao Nguyen, and Elwood T. Olsen. 1992. Copulas and Markov processes. *Illinois Journal of Mathematics* 36: 600–42.

Fengler, Matthias R., and Ostap Okhrin. 2016. Managing risk with a realized copula parameter. *Computational Statistics & Data Analysis* 100: 131–52.

Górecki, Jan, Marius Hofert, and Martin Holeňa. 2016. An approach to structure determination and estimation of hierarchical archimedean copulas and its application to bayesian classification. *Journal of Intelligent Information Systems* 46: 21–59.

Krupskii, Pavel, and Harry Joe. 2013. Factor copula models for multivariate data. *Journal of Multivariate Analysis* 120: 85–101.

McNeil, Alexander J. 2008. Sampling Nested Archimedean Copulas. *Journal of Statistical Computation and Simulation* 78: 567–81.

Murray, Jared S., David B. Dunson, Lawrence Carin, and Joseph E. Lucas. 2013. Bayesian Gaussian Copula Factor Models for Mixed Data. *Journal of the American Statistical Association* 108: 656–65.

Oh, Dong Hwan, and Andrew J. Patton. 2017. Modelling dependence in high dimensions with factor copulas. *Journal of Business & Economic Statistics* 35: 139–54.

Okhrin, Ostap, Yarema Okhrin, and Wolfgang Schmid. 2013. On the structure and estimation of hierarchical archimedean copulas. *Journal of Econometrics* 173: 189–204.

Segers, Johan, and Nathan Uyttendaele. 2014. Nonparametric estimation of the tree structure of a nested archimedean copula. *Computational Statistics & Data Analysis* 72: 190–204.

Sklar, A. 1959. Functions de Répartition à n dimensions et leurs Marges. *Publications Institut de Statistique de l'Université de Paris* 8: 229–31.