

Article

Computational Analysis of Compressed Stiffened Composite Panels with Impact Damage

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Abstract: A complex modeling technique is presented in this paper for a numerical analysis of compressed stiffened composite panels with impact damage. The numerical technique is based on the LS-Dyna code application, which simulates both the dynamic deformation of the panel subjected to a local impact and the quasi-static uniform compression of the panel within the local damage zone. The technique has been validated by both impact and compression experimental tests of the stiffened composite panel. The obtained numerical results show that impact damage to the composite panel can reduce the carrying capacity in more than 50% of damaged panels compared to undamaged panels.

Keywords: composite stiffened panels; stresses; strains; kinetic energy; carrying capacity; dynamic loading; impact damage; static compression; finite element method; explicit scheme

1. Introduction

Fiber reinforced polymer (FRP), which has high strength characteristics, is widely used in aerospace, shipbuilding, and other industries. Scientific investigations of composite structures have been carried out over several decades [1–12]. General aspects of the numerical modeling of deformations and failure of aircraft structures, made of composite materials are presented in [12]. This paper points out that the increased use of FRP presents new difficulties for stress analysis due to the complicated structure and highly complex failure modes of composite materials. When solving problems with composite materials it is necessary to take into account physical nonlinearity, large displacements, and contact interactions of layers and fibers of FRP laminates. It is important for correct predictions of reciprocal action of the possible buckling and instability of loading process and damage growth, reducing residual strength of composite structure. It is well known that composite plates and shells are very sensitive to local dynamic loads, such as the hail, and the impact of small stones from the runway. It is noted in [11] that, with relatively low levels of kinetic energy, impact damage appears inside composite materials and can be invisible on the external surface of the structure. The experimental results show that the damage occurs within a conical contact zone with the apex at the point of impact [13]. A real damage zone also includes broken fibers, matrix cracks, and delamination of plies. In the compression of thin panels, this local damaged zone with delamination allows the fibers to buckle at much lower strains than in an undamaged region. Therefore, predicting delamination growth due to buckling is very important.

Some results of numerical simulations of the post-buckling delamination growth in compressed composite plates and shells are presented in [14–17]. A finite-element nonlinear analysis of the post-buckling behavior of a compressed rectangular plate with circular delamination was conducted

in [15] with the ABAQUS code and validated by experimental data. There was a discrepancy within 10% between the predicted load for delamination growth and the values measured from the tests. In general, the “compression after impact” problem remains for composite materials, limiting allowable compressive strains, often as low as $\varepsilon = 0.3\%$.

Different approaches for numerical simulations of impulse dynamic loading, predicting impact damage of composite structures and analysis of experimental data, are described in [18–26]. Most of the numerical test results presented in these papers have been obtained on coupons with an impact velocity of less than 5 m/s and a mass of less than 5 kg. The tests [18,19] show that the damage caused by the low-velocity impact with kinetic energy of $E_k \sim 60$ J can reduce the compressive properties of coupons by up to 50% and reduce the compressive properties of stringer stiffened panels by as much as 20% [21]. A series of carbon composite thin plates were tested in the range of impact energy $E_k = 0.60$ –9.34 J in [20] and compared with numerical results obtained with finite-element impact code FE77 with explicit solver and the LS-Dyna code. The developed modeling technique has worked well for carbon composites and allowed the predication of the force history and the onset of delamination. Additionally, different composite plates and stiffened panels subjected to impacts with an energy of $E_k = 13$ –25 J were studied in [22]. It is noted in this paper that the debilitating effects on compression strength limit the permissible design strain levels to 0.3–0.4%. A modeling technique based on the LS-Dyna code was developed in [23] using single-layer shell elements to predict both the in-plane damage and delamination. This technique was applied to a stiffened composite plate loaded with $E_k \sim 15$ J and validated by comparing the numerical and test results of [21], with differences of 16% for force–time response and 20% for damage area being obtained. Delamination threshold load assessment was studied in [24] for 100×100 mm square samples of glass–carbon–epoxy composite. It was experimentally found that there was a threshold impact energy of $E_k = 1.5$ J independent of the glass fiber stacking sequence. Moreover, 150×100 mm rectangular plates of carbon fiber–epoxy were tested in [26] with low-velocity impacts in the range of $E_k = 15$ –35 J. It is pointed out in this paper that delamination area and its shape depend on the stacking sequence.

A brief literature survey shows that all observed publications can be divided into two separate groups: the first one includes investigations of damage in composite structures subjected to local dynamic loading, and the second one presents an analysis of the post-buckling behavior of the initially damaged structures to predict load for delamination growth. In the present paper, an attempt has been made to develop a modeling tool and conduct complex analysis simulating both stages on the base of one platform using the LS-Dyna code: dynamic loading by impactor and static compression of stiffened panel with impact damage obtained in the first stage.

All new aircrafts have to meet very stringent requirements of national and international aviation rules. A designer must demonstrate the static strength and carrying capacity of the units made of composite materials, preliminarily subjected to local dynamic loading with kinetic energy of $E_k = 135$ J [11]. This impulse is transmitted by an impactor with a spherical end. Such loading conditions are investigated here.

2. Statement of Work and Governing Equations

Deformations of a thin-walled stiffened panel are described according to the Lagrange approach [27]. There are two coordinate systems: the first one is a fixed rectangular Cartesian coordinate system, $X = [X_1 X_2 X_3]$, and the second one is a local coordinate system, $x = [x_1 x_2 x_3]$, with directional cosines n_{ij} :

$$x_i = n_{ij} X_j, \quad i, j = \overline{1, 3}. \quad (1)$$

The origin of the local coordinates is located at a midsurface of the panel, axis x_3 is a normal to the midsurface, and x_1, x_2 are orthogonal to x_3 . We assume that transverse shear deformations are small, which allows us to suppose that the local basis is orthogonal during all the process of deformation.

The velocity–strain components in the local basis $\dot{\varepsilon}_{ij}$ are defined by velocity–strain components in the general basis $\dot{\varepsilon}_{ij}$:

$$\dot{\varepsilon}_{ij} = n_{im}n_{jk}\dot{\varepsilon}_{mk}, \quad (2)$$

Velocity–strain components $\dot{\varepsilon}_{ij}$ are defined in a current state metrics:

$$\begin{aligned} \dot{\varepsilon}_{ij} &= (\dot{U}_{i,j} + \dot{U}_{j,i})/2, \quad (i, j = \overline{1,3}) \\ X_i &= X_i|_{t=0} + \int_0^t \dot{U}_i dt \end{aligned} \quad (3)$$

where displacements U_i are defined in Cartesian coordinate system X , comma means a space derivative, and a superposed dot denotes a time derivative.

The equation of motion is written in the form of the variation principle of balance of virtual powers of work [28,29]:

$$\int_{\Omega} \sigma_{ij} \delta \dot{\varepsilon}_{ij} dV + \int_{\Omega} \rho \ddot{U}_i \delta \dot{U}_i dV = \int_{\Gamma_p} P_i \delta \dot{U}_i d\gamma + \int_{\Gamma_q} P_i^q \delta \dot{U}_i d\gamma, \quad (i, j = \overline{1,3}) \quad (4)$$

where: σ_{ij} —components of stress tensor, ρ —density, P_i^q —contact pressure, P_i —distributed load, Ω —volume of structure, Γ_q —contact surface, Γ_p —surface of external pressure is applied, $\delta \dot{\varepsilon}_{ij}$, $\delta \dot{U}_i$ —variations of $\dot{\varepsilon}_{ij}$, and $\dot{U}_i$ at the surface with given boundary conditions $\delta \dot{U}_i = 0$.

Equations of state for polymer composite material with sandwich structure of fiber plies are formulated as for orthotropic material model Chang-Chang [30], which can be written for 2D in-plane state of stress, typical for layers of stiffened panel, in the following form:

$$\begin{aligned} \varepsilon_{11} &= \frac{1}{E_1} (\sigma_{11} - \nu_1 \cdot \sigma_{22}) \\ \varepsilon_{22} &= \frac{1}{E_2} (\sigma_{22} - \nu_2 \cdot \sigma_{11}) \\ 2\varepsilon_{12} &= \frac{1}{G_{12}} \tau_{12} + \alpha \cdot \tau_{12}^3 \end{aligned} \quad (5)$$

where: E_1 , ν_1 —elastic moduli and Poisson ratio, respectively, in the longitudinal direction; E_2 , ν_2 —elastic moduli and Poisson ratio, respectively, in transverse direction; and G_{12} —shear modulus of monolayer. The third equation in (4) defines the nonlinear shear-stress parameter α . A fiber matrix shearing term augments each damage mode:

$$\bar{\tau} = \frac{\frac{\tau_{12}^2}{2G_{12}} + \frac{3}{4}\alpha\tau_{12}^4}{\frac{S_{12}^2}{2G_{12}} + \frac{3}{4}\alpha S_{12}^4}, \quad (6)$$

which is the ratio of the shear stress to the shear strength S_{12} . The failure criteria for matrix cracking F_{matrix} , compression F_{comp} , and fiber breakage F_{fiber} are given by the following equations:

$$F_{matrix} = \left(\frac{\sigma_{22}}{S_2} \right)^2 + \bar{\tau}, \quad (7)$$

$$F_{comp} = \left(\frac{\sigma_{22}}{2S_2} \right)^2 + \left[\left(\frac{C_2}{2S_{12}} \right)^2 - 1 \right] \frac{\sigma_{22}}{C_2} + \bar{\tau}, \quad (8)$$

$$F_{fiber} = \left(\frac{\sigma_{11}}{S_1} \right)^2 + \bar{\tau}, \quad (9)$$

where: S_1 , S_2 —longitudinal and transverse tensile strength, S_{12} —shear strength, and C_2 —transverse compressive strength. Failure is assumed if any criteria $F_{matrix, (comp, fiber)} > 1$ and constants E_1 , E_2 , G_{12} , ν_1 , and ν_2 are set to zero.

Modeling of delaminations: In the suggested approach, every composite layer is simulated as a separate domain. For every pair of adjacent domains, the specific contact algorithm, CONTACT...TIEBREAK [31], is applied. Delamination between domains appears if:

$$\left(\frac{\sigma_n}{NFLS}\right)^2 + \left(\frac{\sigma_s}{SFLS}\right)^2 \geq 1 \quad (10)$$

where: σ_n, σ_s —normal and shear stresses in the connection, respectively NFLS, SFLS—ultimate normal and shear stresses, respectively. If delamination appears between domains, usual contact interaction is applied for further simulation.

An explicit scheme for time discretization and the finite element method [28] are used to numerically solve Equations (1)–(10).

The problem of local dynamic loading and consequent uniform static compression of the stiffened composite rectangular flat panel of constant thickness h_1 with sizes of $a \times 2b$ is considered in this paper (see Figure 1). At the first stage, the panel is locally impacted by dynamic force P caused by steel impactor. Then, at the second stage, the panel is statically loaded by longitudinal compression applied as uniform displacements at the free edge of the panel. The opposite edge of the panel is rigidly restrained, and symmetrical boundary conditions are applied at the side edges of the panel.

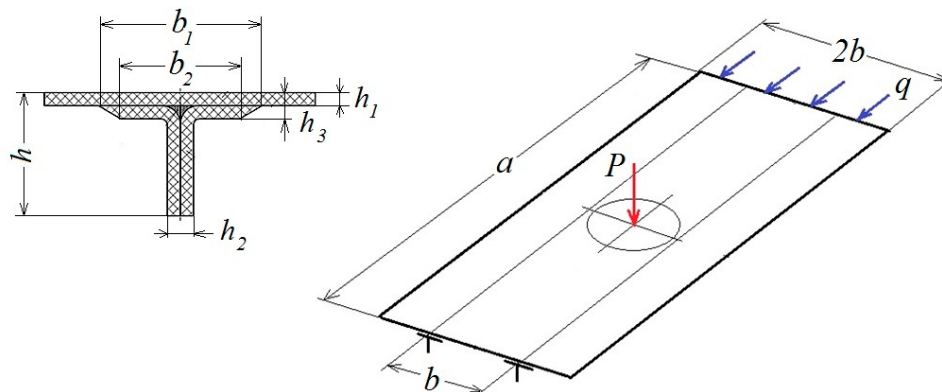


Figure 1. Stiffened panel with impact damage. a —panel length; b —half panel width or the distance between the stringers; h_1 —the composite plating thickness; b_1, b_2, h_2 —the stringer geometric parameters; h_3 —the sum of the stringer shelf thickness and the composite plating thickness; h —the sum of stringer height and the composite plating thickness; P —force caused by impactor.

The panel is made of carbon/epoxy FRP with laminate stacking sequence $(+45^\circ, 0^\circ, -45^\circ, 0^\circ, 90^\circ, 0^\circ, -45^\circ, 0^\circ, \text{ and } +45^\circ)_{3s}$. The normalized dimensions of the panel are presented in Table 1.

Table 1. Main normalized dimensions of the carbon/epoxy fiber reinforced polymer pane ¹.

h	h_2	h_3	a	b	b_1	b_2
7.5	2.0	2.0	66	23.3	17.0	11.4

¹ Dimensions are normalized by h_1 value.

3. Local Dynamic Deformation of the Panel

The finite-element model based on 8-node brick elements SOLID_ORTHO [31] was developed to simulate dynamic deformation of the panel subjected to local impact. In this computing model, every fiber layer is simulated by a separate layer of solid finite elements (Figure 2). The number of finite elements in this model is about 1.25 million.

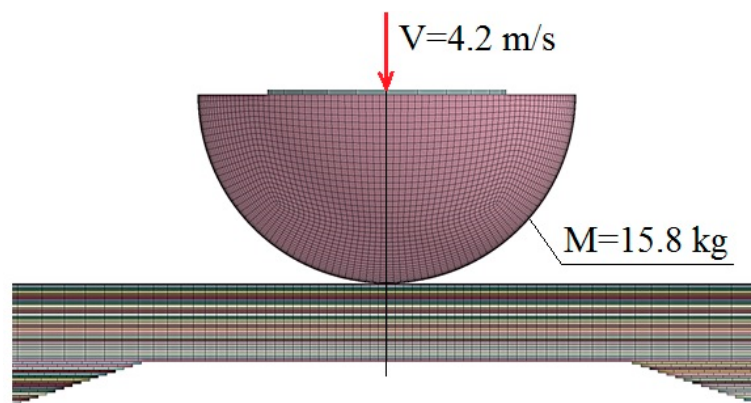


Figure 2. Fragment of the panel finite-element model. V —the impactor initial velocity, M —the impactor weight.

The weight M and initial velocity V of the dropping pin impacting the panel between stringers with kinetic energy $E_k = 140$ J are presented in Figure 2. Some results of numerical simulations obtained with LS-Dyna code are shown in Figures 3 and 4. A comparison of contact interaction force P between rod and panel via time obtained in experiment (curve 1) and in simulation (curve 2) is shown in Figure 3. A Dynatup INSTRON 9250 HV (Instron, High Wycombe, UK) pile driver with vertical dropping pin, having a spherical end with a diameter of 25.4 mm, was used to inflict impact damage. It can be seen that there is a small reduction of the contact force at $P_{exp} \sim 6.5$ kN and $t_1 \sim 0.6$ ms in the experiment, perhaps due to partial failure of fibers and matrix. The numerical curve also demonstrates some reduction of the contact force at $P_{num} \sim 12.3$ kN at $t_2 \sim 1.2$ ms. In general, numerical and experimental force time histories are close to each other when they grow. At the same time, the maximal level of the numerical force, $P_{num\ max} \sim 19.3$ kN, exceeds the experimental level, $P_{exp\ max} \sim 16.1$ kN, by $\sim 20\%$. The numerical duration of the impulse also exceeds the experimental one. There is one more curve 3 in Figure 3, obtained for the same level of kinetic energy, $E_k = 140$ J, and impactor weight of $M = 1.7$ kg. It can be seen from Figure 3 that the same level of kinetic energy provides a different impulse of the contact force.

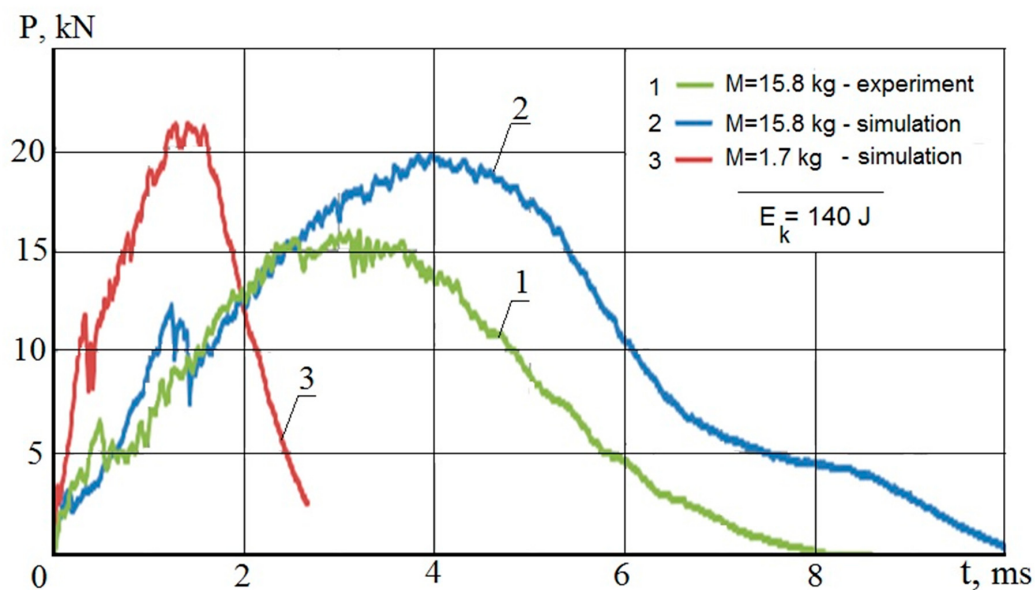


Figure 3. Contact force between impactor and panel. P —force caused by impactor; t —time; E_k —kinetic energy.

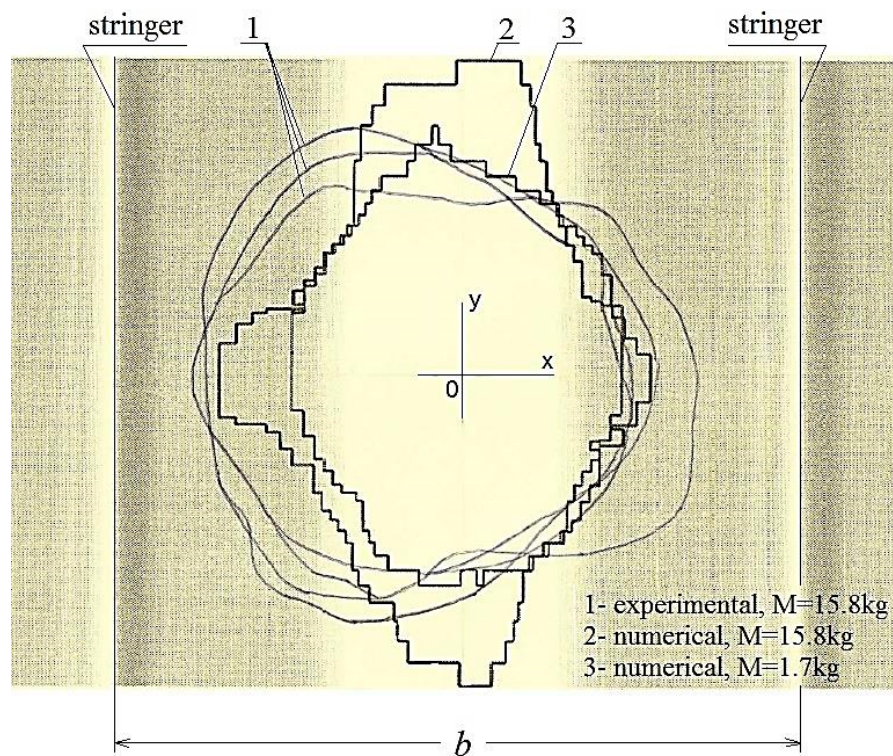


Figure 4. Experimental and numerical damage zones.

Figure 4 shows delamination zones defined with acoustic ultrasonic control in four tests (curve 1) and corresponding numerical damage zones obtained for an impactor of $M = 15.8$ kg (curve 2) and for an impactor of $M = 1.7$ kg (curve 3). The quantity b is the distance between stringers. A comparison of the test sizes and numerical dimensions of the damage zones are presented in Table 2.

Table 2. The sizes of the damage zone.

Results	Experiment	Simulation *
Size in X direction	0.64–0.75b	0.65b
Size in Y direction	0.63–0.70b	0.88b

* Simulations with $M = 15.8$ kg.

Curves of dimensionless stresses in the longitudinal direction along fibers ($\bar{\sigma}_1$) and in the transversal direction ($\bar{\sigma}_2$) in the top monolayer (dotted line) and bottom monolayer (solid line) are shown in Figure 5 for two impacts with the same kinetic energy, $E_k = 136$ J, for two impactors having masses of $M_1 = 1$ kg (curve 1) and $M_2 = 10$ kg (curve 2). Top and bottom points in which stresses have collected are located on the axis of impact. Dimensionless stresses are normalized with corresponding ultimate stresses of tension (+) and compression (−) of fibers and matrix. Numerical results show that bottom fibers maintain strength and top fibers do not for both impactor masses. Failure of the top compressed fibers for both masses begins at approximately $t \sim 0.4$ – 0.5 ms. Stress $\bar{\sigma}_2$ in the matrix of the top monolayer does not reach limit stress of compression, however in the bottom monolayer it exceeds this limit slightly only for mass $M_2 = 10$ kg.

Due to impact loading damage appears in the layers of fibers and between layers. Any fibers can be ruptured due to tension or destroyed by bending and compression in the layers, and the matrix also can be damaged. Modeling technique allows the prediction of sizes of damage zones for both matrix failure and delamination of layers.

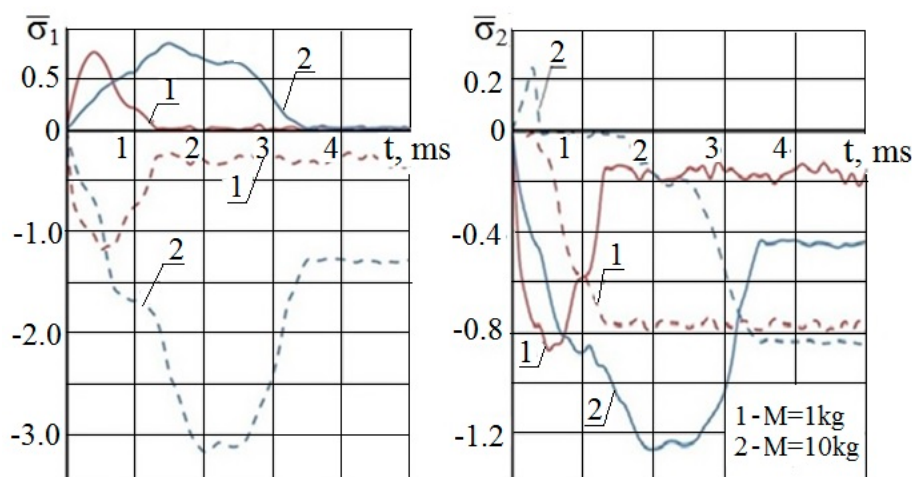
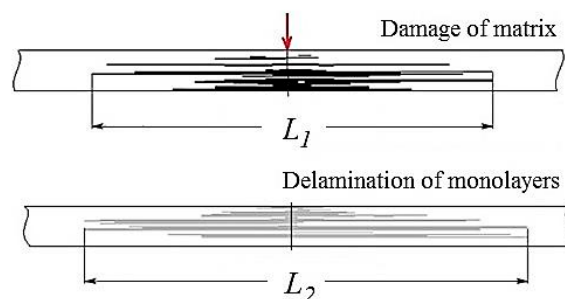


Figure 5. Stresses in fibers ($\bar{\sigma}_1$) and matrix ($\bar{\sigma}_2$).

Cross-sections of calculated impact damage zones in the longitudinal direction are presented in Figure 6. The dimensionless sizes of the damage zones are normalized by the distance between stringers— b . It can be seen that loading with the same kinetic energy, $E_k = 136 \text{ J} = \text{const.}$, in two loading cases with rod masses $M_1 = 1 \text{ kg}$ and $M_2 = 10 \text{ kg}$ provides different sizes of damage zones, with the impact of the heavy rod causing an increase of damage areas of 15–20%.



$$\text{For } M_1 = 1 \text{ kg } L_1 = 0.524b \quad L_2 = 0.611b$$

$$\text{For } M_2 = 10 \text{ kg } L_1 = 0.635b \quad L_2 = 0.698b$$

b - distance between stringers

Figure 6. Impact damage zones in the longitudinal direction. M_1, M_2 —the weights of different impactors; L_1 —the size of matrix damage area; L_2 —the size of delamination area.

4. Quasi-Static Compression of the Panel

The impact damage zone is specified as a delaminated area with contact interaction between composite layers with overall sizes preliminarily defined by numerical simulations and ultrasonic control of the panel. The broken fibers and matrix cracks in layers are not included in the computing model. Solving of the problem is carried out with explicit method [27] by slow growing of the edge uniform displacements to exclude dynamic effects. Some numerical results are presented in Figures 7–9.

Dimensionless load–displacement curves are shown in Figure 7 for undamaged panel (curve 1) and for the panel with impact damage (curve 2). The dimensionless load $\bar{q}$ is normalized with a critical level of compressive load and dimensionless displacement is normalized with the thickness of the plate. The maximal displacement of the undamaged panel grows monotonically. In the panel with local impact damage, displacement demonstrates complicated behavior, caused by the local buckling in the damage zone. As can be seen from Figures 7 and 8, weak bending of the layers with small

hardening occurs in the damage area at a load of about $\bar{q} = 0.4$. General delamination of the layers and separation of the stringers begin at a load of $\bar{q} = 1.0$, and it follows to the formation of general transversal buckling folding and defines a carrying capacity of the damaged panel.

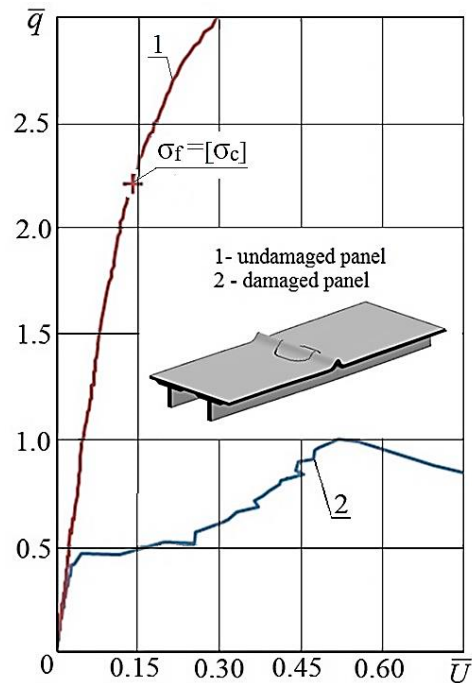


Figure 7. Displacements of undamaged and damaged panels. $\bar{q}$ —dimensionless load; $\bar{U}$ —dimensional displacement; σ_f —actual compression stress in the fibers, $[\sigma_c]$ —ultimate compression stress of the fibers.

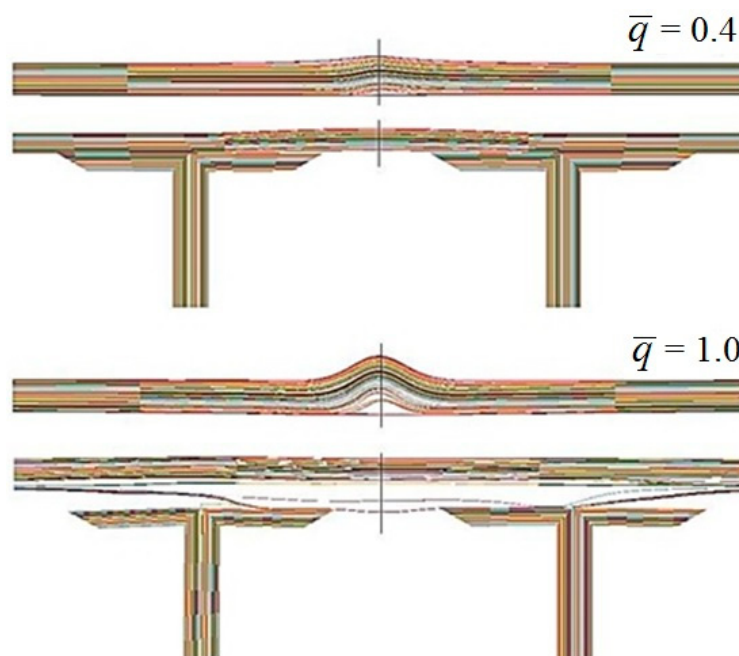


Figure 8. Post-buckling deformations of the panel in the transversal and longitudinal directions near the damage area.

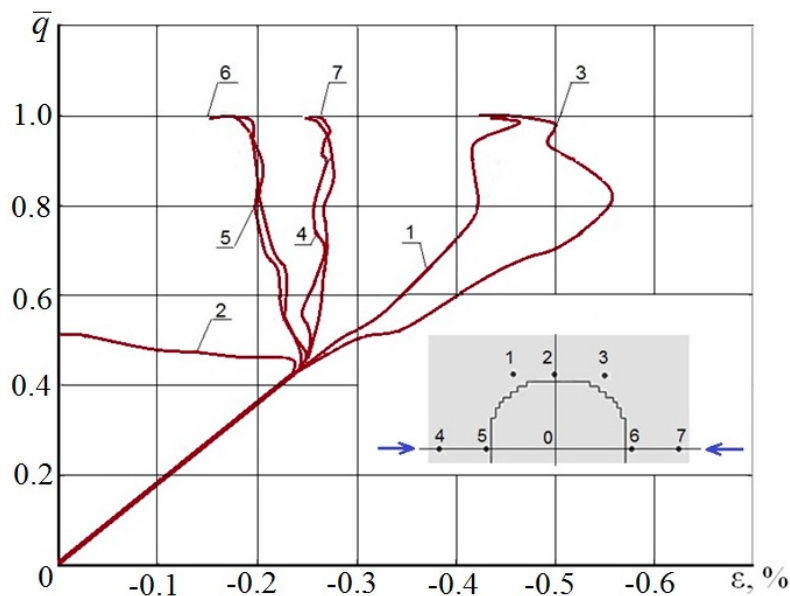


Figure 9. Strains in tested points near impact damage zone.

The level of the load at which the ultimate compression stress of the fibers is reached in the undamaged panel, $\bar{q} = 2.2$, is marked by a small cross on curve 1. It can be concluded that the local impact damage zone reduces the carrying capacity of composite panel by more than 50%.

The curves of longitudinal strains specific points on the external surface of the panel near the impact damage zone are shown in Figure 9. The numbers of the curves corresponding to the point numbers are presented in Figure 9. The panel is compressed along the line crossing gauges located at points 4–7. Numerical results show that deformations at all points grow nearly linearly to a level of $\varepsilon \sim 0.26\text{--}0.27\%$, up to a load of $\bar{q} = 0.4\text{--}0.45$. Then, the deformations are changed significantly with load growing up to the level of $\bar{q} = 1.0$. Gauge 2 is located just on the top of the buckling fold crossing the panel in the transverse direction (see Figure 7). That is why, due to extensive bending compression, strain at point 2 is reduced rapidly, passing into tension deformation. A small reduction of compression strains takes place at points 5 and 6 closing to the damage delaminated zone. In the adjacent points 4 and 7 located at the same axis, compression deformations remain close to the constant level by the load of $\bar{q} = 1.0$. Compression strains at points 1 and 3 continue to grow due to additional bending during the formation of the buckle fold.

A comparison of the obtained numerical results and experimental data is presented in Table 3. Compression tests of the panels were implemented with MTS500 automatic hydraulic test bench (PCE Group CO KG, Meschede, Germany) with a maximal load of 5000 kN and measuring accuracy of 1%. Strain measurement was conducted with CTMM strain gage equipment.

Table 3. Longitudinal strains ε near damage zone ($\bar{q}$ —dimensionless load).

Load $\bar{q}$	Points	Experimental ε , %	Numerical ε , %
1.0	1	−0.45	−0.44
	3	−0.48	−0.45
	4	−0.26	−0.25
	7	−0.30	−0.27

One can see that the calculated strains agree with measured ultimate deformations corresponding to the carrying capacity level of the damaged panel.

5. Conclusions

1. A complex modeling technique has been developed on the basis of the LS-Dyna code for the prediction of local impact damage and its influence on the carrying capacity of a compressed flat stiffened composite panel. The numerical analysis shows that local buckling of delaminated fibers in impact damage zone is one of the main reasons for a significant reduction of the carrying capacity of the compressed FRP stiffened panel.
2. The validity of the modeling technique was confirmed by the agreement between the numerical and experimental results. The maximal dynamic force and impulse duration in the simulation of local impact exceed experimental data by no more than ~20%. The numerical and experimental sizes of the impact damage zone are $\sim 0.7b$ and close to each other. These predictions could be quite acceptable for preliminary engineering analysis.
3. The fixed level of the kinetic energy of $E_k = \text{const.}$ is not a fully correct measure of local damage for FRP stiffened panel, since the decrease of the impactor weight and keeping $E_k = \text{const.}$ cause decreasing sizes of damage zone.
4. Quasi-static simulation of the damaged panel at the stage of longitudinal compression allows the prediction of the ultimate longitudinal strains near the impact damage zone: $\varepsilon_{\text{num}} = -0.25$ to -0.48% , which are close to the experimental levels: $\varepsilon_{\text{exp}} = -0.27\%$ to -0.45% , corresponding to the carrying load of the FRP panel.
5. The obtained numerical results show that the local impact damage zone can reduce the carrying capacity of the compressed composite panel by more than 50% compared to the undamaged composite panel.

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