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Markowitz Mean-Variance Portfolio Selection and Optimization under a Behavioral Spectacle: New Empirical Evidence

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Abstract: This paper investigates the robustness of the conventional mean-variance (MV) optimization model by making two adjustments within the MV formulation. First, the portfolio selection based on a behavioral decision-making theory that encapsulates the MV statistics and investors psychology. The second aspect involves capturing the portfolio asset dependence structure through copula. Using the behavioral MV (BMV) and the copula behavioral MV (CBMV), the results show that stocks with lower behavioral scores outperform counterpart portfolios with higher behavioral scores. On the other hand, in the Forex market, the reverse is observed for the BMV approach, while the CBMV remains consistent.

Keywords: mean-variance; dependence structure; portfolio optimization; cumulative prospect theory; differential evolution

JEL Classification: C02; G11; G17

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1. Introduction

Portfolio construction is fundamental to the investment management process. The Nobel Prize winner Harry Markowitz with his innovative work in (Markowitz 1952) established the underpinnings for the modern portfolio theory. This is an investment framework for the selection and construction of investment portfolios based on the maximization of expected portfolio returns and simultaneous minimization of investment risks that constitute the original mean-variance (MV) framework. This framework is appealing because it is efficient from a computational point of view. However, it also has well-established failings that can lead to portfolios that are not optimal from a financial point of view, see (Michaud 1989). Among these criticisms of MV optimization are: concentrated asset class allocation, inability to account for skewness and kurtosis, and lack of risk diversification. To address these shortcomings, many extensions of MV have been proposed in the literature. Zhou and Li (2000) introduced the stochastic linear quadratic (LQ) control as a general framework to study the mean-variance optimization, and found an analytical optimal portfolio policy and an explicit expression of the efficient frontier for a continuous-time mean-variance portfolio selection problem. Fahmy (2020) proposed a theoretical extension of the MV framework by adding a time dimension, so that the construction of a portfolio is thought of as an activity that consists of monetary outcomes. This mean-variance time model has the ability to explain many of the observed time-related anomalies of stock returns. Ötken et al. (2019) proposed another extension of MV in which the problem specification has three additional sets of constraints: cardinality, sector capitalization, and tracking error, on top of the Markowitz model and other diversification constraints regarding the portfolio. Each one of these MV extensions addressed a particular aspect of portfolio optimization.

In this study, an extension of the classical MV approach was achieved by first incorporating investor psychology through the cumulative prospect theory, then by coupling it to the copula model. These models will be applied to two different classes of assets, namely, the Johannesburg Stock Exchange (JSE) traded assets and the Forex market. Another important aspect that we aim to investigate is the performance ability of these models on each asset class.

According to Markowitz, investors always make rational decisions in order to maximize their utility. Thus, the prime objective of an investor is to maximize his/her utility by either maximizing the portfolio mean (i.e., return) or minimizing the portfolio standard deviation (i.e., risk) or vice-versa. Still on this line of utility maximization, in the context of insurance contract design, Xu et al. (2019) presented the optimal insurance model under the rank-dependent utility (RDU) framework and derived a general necessary and sufficient condition for optimal solutions. However, other researchers working in the field of behavioral and experimental economics hold contrary views about some of the assumptions underlying Markowitz's theory. They argue that investors are not fully rational and occasionally make sub-optimal decisions. In particular, investors are 'risk seeking' in the region of gains, but 'risk averse' in the region of losses (see, for example, Kahneman and Tversky 1979, 1992). The goal of this paper is to show how mean-variance portfolio allocation can benefit, on the one hand, from the development of the cumulative prospect theory (CPT) introduced by (Kahneman and Tversky 1992), and on the other hand, from copula, which embodies all of the information about the dependence between the components of a random vector. We will also assess the robustness and the sensitivity of these models to asset classes. As we will illustrate in this study, sensitivity of a model over asset classes should not be ignored in model building and validation. This assessment will be conducted in various portfolios constructed from the two asset classes (JSE and Forex) using their CPT scores.

The CPT theory is rooted in behavioral psychology and was demonstrated to possess sufficient explanatory power for use in actual decision-making problems. Few studies (see, for example, Ababio et al. 2020; Omane-Adjepong et al. 2019; Simo-Kengne et al. 2018; He and Zhou 2011) have delved into this area attempting to adopt some behavioral decision-making theories, to take a re-look at the Markowitz strategy. In these studies, the authors adopted the cumulative prospect theory with different probability weighting functions, such as the portfolio asset selection technique. He and Zhou (2011), in particular, introduced a new measure of loss aversion for large payoffs and investigated the sensitivity of the CPT value function with respect to the stock allocation. Jin and Yu Zhou (2008) established a continuous-time behavioral portfolio selection model-based cumulative prospect theory, featuring very general S-shaped utility functions and probability distortions, and obtained closed-form solutions for an important special case. With this approach, they were able to examine how the allocations to equity were influenced by behavioral criteria.

In this study, the impact of behavioral criteria on portfolio allocation is assessed empirically through two adjustments. The first one combines the traditional mean-variance, in what we will name behavioral mean-variance (BMV). A second adjustment to BMV with the copula function will be considered, and the resulting approach will be named copula behavioral mean-variance (CBMV).

The copula function was introduced by (Sklar 1959). It was designed to provide an idiosyncratic description of the dependence structure between random variables, irrespective of their marginal distributions. Sklar's theorem in (Sklar 1959) shows that copula fits well into a portfolio selection context, where various assets in the portfolio have different distributional characteristics. The interactions between assets have to be assessed in order to choose the copula that best models the dependence structure in the portfolio. One can immediately employ the multivariate copulae, such as t copula and Gaussian copula, thus discarding many available existing bivariate copulae with interesting properties. Thanks to vine copula, it is possible to simultaneously use different copulae in modeling the dependence structures in a multivariate setting. The first regular vine copula was introduced by (Joe 1994) to extend parametric bivariate extreme value copula families to higher dimensions. In the regular vine class, we have the C-vine and the D-vine. A preliminary check to the study conducted in this paper has seen C-vine outperforming D-vine in terms of portfolio risk and return. More illustrations of copula in the portfolio selection context can be seen in (Mba et al. 2018; Mba and Mwambi 2020, 2021; Ababio et al. 2020). Ababio et al. (2020) used CPT scores as portfolio selection criteria among two asset classes (indices and cryptocurrency) and employed t copula in the optimization process. Their results showed consistency throughout the various portfolios constructed: portfolios constructed from lower CPT scores outperformed those obtained from higher CPT scores irrespective of the asset class. Can we generalize these findings? The results we will obtain in this paper will show that such a generalization can be misleading. To assess this, we considered two different asset classes: JSE traded stocks on the Forex market. Instead of the t copula, as in the previous study, we use the vine copula, which is flexible enough to auto-select suitable bivariate copulae for dependence structure modeling. We can summarise the approach used in this paper as follows.

In this paper, we investigate the robustness of the conventional mean-variance (MV) optimization model by making two adjustments based, on the one hand, on a behavioral decision-making theory and investor psychology, called the behavioral mean-variance (BMV) approach, and on the other hand, by using the copula theory to extract the portfolio asset dependence structures, called the copula behavioral mean-variance (CBMV) approach. For this assessment, two markets will be considered: the Johannesburg Stock Exchange (JSE) and the Forex markets. The findings illustrate that CBMV is consistent with those in the literature; that is, portfolios with lower CPT scores outperform those with higher CPT scores. Whereas the BMV shows the reverse in the Forex market. This may be attributed to the lack of the classical MV to capture a nonlinear dependence structure.

The rest of the paper is organised as follows: Section 2 describes the data, the behavioral selection process, the copula theory, and the optimization algorithm. Section 3 presents the results and discussion. Section 4 concludes the study. Appendix A presents some basics of the JGR-GARCH model, introduced by (Glosten et al. 1993), which is used in this study to simulate the dynamics of the conditional variance.

2. Material and Methods

This study uses the cumulative prospect theory (CPT) as a portfolio assets selection technique before the mean-variance (MV) portfolio setting is implemented. This combination blends investor psychology into the traditional MV approach, which focuses mainly on the first two moments of asset return distribution. Let us start with the cumulative prospect theory.

2.1. Cumulative Prospect Theory

The study focuses on the extreme assets classified by a behavioral decision-making theory called the cumulative prospect theory proposed by (Kahneman and Tversky 1992). This decision-making theory helps capture investors' cognitive biases in their quests to select assets with the sole objective of adding value to their investments. Expected utility theory (EUT), a pioneer normative model of decision-making theory proposed by (Von Neumann and Morgenstern 1947), is estimated by finding the product of utility and its objective probability. On the other hand, the CPT, a descriptive model of decision-making, is a product of a value function (VF) and a non-linear decision weight estimated using an inverted S-shaped probability weighting function (PWF). The S-shaped PWF captures the deviations from rational thinking as prescribed by the EUT model. The PWF reflects probability distortions or biases and primarily governs the amplification of low probabilities and attenuates mid-range and high probabilities. In addition, the CPT provides several new features that depart from the EUT and broaden the decision-making process of investors. The behavioral model that describes the investor's decision-making process and characteristics in uncertain situations, is specified as follows:

$$CPT(x) = \sum(\nu_-\omega_- + \nu_+\omega_+)$$

where $\nu_-\omega_-$ and $\nu_+\omega_+$ represent the product of the VF function and the PWF for gains and losses. $CPT(x)$ denotes the behavioral numerical score for any given asset. For example, VF is mathematically estimated as follows:

$$\begin{cases} \nu_+ = x^\alpha, x \geq 0, \\ \nu_- = -\lambda(-x)^\beta, x < 0 \end{cases}$$

where $\nu(x)$ represents the value function over gain or loss x relative to a reference point; α, on the other hand, is the coefficient of risk-aversion $(0 < \alpha < 1)$; β is the coefficient of risk-seeking in the domain of losses $(0 < \beta < 1)$, and $\lambda > 0$ is the coefficient of the scaling factor called the loss-aversion. The estimated parameter values, according to (Kahneman and Tversky 1992), are $\alpha = 0.88$ and $\lambda = 2.25$. On the contrary, the PWF of the CPT model takes inverse S-shaped weighting functions, with separate functions for gains and losses mathematically expressed below:

$$\pi(p) = \begin{cases} \omega_-(p) = \dfrac{p^\nu}{(p^n u + (1-p)^\nu)^{\frac{1}{\nu}}}, 0 < \nu \leq 1 \\ \omega_+(p) = \dfrac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}}, 0 < \delta \leq 1 \end{cases}$$

for $p \in [0,1]$, where $(\omega_+(p))$ and $(\omega_-(p))$ represent the weighting functions for gains and losses, respectively. Kahneman and Tversky (1992) suggested the value of parameter $\nu = 0.69$ for ω_- and $\delta = 0.61$ for ω_+.

BMV Mathematical Formulation

For the behavioral mean-variance (BMV) portfolios, the first step involves the asset selection based on CPT scores; then the optimization is carried out according to the following formulation: $\begin{cases} \arg\min_{\omega} \omega^T\Sigma\omega - \omega^T\mu, \\ \omega^T i = 1, \\ \omega_i \geq 0,\ i = 1, \cdots, n \end{cases}$ where $\omega = (\omega_1, \cdots, \omega_n)$ is the weight vector; $\mu = (\mu_1, \cdots, \mu_n)$ the return vector, and Σ the variance-covariance matrix of the portfolio's assets; i is a column vector of ones.

2.2. Dependence Measure

The analysis of the dependence structure between random variables has gained much attention in probability and statistics. Various concepts and measures of statistical dependence have been introduced, including Pearson's Rho, Spearman's Rho, copula, distribution-based measures, the distance covariance, the HSIC measure popular in machine learning, and the local Gaussian correlation, which is a local version of Pearson's Rho, among others. On the one hand, Pearson's correlation coefficient is the most used measure of statistical dependence. It gives a complete characterization of dependence in the Gaussian case, and it also works well in some non-Gaussian situations. Copula, on the other hand, seems to provide a better way to model dependence, especially in heavy-tailed distributions and in nonlinear situations.

2.2.1. Copula

From the literature, we distinguished two groups of copula families: elliptical (such as Gaussian or Student's t copula) and Archimedean copula (such as Clayton, Gumbel, or

Frank copula). To model the dependence structure between three or more random variables, multivariate copulas have been used, but they lack flexibility in higher dimensions. To overcome this issue of flexibility, vine copula models were introduced. They use bivariate conditional copulas as building blocks, making them flexible enough to capture the underlying dependence structure. In dimension d (i.e., d random variables), a multivariate density is constructed by $d(d-1)/2$ bivariate (conditional) copulas (see, Bedford and Cooke 2001) as building blocks; thus, the name pair-copula construction (PCC) is given to this construction process.
For example, let x_1, x_2, and x_3 be three random variables with distribution functions F_1, F_2, and F_3, respectively. The joint density can be decomposed as

$$f(x_1, x_2, x_3) = f_{3|12}(x_3|x_1, x_2) f_{2|1}(x_2|x_1) f_1(x_1)$$

where

$$\begin{aligned} f_{2|1}(x_2|x_1) &= c_{12}\Big(F_1(x_1), F_2(x_2)\Big) f_2(x_2) \\ f_{3|12}(x_3|x_1, x_2) &= c_{13;2}\Big(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)\Big) f_{3|2}(x_3|x_2) \\ f_{3|2}(x_3|x_2) &= c_{23}\Big(F_2(x_2), F_3(x_3)\Big) f_3(x_3) \end{aligned}$$

with

$$F(x|\mathbf{v}) = \frac{\partial C_{x,v_j;\mathbf{v}_{-j}}\{F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})\}}{\partial F(v_j|\mathbf{v}_{-j})}$$

for every v_j of the vector $\mathbf{v}$ with $\mathbf{v}_{-j} = \mathbf{v} - \{v_j\}$ in the general case.

Note that the construction is not unique. All of the possible constructions are illustrated by a set of nested trees $T_i = (V_i, E_i)$ where V_i are the nodes and E_i the edges. This set of trees is called a Vine (Bedford and Cooke 2001).

A nested set of trees is a regular vine if and only if the trees fulfill the following conditions (Bedford and Cooke 2001):

1. T_1 is a tree with nodes $V_1 = \{1, \cdots, d\}$ and edges E_1;
2. For $i \geq 2$, T_i is a tree with nodes $V_i = E_{i-1}$ and edges E_i;
3. If two nodes in T_{i+1} are joint by an edge, the corresponding edge in T_i must share a common node (proximity condition).

A PCC is called a regular vine (R-vine) copula if all marginal densities are uniform. The class of regular vines is still very general and embraces a large number of possible pair-copula decompositions. We will concentrate here on a special case of regular vines called the **canonical vine**, also known as the (*C-vine*). Aas et al. (2009) specialize to a C-vine the density of an n-dimensional distribution given by (Bedford and Cooke 2001) in terms of a regular vine. The d-dimensional density $f(x_1, \cdots, x_d)$ corresponding to a C-vine is given by

$$\prod_{k=1}^{d} f(x_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{j,j+i|1,\cdots,j-1}\big(F(x_j|x_1, \cdots, x_{j-1}), F(x_{j+i}|x_1, \cdots, x_{j-1})\big). \tag{1}$$

CBMV Mathematical Formulation

For the copula behavioral mean-variance (CBMV) portfolios, the first step is the asset selection based on CPT scores, then the below steps are followed:

i. Use the GJR-GARCH model to filter the asset returns and obtain standardized residuals;
ii. Convert these residuals to a chosen marginal distribution for the estimation of the copula;

iii. Fit the vine copula to the data obtained in (ii);
iv. Simulate new data from the vine copula;
v. Use the inverse transform of the marginal distribution to convert the new data obtained in (iv) to new returns ready to be used in the optimization algorithm.

The optimization is again carried out according to the following formulation:

$$\begin{cases} \arg\min\limits_{\omega} \omega^T \Sigma \omega - \omega^T \mu, \\ \omega^T i = 1, \\ \omega_i \geq 0, \ i = 1, \cdots, n \end{cases}$$

where $\omega = (\omega_1, \cdots, \omega_n)$ is the weight vector; $\mu = (\mu_1, \cdots, \mu_n)$ the simulated return vector, Σ the variance–covariance matrix of the portfolio assets computed from the simulated returns, and i is a column vector of ones.

Portfolio Optimization

In the portfolio optimization process, the main challenge resides in designing a proper model that empirically best fits the data and, at the same time, is feasible and robust enough to generate simulation-based inference for risk evaluation.

In this study, we used an evolutionary algorithm called differential evolution (DE). Evolutionary algorithms (EAs) are search methods that take their inspiration from natural selection and survival of the fittest in the biological world. EAs differ from more traditional optimization techniques in that they involve a search from a "population" of solutions, not from a single point. Each iteration of an EA involves a competitive selection that eliminates poor solutions. The solutions with high *fitness* are *recombined* with other solutions by swapping parts of a solution with another. Solutions are also *mutated* by making a small change to a single element of the solution. What is appealing in these types of algorithms is that they use recombination and mutation to generate new solutions that are biased towards regions of the space for which good solutions have already been seen.

The DE algorithm introduced by (Storn and Price 1997) has the capability of solving nonlinear optimization problems. This algorithm uses biology-inspired operations of **initialization**, **mutation**, **recombination**, and **selection** on a population to minimize an objective function through successive generations (Holland 1975). Similar to other evolutionary algorithms, to solve optimization problems, DE uses alteration and selection operators to evolve a population of candidate solutions. In this process, a global solution is reachable. It has become a powerful and flexible tool to solve optimization problems arising in finance. DE has been successfully used in finding optimal portfolio weights and returns in (Ababio et al. 2020; Krink et al. 2009; Krink and Paterlini 2011; Maringer and Oyewumi 2007; Mba and Mwambi 2020, 2021; Mba et al. 2018; Yollin 2009).

3. Results and Discussion

3.1. Data

Model performance is particularly tested during normal and stressful (extreme) market conditions. However, while many models perform well during normal market conditions, the reverse is not always the case and poses model robustness problems. Thus, a robust model is expected to perform during normal and turbulent market periods. Therefore, the study period was chosen to test the robustness of the models adopted in analysing the empirical data. The 2008 global financial crisis is one good example from recent years and could serve the study's purpose.

Data comprise two different asset types. The first one constitutes one hundred and eight (108) stocks from the Johannesburg Stock Exchange (JSE) spanning from January 2007 to December 2009. The second constitute 95 Forex data (US dollar (USD) against

95 other currencies) from August 2007 to April 2007. The daily stock and Forex returns were estimated as

$$r_i = \ln\left(\frac{p_t}{p_{t-1}}\right) \times 100\%$$

where r_i represents daily stock returns and $p_t(p_{t-1})$ denotes the stock price for day $t(t-1)$.

3.2. Behavioral Asset Selection Criterion within the MV Framework

The MV model is a normative model of decision-making that focuses on how investors are expected to behave in financial markets. Thus, according to (Markowitz 1952)investors are more particular about the first two moments of asset return distribution in taking market-related decisions. However, it has become obvious that many investors have departed from (Markowitz 1952) rational norms. Thus, other factors apart from market returns' mean and standard deviations influence investors' decision-making. One key highlight in behavioral economics is that investors are not fully rational and depart from many rational norms due to cognitive biases that impact decision-making. Hence, the cumulative prospect theory (CPT) has become a golden tool to encapsulate Markowitz's proposed norms and other factors that influence investors' decision-making processes. The CPT model is a descriptive model of decision-making; however, it dwells on what investors do in real-life as opposed to normative models. Investors' cognitive biases become evident in their decision-making processes on a daily basis; however, normative models of decision-making fail to capture these biases and deviations from rationality, as depicted in Figure 1 below.

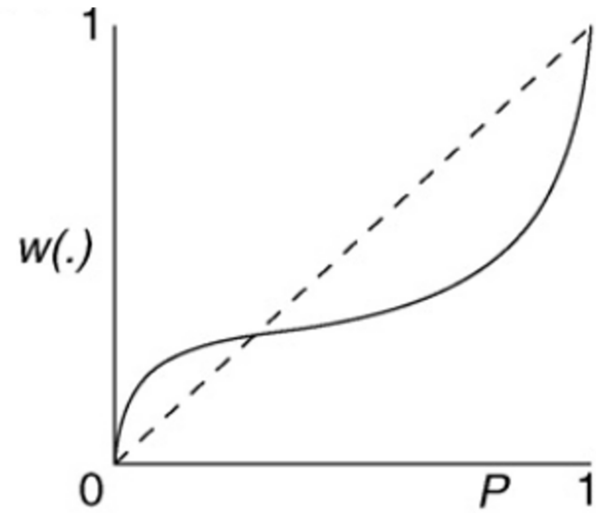

Figure 1. Nonlinear function of probability (P) in the behavioral decision theory.

The probability weighting function expresses that investors overreact to prospects with small probabilities, but under-react to medium and large probabilities.

The behavioral score estimated using the CPT model for 108 stocks ranges from 0.0678 to 0.6154. The first and last eight (8) ranked stocks were selected and classified based on their behavioral scores. Stocks with lower (higher) behavioral scores ranged between 0.0678 and 0.1043 (0.3505 and 0.6154). Likewise, the Forex data with lower (higher) behavioral scores were estimated between 0.0032 and 0.0161 (0.1665 and 0.7239) thresholds, indicating min (max) behavioral scores as 0.0032 (0.7239). Four distinct portfolios made up of assets with extreme behavioral scores were selected for optimization analyses using the conventional MV model. The MV asset selection criteria focuses mainly on the first two moments of asset return distribution, disregarding investors' psychology. On the other hand, the behavioral asset selection criterion encapsulates the distribution of asset returns and the behavioral scores, primarily capturing investors' cognitive biases in their decision-making processes.

3.3. Asset Classification and Selection

Eight assets with extreme behavioral scores were selected for portfolio analysis and optimization to form a portfolio in both asset classes. Thus, two portfolios were constructed for each asset class. The first (second) portfolio was composed of eight assets with extremely low (high) behavioral scores in both cases. Thus, eight assets were carefully selected to form the different portfolios with the sole objective of benefiting from portfolio diversification. The stock (Forex) portfolio has two distinct portfolios: the stock portfolio 1L and 1H (Forex portfolio 1L and 1H), respectively. Assets in the four different portfolios with corresponding behavioral scores are presented below:

3.3.1. CPT Scores for Stocks

JSE Stock Portfolio 1L: RES (0.0678), EMI (0.0790), CLH (0.0839), SPP (0.0873), TKG (0.0975), VKE (0.1005), CML (0.1017), JSE (0.1043).

JSE Stock Portfolio 1H: CFR (0.3505), ITE (0.3516), ACT (0.3521), RCL (0.3615), KIO (0.3689), LON (0.4748), MTA (0.5311), TDH (0.6154).

3.3.2. CPT Scores for Dollar Exchange Rate

Forex Portfolio 1L: Qatari Rial/US Dollar (0.0032), Oman Rial/US Dollar (0.0033), Hong Kong/US Dollar (0.0046), Bahraini Dinar/US Dollar (0.0050), UAE Dirham/US Dollar (0.0084), Jordanian Dinar/US Dollar (0.0087), Egyptian Pound/US Dollar (0.0152), Chinese Yuan/US (0.0161).

Forex Portfolio 1H: Swaziland Lilangeni/US Dollar (0.1665), Georgian Lari/US Dollar (0.1717), Ukraine Hryvnia/US Dollar (0.1794), Icelandic Krona/US Dollar (0.1870), South Africa Rand/US Dollar (0.2128), Kazakhstan Tenge/US Dollar (0.2387), Seychelles Rupee/US Dollar (0.5559), Cape Verde Escudo/US Dollar (0.7239).

The paper adopted three key portfolio statistics in comparing the performances of the four formulated portfolios. These statistics include the portfolio return, risk, and the Sharpe ratio, respectively. Apart from the two moments of the portfolio return distribution, the Sharpe ratio, on the other hand, makes it possible to evaluate portfolio performance easily. The higher the Sharpe ratio, the better, and portfolios with such characteristics attract many investors. The optimization results of the Forex portfolios were used to validate the results of the JSE stock portfolios.

3.4. Descriptive Statistics

Tables 1 and 2 present the summary results of the classified stocks with extreme low and high behavioral stocks estimated using the proposed metric by (Kahneman and Tversky 1992) respectively. A cursory inspection of these tables indicates that the classified stocks depart significantly from normality. The skewness and the kurtosis estimates of all stocks in the two behaviorally classified stocks exhibit non-normality characteristics. In both cases, the results showed a non-zero skewness signifying the non-normality of the stock returns distribution. Thus, the skewness estimates for stocks with lower (higher) behavioral scores exhibited strictly positive skewness. Likewise, on the other hand, the kurtosis also indicates either lighter (i.e., kurtosis less than 3) or heavier (i.e., kurtosis more than 3) tails than the normal distribution. The eight stocks classified as having lower behavioral scores all had kurtosis estimates of less than three (3), with six (two) estimates being negative (positive), see Table 1. Likewise, the classified stocks with higher behavioral scores, see Table 2. Table 3 also presents the same results as in Tables 1 and 2 but for the classified foreign exchange rate of the US Dollar against a universe of other currencies. Similarly, the classified Forex with extreme lower and higher behavioral scores depicted similar summary statistics as Tables 1 and 2.

Table 1. Summary statistics of classified JSE stocks with lower behavioral scores.

	JSE Stocks							
	RES	**EMI**	**CLH**	**SPP**	**TKG**	**VKE**	**CML**	**JSE**
Mean	0.090	−0.0005	0.006	0.037	−0.121	0.016	0.239	0.113
min	−3.680	−5.190	−17.600	−4.990	−8.710	−6.700	−5.830	−7.750
max	4.270	4.950	5.200	5.480	6.400	6.860	6.520	6.770
sd	0.796	1.180	1.580	1.300	1.560	1.170	1.540	1.500
asd	12.600	18.700	25.000	20.600	24.700	18.500	24.300	23.700
Skewness	4.230	3.770	20.400	1.270	3.780	4.840	2.420	1.930
Kurtosis	−0.155	−0.172	−1.660	0.161	−0.570	0.048	−0.166	−0.093
JB	570.000	454.000	13,452.000	55.000	494.000	741.000	190.000	120.000
Q10	12.800	49.100	18.300	15.800	8.240	9.520	11.100	5.310
Q102	34.400	179.000	7.050	9.410	62.200	77.800	25.400	135.000
ACF	0.149	0.197	0.015	0.052	0.198	0.277	0.149	0.109

Table 2. Summary statistics of classified JSE stocks with higher behavioral scores.

	JSE Stocks							
	CFR	**ITE**	**ACT**	**RCL**	**KIO**	**LON**	**MTA**	**TDH**
Mean	0.037	−0.031	−0.096	0.017	−0.108	−0.065	−0.104	−0.088
min	−8.151	−23.180	−26.788	−9.496	−20.373	−18.427	−46.801	−63.178
max	28.996	28.768	28.768	30.449	29.725	40.547	46.801	57.536
sd	2.407	3.269	5.321	2.249	4.079	3.955	4.106	6.257
asd	38.061	51.691	84.126	35.558	64.493	62.527	64.918	98.933
Skewness	27.680	19.429	7.022	45.764	5.417	16.450	63.627	40.428
Kurtosis	2.153	0.515	0.213	3.301	0.370	0.964	−0.385	−0.652
JB	24,731.780	11,934.540	1562.512	67,365.850	944.345	8649.373	127,575.800	51,558.460
Q10	9.272	12.337	59.195	15.397	14.284	11.993	140.625	40.508
Q102	6.894	19.195	89.952	0.433	155.785	4.640	185.669	27.276
ACF	0.004	0.152	0.215	0.015	0.315	0.026	0.470	0.154

Table 3. Summary statistics of classified Forex with lower and higher behavioral scores.

	Currencies															
	QATARI.RIAL	OMAN.RIAL	HONG.KONG	BAHRAINI. DINAR	UAE.DIRHAM	JORDANIAN. DINAR	EGYPTIAN. POUND	CHINESE. YUAN	SWAZILAND. LILANGENI	GEORGIAN. LARI	UKRAINE. HRYVNIA	ICELANDIC. KRONA	SOUTH. AFRICA.RAND	KAZAKHSTAN. TENGE	SEYCHELLES. RUPEE	CAPE. VERDE.ESCUDO
Mean	0.0001	−0.00001	−0.002	−0.00003	0.00001	−0.0001	−0.001	−0.024	0.055	0.0004	0.111	0.141	0.055	0.044	0.212	0.009
min	−0.137	−0.138	−0.235	−0.293	−0.339	−0.304	−0.857	−0.421	−6.906	−0.817	−16.448	−14.461	−6.776	−1.338	−6.340	−69.253
max	0.137	0.151	0.202	0.236	0.431	0.432	0.769	0.861	12.093	13.386	13.231	13.735	16.172	19.546	50.499	68.944
sd	0.032	0.025	0.044	0.041	0.037	0.092	0.202	0.126	1.698	0.680	2.027	2.245	1.759	0.955	2.667	4.844
asd	0.500	0.398	0.701	0.643	0.582	1.449	3.199	1.991	26.841	10.746	32.057	35.497	27.814	15.099	42.170	76.597
Skewness	4.911	12.296	5.354	19.613	71.580	2.917	2.507	5.603	8.139	347.044	21.712	17.166	17.768	401.594	292.018	189.641
Kurtosis	−0.092	0.113	−0.221	−0.608	0.967	0.626	0.180	0.714	0.979	17.755	−0.052	−0.687	1.897	19.800	15.758	−0.138
JB	439.217	2741.502	524.643	6993.046	92,762.000	183.625	117.292	607.449	1271.173	2,200,989.000	8536.129	5371.844	5977.720	2,945,088.000	1,560,195.000	650,475.800
Q10	80.245	47.611	19.963	45.141	234.026	82.264	11.198	8.112	17.518	4.448	13.919	23.101	26.243	2.662	18.185	96.822
Q102	102.499	21.124	82.127	61.386	222.140	22.936	26.124	1.504	242.653	0.023	34.902	548.066	95.065	0.018	1.010	107.313
ACF	0.358	0.198	0.269	0.298	0.515	0.100	0.080	0.013	0.247	−0.003	0.210	0.797	0.164	0.001	0.046	−0.004

3.5. Empirical Results

This section presents the optimization result of all portfolios described in the previous section and intuitively explains the results' direction. The traditional MV optimization technique and the proposed copula-based C-vine were adopted to optimize the various portfolios constituting the two asset classes. For each portfolio, the two estimation techniques were used to analyze the portfolios. The portfolio optimization results are discussed below.

3.5.1. Optimization of Classified Stocks

The optimization of the classified stocks that consist of two types of portfolios: portfolio with lower CPT scores and portfolio with higher CPT scores, were optimized using the traditional mean-variance (MV) approach as well as the C-vine approach. While the MV approach adopts the conventional Pearson correlation as a measure of dependence between portfolio assets, the C-vine uses copula as a measure of dependence. Copula captures the linear and nonlinear dependence structures between two variables (for bivariate copula) or more than two variables (for multivariate copula), whereas the Pearson correlation captures only linear dependence between two variables. Below are the optimization results.

From the return and sharp ratio resulting from the mean-variance (MV) approach, as displayed in Table 4, it is evident that the portfolio of stocks with lower CPT scores outperforms that of stocks with higher CPT scores.

Table 4. Optimization results of JSE stock portfolios using BMV approach.

	Lower CPT Stock Portfolios									Higher CPT Stock Portfolios					
Tickers	**RES**	**EMI**	**CLH**	**SPP**	**TKG**	**VKE**	**CML**	**JSE**	**CFR**	**ITE**	**ACT**	**RCL**	**KIO**	**LON**	**MTA**	**TDH**
Weights	30.04	12.48	9.23	10.14	9.31	13.56	6.36	8.89	8.19	14.36	8.12	17.26	15.12	13.12	16.76	7.08
Return	0.0476								−0.0548							
Risk	0.0875								0.9924							
SR	0.5440								−0.0552							

This is also supported by the results in Table 5 obtained from the C-vine approach.

Table 5. Optimization results of JSE stock portfolios using the CBMV approach.

	Lower CPT Stock Portfolios								Higher CPT Stock Portfolios							
Tickers	**RES**	**EMI**	**CLH**	**SPP**	**TKG**	**VKE**	**CML**	**JSE**	**CFR**	**ITE**	**ACT**	**RCL**	**KIO**	**LON**	**MTA**	**TDH**
Weights	12.50	12.50	12.50	12.50	12.50	12.50	12.50	12.50	3.00	21.00	11.00	10.00	10.00	2.00	8.00	35.00
Return	0.0176								0.0079							
CVaR	0.5840								0.3060							
SR	0.0301								0.0258							

While MV outperforms C-vine in these individual portfolios, Tables 4 and 5, the reverse is observed when both classified stocks are merged into one portfolio, as displayed in Tables 6 and 7. In the merged portfolio using the MV approach, 83.95% of capital were mainly allocated to stocks with lower CPT scores. However, with the C-vine approach, in the merged portfolio, 77% of capital were allocated to stocks with higher CPT scores.

Table 6. Optimization results of the portfolio consisting of a combination of JSE stocks with lower and higher behavioral scores using the BMV approach.

Combined (Lower and Higher) Stock Portfolio																
Tickers	**RES**	**EMI**	**CLH**	**SPP**	**TKG**	**VKE**	**CML**	**JSE**	**CFR**	**ITE**	**ACT**	**RCL**	**KIO**	**LON**	**MTA**	**TDH**
Weights	15.03	15.79	9.53	9.45	17.18	14.27	0.00	2.70	2.22	2.15	0.84	4.44	c1.44	1.70	c2.03	1.24
Return	−0.0036															
Risk	0.1133															
SR	−0.0318															

Table 7. Optimization results of the portfolio consisting of a combination of JSE stocks with lower and higher behavioral scores using the CBMV approach.

Combined (Lower and Higher) Stock Portfolio																
Tickers	**RES**	**EMI**	**CLH**	**SPP**	**TKG**	**VKE**	**CML**	**JSE**	**CFR**	**ITE**	**ACT**	**RCL**	**KIO**	**LON**	**MTA**	**TDH**
Weights	2	2.00	2.00	5.00	2.00	2.00	2.00	6.00	2.00	24.00	2.00	2.00	3.00	5.00	3.00	36.00
Return	0.0080															
CVaR	0.2360															
SR	0.0338															

While with the MV approach, a higher percentage of funds were allocated to stocks with lower CPT scores, the portfolio optimization resulted in a negative return with commensurable risk. This may be as a result of investing in false positive stocks (by positive, we mean assets with the potential to add value to the portfolio as selected by the MV approach. However, this leads to a negative return and the corresponding negative Sharpe ratio. Thus, the name *false positive stocks*). The C-vine, on the other hand, achieved a positive return and a positive Sharpe ratio by allocating more that $\frac{3}{4}$ of funds to stocks with higher CPT scores. Hence, these stocks can be called true-positive stocks.

For the Forex market, while higher CPT portfolio outperforms the lower CPT one for the MV approach (Table 8), the reverse is observed with C-vine (Table 9). The fact that the C-vine results are consistent for both markets may be attributed to its ability of capturing both linear and nonlinear dependence structures through suitable bivariate copulas. On the other hand, the inconsistency of the MV results for both markets may be a result of the market characteristics. While the stocks are country specific markets, the Forex on the other hand is a global and highly liquid market. Nonlinearity has been observed in many highly liquid developed markets, see (Antoniou et al. 1997). The contradictory results on both markets from the MV approach may be as a result of the failure to capture the nonlinearity in the highly liquid Forex market.

Table 8. Optimization results of Forex portfolios using the BMV approach.

Forex Portfolio with Lower Behavioral Scores					Forex Portfolio with Higher Behavioral Scores				
Tickers	Weights	Return	Risk	SR	Tickers	Weights	Return	Risk	SR
QATARI.RIAL.TO.USD	18.20	−0.0033	0.0002	−16.5000	SWAZILAND.LILANGENI.TO.USD	5.45	0.0784	0.2236	0.3506
OMAN.RIAL.TO.USD	24.28				GEORGIAN.LARI.TO.USD	20.86			
HONG.KONG.TO.USD	17.71				UKRAINE.HRYVNIA.TO.USD	13.64			
BAHRAINI.DINAR.TO.USD	6.33				ICELANDIC.KRONA.TO.USD	11.92			
UAE.DIRHAM.TO.USD	17.48				SOUTH.AFRICA.RAND.TO.USD	3.94			
JORDANIAN.DINAR.TO.USD	3.50				KAZAKHSTAN.TENGE.TO.USD	31.21			
EGYPTIAN.POUND.TO.USD	0.04				SEYCHELLES.RUPEE.TO.USD	12.99			
CHINESE.YUAN.TO.USD	12.45				CAPE.VERDE.ESCUDO.TO.USD	0.00			

Table 9. Optimization results for Forex portfolios using the CBMV approach.

Forex Portfolio with Lower Behavioral Scores					Forex Portfolio with Higher Behavioral Scores				
Tickers	**Weights**	**Return**	**Risk**	**SR**	**Tickers**	**Weights**	**Return**	**Risk**	**SR**
QATARI.RIAL.TO.USD	12.50				SWAZILAND.LILANGENI.TO.USD	2.00			
OMAN.RIAL.TO.USD	12.50				GEORGIAN.LARI.TO.USD	7.00			
HONG.KONG.TO.USD	12.50				UKRAINE.HRYVNIA.TO.USD	15.00			
BAHRAINI.DINAR.TO.USD	12.50				ICELANDIC.KRONA.TO.USD	2.00			
UAE.DIRHAM.TO.USD	12.50	0.0068	0.5820	0.0116	SOUTH.AFRICA.RAND.TO.USD	10.00	0.0044	0.4650	0.0094
JORDANIAN.DINAR.TO.USD	12.50				KAZAKHSTAN.TENGE.TO.USD	19.00			
EGYPTIAN.POUND.TO.USD	12.50				SEYCHELLES.RUPEE.TO.USD	5.00			
CHINESE.YUAN.TO.USD	12.50				CAPE.VERDE.ESCUDO.TO.USD	40.00			

For the Forex merged portfolio, MV outperforms the C-vine (Table 10), as opposed to the observations made for stocks. The results exhibited by C-vine are consistent with those obtained by (Ababio et al. 2020) using R-vine on indices and cryptocurrency portfolios.

3.5.2. Robustness of Optimization Results

In the JSE stock market, using the BMV and the CBMV approaches, the stocks with lower CPT scores were found to outperform the counterpart portfolios with higher CPT scores. The CBMV results in the JSE stock market are consistent with those of the Forex market. However, in the Forex market, the BMV portfolio with higher CPT scores outperformed the counterpart portfolio with lower CPT scores, which is different from the BMV results in the JSE stock market.

Merging the two portfolios using the BMV and CBMV in each market, the CBMV outperforms that of the BMV in the JSE stock market, while in the Forex market, the reverse was observed.

Based on the Sharpe ratio, the BMV appears to outperform the CBMV in almost all settings. It appears that investing in the Forex market using the behavioral MV approach is more promising and could add value to the invested portfolio.

Table 10. Optimization results of Forex portfolios using BMV and CBMV, respectively.

Forex Portfolio Obtained by Merging Currencies with Lower and Higher Behavioral Scores (BMV Approach)					Forex Portfolio Obtained by Merging Currencies with Lower and Higher Behavioral Scores (CBMV Approach)				
Tickers	**Weights**	**Return**	**Risk**	**SR**	**Tickers**	**Weights**	**Return**	**Risk**	**SR**
QATARI.RIAL.TO.USD	0.00				QATARI.RIAL.TO.USD	2.00			
OMAN.RIAL.TO.USD	29.66				OMAN.RIAL.TO.USD	8.00			
HONG.KONG.TO.USD	0.00				HONG.KONG.TO.USD	2.00			
BAHRAINI.DINAR.TO.USD	20.89				BAHRAINI.DINAR.TO.USD	7.00			
UAE.DIRHAM.TO.USD	11.14				UAE.DIRHAM.TO.USD	4.00			
JORDANIAN.DINAR.TO.USD	0.00				JORDANIAN.DINAR.TO.USD	4.00			
EGYPTIAN.POUND.TO.USD	2.32				EGYPTIAN.POUND.TO.USD	2.00			
CHINESE.YUAN.TO.USD	0.00				CHINESE.YUAN.TO.USD	4.00			
SWAZILAND.LILANGENI.TO.USD	1.64	0.0376	0.0488	0.7705	SWAZILAND.LILANGENI.TO.USD	2.00	0.0134	0.3250	0.0412
GEORGIAN.LARI.TO.USD	0.87				GEORGIAN.LARI.TO.USD	8.00			
UKRAINE.HRYVNIA.TO.USD	6.54				UKRAINE.HRYVNIA.TO.USD	2.00			
ICELANDIC.KRONA.TO.USD	6.08				ICELANDIC.KRONA.TO.USD	7.00			
SOUTH.AFRICA.RAND.TO.USD	1.55				SOUTH.AFRICA.RAND.TO.USD	3.00			
KAZAKHSTAN.TENGE.TO.USD	12.46				KAZAKHSTAN.TENGE.TO.USD	2.00			
SEYCHELLES.RUPEE.TO.USD	6.86				SEYCHELLES.RUPEE.TO.USD	5.00			
CAPE.VERDE.ESCUDO.TO.USD	0.00				CAPE.VERDE.ESCUDO.TO.USD	38.00			

4. Conclusions

In this paper, we investigated the robustness of the conventional mean-variance (MV) optimization model by making two adjustments, based, on the one hand, on a behavioral decision-making theory and investor psychology, called the BMV approach, and on the other hand, by using the copula theory to extract the portfolio asset dependence structures, called CBMV. Applying the BMV and the CBMV on the JSE stock market, the results show that portfolios of stocks with lower behavioral scores outperform counterpart portfolios with higher behavioral scores. Whereas on the Forex market, the reverse is observed for the BMV, while the CBMV remains consistent. More specially, in the Forex market, the BMV portfolio with higher CPT scores was found to outperform the counterpart portfolio with lower CPT scores. This could be due to the failure of the classical MV to capture the non-linearity exhibited by a highly liquid market as the Forex. In the future, other markets, such as commodities, bonds, or derivatives, will be assessed. Applying the BMV and the CBMV on the combined portfolios within the same asset classes, the CBMV outperforms that of the BMV in the stock market, while in the Forex market, the reverse was observed. Based on the Sharpe ratio, the BMV appears to outperform the CBMV in almost all settings. It appears that investing the Forex market using the behavioral BMV approach is more promising and could add value to investors' portfolios.

The approach presented in this study has the advantage of incorporating investor psychology in the portfolio selection since investors are not fully rational in their decision making. Previous studies on world indices and cryptocurrency markets have shown consistency with the indices/cryptoassets with lower CPT scores outperforming those with higher CPT scores. This was based on a t-copula type model. The same conclusion is drawn here with the vine copula type model (CBMV), whereas the BMV model does not align to such conclusions in the Forex market. Thus, is such a claim market-related or model-specific? We will investigate this in future work.

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Appendix A

Appendix A.1. JGR-GARCH Model

It is useful in the portfolio selection to take into account the dependence structure, the stylized facts related to the fat tail distributions, as well as the phenomenon of volatility clustering. This explains the use of GJR-GARCH and vine copula in this study to model these features.

Appendix A.2. GARCH Specifications

Let $\mathbf{r}_t = (r_{1,t}, r_{2,t}, \cdots, r_{d,t})$ be a d-dimensional vector of asset returns. The standard GARCH specification is given by:

$$\begin{aligned} r_{i,t} &= \mu_{i,t} + \epsilon_{i,t} \\ \epsilon_{i,t} &= \sigma_{i,t}\nu_{i,t} \\ \sigma_{i,t} &= \omega_{i,t} + \alpha_{i,t}\epsilon_{i,t}^2 + \beta_{i,t}\sigma_{i,t-1} \end{aligned} \tag{A1}$$

where $\sigma_{i,t}$ is the conditional variance of the returns series $r_{i,t}$ and $\nu_{i,t} \sim N(0,1)$ the standardized innovations/residuals, which can be assumed to follow a normal, Student's t, skewed-Student's t, or any other distribution. Giving the lack of a standard GARCH model to capture the leverage effect, we opted for an asymmetric GARCH, namely GJR-GARCH (Glosten et al. 1993) to simulate the dynamics of the conditional variance given by:

$$\sigma_{i,t}^2 = \omega_i + \sum_{j=1}^{p} \beta_{i,j}\sigma_{i,t-j}^2 + \sum_{k=1}^{q} (\alpha_{i,k} + \gamma_{i,k}\mathbb{I}_{i,t-k})\epsilon_{i,t-k} \tag{A2}$$

where $\mathbb{I}_{i,t-k} = \begin{cases} 1, & \text{if } \epsilon_{i,t-k} < 0; \\ 0, & \text{if } \epsilon_{i,t-k} \geq 0 \end{cases}$ Financial return time series are known to display fat tail features. Giving that the heteroskedasticity does not explain all characteristics of fat-tailed distributions, we need to specify such distributions to model innovations. Bollerslev and Wooldridge (1992) used the t-distribution while Engle and Gonzalez-Rivera (1991) used non-parametric modeling. To control the higher order moments (Kurtosis and Skewness), Hansen (1994) constructed a new distribution called *skewed Student's t*, which is a generalization of the Student's-t distribution with an additional parameter to control the skewness. Its density is given by:

$$d(x;\eta,\lambda) = \begin{cases} bc\left(1 + \dfrac{1}{\eta-2}\left(\dfrac{bx+a}{1-\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & \text{if } x > -\dfrac{a}{b} \\ bc\left(1 + \dfrac{1}{\eta-2}\left(\dfrac{bx+a}{1+\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & \text{if } x \leq -\dfrac{a}{b} \end{cases}$$

where $a = 4\lambda c\dfrac{\eta-2}{\eta-1}$, $b = 1 + 3\lambda^2 - a^2$, $c = \dfrac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)\Gamma\left(\frac{\eta}{2}\right)}}$ and Γ is the gamma function. What is interesting about this density function is that it encompasses a wide range of conventional densities. For example, when $\lambda = 0$, it reduces to Student's-t distribution. If $\lambda = 0$ and $\eta \to \infty$, it reduces to the normal distribution. This justifies our choice for the skewed Student's-t distribution as our margins in this study.

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