

Article

An Improved Valuation Model for Technology Companies

Ako Doffou

The Institute of International Studies, Ramkhamhaeng University, Bangkok 10240, Thailand;
E-Mail: adoff21@gmail.com; Tel.: +66-(0)-2310-8895

Academic Editor: Nicholas Apergis

Received: 4 April 2015 / Accepted: 20 May 2015 / Published: 1 June 2015

Abstract: This paper estimates some of the parameters of the Schwartz and Moon (2001) model using cross-sectional data. Stochastic costs, future financing, capital expenditures and depreciation are taken into account. Some special conditions are also set: the speed of adjustment parameters are equal; the implied half-life of the sales growth process is linked to analyst forecasts; and the risk-adjustment parameter is inferred from the company's observed stock price beta. The model is illustrated in the valuation of Google, Amazon, eBay, Facebook and Yahoo. The improved model is far superior to the Schwartz and Moon (2001) model.

Keywords: valuation; cross-sectional data; stochastic costs; speed of adjustment; implied half-life; risk-adjustment parameter

JEL Classifications: G12

1. Introduction

The basic model used in Doffou [1] to price Internet companies or technology companies is an improvement of the Schwartz and Moon [2] model based on real options theory and capital budgeting techniques and shows that uncertainty about some specific variables (the changes in sales and the expected rate of growth in sales) significantly affects the pricing of technology companies. Doffou [1] also shows that estimating the parameters of the model using cross-sectional quarterly data improves the pricing accuracy of the model.

The attempt of this paper is to improve this basic model further in many ways to make it more realistic. The variable costs function is now allowed to move around randomly under a mean reverting

process with its own volatility that also exhibits mean reversion with deterministic trends. This stochastic behavior of the variable costs function captures the fact that many technology companies operate at a loss for many years before they are expected to generate profits in the future.

In computing the net after tax cash flows, capital expenditures and depreciation are accounted for through the introduction of a third path dependent but deterministic variable which is property, plant and equipment. This new variable decreases with depreciation, increases with capital expenditures and allows the model to fit and captures the technology companies that undertake huge investments in fixed assets.

In the basic model, the technology company is bankrupt when it has zero cash. This occurs when its cash position reaches zero for the first time. In reality, the cash balance could very well reach zero or be negative while the prospects of the company could be strong enough to allow the management to raise new fresh cash or merge with another company. Hence, this paper improves the bankruptcy condition in the model and allows the cash balances to become negative, opening the possibility of additional financing in the future (equity or debt financing or possible sale of the company). The optimal financing is the one that maximizes the value of the firm.

The market price of risk in the basic model is unobservable and its estimation was quite challenging. Rather than estimating this parameter, it can be inferred from the beta of the technology company stock. The theoretical framework allows the computation of the beta of the stock as a way to infer the risk premium in the model.

To facilitate the practical implementation of the model, a number of simplifying assumptions are introduced. First, all of the speeds of adjustment in the model are set to be equal and are derived from the half-life of the company to being a normal firm. Second, the variable costs and the growth rates in sales are both assumed to be orthogonal to the market returns. Hence, to implement the model, only the risk premium associated with the sales process must be estimated.

Because the speed of mean reversion for the rate of growth in sales process has a supreme importance in the valuation model, the half-life of the deviations is related to analyst expectations about future sales.

The computation of the value of the stock starts from the total value of the firm and takes into account the market prices of the straight debt issues when these market prices are readily available.

All of the above improvements lead to an expanded model with six state variables. Three of the state variables are deterministic and path dependent and the remaining three are stochastic. The state variables sales, growth rates in sales and variable costs are stochastic while the loss carry-forward, the amount of cash available and the accumulated property, plant and equipment are deterministic and path dependent. This large number of state variables and the complexity of these path dependencies can be easily accounted for by using a Monte Carlo simulation to solve this model and derive the value of the technology company. To implement the Monte Carlo simulation, a discrete time approximation of the model is used because the simulation runs on an interval of time of one quarter, six months or even one year which is quite long.

Section 2 introduces the improved model with all of its characteristics described above. The model parameters are estimated in Section 3 while the simulations are carried out in Section 4. Section 5 discusses the simulation results and the derivation of the share price. Some sensitivity analysis is carried out on some critical parameters in Section 6. Finally, Section 7 concludes the paper.

2. The Improved Model

Suppose the technology company generates sales at time t given by S_t . The sales at time t are also the instantaneous rate of revenues at time t . The dynamics of these revenues are assumed to follow the stochastic differential equation:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dZ_1 \quad (1)$$

where the drift term μ_t represents the expected rate of growth in sales, σ_t the volatility of the rate of sales growth and dZ_1 is an increment of a Brownian motion. The drift is assumed to exhibit mean reversion and is pulled over time to its long-term mean value $\bar{\mu}$ at a certain speed. This speed of adjustment is dictated by the competitiveness within each industry. The initial super high growth rates of the technology company will converge to a more reasonable and lower sustainable rate of growth characteristic of each industry. Hence,

$$d\mu_t = K(\bar{\mu} - \mu_t)dt + \delta_t dZ_2 \quad (2)$$

where the initial volatility of expected rates of growth in sales is given by δ_0 and K is the speed of adjustment. This speed of adjustment or speed of mean reversion translates the rate at which the growth rate in sales is expected to converge to its long-term mean value $\bar{\mu}$, and $\ln(2)/K$ can be seen as the “half-life” of the deviations. Any deviation of the growth rate in sales from its long-term mean value will be reduced to half that deviation value within the time period $\ln(2)/K$. The unanticipated changes in the drift and the unanticipated changes in sales are assumed to converge deterministically to zero and to a more normal level, respectively:

$$d\delta_t = -K_2\delta_t dt; \quad d\sigma_t = K_1(\bar{\sigma} - \sigma_t)dt \quad (3)$$

Furthermore, the unanticipated changes in the drift and those in the growth rate of sales may be correlated:

$$dZ_1 dZ_2 = \rho_{12} dt \quad (4)$$

The total costs, C_t , to the technology company at any given time t are composed of a variable component assumed to be proportional to sales and a fixed component:

$$C_t = \omega_t S_t + F \quad (5)$$

The variable costs parameter ω_t , assumed to be stochastic to reflect the uncertainty related to future market share, new developments in technologies and future behaviors of potential competitors, follows the stochastic differential equation:

$$d\omega_t = K_3(\bar{\omega} - \omega_t)dt + \vartheta_t dz_3 \quad (6)$$

where the speed of adjustment coefficient “ K_3 ” translates the rate at which the variable costs are expected to converge to their long-term mean value, ϑ_t is the unanticipated changes in the variable costs and dz_3 is an increment of a Brownian motion. Any deviation of the variable costs from their long-term mean value is expected to be reduced to half of that deviation value within the time period $\ln(2)/K_3$. Hence, $\ln(2)/K_3$ translates the “half-life” of the deviations. The unanticipated changes in

variable costs are assumed to converge deterministically to a more normal level at the speed K_4 , that is:

$$d\vartheta_t = K_4(\bar{\vartheta} - \vartheta_t)dt \quad (7)$$

Further, the unanticipated changes in variable costs may be correlated with both sales and growth rates in sales:

$$dZ_1dZ_3 = \rho_{13}dt; \quad dZ_2dZ_3 = \rho_{23}dt \quad (8)$$

The after tax rate of net income to the technology company, RNI_t , can be expressed as follows:

$$RNI_t = (S_t - C_t - D_t)(1 - TR_c) \quad (9)$$

where D_t is the rate of depreciation at time t and TR_c is the corporate tax rate. The technology company pays taxes only when there is no loss carry-forward and the net income is positive. When the loss carry-forward is positive, the tax rate is zero, and no tax is paid. The dynamics of the loss carry-forward, LCF_t , can be formulated as follows:

$$dLCF_t = -RNI_t dt \quad \text{if } LCF_t > 0 \quad (10a)$$

or

$$dLCF_t = \text{Max}(-RNI_t dt, 0) \quad \text{if } LCF_t = 0 \quad (10b)$$

Property, plant and equipment accumulated at time t , PPE_t , is a function of the rate of capital expenditures for the period, CE_t , and the associated rate of depreciation, D_t . Capital expenditures planned for an initial period " $\bar{\tau}$ ", PCE_t , are well characterized and after that period, are assumed to be a portion " ϵ " of sales. Depreciation is assumed to be a portion " ϵ " of the value of the accumulated property, plant and equipment. Hence,

$$dPPE_t = (CE_t - D_t)dt \quad (11a)$$

$$CE_t = PCE_t \text{ for } t \leq \bar{\tau} \quad (11b)$$

$$D_t = \epsilon * PPE_t \quad (11c)$$

The technology company has a certain amount of cash available at time t , $CASH_t$, that follows the process:

$$dCASH_t = (\pi CASH_t + RNI_t + D_t - CE_t)dt \quad (12)$$

where π is the untaxed interest earned on the cash available. Because this untaxed interest earned is integrated in the dynamics of the cash available, the results of the valuation of the technology company will not be sensitive to the exact time its cash flows are distributed to its security holders. To be consistent with the risk neutral framework, this untaxed interest rate is the risk-free rate of return. In the absence of bankruptcy, the cash flows generated by the technology company are discounted when they occur at the risk-free rate.

This improved model precludes the formulation of any dividend policy as the operating cash flow generated by the technology company is retained in the firm to earn the risk-free rate of return and is available to shareholders in the long run at time T when the firm reverts to a regular company with its

industry growth rate. Bankruptcy occurs when the cash available reaches a specified negative amount, $CASH^*$. The cash available can be negative and the prospects of the technology company still good, which will allow the firm to raise new fresh cash or merge with another company to survive. Therefore, in this improved model, future financing is allowed and the optimal amount of new financing is the choice that maximizes the current value of the firm. Practically, the firm may go bankrupt even before its value becomes zero. The optimal amount of new financing is obtained by decreasing $CASH^*$ until the technology company value is maximized. It should be noted that the correct optimal amount of new financing is state dependent and must be obtained jointly with the valuation of the firm via dynamic programming methodologies. This problem can be solved using cross-sectional data from the simulation in least-squares regressions (Schwartz and Longstaff [3]).

Under the equivalent martingale measure (risk-neutral measure) Θ , the value V_0 of the technology company at the current time zero is the expectation of the discounted net cash flow to the company at the risk-free rate assumed to be constant:

$$V_0 = E^\Theta(CASH_T + N * (S_T - C_T))e^{-\pi T} \quad (13)$$

where T is the valuation horizon, π is the risk-free rate and N is a multiple. The value of the technology company at the horizon T is composed of the remaining cash balance and the value of the firm as a going concern which is a multiple of EBITDA (earnings before interest, tax, depreciation and amortization).

The model is characterized by three sources of uncertainty: the uncertainty related to the change in sales, the uncertainty related to the expected rate of growth in sales and the uncertainty related to the variable costs. Assuming the processes for the expected rate of growth in sales and variable costs do not have risk premiums attached to them, their risk-adjusted processes are the same as their true processes. Only the changes in sales are assumed to carry a risk premium which can be related to the systematic risk (the beta) of the stock. Using Brennan and Schwartz [4] simplifying assumptions leading to the Merton [5] intertemporal capital asset pricing model, the risk-adjusted processes for sales under the risk-neutral probability measure Θ are:

$$\frac{dS_t}{S_t} = (\mu_t - \gamma_t)dt + \sigma_t dZ_t^\Theta \quad (14)$$

The risk premium in Equation (14), γ_t , is associated with the covariance between the sales process and the return on the market portfolio.

The derivation of the share price from the total value of the technology company obtained from the model requires a detailed examination of the firm's capital structure. More specifically, the number of shares currently outstanding, the number of shares likely to be issued to employees who hold stock options and convertible bonds must be clearly accounted for. The cash flow that will be available to shareholders after principal and coupon payments are made to the bondholders must be known. It is reasonable to assume, in the case the technology company does not go bankrupt, that options will be exercised and convertible bonds will be converted into shares of common stocks. Consequently, the number of shares is adjusted to translate the exercise of options and convertibles in the no-default scenarios of the simulations. The cash flow available to all security holders determines the total value of the technology company. To compute the share price, the cash flow available to shareholders is

needed. It is obtained by subtracting the principal and after-tax coupon payments on the debt from the cash flow available to all security holders and adding the payments by option holders at the exercise of the options. The exercise of options and convertibles take place at their maturity dates because each technology company considered is assumed to pay no dividends. This approach undervalues the options and convertibles and overvalues the stock when option holders exercise their options optimally. Moreover, to be able to sell the underlying stock to diversify their portfolios, employees more often exercise stock options prior to the maturity date if the options are exercisable. It is also important to note that many employees leave the company before they are vested in their stock options. Consequently, not all of the options will be exercised even if they are in the money. The impact on the share value of the number of new shares to be issued at exercise and conversion is likely to be small if the number of these new shares is small relative to the total number of shares outstanding.

The improved model described above implicitly dictates that the value of the technology company (*i.e.*, the value of its stock), at any time, depends on the state variables and time. The state variables are sales, expected growth in sales, variable costs, loss carry-forward, cash available and accumulated property, plant and equipment. Hence, the value of the technology company stock can be expressed as follows:

$$V \equiv V(S, \mu, \omega, LCF, CASH, PPE, t) \quad (15)$$

The dynamics of the stock value provided in Equation (16) below is obtained by applying Ito's lemma to Equation (15):

$$\begin{aligned} dV = & V_S dS + V_\mu d\mu + V_\omega d\omega + V_{LCF} dLCF + V_{CASH} dCASH + V_{PPE} dPPE + V_t dt \\ & + \frac{1}{2} V_{SS} dS^2 + \frac{1}{2} V_{\mu\mu} d\mu^2 + \frac{1}{2} V_{\omega\omega} d\omega^2 + V_{S\mu} dS d\mu + V_{S\omega} dS d\omega + V_{\mu\omega} d\mu d\omega \end{aligned} \quad (16)$$

The volatility σ_V^2 of the stock is derived from Equation (16) as follows:

$$\begin{aligned} \sigma_V^2 = & \frac{1}{dt} \text{var} \left(\frac{dV}{V} \right) = \left(\sigma_S \frac{V_S}{V} \right)^2 + \left(\delta \frac{V_\mu}{V} \right)^2 + \left(\vartheta \frac{V_\omega}{V} \right)^2 + 2 \frac{V_S V_\mu}{V^2} S \sigma \delta \rho_{12} \\ & + 2 \frac{V_S V_\omega}{V^2} S \sigma \vartheta \rho_{13} + 2 \frac{V_\mu V_\omega}{V^2} \delta \vartheta \rho_{23} \end{aligned} \quad (17)$$

The improved model can be used, as shown in Equations (16) and (17) above, to assess the price of the stock and its volatility. The partial derivatives of the stock price with respect to the level of sales, the expected rate of growth in sales and the variable costs are generated through simulation. Basically, to obtain new values of the equity from which these partial derivatives are computed, a perturbation is created on the initial value of sales, rate of growth in sales and variable costs. A perturbation often here is a 10 percent increase in the value of a given parameter while leaving the remaining parameters unchanged. The initial volatility δ_0 of the expected growth rate in sales is critically important in the valuation model and is very challenging to estimate. Consequently, δ_0 is implied from the observed volatility of the stock price using Equation (17).

The systematic risk or beta of the stock, β_V , can be expressed as a function of the beta of sales, β_S , using the continuous time return on the market portfolio and Equation (16) and taking into account the fact that only the sales process is correlated with the market return:

$$\beta_V = \frac{\sigma_{VM}}{\sigma_M^2} = \left(\frac{SV_S}{V} \right) \frac{\rho_{SM} \sigma_M \sigma_t}{\sigma_M^2} = \left(\frac{SV_S}{V} \right) \beta_S \quad (18)$$

Based on the intertemporal capital asset pricing model, the expected return on the stock, r_v , can be expressed as follows:

$$r_v = \pi + \beta_v (r_M - \pi) = \pi + \frac{SV_s}{V} \beta_s (r_M - \pi) \quad (19)$$

where r_M is the market return and π is the risk-free rate of return. The risk premium in this improved model, γ_t , can be obtained by stating that the expected return on the stock given by Equation (19) is equal to the one derived from Equation (16):

$$\gamma_t = \beta_s (r_M - \pi) \quad (20)$$

Combining Equations (18) and (20) leads to the expression of the beta of the stock as a function of the risk premium in the improved model:

$$\beta_v = \left(\frac{SV_s}{V} \right) \frac{\gamma_t}{r_M - \pi} \quad (21)$$

The risk premium can then be inferred from the beta of the stock using Equation (21). Because the volatility of sales changes with time and the risk premium is proportional to the beta of sales, γ_t is set equal to $\bar{\gamma}\sigma_t$ in the implementation of the model.

The model described above is path dependent because the cash available at any time t , the loss carry-forward and the depreciation tax shields are path dependent. These path dependencies can be easily accounted for using a Monte Carlo simulation to derive the value of the technology company. To implement the model, all the mean reversion parameters are set equal to one unique value K inferred from the expected half-life of the deviations in growth rates in sales. To implement the Monte Carlo simulation, the discrete versions of the risk-neutral processes given in Equations (1), (2) and (6) are used:

$$S_{t+\Delta t} = S_t e^{\left\{ \left(\mu_t - \bar{\gamma}\sigma_t - \frac{\sigma_t^2}{2} \right) \Delta t + \sigma_t \sqrt{\Delta t} \xi_1 \right\}} \quad (22)$$

$$\mu_{t+\Delta t} = e^{-K\Delta t} \mu_t + (1 - e^{-K\Delta t}) \bar{\mu} + \sqrt{\frac{1 - e^{-2K\Delta t}}{2K}} \delta_t \xi_2 \quad (23)$$

$$\omega_{t+\Delta t} = e^{-K\Delta t} \omega_t + (1 - e^{-K\Delta t}) \bar{\omega} + \sqrt{\frac{1 - e^{-2K\Delta t}}{2K}} \vartheta_t \xi_3 \quad (24)$$

where:

$$\sigma_t = \sigma_0 e^{-Kt} + \bar{\sigma}(1 - e^{-Kt}) \quad (25a)$$

$$\delta_t = \delta_0 e^{-Kt} \quad (25b)$$

$$\vartheta_t = \vartheta_0 e^{-Kt} + \bar{\vartheta}(1 - e^{-Kt}) \quad (25c)$$

Integrating Equations (3) and (7) with the initial conditions σ_0 , δ_0 and ϑ_0 leads to Equations (25a)–(25c) which are not approximations but exact solutions. The ξ_j , $j=1,2,3$ are standard correlated normal variates.

The closed form solution for the future distribution of the rate of growth in sales, Equation (2), and that of the variable costs, Equation (6), can be easily assessed because these variables exhibit mean reversion and have a decreasing volatility with time as shown in Equations (25b) and (25c). The variable costs at time t can be shown to be normally distributed with mean and variance provided below:

$$E(\omega_t) = \omega_0 e^{-Kt} + \bar{\omega}(1 - e^{-Kt}) \quad (26)$$

$$Var(\omega_t) = (\vartheta_0^2 - 2\vartheta_0\bar{\vartheta} + \bar{\vartheta}^2)te^{-2Kt} + 2(\vartheta_0\bar{\vartheta} - \bar{\vartheta}^2)\left(\frac{1 - e^{-Kt}}{K}\right)e^{-Kt} + \bar{\vartheta}^2\left(\frac{1 - e^{-2Kt}}{2K}\right) \quad (27)$$

assuming that the speed of adjustment in the variable cost process is identical to that in its volatility process. When time t increases to infinity, this distribution converges to a normal distribution with mean and variance given by:

$$E(\omega_\infty) = \bar{\omega} \quad (28)$$

$$Var(\omega_\infty) = \frac{\bar{\vartheta}^2}{2K} \quad (29)$$

Because the variable costs given by Equation (6) and the growth rate in sales given by Equation (2) have similar expressions, the distribution of the growth rate in sales is equivalent to that of the variable costs with the exception that its long-term volatility is zero. This is because the unanticipated changes in the expected growth rate in sales are assumed to converge deterministically to zero (see Equation (3)). In the limit as time increases to infinity, the volatilities in the improved model are:

$$\sigma_\infty = \bar{\sigma} \quad (30a)$$

$$\delta_\infty = 0 \quad (30b)$$

$$\vartheta_\infty = \bar{\vartheta} \quad (30c)$$

It should be noted that taking the limits of Equations (25a)–(25c) as time goes to infinity gives Equations (30a)–(30c). As time grows to infinity, Equations (23) and (30b) indicate that the growth rate in sales converges to:

$$\mu_\infty = \bar{\mu} \quad (31)$$

In the limit when time t goes to infinity, the dynamics of the sales or revenues converge to:

$$\frac{dS_\infty}{S_\infty} = \bar{\mu}dt + \bar{\sigma}dZ_1 \quad (32)$$

In this discrete approximation, the after tax net income is still provided by Equation (9) in which all the variables are measured within the time increment Δt . The discrete versions of the loss carry-forward process, the accumulated property, plant and equipment process and the amount of cash available process can also be easily obtained in the same manner. The time increment Δt is usually quarterly or annual and is dictated by data availability. An annual time increment can smooth the seasonal effects and is more appropriate for longer multi-year analyst projections.

3. Parameters Estimation

To implement the improved valuation model, its parameters need to be estimated. The parameters are estimated each time to value each of the following technology companies: Google, eBay, Amazon, Facebook and Yahoo. Some of the parameters are observable, and those not observable are estimated using cross-sectional data from a sample of twenty technology companies which includes the five firms listed above. The cross-sectional estimation approach is exactly the same as in Doffou [1]. The parameters estimated are those needed to run the Monte Carlo simulations from the past and present data available for Google, Amazon, eBay, Facebook and Yahoo and from analysts' forecasts pertaining to these companies. Because corporate forecasts of capital expenditures, sales and costs are done in general on an annual basis, the simulations are run on an annual basis. The parameters needed are those of sales, growth rate in sales and variable cost dynamics as well as those related to the half-life of deviations, correlations, balance sheets data, capital expenditures, depreciation and other environmental and risk factors (corporate tax rate, risk-free rate, risk premium, market price of risk and the beta of the stock).

In the sales simulation, the initial value for sales is the actual sales value for 2013 which is \$59.825 billion for Google, \$74.452 billion for Amazon, \$16.047 billion for eBay, \$7.872 billion for Facebook and \$4.680 billion for Yahoo. The initial annualized volatility of sales of 0.170, 0.628, 0.169, 0.257 and 0.173, respectively, for Google, Amazon, eBay, Facebook and Yahoo are derived from quarterly data from the second quarter of 2011 to the first quarter of 2014. These volatilities are assumed to decrease with time to their long-term volatility of sales of 0.085, 0.314, 0.084, 0.128 and 0.087, respectively, for Google, Amazon, eBay, Facebook and Yahoo. The half-life of the deviation is the length of time necessary for the analysts' sales growth prediction for 2014 to closely match the model expected sales growth. Its numerical value will be provided later in the paper.

The initial growth rates in sales of 10.34% for Google, 21.15% for Amazon, 13.81% for eBay, 54.89% for Facebook and 1.72% for Yahoo are the consensus analyst forecasts for the growth rates from 2013 to 2014. In the long run, as these technology companies become normal firms, these growth rates are assumed to converge to the average industry sales growth rate of 5% per year. The initial annual volatility of expected rates of growth in sales, a critical unobservable parameter in the model, is implied from the volatility of the stock. Average implied volatilities of 67% for Google, 74% for Amazon, 65% for eBay, 84% for Facebook and 72% for Yahoo are computed from the daily implied volatility of each technology company options from 1 February 2012 to 30 April 2014. Moreover, the distribution of the rate of growth in sales implied from these parameters gets smaller and shifts to the left as time increases to converge at infinity to 5% which is the average industry sales growth per year.

The variable cost dynamics are estimated using actual data from 2012 and 2013 as well as analyst estimates for 2014. Annual cash costs in EBITDA are regressed over annual sales. The results of these regressions give the initial variable cost as a fraction of sales of 0.15, 0.85, 0.61, 0.34 and 0.55 and the annual fixed cost of \$29 billion, \$51 billion, \$8 billion, \$3 billion and \$4 billion, respectively, for Google, Amazon, eBay, Facebook and Yahoo. In the long run, variable costs are assumed to remain at \$0.15, \$0.85, \$0.61, \$0.34 and \$0.55 per dollar of sales, respectively, for each of the companies listed above. The initial volatilities of variable costs of 0.07, 0.09, 0.065, 0.08 and 0.075, respectively, for

Google, Amazon, eBay, Facebook and Yahoo are assumed to revert to their long-run values of 0.035, 0.045, 0.0325, 0.04 and 0.0375.

As identified in Doffou [1], the speed of adjustment parameter for the rate of growth in the sales process has a more potent effect on the valuation model. For simplicity, all the mean reverting processes in this improved model have the same speed of adjustment coefficient. To derive the half-life of deviations, actual and expected growth rate in sales predicted by analysts and implied by the model parameters are drawn over the years 2011–2020. The half-life is the number of years during which the model expected sales growth closely matches the analysts sales growth prediction. This match occurs in 2015 to give a half-life (in years) of 2.68 for Google, 2.95 for Amazon, 2.79 for eBay, 2.91 for Facebook and 2.83 for Yahoo when the three stochastic processes in the model with mean reversion are uncorrelated.

4. Simulation

The proposed improved model is used to value five of the best-known, well-managed, largest technology companies in the world: Google, Amazon, eBay, Facebook and Yahoo. In the year 2014, analysts have forecasted sales to be \$66.07 billion and capital expenditures to be \$8.44 billion or 12.77% of sales for Google. The sales forecasted in 2014 for Amazon, eBay, Facebook and Yahoo are, respectively, \$90.812 billion, \$18.274 billion, \$11.843 billion and \$4.503 billion. The capital expenditures forecasted by analysts in 2014 are \$4.464 billion or 4.92% of sales for Amazon, \$1.316 billion or 7.2% of sales for eBay, \$2.231 billion or 18.84% of sales for Facebook and \$0.491 billion or 10.90% of sales for Yahoo. Therefore, starting from 2015, capital expenditures are assumed to be 12.77% of sales for Google, 4.92% of sales for Amazon, 7.2% of sales for eBay, 18.84% of sales for Facebook and 10.90% of sales for Yahoo. Based on the practice of each firm in the past four years, the annual depreciation allowance is assumed to be 32.76%, 25.48%, 32.47%, 27.69% and 52.98% of the accumulated property, plant and equipment of the previous year. Each of these numbers is the average of the annual depreciation allowances from 2010 to 2013 derived from each company's balance sheets and income statements.

The simulations are run with a risk-free rate of return of 5% and a corporate tax rate of 35%. The market portfolio inherent in the model that links the market price of risk to the beta of the stock is assumed to have a risk premium of 6%. Because the market price of the risk parameter in the model associated with the sales dynamics is not observable, it is implied from the beta of the stock. To estimate beta, weekly stock returns for each of the technology companies considered are regressed over the S&P 500 Index returns from 5 June 2011 to 5 June 2014. The beta derived from this regression is the raw beta or historical beta. The raw betas are 1.14, 0.77, 0.84, 1.08 and 1.07, respectively, for Google, Amazon, eBay, Facebook and Yahoo. The simulations are run using the Bloomberg adjusted beta which is an estimate of each security's future beta. The adjusted beta is initially derived from historical data, but modified by the assumption that a security's true beta will move overtime towards the market average beta of one. The Bloomberg adjusted beta is given by:

$$\text{Adjusted Beta} = (0.67)(\text{RawBeta}) + (0.33)(1.0) \quad (33)$$

Using Equation (33), the Bloomberg adjusted betas are 1.09, 0.84, 0.89, 1.05 and 1.04, respectively, for Google, Amazon, eBay, Facebook and Yahoo. The simulations are run using the Bloomberg adjusted betas to correct the measurement error in the raw beta estimation and to take into account the fact that the beta will converge to one over the ten year horizon considered in this study.

A total of 50,000 simulations are run (10,000 for each technology company examined) with increments of one year and up to a ten-year horizon. The terminal value of each company at the end of the horizon is assumed to be ten times operating profit or EBITDA (earnings before interest, taxes, depreciation and amortization), a multiple very often used by finance practitioners. The simulations are carried out on 5 June 2014.

5. Simulation Results and Derivation of the Share Price

To be able to determine the price of each stock given the total value of each Internet or technology company, the following pieces of information are needed: the cash and marketable securities available at the end of year 2013, the loss carry-forward, the accumulated property, plant and equipment, the number of shares outstanding, the number of employee stock options outstanding with their weighted average life in years and their weighted average exercise price in dollars and the dollar value of the mortgage debt outstanding for each firm.

The cash and marketable securities available at the end of the year 2013 was \$58.717 billion, 12.447 billion, 9.025 billion, 11.449 billion and \$3.408 billion, respectively, for Google, Amazon, eBay, Facebook and Yahoo. None of these companies had a loss carry-forward at this point in time. The accumulated property, plant and equipment was \$16.524 billion, \$10.949 billion, \$2.76 billion, \$2.882 billion and \$1.489 billion, respectively, for Google, Amazon, eBay, Facebook and Yahoo.

The number of shares outstanding was 671.01 million, 459 million, 1.294 billion, 2.547 billion and 1.01434 billion, respectively, for Google, Amazon, eBay, Facebook and Yahoo. The input data related to each company's employee stock options outstanding are captured in Table 1 below:

Table 1. Input data to the share price derivation as of 31 December 2013.

Technology Firm	Google	Amazon	eBay	Facebook	Yahoo
<i>Input Data</i>					
Number of Employee Stock Options Outstanding	5,032,863	16,300,000	14,000,000	22,102,000	20,968,000
Stock Options Weighted Average Exercise Price	\$431.00	\$233.00	\$29.79	\$3.56	\$20.43
Stock Options Weighted Average Remaining Life	5 years	1.2 years	3.54 years	4.66 years	4.20 years
Mortgage Debt Outstanding	\$0.00	\$385,000,000	\$28,000,000	\$0.00	\$0.00

The other remaining parameters of the improved model are estimated using cross-sectional quarterly data available from a sample of twenty internet or technology companies which include Google, Amazon, eBay, Facebook and Yahoo, exactly as in Doffou [1]. Using the parameters above and those estimated via a cross-sectional data analysis, the improved model stock prices generated by the simulations run on 5 June 2014 are \$530.36, \$305.55, \$47.53, \$60.58 and \$33.26, respectively, for

Google, Amazon, eBay, Facebook and Yahoo. The model price represents 93.88% of the Google market price of \$564.93 at the close of 5 June 2014. Similarly, the model price represents 94.43%, 93.97%, 95.87% and 95.19% of the market price of \$323.57, \$50.58, \$63.19 and \$34.94, respectively, for Amazon, eBay, Facebook and Yahoo at the close of 5 June 2014. Overall, the improved model pricing accuracy is acceptable. Furthermore, the pricing accuracy achieved here is superior to the one obtained in Doffou [1] and far better than the one obtained in Schwartz and Moon [6]. This superior performance is dictated by: (1) the introduction of three sources of uncertainty, the uncertainty about the changes in sales, the uncertainty about the expected rate of growth in sales and the uncertainty about the variable costs; and (2) estimating some of the model parameters using cross-sectional data. These simulation results are summarized in Table 2 below.

Table 2. Pricing accuracy of the three models as of 5 June 2014.

Technology Company	Google	Amazon	eBay	Facebook	Yahoo
Actual Stock Market Price	\$564.93 (100%)	\$323.57 (100%)	\$50.58 (100%)	\$63.19 (100%)	\$34.94 (100%)
Improved Model Price	\$530.36 (93.88%)	\$305.55 (94.43%)	\$47.53 (93.97%)	\$60.58 (95.87%)	\$33.26 (95.19%)
Doffou (2014) Model Price	\$528.21 (93.50%)	\$297.68 (92.08%)	\$46.48 (91.89%)	\$57.24 (90.59%)	\$32.05 (91.74%)
Schwartz and Moon (2001) Model Price	\$322.29 (57.05%)	\$188.28 (58.19%)	\$28.94 (57.21%)	\$38.11 (60.31%)	\$20.92 (59.87%)

Each number in parenthesis gives the value of the model price as a percentage of the actual market price of the stock for each of the technology companies examined. The improved model offers a pricing accuracy superior to that of the Doffou [1] model and far superior to that of Schwartz and Moon [6].

The net present value (NPV) of the cash flows can be obtained using the same data as in the previous analysis by setting all of the volatilities in the improved model to zero. To effectively compute the NPV, the risk-adjusted discount rate is needed. As of March 2014, all of the companies examined use both debt and equity financing except Facebook which is all equity financed. The capital structure of Google is 6.4% debt and 93.6% equity, Amazon uses 23.4% debt and 76.6% equity, eBay uses 17.4% debt and 82.6% equity and Yahoo uses 8.7% debt and 91.3% equity. The risk adjusted discount rate is the weighted average cost of debt and equity. Given that Facebook is 100% equity financed, its risk adjusted discount rate is the cost of equity capital obtained by application of the capital asset pricing model (CAPM). Using the same parameters for the risk-free rate, the market risk premium and the beta of each stock, the costs of equity capital are 11.54%, 10.04%, 10.34%, 11.30% and 11.24%, respectively, for Google, Amazon, eBay, Facebook and Yahoo. The cost of debt must be factored in to derive the risk-adjusted discount rate for Google, Amazon, eBay and Yahoo. The cost of debt is the average or aggregate interest each firm must pay to borrow funds. The weighted average of various costs of debt is derived from each company's 10-K filing for 2013 with the U.S. Securities and Exchange Commission. The cost of debt is 2.42%, 2.55%, 2.37% and 2.57%, respectively, for Google, Amazon, eBay and Yahoo. Given the cost of debt, the cost of equity and the capital structure of each firm, the weighted average cost of capital is computed to be 10.90%, 8.08%, 8.81%, 11.30% and

10.41%, respectively, for Google, Amazon, eBay, Facebook and Yahoo. Using this weighted average cost of capital as the risk-adjusted discount rate in the NPV approach leads to a stock value of \$290.90, \$159.98, \$26.92, \$31.36 and \$18.45, respectively, for Google, Amazon, eBay, Facebook and Yahoo. The difference between the improved model price and the NPV price is \$239.46, \$145.57, \$20.61, \$29.22 and \$14.81, respectively, for Google, Amazon, eBay, Facebook and Yahoo. The NPV computation uses exactly the same data or parameters as the improved model except the volatilities and correlations. The complete distributions of the state variables included in the analysis offer two sources of justification for this difference in the share prices. The abandonment option is the first source of value that permits the technology company to exit via bankruptcy if performances are very poor. The second source of value has to do with the fact that the expected value of a convex non-linear function of several variables is greater than that function of the expected values of those variables (Jensen's inequality). This second source of value was also recognized in Schwartz and Moon [6] and is the dominant effect because of the very low probability of bankruptcy.

6. Sensitivity Analysis on Critical Parameters

The first critical parameters examined in this section are long-term variable costs which are expected to have a strong influence on prices. An increase in long-term variable costs from 0.15 to 0.16, 0.85 to 0.89, 0.61 to 0.64, 0.34 to 0.36 and 0.55 to 0.58 leads to a decrease in the stock price given by the model to \$487.29, \$277.99, \$43.61, \$55.21 and \$30.49, respectively, for Google, Amazon, eBay, Facebook and Yahoo. A decrease in long-term variable costs to 0.12, 0.82, 0.58, 0.31 and 0.52 leads to an increase in the model stock price to \$579.58, \$336.14, \$52.00, \$66.57 and \$36.41, respectively, for Google, Amazon, eBay, Facebook and Yahoo. To arrive at these results in both cases, the market price of risk is slightly adjusted to match the beta of each stock.

The second critical parameters investigated are the correlations. Suppose the correlation between variable costs and growth rates in sales is positive. This is the case when the business environment in which the technology company operates is competitive such that decreases in variable cost or increases in profit margin are associated with a decrease in growth rates in sales. The simulations are run with the same data described before but with a correlation set at 0.6. The model price decreased to \$518.69, \$296.44, \$46.41, \$58.87 and \$32.45, respectively, for Google, Amazon, eBay, Facebook and Yahoo. To arrive at these results, the market price of risk and the volatility of the growth rate were slightly adjusted to match the volatility and the beta of the stock for each company. However, the effect of this correlation on prices increases with the volatility of variable costs.

The third critical parameters analyzed are the half-life of deviations. The speed of mean reversion, which characterizes how fast the growth rate of sales and other variables decrease, has a potent effect on expected future sales and, therefore, on the valuation of the technology company. An increase of the half-life parameter from 2.68 to 2.88, 2.95 to 3.15, 2.79 to 2.99, 2.91 to 3.11 and 2.83 to 3.03 years increases the model stock price to \$572.31, \$332.89, \$51.33, \$65.82 and \$35.95, respectively, for Google, Amazon, eBay, Facebook and Yahoo. A decrease of the half-life parameter to 2.48, 2.75, 2.59, 2.71 and 2.63 years decreases the model stock price to \$487.83, \$278.29, \$43.66, \$55.27 and \$30.53, respectively, for Google, Amazon, eBay, Facebook and Yahoo.

Finally, the simulation horizon and the terminal value parameters are examined. An increase in the horizon of the simulation from 10 to 11 years causes the model price to move from \$530.36 to \$530.41, \$305.55 to \$305.59, \$47.53 to \$47.59, \$60.58 to \$60.66 and \$33.26 to \$33.34, respectively, for Google, Amazon, eBay, Facebook and Yahoo. These model price changes are quite negligible and very well within the standard errors of the simulations. Consequently, increasing the horizon of the simulations has no impact on the model price. When the multiple of earnings used to assess the terminal value of each stock was increased from 10 to 11, the model price increased to \$563.35, \$324.71, \$50.48, \$64.37 and \$35.34, respectively, for Google, Amazon, eBay, Facebook and Yahoo.

7. Conclusions

This paper substantially improves the model developed in Schwartz and Moon [6] for pricing Internet companies by estimating some of the parameters using cross-sectional data of technology companies. The parameter estimation technique using cross-sectional data is the same as in Doffou [1]. In addition, the uncertainty in costs and the tax effects of depreciation are taken into account. Because the market price of risk and the volatility of the growth rate in sales are two critical parameters of the model that are not observable, they are inferred from the beta and the volatility of the stock of each technology company. An alternative to this approach is to use the credit rating of each firm to better estimate some of the unobservable parameters of the model. The credit rating of each firm can be inferred from the probabilities of bankruptcy derived from the improved model. A flexible bankruptcy condition is introduced by allowing the firm to seek future financing. The practical implementation of the model is simplified by setting the half-life of all processes to be equal within each firm. Further, the half-life is estimated from the expectations of future sales.

This improved model developed for technology or Internet companies can also be used to value the stock of any high-growth company. Once the parameters are estimated, the model is stable and flexible enough to be used to price technology companies and to incorporate large changes in the technology firm's growth prospects. The improved model can easily incorporate more complex cost functions because it is solved by simulation. Stochastic interest rates can also be incorporated into this framework. The technique developed here largely depends on the parameters used in the estimation. Further, because the growth rate in sales is unobservable, it can be updated in the model using "learning models" when new pieces of information on sales are revealed. The results show that the use of cross-sectional data to estimate some of the parameters greatly improves the performance of the model, and its pricing accuracy is far superior to that of the model in Schwartz and Moon [6].

Conflicts of Interest

The author declares no conflict of interest.

References

1. Doffou, A. The Valuation of Internet Companies. *J. Appl. Financ. Res.* **2014**, *1*, 71–86.
2. Schwartz, E.S.; Moon, M. Rational Pricing of Internet Companies. *Financ. Anal. J.* **2000**, *56*, 62–75.

3. Longstaff, F.A.; Schwartz, E.S. Valuing American Options by Simulation: A Simple Least-Square Approach. *Rev. Financ. Stud.* **2001**, *14*, 113–147.
4. Brennan, M.J.; Schwartz, E.S. Consistent Regulatory Policy under Uncertainty. *Bell J. Econ.* **1982**, *13*, 507–521.
5. Merton, R.C. An Intertemporal Capital Asset Pricing Model. *Econometrica* **1973**, *41*, 867–888.
6. Schwartz, E.S.; Moon, M. Rational Pricing of Internet Companies Revisited. *Financ. Rev.* **2001**, *36*, 7–26.

© 2015 by the author; licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by/4.0/>).