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The Valuation of Equities and the GDP Growth Effect: A Global Empirical Study

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Abstract: One of the main characteristics of the (recently proposed) non-arbitrage valuation of equities framework is the reduction in pricing subjectivity. This is evidenced in terms of the dividends discount rate and the outlook of future performance (dividends projection) of the company that is being valued. Under this framework, as in the case of derivatives pricing, the discount rate is the risk-free interest rate (not the cost of equity), and the subjectively-determined drift of the stochastic process that drives the operating profits of the company is eliminated. The challenge that emerges is that the structure of the new drift of the operating profits process is undetermined under the methodology (this is a similar feature that is observed in the case of derivatives related to non-tradable assets). This paper proposes that the structure of this new drift is represented by the (country-specific) GDP nominal growth effect. This proposition is tested through an empirical study that involves several companies of 10 equity indices worldwide, for two different periods (1995–2004 and 2005–2014). The results of the test are reasonably successful, meaning that further research related to the framework could provide useful information for the understanding of financial assets and their links to the macro-economy.

Keywords: non-arbitrage valuation; equities pricing; market price of earnings risk; macro-finance linkages

JEL Classification: G1; G12

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1. Introduction

One of the most influential advances in the modern financial theory has been the introduction of the non-arbitrage valuation framework developed by Black and Scholes [1] and Merton [2]. The framework has been further generalized by Ross [3], Rubinstein [4], Harrison and Kreps [5] (among others) and it is mainly applied to the valuation of derivatives (and not necessarily to other asset classes). As stated by Cochrane [6], it seems that there is a pricing theory depending on the type of asset (equities, bonds and options). This evidences that there is space for further developments in the area of asset pricing under non-arbitrage conditions.

Rey [7] has recently proposed a non-arbitrage valuation framework for equities, opening a new area of research in this topic. The results of the proposed method provide similar conclusions as the ones provided by the non-arbitrage valuation method applied to derivatives. In fact, under the framework, the equity of a specific company is interpreted as a set of path-dependent financial derivatives; considering the EBITDA (or any other representative measure of operating profits) of the Company as underlying, each dividend payment date as maturity date, and a pay-off that depends on the dynamics of the company (business performance, financial structure and dividend strategy). The framework assumes that the operating profits process is continuously observed, and each future dividend (under the path-dependent financial derivative interpretation) is traded. Of course, these last two assumptions represent conditions that are difficult to encountered in the real world (the profits of companies are observed once a quarter, and derivatives related to dividends are difficult to find), but for the purpose of this paper those two assumptions are maintained. This abstraction is useful for the improvement of the theoretical framework. Future research should address the implications of relaxing both assumptions. In this sense, estimation (expectation) of operating profits could be incorporated as a continuous time process. Furthermore, a bond and equity related to the corresponding company could be used as hedging instruments (offsetting the risk factor provided by the operating profits process).

As part of the methodology, for valuation purposes, the original drift of the instantaneous level of EBITDA process is eliminated. As in the case of derivatives pricing, this generates a more objective valuation. The issue that emerges is that the structure of the new drift is undetermined under the methodology. This is also observed in the pricing of derivatives related to non-traded assets (for example Pelsser [8], in the case of interest rates derivatives), as EBITDA, in this case. In the original formulation of the framework, Rey [7] proposed that (for valuation purposes) the instantaneous level of EBITDA process is driven by the (country-specific) inflation effect (defined as the impact in the drift of the process due to inflation), supporting the proposal by an empirical analysis for a particular market (S&P 100, for the period 2000–2014), but stating that the proposition should be further validated.

This paper presents a different proposition for the mentioned drift, based on the GDP nominal growth effect (defined as the impact in the drift of the process due to GDP nominal growth), instead of the inflation effect. Moreover, an empirical test (applying standard econometric technics, Greene [9], and non-standard econometric technics based on stochastic simulation), for the validation of the proposition, is presented. The study is based on financial information of several companies related to 10 equity indices worldwide, for two different periods (from 1995 to 2004, and form 2005 to 2014). The results of the test are reasonably successful.

The valuation approach, complemented by the proposed and tested hypothesis presented in this paper, offers some advantages in relation to the traditional methods that are used for the valuation of equities (based on Sharpe [10], Ross [11] and Fama and French [12], among many others). This means that further research related to the framework could provide useful material for the understanding of financial assets and their links to the macro-economy. This could be an interesting contribution, since the macro-finance models (represented by Hansen [13]), in practice, take into account the implicit but not the explicit link between the real economy and the financial asset prices.

Future research should be focused on the incorporation of the GDP growth rate as a (potential risk-neutral) stochastic process. This is consistent with Vassalou [14] that, using a factor model, considers changes in GDP a crucial risk factor in equity returns.

2. Theoretical Framework

The theoretical framework that gives space to the empirical analysis that is developed in this paper is based on the non-arbitrage valuation of equities methodology. This methodology states that, under non-arbitrage conditions, the value of the equity of a *company* (at time $t \geq 0$) is given by:

$$S\left(t\right) = E_{\mathbb{Q}}^{(t)}\left[\sum_{i=1}^{\infty} exp\left[-\delta\left(t_i - t\right)\right]\ f\left(\Upsilon\left(t_i\right)\right)\right], \tag{1}$$

where the expectation is conditional to the information available at valuation time t, under the measure $\mathbb{Q}$. The parameter δ is the *risk-free short-term interest rate* and the function $f(\Upsilon(t_i))$ determines the dividend amount that is distributed at each predefined future time t_i ($t_i \geq t$; $i = 1, 2, \ldots$).

The *dividend function* depends on the *amount of distributable cash-flows* $\Upsilon\left(u\right)$, which presents path-dependent features and it is determined, depending on the characteristics of the company, by the interaction between the evolution of *EBITDA*, the *long-term debt*, the *capital requirements*, the *short-term debt*, the *corporate taxes*, and the *dividend strategy*. Moreover, the amount of distributable cash-flows is a *Markov* process in which the stochastic component is provided by the EBITDA process $E_{t_{i-1}}(u) = \int_{t_{i-1}}^{u} \varepsilon(y)\, dy$, being $\varepsilon(y)$ its instantaneous level.

The *instantaneous level of EBITDA*, $\varepsilon(y)$, under the measure $\mathbb{Q}$, is driven by:

$$d\varepsilon(y) = \zeta(\varepsilon, y)\ du + \nu(\varepsilon, y)\ dW^{\mathbb{Q}}(y), \tag{2}$$

where $\zeta(\varepsilon, y)$ and $\nu(\varepsilon, y)$ represent the drift and the volatility of the process, and $W^{\mathbb{Q}}(y)$ is a $\mathbb{Q}$-Brownian motion. Based on the *Radon-Nikodym* and the *Cameron-Martin-Girsanov* theorems (change of measure procedure), the drift of the process is given by:

$$\zeta(\varepsilon, y) = \xi(\varepsilon, y) - \gamma(\varepsilon, y)\ \nu(\varepsilon, y), \tag{3}$$

where $\xi(\varepsilon, y)$ represents the actual drift of the instantaneous level of EBITDA process (under the original physical measure $\mathbb{P}$, not under the measure $\mathbb{Q}$) and $\gamma(\varepsilon, y)$ is the *market price of earnings risk* (interpreted as the additional return of the equity in excess of the short-term risk-free interest rate, for taking an additional unit of earnings risk). This change of measure is possible if the *Novikov* condition is in place ($E_{\mathbb{P}}\left[(1/2)\, exp\left[\int_0^t \gamma(\varepsilon, y)^2\ dy\right]\right] < \infty$).

As it can be observed, the drift in Equation (2) does not equal the risk-free short-term interest rate. In this case, as in the case of derivatives pricing related to non-tradable quantities, the drift is a function in which the market price of risk is explicitly involved.

Even when $\xi(\varepsilon, y)$ and $\nu(\varepsilon, y)$ could be observed or estimated, this is not the case neither for $\zeta(\varepsilon, y)$ nor $\gamma(\varepsilon, y)$, which are undetermined under the methodology. The next section presents a proposal for the structure of these two important components of the methodology.

3. A Proposal for the Structure of ζ and γ

The original formulation of the non-arbitrage valuation of equities methodology proposed the (country-specific) inflation effect as the drift of the instantaneous level of EBITDA process, under the measure $\mathbb{Q}$, presenting an empirical analysis for a particular market (S&P 100, for the period 2000–2014), and stating that the proposition should be further validated. This paper presents a different proposal for the mentioned drift, based on the GDP nominal growth effect, instead of the inflation effect.

Proposition 1 (the GDP nominal growth effect): *The relationship between $\xi(\varepsilon, y)$ and $\nu(\varepsilon, y)$ is given by:*

$$\xi(\varepsilon, y) = \zeta(\varepsilon, y) + \gamma(\varepsilon, y)\, \nu(\varepsilon, y), \tag{4}$$

in which $\zeta(\varepsilon, y)$ corresponds to the GDP nominal growth effect.

Equation (4) is a re-expression of Equation (3), which emerges as a result of the change of measure. The proposition states that EBITDA nominal growth is explained by: (1) a base growth component that is provided by the GDP nominal growth effect; and (2) an additional growth component that depends on the risk (volatility) characteristics of the business.

An alternative interpretation of Equation (4) is based on the following expression:

$$\xi(\varepsilon, y) - \zeta(\varepsilon, y) = \gamma(\varepsilon, y)\, \nu(\varepsilon, y).$$

This indicates that the EBITDA nominal growth in excess of the GDP nominal growth effect ($\xi(\varepsilon, y) - \zeta(\varepsilon, y)$) is proportional to the risk characteristics of the business ($\gamma(\varepsilon, y)\, \nu(\varepsilon, y)$).

It is clear that the market price of earnings risk ($\gamma(\varepsilon, y)$) is a signed quantity, it could take positive or negative values. But, independent of its sign the stated proportionality would remain:

$$|\xi(\varepsilon, y) - \zeta(\varepsilon, y)| = |\gamma(\varepsilon, y)|\, \nu(\varepsilon, y).$$

More important, independent of the sign of the market price of earnings risk, the structure of $\zeta(\varepsilon, y)$ (the drift involved in Equation (2)) would not be altered. This is very important for the framework since, if the proposition is validated, it means that the new drift (under the measure $\mathbb{Q}$) would be unique.

The empirical validation of the presented proposition for $\zeta(\varepsilon, y)$ and $\gamma^{\circ}(\varepsilon, y) = |\gamma(\varepsilon, y)|$ motivates the study developed in the next section.

4. Empirical Study

The empirical study developed in this paper is oriented to validate the proposition presented in the Section 3. This proposition is divided in two pieces (hypotheses):

- *Hypothesis 1 (the market price of earnings risk):* $\gamma^{\circ}(\varepsilon, y) = |\gamma(\varepsilon, y)|$ drives a linear relationship between $|\xi(\varepsilon, y) - \zeta(\varepsilon, y)|$ and $\nu(\varepsilon, y)$,
- *Hypothesis 2 (the GDP nominal growth effect):* $\zeta(\varepsilon, y)$ corresponds to the GDP nominal growth effect.

For the study, it is assumed that $\xi(\varepsilon, y) = \epsilon(y)\,\xi$ and $\upsilon(\varepsilon, y) = \epsilon(y)\,\nu$. Then, $\hat{\xi}$ and $\hat{\nu}$ are defined as empirical parameters constructed over a 10-year period of annual EBITDA for each specific company:

$$\hat{\xi} = \frac{1}{10}\sum\nolimits_{z=y_0}^{y_0+9} \frac{EBITDA_z - EBITDA_{z-1}}{EBITDA_{z-1}}, \tag{5}$$

$$\hat{\nu} = \sqrt{\frac{1}{9}\sum\nolimits_{z=y_0}^{y_0+9}\left(\frac{EBITDA_z - EBITDA_{z-1}}{EBITDA_{z-1}} - \xi\right)^2}. \tag{6}$$

The determination of the 10-year period is arbitrary, but takes into account that: (1) for a shorter sample period, the parameters would not be completely stabilized; and (2) for a longer sample period, structural changes could distort the parameter estimation or availability of information would not permit to analyze more than one period (as it is analyzed in the presented study).

The presented empirical parameters are estimated (based on financial information) for the companies related to 10 equity indices worldwide. Table 1 contains the markets (indices) under analysis.

Table 1. Markets under analysis.

Index	Country	Number of Companies
INDU	USA	30
UKX	GBR	100
TPXC30	JPN	30
HSFML25	HKG	25
SPTSX60	CAN	60
DAX	DEU	30
SENSEX	IND	30
SMI	CHE	20
ASX50	AUS	50
KOSPI50	KOR	50

Appendix A contains the estimated parameters $\hat{\xi}$ and $\hat{\nu}$ for each of the companies, for two separate periods (from 2005 to 2014 and from 1995 to 2004). The estimation is only included for the companies for which information is available during each of the complete period under analysis (for this reason, not all the companies are included).

In line with the presented proposition, the first step is to estimate the coefficients related to the following regression equation, for each of the markets and the periods under analysis:

$$\left|\hat{\xi}_{h,\,j} - \hat{\zeta}_h\right| = \rho_h + \gamma_h^{\circ}\,\hat{\nu}_{h,\,j} + e_{h,\,j}, \tag{7}$$

where $\hat{\xi}_{h,\,j}$ is the EBITDA growth rate of the company j (included in the index h), as defined in Equation (5), $\hat{\zeta}_h$ is the 10-year average GDP nominal growth rate of the country in which the companies included in the index h operate, $\hat{\nu}_{h,\,j}$ is the EBITDA volatility for the company j (included in the index h) as defined in Equation (6), $e_{h,\,j}$ is a residual component, ρ_h and $\overset{\circ}{\gamma}_h$ are the coefficients involved in the regression.

The relevant information related to the regression proposed in Equation (7) is included in Tables 2 and 3, for each of the markets and the periods under analysis.

Table 2. Regression Results (Equation (7)): 2005–2014.

h	Index	$\hat{\zeta}_h$ (%)	$\hat{\rho}_h$ (%) (se_h%)	$\hat{\overset{\circ}{\gamma}}_h$ (%) (se_h%)	$\hat{\overset{\circ}{\gamma}}_h^*$ (%) (se_h^*%)	N_h	R_h^2 (%)	R_h^{2*} (%)	$H.C.$
1	INDU	+3.59	+1.38 (1.04)	+13.84 (4.59)	+19.15 (2.36)	26	74.9	73.1	No
2	UKX	+3.79	+1.44 (0.81)	+25.54 (2.61)	+28.31 (2.12)	73	72.4	71.2	Yes
3	TPXC30	−0.30	+0.31 (1.43)	+27.61 (3.61)	+28.20 (2.29)	15	91.6	91.5	No
4	HSFML25	+5.58	+6.78 (10.87)	+36.47 (6.58)	+39.22 (4.57)	6	94.2	93.6	No
5	SPTSX60	+4.12	+0.96 (0.75)	+34.12 (2.99)	+35.96 (2.65)	41	82.9	82.1	Yes
6	DAX	+2.56	+2.84 (1.65)	+14.61 (6.64)	+24.65 (3.30)	24	74.2	70.7	Yes
7	SENSEX	+14.51	+6.64 (2.79)	+16.65 (14.70)	+47.04 (8.52)	13	81.3	71.8	Yes
8	SMI	+2.78	+1.28 (1.28)	+29.43 (4.87)	+32.68 (3.62)	16	85.5	84.4	Yes
9	ASX50	+6.31	+1.29 (1.81)	+26.40 (1.25)	+26.86 (1.06)	26	96.3	96.3	Yes
10	KOSPI50	+5.44	+3.17 (1.69)	+19.48 (2.61)	+22.81 (2.00)	30	83.8	81.8	No

Table 3. Regression Results (Equation (7)): 1995–2004.

h	Index	$\hat{\zeta}_h$ (%)	$\hat{\rho}_h$ (%) (se_h%)	$\hat{\overset{\circ}{\gamma}}_h$ (%) (se_h%)	$\hat{\overset{\circ}{\gamma}}_h^*$ (%) (se_h^*%)	N_h	R_h^2 (%)	R_h^{2*} (%)	$H.C.$
1	INDU	+5.33	+2.03 (2.60)	+34.60 (8.90)	+40.33 (4.98)	25	73.9	73.2	No
2	UKX	+5.35	+4.54 (1.85)	+29.55 (6.62)	+42.29 (4.32)	46	71.9	68.1	No
3	TPXC30	+0.17	+0.02 (6.32)	+31.56 (5.69)	+31.57 (3.26)	4	96.9	96.9	No
4	HSFML25	-	-	-	-	-	-	-	-
5	SPTSX60	+5.37	+1.08 (1.65)	+45.03 (6.61)	+48.12 (4.54)	23	83.9	83.6	Yes
6	DAX	+2.18	+5.87 (2.76)	+21.30 (8.68)	+36.88 (5.15)	16	82.9	77.4	No
7	SENSEX	-	-	-	-	-	-	-	-
8	SMI	+2.07	-0.97 (3.10)	+29.82 (26.07)	+21.79 (3.38)	5	91.5	91.2	Yes
9	ASX50	+6.32	+1.93 (3.52)	+32.06 (2.31)	+32.74 (1.90)	15	95.6	95.5	No
10	KOSPI50	-	-	-	-	-	-	-	-

Each table shows, for each of the markets (indices) under analysis ($h = 1, 2, \ldots, 10$), and for each of the periods under analysis (2005–2014 and 1995–2004), the GDP nominal growth rate ($\hat{\zeta}_h$) of the country in which operate the companies included in each index. The detailed information that supports the GDP nominal growth rate is included in Appendix B. In addition, the tables contain the estimated parameters of Equation (7): $\hat{\rho}_h$ and $\hat{\overset{\circ}{\gamma}}_h$. Alternatively, the estimated parameter of Equation (7) without the intercept is also included: $\hat{\overset{\circ}{\gamma}}_h^*$. The tables include the standard error related to each estimation (se_h and se_h^*). As complementary information, the size of the sample (N_h) and the coefficients of determination (R_h^2 and R_h^{2*}) for each regression are also presented. Finally, it is informed if heteroscedasticity correction ($H.C.$) has taken place. A Test for heteroscedasticity (White test) is contained in Appendix C. In the cases in which heteroscedasticity correction is needed, the correction is based weighted least squares, assuming a quadratic variance.

For the 2005–2014 period, all the markets contain sufficient information for the analysis. In contrast, for the 1995–2004 period, not all the markets contain all the sufficient information for the analysis (due to the lack of data). For this reason, for that period, Table 3 is not completed for HSFML25, SENSEX and KOSPI50.

4.1. Testing Hypothesis 1: The Market Price of Earnings Risk

The market price of earnings risk, $\overset{\circ}{\gamma}(\varepsilon, y)$, that emerges from the non-arbitrage valuation of equities methodology, is interpreted as the additional return of the equity in excess of the short-term risk-free interest rate, for taking an additional unit of earnings risk. This quantity plays a significant role in the proposition presented in Section 3, as it drives the relationship between $|\xi(\varepsilon, y) - \zeta(\varepsilon, y)|$ and $\nu(\varepsilon, y)$. Moreover, the hypothesis states that this relationship is linear.

Using the coefficient of determination (from Tables 2 and 3) as a measure of goodness of fit, it could be observed that the presented relationship is clearly linear and it is driven by $\gamma^{\circ}(\varepsilon, y)$. For the period 2005–2014, the average coefficient of determination is 83.7% (minimum of 72.4%) for the regression including the intercept, and 76.3% (minimum of 70.7%) for the regression excluding the intercept. For the period 1995–2004, the average coefficient of determination is 85.2% (minimum of 71.9%) for the regression including the intercept, and 83.7% (minimum of 68.1%) for the regression excluding the intercept. This results support the proposed hypothesis.

4.2. Testing Hypothesis 2: The GDP Growth Effect

The GDP nominal growth effect is proposed to be the drift of the instantaneous level of EBITDA process, under the measure $\mathbb{Q}$. In order to test this hypothesis, it is performed an analysis of the individual significance of the parameter ρ_h, for each market (h). In addition, it is presented a global significance test for the parameter ρ, taking into consideration all the markets together.

The economic interpretation of the non-significance of this parameter is that the GDP growth effect hypothesis (as a deflactator of the EBITDA growth) is presumably correct, since the intercept in Equation (7) does not provide additional information (i.e., the relevant information is already provided by the GDP nominal growth).

In this sense, for each market (h) and for each period under analysis, the following hypotheses are presented:

$$H_0^h : \rho_h = 0,$$

$$H_A^h : \rho_h \neq 0,$$

The results of the test (at a 95% confidence level), for each market and period under analysis, are shown in Tables 4 and 5.

Table 4. Individual significance test (95%) of the parameter ρ_h: 2005–2014.

h	Index	t_h	t_h^c	p-Value (%)	Reject H_0^h
1	INDU	+1.33	±2.06	19.47	No
2	UKX	+1.78	±1.99	7.93	No
3	TPXC30	+0.21	±2.16	83.32	No
4	HSFML25	+0.62	±2.78	56.64	No
5	SPTSX60	+1.29	±2.02	20.37	No
6	DAX	+1.72	±2.07	9.95	No
7	SENSEX	+2.37	±2.20	3.68	Yes
8	SMI	+1.00	±2.14	33.47	No
9	ASX50	+0.71	±2.06	48.41	No
10	KOSPI50	+1.87	±2.05	7.15	No

Table 5. Individual significance test (95%) of the parameter ρ_h: 1995–2004.

h	Index	t_h	t_h^c	p-Value (%)	Reject H_0^h
1	INDU	+0.78	±2.07	44.35	No
2	UKX	+2.45	±2.01	1.82	Yes
3	TPXC30	+0.00	±4.30	99.77	No
4	HSFML25	-	-	-	-
5	SPTSX60	+0.65	±2.08	52.13	No
6	DAX	+2.12	±2.14	5.19	No
7	SENSEX	-	-	-	-
8	SMI	−0.31	±3.18	77.59	No
9	ASX50	+0.55	±2.16	59.36	No
10	KOSPI50	-	-	-	-

Each table shows, for each of the markets and periods under analysis, the t-statistic ($t_h = \hat{\rho}_h / se\left(\hat{\rho}_h\right)$), the critical value ($t_h^c$) at a 95% level of confidence, and the p-value. Finally, it is informed if H_0^h is rejected or not.

The results of the test support the non-significance of the parameter ρ_h. For the period 2005–2014, one of the ten markets shows an intercept that is statistical significant, this is the case for SENSEX (p-value: 3.68%). For the period 1955–2004, one of the seven markets shows an intercept that is statistical significant, this is the case for UKX (p-value: 1.82%).

In addition to the significance test for the parameter ρ_h, for each individual market (h), it is presented a global significance test, taking into consideration all the markets together. For each period under analysis, the following hypotheses are presented:

$$H_0 : \rho_1 = \rho_2 = \ldots = \rho_H = 0,$$

$$H_A : H_0 \; is \; false.$$

For the purpose of the test, a global statistic (g) is constructed;

$$g = \frac{1}{H} \sum_{h=1}^{H} t_h^2,$$

in which each t_h corresponds to the statistic of the individual significance test of each market (h) and H corresponds to the number of markets under analysis. It is clear that g has not a known distribution, but for the purpose of obtaining the critical values and the p-values a simulation of 10,000 scenarios is generated. This consists in simulating each t_h and then constructing g, according to the definition presented above.

The results of the test (at a 95% confidence level), for each period, are shown in Table 6.

Table 6. Global significance test (95%) of the parameter $\boldsymbol{\rho_h}$.

Period	g	g^c	p-Value (%)	Reject H_0
2005–2014	+2.07	+2.40	8.85	No
1995–2004	+1.71	+4.96	29.00	No

The table shows, for periods under analysis, the statistic (g), the critical value (g^c) at a 95% level of confidence, and the p-value. Finally, it is informed if H_0 is rejected or not. Similar to the conclusions that emerge from the individual test, the results of the global test support the non-significance of the parameter ρ.

The proposed tests (individually and globally considered) provide sufficient evidence in favor of not rejecting H_0 (i.e., $\rho = 0$). Nevertheless, this is an hypothesis that should be continuously tested.

5. The Stability of the Market Price of Earnings Risk

The aim of this section is to test the stability of the market price of earnings risk. In this sense, it is analyzed if the market price of earnings risk is the same for all the markets (geographical stability), at a point in time. Furthermore, it is analyzed if the market price of earnings risk is the same for each period (time stability). In both cases, an individual test (for each market (h)) and a global test (taking into consideration all the markets together) are performed.

Due to the results obtained in the previous section, the model (Equation (7)) that is utilized for the tests does not include the intercept (ρ_h).

5.1. Geographical Stability of the Market Price of Earnings Risk

For testing the geographical stability of the market price of earnings risk, the following hypotheses for each individual market are presented:

$$H_0^h : \gamma_h^{\circ} = \gamma^{\circ},$$

$$H_A^h : \overset{\circ}{\gamma}_h \neq \overset{\circ}{\gamma},$$

Under H_0 it is assumed that the geographically-stable $\overset{\circ}{\gamma}$ is the average of $\overset{\circ}{\gamma}_h$ ($h = 1, 2, \ldots, 10$). For the period 2005–2014, the average is 30.49%; and for the period 1995–2004, the average is 36.25%.

The results of the test (at a 95% confidence level), for each market and period under analysis, are shown in Tables 7 and 8.

Table 7. Geographical stability test (95%) for $\overset{\circ}{\gamma}$: 2005–2014.

h	Index	t_h	t_h^c	p-Value (%)	Reject H_0^h
1	INDU	−4.87	±2.06	0.01	Yes
2	UKX	−1.02	±1.99	30.90	No
3	TPXC30	−1.00	±2.14	33.50	No
4	HSFML25	+1.91	±2.57	11.41	No
5	SPTSX60	+2.06	±2.02	4.55	Yes
6	DAX	−1.77	±2.07	9.03	No
7	SENSEX	+1.94	±2.18	7.58	No
8	SMI	+0.60	±2.13	55.46	No
9	ASX50	−3.43	±2.06	0.21	Yes
10	KOSPI50	−3.84	±2.05	0.06	Yes

Table 8. Geographical stability test (95%) for $\overset{\circ}{\gamma}$: 1995–2004.

h	Index	t_h	t_h^c	p-Value (%)	Reject H_0^h
1	INDU	+0.82	±2.06	41.96	No
2	UKX	+1.40	±2.01	16.85	No
3	TPXC30	−1.44	±3.18	24.67	No
4	HSFML25	-	-	-	-
5	SPTSX60	+2.61	±2.07	1.59	Yes
6	DAX	+0.12	±2.13	90.35	No
7	SENSEX	-	-	-	-
8	SMI	−4.28	±2.78	1.29	Yes
9	ASX50	−1.85	±2.14	8.61	No
10	KOSPI50	-	-	-	-

Each table shows, for each of the markets and periods under analysis, the t-statistic ($t_h = (\hat{\overset{\circ}{\gamma}}{}_h^{*} - \overset{\circ}{\gamma}_h)/se(\hat{\overset{\circ}{\gamma}}{}_h^{*})$), the critical value ($t_h^c$) at a 95% level of confidence, and the p-value. Finally, it is informed if H_0^h is rejected or not.

The results of the test are not sufficiently conclusive with respect to the geographical stability of the parameter $\overset{\circ}{\gamma}$. For the period 2005–2014, four of the ten markets show instability, this is the case for INDU (p-value: 0.01%), SPTSX60 (p-value: 4.55%), ASX50 (p-value: 0.21%) and KOSPI50 (p-value: 0.06%). For the period 1955–2004, two of the seven markets show instability, this is the case for SPTSX60 (p-value: 1.59%) and SMI (p-value: 1.29%).

In addition to the individual geographical stability test for each parameter $\overset{\circ}{\gamma}_h$, for each market ($h$), it is presented a global test, taking into consideration all the markets together. For each period under analysis, the following hypotheses are presented:

$$H_0 : \overset{\circ}{\gamma}_1 = \overset{\circ}{\gamma}_2 = \ldots = \overset{\circ}{\gamma}_H = \overset{\circ}{\gamma},$$

$$H_A : H_0 \; is \; false.$$

Similar to the case of the individual geographical stability test, under H_0 it is assumed that the geographically-stable $\overset{\circ}{\gamma}$ is the average of $\overset{\circ}{\gamma}_h$ ($h = 1, 2, \ldots, 10$).

For the purpose of the test, a global statistic (g) is constructed;

$$g = \frac{1}{H} \sum_{h=1}^{H} t_h^2,$$

in which each t_h corresponds to the statistic of the individual geographical stability test of each market (h) and H corresponds to the number of markets under analysis. As in the global test included in the previous section, it is clear that g has not a known distribution, but for the purpose of obtaining the critical values and the p-values a simulation of 10,000 scenarios is generated. This consists in simulating each t_h and then constructing g, according to the definition presented above.

The results of the test (at a 95% confidence level), for each period, are shown in Table 9.

Table 9. Global geographical stability test (95%) of the parameter $\mathring{\gamma}_h$.

Period	g	g^c	p-Value (%)	Reject H_0
2005–2014	+6.75	+2.30	0.08	Yes
1995–2004	+4.75	+3.45	2.25	Yes

The table shows, for the periods under analysis, the statistic (g), the critical value (g^c) at a 95% level of confidence, and the p-value. Finally, it is informed if H_0 is rejected or not. The results of the global test are conclusive. The geographical stability hypothesis for the parameter $\mathring{\gamma}_h$ is rejected. This means that each market would have its own market price of earnings risk.

5.2. Time Stability of the Market Price of Earnings Risk

For testing the time stability of the market price of earnings risk, the following hypotheses for each individual market are presented:

$$H_0^h : \mathring{\gamma}_{h,t_i} = \mathring{\gamma}_{h,t_{i+1}} = \mathring{\gamma}_h,$$

$$H_A^h : H_0^h \ is \ false,$$

where t_i and t_{i+1} correspond to each of the periods. Under H_0 it is assumed that the time-stable $\mathring{\gamma}_h$ is the average $\left(\mathring{\gamma}_{h,t_i} + \mathring{\gamma}_{h,t_{i+1}}\right)/2$ (for each $h = 1, 2, \ldots, 10$). This average is only possible in the cases of the markets that information is available for both periods under analysis. For this reason, only seven markets are analyzed.

For the purpose of the test, the following statistic (g_h) is constructed;

$$g_h = \frac{1}{2}\left(t_{h,t_i}^2 + t_{h,t_{i+1}}^2\right),$$

in which t_{h,t_i} and $t_{h,t_{i+1}}$ correspond to the statistic of an individual test (i.e., $t_{h,t_i} = (\hat{\mathring{\gamma}}_{h,t_i}^* - \mathring{\gamma}_h)/se(\hat{\mathring{\gamma}}_{h,t_i}^*)$)). Again, it is clear that g_h has not a known distribution, but for the purpose of obtaining the critical values and the p-values a simulation of 10,000 scenarios is generated. This consists in simulating t_{h,t_i} and $t_{h,t_{i+1}}$, and then constructing g_h, according to the definition presented above.

The results of the test (at a 95% confidence level), for each period, are shown in Table 10.

Table 10. Time stability test (95%) for $\mathring{\gamma}$.

h	Index	g_h	g_h^c	p-Value (%)	Reject H_0^h
1	INDU	+12.63	+3.28	0.05	Yes
2	UKX	+6.73	+3.18	0.25	Yes
3	TPXC30	+0.40	+6.29	70.78	No
4	HSFML25	-	-	-	-
5	SPTSX60	+3.53	+3.35	4.31	Yes
6	DAX	+2.42	+3.54	11.96	No
7	SENSEX	-	-	-	-
8	SMI	+2.42	+5.09	17.25	No
9	ASX50	+5.06	+3.60	1.79	Yes
10	KOSPI50	-	-	-	-

The table shows, for each of the markets under analysis, the statistic (g_h), the critical value (g_h^c) at a 95% level of confidence, and the *p*-value. Finally, it is informed if H_0^h is rejected or not.

The results of the test are not sufficiently conclusive with respect to the time stability of the parameter γ°. Four of the seven markets show instability, this is the case for INDU (*p*-value: 0.05%), UKX (*p*-value: 0.25%), SPTSX60 (*p*-value: 4.31%) and ASX50 (*p*-value: 1.79%).

In addition to the individual test for each parameter $\mathring{\gamma}_h$, for each market (h), it is presented a global test, taking into consideration all the markets together. In this sense, the following hypotheses are presented:

$$H_0 : \mathring{\gamma}_{1,t_i} = \mathring{\gamma}_{1,t_{i+1}} = \mathring{\gamma}_1 \; and \; \mathring{\gamma}_{2,t_i} = \mathring{\gamma}_{2,t_{i+1}} = \mathring{\gamma}_2 \; \ldots \; and \; \mathring{\gamma}_{H,t_i} = \mathring{\gamma}_{H,t_{i+1}} = \mathring{\gamma}_H,$$

$$H_A : H_0 \; is \; false.$$

Similar to the case of the individual time stability test, under H_0 it is assumed that the time-stable $\mathring{\gamma}_h$ is the average $\left(\mathring{\gamma}_{h,t_i} + \mathring{\gamma}_{h,t_{i+1}}\right)/2$ (for each $h = 1, 2, \ldots, 10$).

For the purpose of the test, a global statistic (g) is constructed;

$$g = \frac{1}{H} \sum_{h=1}^{H} g_h,$$

in which each g_h corresponds to the statistic of the individual time stability test of each market (h) and H corresponds to the number of markets under analysis. As in the global test included in the previous section, it is clear that g has not a known distribution, but for the purpose of obtaining the critical value and the *p*-value a simulation of 10,000 scenarios is generated. This consists in simulating each g_h and then constructing g, according to the definition presented above.

The results of the test (at a 95% confidence level) are shown in Table 11.

Table 11. Global time stability test (95%) of the parameter γ°.

g	g^c	*p*-Value (%)	Reject H_0
+4.74	+2.44	0.75	Yes

The table shows, the statistic (g), the critical value (g^c) at a 95% level of confidence, and the *p*-value. Finally, it is informed if H_0 is rejected or not. The results of the global test are conclusive. The time stability hypothesis for the parameter γ° is rejected. This means that as time evolves the market price of earnings risk would not remain constant.

6. Discussion

As originally presented, the non-arbitrage valuation of equities methodology is not oriented to predict how equities market prices behave, but it is oriented to generate an equity valuation method as objective as possible. Nevertheless, if actual market prices do not allow arbitrage opportunities, it is expected that those prices would converge to the values that emerge from the methodology.

The main contribution of this paper is to generate and test a proposition that allows defining the undetermined components of the methodology: the drift of the instantaneous level of EBITDA under the measure $\mathbb{Q}$ (as the GDP nominal growth effect) and, consequently, the market price of earnings risk.

The non-arbitrage valuation framework (complemented by the proposed and tested hypothesis) presents some advantages in relation to the traditional methods that are used for the valuation of equities:

- The non-arbitrage approach does not use (historical or projected) equities prices information (or equities returns) for the derivation of current equity values, avoiding the absorption of market sentiments or market mispricing. The traditional method applies discount rates (cost of equities) that are based on equilibrium models (i.e., factor models), in which observed market prices and returns intervene, assuming that those prices are correctively determined.

- The non-arbitrage approach does not require a subjective outlook for the performance of the company that is being valued, as it is the case of the traditional method that requires a subjective estimation of future dividends. Under the non-arbitrage approach, the subjectively-determined drift for the instantaneous level of EBITDA is eliminated.
- Under the traditional method, for the cost of equity derivation, it is very important to carefully select a market portfolio (or reference index). This provides subjectivity to the valuation since it is not completely clear which the components of that portfolio (or index) should be. In this sense, for the same equity, different valuations could emerge depending on the selected market portfolio. In contrast, under the proposed method, this potential inconsistency is avoided.
- In the case of the factor models, the issue of selecting the appropriate factors is an important challenge, since they could change over time. It is usual to observe that the factors that were useful in the past, are not necessarily useful for the future. Moreover, the parameters (coefficients related to each factor) involved in the regression equation are not stable over time. Under the non-arbitrage approach, the only factor that is involved is the GDP nominal growth rate.
- The macro-finance models consider market prices as a response variable, depending on the impulse (behavior) of several macro-variables. This relationship is implicit in the impulse-response function. Instead, under the non-arbitrage framework, the link between the real economy and the financial assets prices is explicit.

Beside these points, a new challenge for the proposed approach emerges. Under the non-arbitrage valuation method, the future GDP nominal growth rate is involved. It is clear that this is a variable that is difficult to predict, especially in the long-run. In this sense, it would be necessary to incorporate it, as part of the framework, as an additional stochastic process. This gives space to an area of future research, in which a risk-neutral process for the GDP growth rate could be introduced.

7. Conclusions

This paper complements the non-arbitrage valuation of equities framework by proposing the GDP nominal growth effect as an important driver of equities values. Moreover, the proposition is tested through an empirical analysis based on several companies worldwide, for two different periods. The test provides sufficient evidence in favor of the proposition. Furthermore, additional conclusions for the behavior of the market price of earnings risk have been generated: it would not be the same value for all the markets, and it would not be constant over time.

The non-arbitrage valuation approach, complemented by the proposed and tested hypothesis presented in this paper, offers some advantages in relation to the traditional methods that are used for the valuation of equities. This means that further research related to the framework could provide useful information for the understanding of financial assets. In particular, future research could be oriented to the incorporation of the GDP nominal growth rate as an additional (potential risk-neutral) stochastic process into the framework.

Conflicts of Interest: The author declares no conflict of interest.

Appendix A

This appendix contains the estimated parameters $\hat{\xi}$ and $\hat{\upsilon}$ for each of the companies (in line with Equations (5) and (6)), for two separate periods (from 2005 to 2014 and from 1995 to 2004). The estimation is only included for the companies for which information is available during each of the complete period under analysis (for this reason, not all the companies are included).

Table A1. Estimated parameters ξ (%) and ν (%), for each company of INDU index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
AAPL	-	-	-	-	KO	6.2	9.1	5.7	15.4
AXP	7.9	28.0	3.7	8.0	MCD	7.5	7.2	6.3	13.8
BA	14.3	31.7	12.1	44.2	MMM	4.6	7.0	7.1	11.3
CAT	15.8	45.9	8.0	16.3	MRK	8.5	39.2	6.6	11.8
CSCO	5.3	15.4	40.6	46.1	MSFT	6.0	17.2	23.7	27.2
CVX	7.9	26.9	30.9	76.9	NKE	9.0	12.3	13.1	21.3
DD	6.4	21.8	−5.1	15.4	PFE	−0.3	17.6	31.5	47.8
DIS	10.8	13.1	8.8	32.4	PG	2.6	13.2	8.9	10.5
GE	−4.1	18.1	10.1	14.1	TRV	-	-	-	-
GS	8.1	40.0	-	-	UNH	11.5	17.1	23.7	14.3
HD	4.1	16.0	23.2	8.7	UTX	8.4	9.9	14.7	31.8
IBM	4.6	7.8	4.3	20.4	V	-	-	-	-
INTC	8.7	33.3	17.0	27.2	VZ	4.9	23.0	22.6	41.6
JNJ	5.4	6.5	15.7	5.6	WMT	5.4	3.7	15.9	10.7
JPM	-	-	-	-	XOM	5.0	27.2	17.6	31.0

Table A2. Estimated parameters ξ (%) and ν (%), for each company of UKX index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
AAL	-	-	-	-	LAND	32.2	185.9	3.4	7.8
ABF	10.3	8.3	3.3	7.6	LGEN	-	-	-	-
ADM	-	-	-	-	LLOY	-	-	-	-
ADN	59.7	120.3	25.1	48.8	LSE	-	-	-	-
AHT	19.5	21.0	24.9	30.8	MERL	-	-	-	-
ANTO	10.6	38.3	-	-	MKS	3.9	12.0	−0.3	14.5
ARM	−22.5	114.9	-	-	MNDI	5.9	23.3	-	-
AV	-	-	-	-	NG	5.4	10.2	-	-
AZN	1.9	23.3	20.2	32.4	NXT	6.4	6.2	17.3	11.7
BA	4.1	18.3	14.2	22.5	OML	-	-	-	-
BAB	25.9	27.7	−29.3	113.2	PFG	3.6	14.7	11.2	5.2
BARC	-	-	-	-	PRU	-	-	-	-
BATS	2.7	15.9	-	-	PSN	−14.6	85.7	36.6	28.0
BDEV	−13.6	102.4	22.1	9.3	PSON	7.0	25.9	10.4	19.4
BKG	16.8	29.6	17.4	22.5	RB	11.5	13.9	-	-
BLND	−57.0	205.5	-	-	RBS	-	-	-	-
BLT	15.7	45.2	-	-	RDSA	5.6	28.8	13.6	29.7
BNZL	9.1	5.9	7.2	13.4	RDSB	5.6	28.8	13.6	29.7
BP	195.7	663.6	-	-	REL	-	-	-	-
BRBY	60.6	184.4	-	-	REX	0.7	18.9	7.5	19.7
BTA	3.7	25.0	1.7	7.4	RIO	56.0	122.4	10.9	30.2
CCH	-	-	-	-	RMG	-	-	-	-
CCL	1.9	11.4	-	-	RR	14.6	28.8	17.5	31.4
CNA	12.1	51.3	-	-	RRS	57.2	99.5	-	-
CPG	6.4	21.7	-	-	RSA	-	-	-	-
CPI	12.7	8.2	36.5	15.8	SAB	11.0	20.8	-	-
CRH	4.7	37.9	23.9	17.0	SBRY	5.0	26.0	−4.3	15.3
DC	-	-	-	-	SDR	32.4	73.0	-	-
DCC	10.8	10.7	14.5	8.0	SGE	7.9	22.3	31.0	21.4
DGE	4.2	9.4	10.1	34.7	SHP	1.4	133.5	-	-
DLG	-	-	-	-	SKY	7.4	11.8	20.0	38.1
EXPN	9.7	8.9	-	-	SL	-	-	-	-
EZJ	28.1	40.4	-	-	SMIN	2.1	14.5	16.6	37.9
FRES	-	-	-	-	SN	7.8	11.2	7.7	14.4
GKN	30.7	73.7	−2.0	26.9	SPD	-	-	-	-

Table A2. *Cont.*

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
GLEN	-	-	-	-	SSE	37.5	134.9	19.5	33.7
GSK	2.0	31.4	-	-	STAN	-	-	-	-
HIK	23.3	25.9	-	-	STJ	-	-	-	-
HL	33.0	26.0	-	-	SVT	2.0	3.6	3.8	7.2
HMSO	−56.5	157.0	-	-	TPK	8.2	24.6	20.4	11.9
HSBA	-	-	-	-	TSCO	−14.4	66.9	11.9	4.8
IAG	-	-	-	-	TUI	-	-	10.8	28.7
IHG	44.8	153.4	-	-	TW	−16.9	147.7	26.3	22.9
III	-	-	-	-	ULVR	2.2	9.9	6.7	9.9
IMB	9.4	17.5	-	-	UU	0.3	8.9	10.0	13.0
INTU	−44.7	133.2	12.0	7.9	VOD	21.3	87.5	43.1	38.9
ISAT	10.6	21.4	-	-	WOS	5.5	29.1	12.0	9.2
ITRK	14.4	11.3	-	-	WPG	-	-	-	-
ITV	10.3	25.6	-	-	WPP	11.7	9.0	17.9	16.7
JMAT	11.3	10.4	8.4	10.4	WTB	16.7	49.1	3.4	16.9
KGF	4.7	33.2	8.6	14.3					

Table A3. Estimated parameters ξ (%) and ν (%), for each company of TPXC30 index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
2914	-	-	-	-	8031	-	-	-	-
3382	-	-	-	-	8058	-	-	-	-
4063	3.8	20.8	-	-	8306	-	-	-	-
4502	-	-	-	-	8316	-	-	-	-
4503	-	-	-	-	8411	-	-	-	-
6501	-	-	-	-	8604	−20.3	90.2	-	-
6752	4.5	26.9	11.9	60.8	8766	-	-	-	-
6758	19.9	69.6	68.9	211.0	8801	5.5	11.4	-	-
6902	-	-	-	-	8802	3.7	12.4	-	-
6954	23.0	69.3	-	-	9020	1.5	4.4	-	-
6981	18.4	37.3	6.2	30.7	9022	2.9	7.8	-	-
7201	4.9	29.2	-	-	9432	−1.3	4.0	-	-
7203	14.1	39.5	-	-	9433	-	-	-	-
7267	-	-	-	-	9437	−1.4	5.9	-	-
7751	0.2	19.1	13.8	14.2	9984	-	-	-	-

Table A4. Estimated parameters ξ (%) and ν (%), for each company of HSFML25 index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
1044	-	-	-	-	3968	-	-	-	-
1088	14.5	10.6	-	-	3988	-	-	-	-
1109	-	-	-	-	688	-	-	-	-
1288	-	-	-	-	700	-	-	-	-
1398	-	-	-	-	728	-	-	-	-
151	23.6	13.0	-	-	762	-	-	-	-
1880	67.7	90.2	-	-	836	-	-	-	-
2318	-	-	-	-	857	6.3	11.0	-	-
2601	-	-	-	-	883	-	-	-	-
2628	-	-	-	-	939	-	-	-	-
267	-	-	-	-	941	-	-	-	-
3328	-	-	-	-	992	146.6	338.0	-	-
386	58.2	202.8	-	-					

Table A5. Estimated parameters ξ (%) and ν (%), for each company of SPTSX60 index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
ABX	−109.6	226.8	5.7	17.4	GIL	21.7	35.1	-	-
AEM	-	-	-	-	HSE	13.1	32.1	-	-
AGU	32.2	77.9	-	-	IMO	6.5	24.3	12.4	26.7
ARX	18.9	37.7	-	-	IPL	16.0	13.2	-	-
ATDB	19.3	19.6	-	-	K	7.2	87.0	29.0	49.5
BAMA	17.2	32.0	-	-	L	0.4	9.2	18.4	12.8
BB	−86.8	314.8	-	-	MFC	-	-	-	-
BBDB	-	-	8.5	21.6	MG	67.0	224.0	-	-
BCE	1.3	4.5	4.6	18.8	MRU	11.2	23.8	10.8	4.2
BMO	-	-	-	-	NA	-	-	-	-
BNS	-	-	-	-	POT	32.9	66.2	25.1	43.2
CCO	12.1	48.2	10.1	20.8	POW	-	-	-	-
CM	-	-	-	-	PPL	20.3	18.7	-	-
CNQ	9.2	12.0	46.4	54.1	QSR	-	-	-	-
CNR	7.6	9.9	16.7	23.3	RCIB	10.9	11.7	10.4	11.4
CP	10.7	19.4	-	-	RY	-	-	-	-
CPG	40.8	23.8	-	-	SAP	10.9	11.5	-	-
CSU	-	-	-	-	SJRB	9.3	7.1	21.6	14.8
CTCA	7.0	5.2	8.9	6.7	SLF	-	-	-	-
CVE	-	-	-	-	SLW	-	-	-	-
DOL	-	-	-	-	SNC	48.0	123.0	15.2	13.4
ECA	−31.4	93.1	28.5	38.0	SU	31.3	76.8	-	-
ELD	-	-	−54.9	171.8	T	3.1	5.3	17.2	42.7
EMA	8.2	14.3	4.9	6.7	TCKB	12.2	38.6	41.7	82.4
ENB	14.6	29.8	30.6	64.6	TD	-	-	-	-
FM	78.4	147.1	-	-	TRI	31.2	109.2	7.5	7.2
FNV	−64.8	231.7	-	-	TRP	5.5	7.2	8.9	13.4
FTS	17.3	17.7	13.4	15.2	VRX	68.2	171.0	93.2	173.2
G	47.7	150.7	-	-	WN	0.9	15.5	15.8	15.5
GIBA	18.2	41.9	78.4	100.1	YRI	-	-	-	-

Table A6. Estimated parameters ξ (%) and ν (%), for each company of DAX index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
ADS	8.8	23.2	22.4	25.8	FRE	13.7	11.3	-	-
ALV	-	-	-	-	HEI	6.2	25.3	13.3	24.7
BAS	4.3	21.3	11.0	17.9	HEN3	5.4	14.2	10.3	25.1
BAYN	8.8	12.4	17.3	69.1	IFX	8.3	63.2	-	-
BEI	4.5	11.4	7.5	7.9	LHA	5.6	19.6	8.2	32.7
BMW	8.3	22.8	19.8	24.2	LIN	11.8	11.6	13.4	26.9
CBK	-	-	-	-	MRK	14.5	32.3	7.8	14.9
CON	15.2	37.9	19.4	28.3	MUV2	-	-	-	-
DAI	18.6	86.4	-	-	RWE	−1.9	15.7	-	-
DB1	6.1	21.3	-	-	SAP	11.2	25.4	25.6	23.2
DBK	-	-	-	-	SDF	35.8	90.0	-	-
DPW	5.1	32.2	-	-	SIE	5.5	19.9	9.0	12.2
DTE	0.2	14.5	1.6	10.7	TKA	-	-	-	-
EOAN	7.5	37.3	24.1	67.6	VNA	-	-	-	-
FME	10.9	9.7	-	-	VOW3	13.7	23.7	8.7	19.2

Table A7. Estimated parameters ζ (%) and ν (%), for each company of SENSEX index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
ADSEZ	-	-	-	-	INFO	-	-	-	-
APNT	22.2	21.1	-	-	ITC	17.2	5.2	-	-
AXSB	-	-	-	-	LPC	41.7	33.0	-	-
BHARTI	-	-	-	-	LT	25.2	17.0	-	-
BHEL	-	-	-	-	MM	25.8	32.7	-	-
BJAUT	-	-	-	-	MSIL	21.4	41.9	-	-
CIPLA	-	-	-	-	NTPC	10.6	14.2	-	-
COAL	-	-	-	-	ONGC	5.9	12.1	-	-
DRRD	-	-	-	-	RIL	13.3	21.4	-	-
GAIL	5.9	18.9	-	-	SBIN	-	-	-	-
HDFC	24.9	11.2	-	-	SUNP	38.5	34.5	-	-
HDFCB	-	-	-	-	TATA	15.6	56.4	-	-
HMCL	-	-	-	-	TCS	-	-	-	-
HUVR	-	-	-	-	TTMT	41.3	59.0	-	-
ICICIBC	-	-	-	-	WPRO	-	-	-	-

Table A8. Estimated parameters ζ (%) and ν (%), for each company of SMI index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
ABBN	13.5	20.5	-	-	NOVN	8.0	9.5	4.6	11.7
ADEN	32.4	109.2	25.5	32.6	RIGN	−24.2	108.4	-	-
ATLN	59.0	137.2	-	-	ROG	9.5	17.3	8.7	13.3
BAER	-	-	-	-	SCMN	0.2	6.2	-	-
CFR	19.5	23.5	0.5	31.2	SGSN	9.7	9.7	6.0	28.0
CSGN	-	-	-	-	SREN	-	-	-	-
GEBN	4.4	10.1	-	-	SYNN	8.5	13.3	-	-
GIVN	7.1	16.0	-	-	UBSG	−2.9	44.1	-	-
LHN	6.8	29.0	3.2	12.8	UHR	11.1	20.9	6.2	15.1
NESN	0.9	9.3	5.1	8.2	ZURN	-	-	-	-

Table A9. Estimated parameters ζ (%) and ν (%), for each company of ASX50 index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
AGL	-	-	-	-	MQG	-	-	-	-
AIO	-	-	-	-	NAB	-	-	-	-
AMC	6.8	20.6	5.3	28.6	NCM	−29.3	142.8	47.7	155.9
AMP	-	-	-	-	ORG	51.1	183.6	-	-
ANZ	-	-	-	-	ORI	21.6	78.1	5.8	24.6
APA	24.1	28.0	-	-	OSH	23.3	51.7	-	-
ASX	77.6	188.7	-	-	QBE	-	-	-	-
AZJ	-	-	-	-	RHC	24.4	25.5	-	
BHP	15.2	44.5	−32.2	109.7	RIO	56.2	122.6	8.7	22.4
BXB	-	-	-	-	S32	-	-	-	-
CBA	-	-	-	-	SCG	-	-	-	-
CCL	2.9	19.1	−31.0	102.1	SEK	35.2	36.7	-	-
CPU	17.7	40.4	42.6	49.1	SGP	−9.9	180.1	182.3	538.5
CSL	28.3	70.7	29.7	37.4	SHL	9.7	10.7	42.9	27.9
CTX	−10.0	125.5	-	-	STO	−3.2	50.8	9.2	22.4
CWN	-	-	-	-	SUN	-	-	-	-
DXS	4.2	19.5	-	-	SYD	-	-	-	-
GMG	21.2	63.3	-	-	TCL	16.8	13.9	-	-
GPT	−9.5	60.7	-	-	TLS	0.6	5.6	-	-
IAG	-	-	-	-	VCX	-	-	-	-
IPL	−120.0	473.0	-	-	WBC	-	-	-	-
JHX	48.3	130.4	-	-	WES	18.7	31.2	17.5	24.3
LLC	−43.9	131.5	7.3	36.9	WFD	-	-	-	-
MGR	−56.7	271.8	-	-	WOW	11.3	11.8	12.9	7.6
MPL	-	-	-	-	WPL	33.1	48.2	13.8	40.5

Table A10. Estimated parameters ξ (%) and ν (%), for each company of KOSPI50 index.

Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}	Company	ξ_{2005}^{2014}	ν_{2005}^{2014}	ξ_{1995}^{2004}	ν_{1995}^{2004}
30	-	-	-	-	23530	10.1	12.8	-	-
270	22.7	43.2	-	-	24110	-	-	-	-
660	35.1	90.4	-	-	28260	-	-	-	-
720	13.3	17.7	-	-	29780	-	-	-	-
810	-	-	-	-	30200	−5.0	11.1	-	-
1800	12.3	27.5	-	-	32830	-	-	-	-
2380	0.7	12.8	-	-	33780	1.5	13.0	-	-
2790	16.3	29.7	-	-	34220	14.6	57.4	-	-
3550	−8.2	33.5	-	-	34730	-	-	-	-
4020	18.8	41.5	-	-	35250	-	-	-	-
5380	9.8	16.5	-	-	35420	-	-	-	-
5490	1.8	23.4	-	-	36570	26.1	87.7	-	-
5930	12.1	23.8	-	-	42660	-	-	-	-
6400	66.2	265.1	-	-	51900	22.6	17.3	-	-
9150	11.8	36.8	-	-	51910	8.1	21.6	-	-
9540	1.0	96.1	-	-	55550	3.1	20.6	-	-
10130	21.0	42.1	-	-	66570	8.0	50.5	-	-
10140	20.3	50.1	-	-	86280	31.1	25.7	-	-
10950	−1.4	67.8	-	-	86790	-	-	-	-
11170	-	-	-	-	88350	-	-	-	-
12330	16.5	17.3	-	-	90430	-	-	-	-
15760	16.0	58.7	-	-	96770	-	-	-	-
17670	1.6	9.0	-	-	105560	-	-	-	-
18260	-	-	-	-	139480	-	-	-	-
21240	-	-	-	-	161390	-	-	-	-

Appendix B

This appendix contains the detailed information that supports the GDP nominal growth rates.

Table B1. GDP nominal growth per country ζ (%): 2005–2014.

Country	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
USA	6.67	5.82	4.49	1.66	−2.04	3.78	3.70	4.16	3.74	3.88
GBR	6.00	5.73	5.52	2.38	−2.23	4.70	4.11	2.82	4.19	4.70
JPN	0.04	0.55	1.24	−2.29	−6.00	2.39	−2.30	0.81	1.06	1.54
HKG	7.23	6.46	9.81	3.44	−2.83	7.06	8.90	5.31	4.99	5.47
CAN	6.47	5.40	5.31	5.11	−4.80	6.11	6.45	3.46	3.41	4.28
DEU	1.33	4.02	5.01	1.93	−3.96	4.87	4.77	1.91	2.39	3.36
IND	13.92	16.28	16.12	12.89	15.06	20.17	13.46	13.09	13.58	10.54
CHE	3.70	6.04	6.50	4.24	−1.73	3.25	2.01	0.91	1.75	1.17
AUS	7.04	8.20	8.92	8.41	6.84	3.05	8.71	5.76	2.24	3.95
KOR	5.00	5.03	7.99	5.87	4.27	9.86	5.32	3.36	3.77	3.89

Table B2. GDP nominal growth per country ζ (%): 1995–2004.

Country	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
USA	4.86	5.69	6.28	5.58	6.29	6.46	3.28	3.35	4.86	6.64
GBR	5.24	6.79	5.49	5.05	4.25	6.20	3.84	5.07	6.16	5.46
JPN	1.20	2.04	2.20	−2.06	−1.47	0.98	−0.85	−1.27	−0.06	0.98
HKG	-	-	-	-	-	-	-	-	-	-
CAN	5.04	3.47	5.44	3.92	6.95	9.61	3.34	4.06	5.32	6.52
DEU	3.75	1.45	2.12	2.60	2.31	2.50	2.99	1.35	0.49	2.28
IND	-	-	-	-	-	-	-	-	-	-
CHE	1.22	0.81	2.10	2.82	1.83	5.35	2.49	−0.19	1.00	3.24
AUS	6.20	6.74	5.22	5.77	5.36	6.53	6.73	6.83	6.28	7.57
KOR	-	-	-	-	-	-	-	-	-	-

Appendix C

This appendix contains a White test for hetersocedasticity detection (at a 95% confidence level). The auxiliary regression equation is given by:

$$\hat{e}^2_{h,j} = \alpha_{h,0} + \alpha_{h,1}\ \hat{v}_{h,j} + \alpha_{h,2}\ \hat{v}^2_{h,j} + u_{h,j}.$$

Then, it is defined $w = N\ .R^2$, where N is the size of the sample, and R^2 is the coefficient of determination of the auxiliary regression equation. If w is larger than 5.9915 (since $w \sim \chi^2_2$) then heteroscedasticity is detected.

In the cases in which heteroscedasticity is detected, then weighted average least squares are applied for correction. This is based on a quadratic variance assumption ($\hat{e}^2_{h,\ j}$). If, for the company j, $\widehat{\hat{e}^2_{h,j}} \leq 0$ then the point (information related to the company j) is eliminated from the sample. This explains, in some cases, that N_h of Tables 2 and 3 contains less elements than the sample that is included in the tables of Appendix A.

Tables C1 and C2 contain the results of the test, indicating in each case if heteroscedasticity is detected.

Table C1. White Test for Heteroscedasticity (95.0%) original data: 2005–2014.

h	Index	$\alpha_{h,0}$	$\alpha_{h,1}$	$\alpha_{h,2}$	R^2 (%)	N	w	Heteroscedasticity
1	INDU	+0.2729	+0.4101	−0.0037	15.51	26	4.0326	No
2	UKX	−4.055	+2.1964	−0.0033	20.4	74	15.096	Yes
3	TPXC30	−3.9504	+0.9606	−0.0084	22.44	15	3.366	No
4	HSFML25	−60.302	+9.1054	−0.0253	98.01	6	5.8806	No
5	SPTSX60	−11.604	+2.5082	−0.0004	18.32	42	7.6944	Yes
6	DAX	+18.299	−0.6352	+0.0172	49.21	24	11.8104	Yes
7	SENSEX	−42.591	+4.4511	−0.0157	43.45	14	6.083	Yes
8	SMI	+3.6203	+0.6082	+0.0012	57.71	16	9.2336	Yes
9	ASX50	−37.932	+2.6843	−0.0054	24.68	30	7.404	Yes
10	KOSPI50	−5.7558	+1.63	−0.0055	11.83	30	3.549	No

Table C2. White Test for Heteroscedasticity (95.0%) original data: 1995–2004.

h	Index	$\alpha_{h,0}$	$\alpha_{h,1}$	$\alpha_{h,2}$	R^2 (%)	N	w	Heteroscedasticity
1	INDU	+8.0553	+2.7225	−0.0284	5.45	25	1.3625	No
2	UKX	+6.8875	+3.3998	−0.0303	4.74	46	2.1804	No
3	TPXC30	+67.688	−0.5307	+0.0011	44.1	4	1.764	No
4	HSFML25	-	-	-	-	-	-	-
5	SPTSX60	−39.551	+3.9221	−0.0126	33.43	27	9.0261	Yes
6	DAX	−4.9956	+2.5425	−0.0325	6.03	16	0.9648	No
7	SENSEX	-	-	-	-	-	-	-
8	SMI	+115.67	−15.478	+0.4786	84.2	8	6.736	Yes
9	ASX50	+147.97	−0.5611	+0.0005	4.46	15	0.669	No
10	KOSPI50	-	-	-	-	-	-	-

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