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Systematic Review

Can Mathematical Modelling Be Taught and Learned in Primary Mathematics Classrooms: A Systematic Review of Empirical Studies

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Abstract: STEM education has been promoted in schools worldwide to cultivate students' 21st-century skills. Mathematical modelling is a valuable method for developing STEM education. However, in this respect, more attention is given to secondary level or above compared with kindergarten or primary level. Teaching mathematics at the primary level is closely related to authentic problems, which is a crucial characteristic of mathematical modelling activities. After screening 239 publications from various databases, we reviewed 10 empirical studies on mathematical modelling at the primary level. In this systematic review, we analysed the following three aspects: (1) the use of professional development intervention methods/strategies to enhance the intervention effects and the competencies of primary teachers to utilize mathematical modelling; (2) the effects of mathematical modelling on primary students and methods of improving their mathematical modelling skills; and (3) methods used to assess the modelling skills of primary school teachers and students. The results indicate that professional development interventions can enhance the teaching quality of mathematical modelling. The components of the interventions should include an introduction to the pedagogy of mathematical modelling, clarifying the role of the teacher and the student in mathematical modelling activities. Through mathematical modelling, students can generate mathematical ideas, explore mathematical theorems independently, develop critical thinking, and improve their metacognitive and communicative skills. The competency of mathematical modelling is often determined using formative assessments of teachers and students. Because limitations still exist in conducting primary-level modelling activities, schools should utilise more standardised assessment methods, provide universal teacher training, and grant more opportunities for primary school students to participate in mathematical modelling activities. The lack of research on cross-cultural contexts should draw the attention of future research.

Keywords: mathematical modelling; primary mathematics education; mathematical modelling competency; systematic review; STEM education

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Citation: Wei, Y.; Zhang, Q.; Guo, J. Can Mathematical Modelling Be Taught and Learned in Primary Mathematics Classrooms: A Systematic Review of Empirical Studies. *Educ. Sci.* **2022**, *12*, 923. https://doi.org/10.3390/educsci12120923

Academic Editors: Ana Barbosa, Isabel Vale and Susana Carreira

Received: 27 October 2022
Accepted: 12 December 2022
Published: 14 December 2022

1. Introduction

In recent years, science, technology, engineering, and mathematics (STEM) education has grown internationally [1]. Merging different subject knowledge and skills, STEM in education goes beyond school subjects. It fosters diverse thinking and creativity, can improve students' critical thinking and problem-solving abilities, and prepares them to compete in their preferred work positions [2]. A necessity has emerged for teachers to educate their students in STEM-related subjects to meet the future needs of individual lives and society. Therefore, how to help students improve their STEM skills is one of the most important topics of interest to educators [3]. In the literature, the most common instructional strategies for conducting STEM education are inquiry-based, project-based, problem-based, and design-based learning [4,5]. All these pedagogical approaches focus on authentic problems in real-world contexts. However, one of the many challenges that educators face is how to teach students complex solutions to unusual problems in the context of STEM education. In recent years, models and modelling have been proposed as a means to increase relevance and authenticity in STEM disciplines, which is a fundamental aspect of STEM pedagogy [6,7]. Compared with other STEM-related subjects, mathematics is taught across all levels in school curricula. Hence, mathematical modelling can be used as an effective pedagogical tool for developing students' STEM skills in primary and secondary education [8,9].

Lesh and Doerr (2003) described mathematical modelling as a process of producing sharable, modifiable, and reusable conceptual tools for describing, predicting, and controlling real-life situations [10]. The characteristics of mathematical modelling are closely connected to all STEM-related applications. Hence, Kertil and Gurel (2016) believed that "all STEM activities are not modelling activities, but students may experience the mathematical modelling process while working on many of them" [9] (p. 46). In general, mathematical modelling is the process of using mathematical methods to analyse real-world situations. During the process, a real-life problem is formulated mathematically, and it is solved with the help of mathematical models, and the solution to the problem is interpreted and evaluated in the real world [11].

A mathematical modelling research base has been established in secondary STEM education [12,13], and mathematical modelling competency (MMC) is a key component of STEM competency [14]. However, traditionally, mathematical modelling is often introduced in schools at the secondary level or above [15], and little attention has been paid to primary classrooms. A common (mis)understanding is that mathematical modelling requires a substantial amount of abstract mathematical knowledge that is not available to primary students. However, the contextual characteristics of mathematical modelling activities are closely related to realistic problems, which are the main situations utilised in primary mathematics. Realistic problems at the primary level are often very relevant to everyday life [15,16]. In English's study (2004), she used a modelling activity about the selection of the Australian swimming team for the 2004 Olympics [15]. This example was in line with a popular topic in Australia at the time and was easy for primary school students to understand and engage their interest [15]. Researchers suggest that it is accessible to younger students, who have "the foundational competencies on which modelling can be developed" [15,16], [17] (pp. 289–296), [18]. Primary students can learn to model, generalise, and justify such scientific and mathematical reasoning; however, these practices are not used in school curricula until high school [19]. For example, in the "Butter Beans" problem, children were asked to use two tables of data recording the weight of butter beans after 6, 8, and 10 weeks of growth under two conditions (sunlight and shade) [19]. This example is not a difficult application problem to understand, nor does it have much knowledge of higher-level maths. Students can easily participate in this modelling task. Although it is difficult for them to identify and explain the pattern, they can still finish it with group discussion and teacher guidance [19].

Therefore, to clarify the possible effects of mathematical modelling on primary teachers and students, it is necessary to pay attention to the current research results of mathematical modelling and provide more empirical evidence about how to implement modelling at the primary school level. However, mathematics education is well-defined across primary and secondary schools worldwide, and not all STEM-related subjects are included in school curricula. For example, science, technology, and engineering design education are usually offered as electives in most secondary school curricula, which is not common in those primary schools [12]. Thus, this systematic review examines studies on mathematical modelling in primary mathematics education. Specifically, this review focuses on three research questions:

(1) What methods/strategies of professional development interventions are used to enhance primary teachers' mathematical modelling competencies, and what are the effects?

(2) What are the effects of mathematical modelling on primary students, and how can their mathematical modelling skills be improved?
(3) How can the modelling skills of primary school teachers and students be assessed?

2. Literature Review

2.1. Mathematical Modelling Competency

Mathematical modelling is an activity described as transforming practical problems into mathematical form. Mathematical modelling competency is defined as a person's insightful readiness to carry out all parts of a mathematical modelling process in a given situation [20]. Some scholars and curricula have provided indicators for success in mathematical modelling. According to Maaβ's framework, six indicators are used to evaluate competency in mathematical modelling: (1) making assumptions, identifying important variables, and creating relations between variables; (2) mathematising quantities and their relations and selecting appropriate representations; (3) using mathematical knowledge to solve problems and utilise heuristics, such as rephrasing; (4) elaborating results and generalising solutions; (5) checking and reflecting on the solution; and (6) reporting [21]. In addition, in this framework, representation and communication are central to mathematical modelling [21].

However, in the syllabus of Common Core State Standards for Mathematics in the United States, modelling with mathematics is treated as a core process and proficiency, which indicates that, in order to model with mathematics, students from kindergarten to high school should be mathematically proficient [22]. The five criteria for measuring MMC proficiency are: (1) identifying and making assumptions for a purpose; (2) making approximations for calculating and estimating quantities; (3) simplifying complex situations and mathematical structures; (4) identifying, relating, and representing ideas and quantities of interest; and (5) interpreting results in context and reflecting on the solution.

In contrast, in early school years, students' mathematical modelling skills tend to be more related to their critical thinking skills and mathematical thinking. These skills enable them to distinguish between personal and task-related knowledge and help them know when and how to apply their respective knowledge in the problem-solving process, rather than emphasizing mathematical knowledge [19,23]. Some modelling problems possess knowledge generation, which means that children can acquire mathematical ideas at different levels [19]. For example, in the "Airplane problem" modelling activity, children can acquire informal notions of rate by considering how time and distance determine the winner of a paper aeroplane race [19]. Also, these modelling activities emphasize that children's mathematical descriptions, explanations, and students' social communication skills are improved through mathematical modelling activities [19,23].

2.2. Mathematical Modelling Activities

Model-Eliciting Activities (MEAs) were established by Lesh and Doerr, who emphasised the process of interpreting and reinterpreting a problem and the cyclic development of mathematical ideas to form an initial model, which must be refined by testing with children [10]. One important characteristic of MEAs is that the problem involves diversely complex statistical information that very young children can manage [10]. Different modelling activities based on the principles of MEAs have been further developed, such as data modelling, the modelling of cultural and community contexts, and STEM-based modelling [10]. Among them, data modelling and the modelling of cultural and community contexts are frequently used in mathematical modelling activities at the primary level [19,23].

2.2.1. Data Modelling

Aside from the principles of MEAs, data modelling also involves statistical information, which requires student engagement. The five steps of data modelling are: (1) posing a statistical problem; (2) designing investigations to solve the problem; (3) generating, selecting, and measuring attributes; (4) organising, structuring, and representing data; and (5) developing a model and drawing informal inferences [24]. Further elaboration was also made regarding the importance of early exposure to data modelling, since today's world has an enormous amount of diverse statistical information [24]. Data modelling benefits students by allowing them to draw inferences from generated models [16] and to make informal inferences, such as recognising uncertainty, identifying variation, and making predictions [25]. Hence, an early foundation in data modelling equips new generations with the ability to deal with statistical information more effectively.

2.2.2. Modelling with Cultural and Community Contexts

Modelling problems that include cultural and community contexts are attractive and motivating to students. Wickstrom and Yates (2021) pointed out that students are more engaged in modelling activities if they discover a connection between mathematics and the real world [26]. More concretely, the lived experiences of students enrich modelling activities by allowing them to form assumptions, make decisions, and apply mathematical operations, analyses, and refinements [24]. The modelling of cultural and community contexts requires critical thinking, which is different from data modelling.

Numerous studies have noted the importance of mathematical modelling competency, what mathematical modelling activities are, and the positive effects they can have on students at the secondary level. In contrast, primary school mathematical modelling activities have received little attention. We created this review to explore the impact of mathematical modelling on primary school teachers and students and to explore methods to measure primary school mathematical modelling skills, thus advancing mathematical modelling activities in the mathematics of primary school education.

3. Search Strategies and Data Analysis Methodology

3.1. Search Strategies

In this section, we explain how we conducted the literature search and present the final search results. A favourable systematic review presupposes that the raw data on which it is based have high quality and are comprehensive [27]. According to the AMSTAR guidelines, at least two databases must be searched in a systematic review [28]. We conducted the final literature search on 30th September 2022 with the following three databases: ERIC (via EBSCOhost), Web of Science (WOS), and Scopus [29]. In the initial search phase, based on the three research questions, we established different search terms for each database to ensure that we obtained accurate and non-missing data. The details of the search terms are presented in Table 1.

After combining the data from the initial search of the three databases, we removed duplicates and obtained 193 documents. Assessing data quality is an indispensable step in literature searches [30]. To ensure the quality of the collected data, we reviewed the 2022 SJR and the subdivisions of the journals in which the 193 publications were published, and we excluded those that were included in journals with lower impact rates (Q3/Q4/NA in SJR). Then, we performed a preliminary screening of 101 higher-quality articles and determined whether they discussed mathematics education or had mathematical-modelling-related titles and abstracts. A total of 81 of these documents contained irrelevant content. For further screening, we checked the eligibility of the full texts of the remaining 30 documents, and we obtained 18 complete documents. We removed articles whose subjects did not include elementary/primary school teachers/students and were not empirical studies and articles whose research content did not answer our research questions, resulting in 10 documents. Details of the selection flow are presented in Figure 1.

Table 1. Initial search terms.

Database	Search Terms	Additional Limiters
ERIC	(TI(modeling) OR TI(model-eliciting) OR TI(modelling)) AND (TI(education) OR AB(education) OR SU(education)) AND (TI(primary) OR TI(elementary) OR AB(primary) OR AB(elementary) OR SU(primary) OR SU(elementary)) AND (TI(math*) OR AB(math*) OR SU(math*)) The asterisk (*) represents any group of characters, including no character.	Full Text Peer Reviewed Language: English Date Published: -20220931 Academic Journals
WOS	(TI = (“modeling”) OR TI = (“model-eliciting”) OR TI = (“modelling”)) AND (TI = (education) OR AB = (education) OR SU = (education)) AND (TI = (primary) OR TI = (elementary) OR AB = (primary) OR AB = (elementary) OR SU = (primary) OR SU = (elementary)) AND (TI = (math*) OR AB = (math*) OR SU = (math*)) The asterisk (*) represents any group of characters, including no character.	English Article
Scopus	(TITLE(“modeling”) OR TITLE(“model-eliciting”) OR TITLE(“modelling”)) AND (TITLE-ABS-KEY(education)) AND ((TITLE-ABS-KEY(primary)) OR (TITLE-ABS-KEY(elementary))) AND (TITLE-ABS-KEY(math*))	English Publication stage: Final Journal article

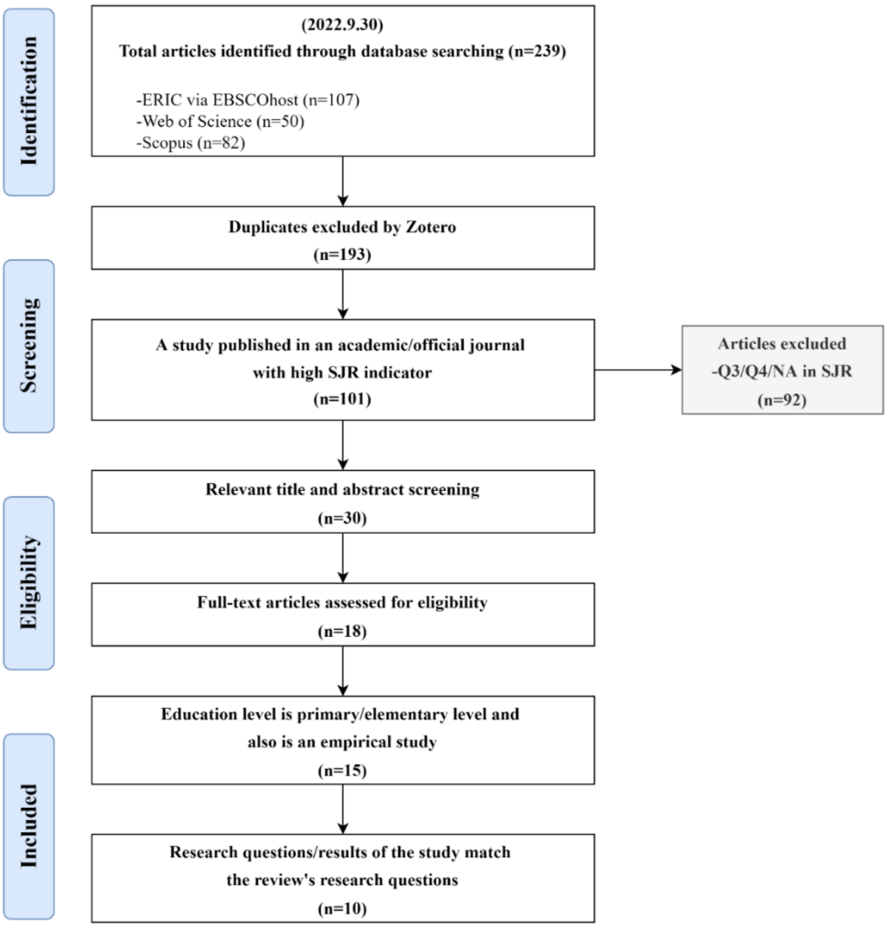

Figure 1. Selection flow of reviewed articles.

3.2. Data Analysis

After the screening, we obtained ten papers that could be used for this systematic review (Details can refer to Appendix A). We first performed a statistical analysis of the data to review the current research on mathematical modelling in elementary/primary-level education. Then, we answered the three research questions.

4. Results

4.1. Statistical Analysis: Participants, Methodology, Experimental Period, and Background

Of the ten studies, three focused only on teachers, two focused only on students, and the remaining five had students as the primary participants and teachers as secondary participants. The current research on mathematical modelling in primary schools is still more focused on the students.

All ten studies used qualitative methods, and three used quantitative methods, as shown in Figure 2. Seven of the ten intervention experiments lasted one semester or more, as shown in Figure 3. Most of the relevant empirical studies required a longer intervention time.

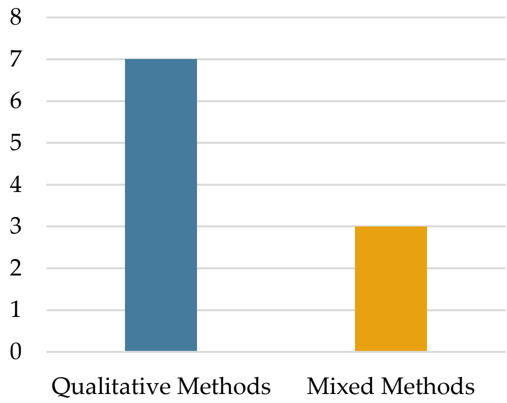

Figure 2. Two methods were used in 10 studies.

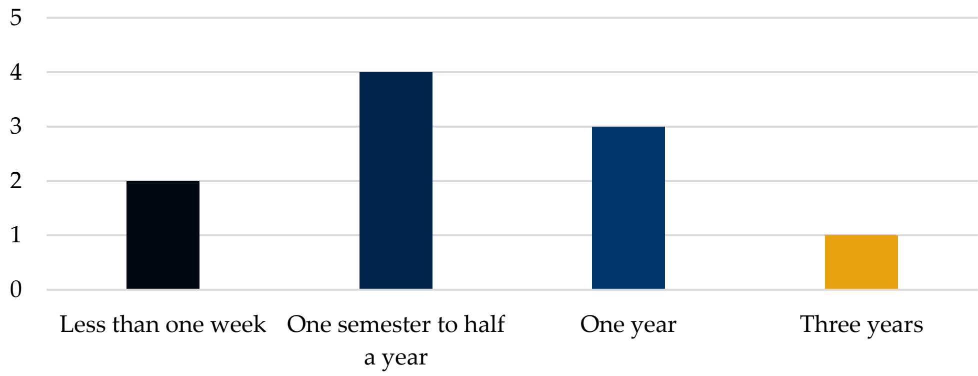

Figure 3. Experimental periods of 10 studies.

Because Professor Lyn D. English is an active scholar in this field, five of the ten studies had an Australian research background, as shown in Figure 4. Because no related research in South America existed, the number of selected articles from South America was zero.

4.2. RQ1: What Methods/Strategies of Professional Development Interventions Are Used to Enhance Primary Teachers' Mathematical Modelling Competencies, and What Are Their Effects?

Three studies [31–33] outlined effective strategies for enhancing teachers' modelling competencies in interventions at the primary level.

An introduction to the pedagogy of mathematical modelling should be a part of the interventions. During the professional development intervention, the intervenor should first introduce the definition or nature of mathematical modelling [31–33]. Then, the diverse and concrete pedagogies of mathematical modelling, such as MEAs and the modelling of cultural and community contexts, should be recommended to the teachers [31]. Because some teachers lack confidence in teaching mathematical modelling and feel discomfort with the use of modelling lessons, resulting in feelings of discouragement regarding the use of mathematical modelling with students, a professional reading related to mathematical modelling should be presented [32]. Not including the pedagogy theory, teachers should also be clear about the role of the teacher and the student in mathematical modelling. After attending the interventions, teachers should realise that a student-centred approach is key to mathematical modelling, whereas a teacher-centred approach may interfere with students' development of critical reasoning [31,32]. Moreover, the teacher should be a facilitator during learning, should focus on observations that assist their students, and should be more able to reflect on their teaching quality [32]. Teachers are encouraged to use mathematical modelling to explore more related knowledge. In a semester-long intervention, the post-test performance of preservice teachers was significantly better than their pre-test performance [33]. Moreover, group work can substantially benefit teachers [31–33].

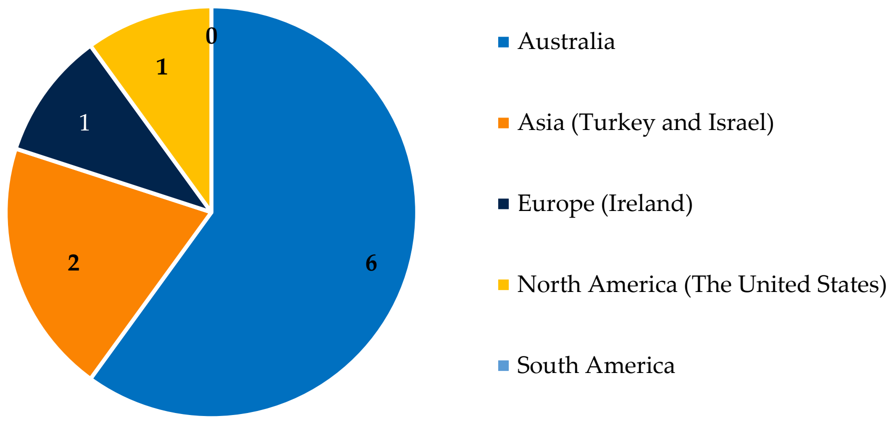

Figure 4. Distribution of research backgrounds.

4.3. RQ2: What Are the Effects of Mathematical Modelling on Primary Students, and Can Their Mathematical Modelling Skills Be Improved?

Eight studies have presented the diverse effects of mathematical modelling in primary contexts for the development of student competency [19,23,32,34–38]. The effects are separated into four aspects. (1) Students can identify and analyse variables [34,35]. Some participants even consider the connection between data contexts and problem contexts, such as "animal welfare" in designing a zoo, to assess the suitability and fit of the model [37]. They apply their knowledge to explain the data, which also helps them to identify the nature of the variables [19]. (2) Modelling problems allow students to explore various mathematical concepts, such as quantitative relationships, changes, identifications, descriptions, and comparisons among data [19]. In addition, by solving modelling problems, students independently develop their mathematical thinking and their metacognitive and critical thinking skills [19,23,34]. (3) Students can collect and record their own data to support their solutions [19]. By continuously expressing ideas, selecting and trialling factors, and creating, testing, and revising models [23], students can create and refine a model by themselves. For deeper learning outcomes, some participants are able to generalise and transfer a model to a new activity [38]. These facts show that mathematical modelling may improve children's independent thinking skills. (4) Modelling problems allow different forms of representation for the development of students' mathematical communication skills, and they allow emerging models to be scrutinised and tested by group members [23]. Modelling problems provide a platform for students to work in a team, which significantly develops their planning, monitoring, constructing, and communication skills [23]. Lower-achieving students are also motivated to engage in challenging tasks and to improve their performance skills [32].

4.4. RQ3: How Can the Modelling Skills of Primary School Teachers and Students Be Assessed?

Participant discussions, written reflections, and samples from the participants' work are evidence that can be used to assess the mathematical modelling competency of teachers [31,33].

One method to assess students' mathematical modelling competency is to analyse their written outcomes, such as teacher assessments. Formative assessments such as notes, workbooks, and reports show how students listen and reflect on their classmates' models during class representation [23]. By analysing students' verbal documentation, valuable student thinking processes can be used to evolve mathematical modelling and concepts. Meta-representation competence has also emerged, especially in the use of inscriptions, structuring and representing data, detecting meaningless information, acting to eliminate unnecessary features, and conserving ideas [35].

5. Discussion

The statistical analysis in Section 4.1 shows that primary school mathematical modelling research is of relatively little concern to teachers. In fact, improving teachers' mathematical modelling skills can help teachers understand a variety of mathematical modelling pedagogies, eliminate discomfort with mathematical modelling, enhance the quality of their teaching, and better teach mathematical-modelling-related knowledge to students [31–33]. Teaching mathematical modelling can be challenging [39]. Many mathematics teachers did not take any systematic courses in mathematical modelling when they were students, and many prospective teachers even have a wrong understanding of the definition of mathematical modelling [40]. Future research in this direction should involve more experiments with interventions for preservice teachers. Although we have emphasised the importance of teachers in mathematical modelling education, importance also lies in noting that excellent mathematical modelling instruction should still be student-centred; otherwise, it can interfere with the growth of students' critical thinking skills [32]. The impact of modelling activities on students not only exists at the mathematical level, such as exploring mathematical definitions, but it also exists in developing students' mathematical thinking, independent thinking, critical thinking, and communication skills [19,23,32,34–38]. In these empirical studies, the methods that were used were relatively fixed, as shown in Figure 2. Most of their experimental procedures were qualitative, including conducting training sessions and teaching reflection discussions, as well as students providing suitable modelling topics to solve. Quantitative analysis research methods are more often perceived as secondary tools. The intervention experiments were mostly one semester or longer, as shown in Figure 3.

Based on the data in Figure 4, many studies have been conducted in the Australian context. Professor Lyn D. English has made many contributions to this field, and most of the intervention experiments she designed lasted three years and had documented literature published at each stage. Her research results and ideas have a significant influence on other researchers. In contrast, very little research in Asian and American contexts exists. The Compulsory Mathematics Curriculum Standards (2022 version) of mainland China established the performance requirement at the primary level, referring to mathematical modelling as "model awareness", which refers to the initial sense of the universality of mathematical models [41] (p. 10). This new concept demonstrates that mathematical modelling at the primary school level has received much attention. However, many gaps in the current research exist in this area [42,43].

Regarding research question 1, several professional development interventions have demonstrated enhancements in teachers' modelling skills [31–33]. The components of interventions should include an introduction to mathematical modelling practices [33] and the pedagogy of mathematical modelling [31–33], clarifying the role of teacher and the student in mathematical modelling [31,32]. To begin, introducing teachers to the nature and pedagogy of mathematical modelling is necessary. Teachers lack pedagogical knowledge about mathematical modelling. Thus, they are less confident when providing feedback or assisting students with learning obstacles [44,45]. Once teachers are equipped with the rich pedagogy of mathematical modelling, they feel more confident to teach. The attitudes of teachers shift from passive to active in discussions during an intervention [32]. Second, a student-centred approach should be the key to learning mathematical modelling. Because schools often rely on teacher-centred approaches, time is required to understand the role and the application of student-centred approaches to mathematical modelling. Munter [46] indicated that most teachers' perspectives of their roles are guiding mathematics in meaningful ways, and four scales were put on a selection list: (1) the teacher is a motivator; (2) the teacher is a monitor; (3) the teacher is a facilitator; and (4) the teacher guides mathematics in meaningful ways.

For research question 2, through mathematical modelling, student competencies are fulfilled as a result of identifying and analysing variables [19,34,35,37], mathematising problems [24,35,41], generalising models [19,24,36,38], representing, and communicating [23,32,34,35]. By identifying and analysing variables, young students can identify and consider the variables' attributes [34,35]. To an extent, they may consider unseen attributes such as "animal welfare" and personal knowledge to explain the data [19,37]. This phenomenon was named "lifting away from the plane activity" [18] (p. 377). When young students face mathematical information, the information can often be mathematised by applying prerequisite mathematics knowledge, such as addition, subtraction, and comparisons between numbers [19,36,38,39]. Young students can successfully find solutions to the given problem and refine their solutions, namely with generalised models [47]. For instance, primary students can collect and analyse data, express opinions, and create, test, and refine these models [19,23,38]. Through multiple reflections and discussions with teammates, students may notice problems with the model, such as one variable being insufficient to fit the model. Hence, they must modify it [36]. As mentioned previously, Maaβ's competency framework (2006) emphasises representation and communication [21]. In several studies [23,32,34,35], pupils have demonstrated outstanding representation and communication during mathematical modelling. For representation, they are talented at presenting data with pictographs, tables, bar charts, drawings, crosses, and ticks, etc. [34,35]. For communication, young students are arranged in a group. Therefore, they may share different opinions on mathematical modelling, construction, and design, and they can then complete the task together [23]. Even low-achieving students can be motivated to engage in the task [32].

For research question 3, formative assessments such as notes, workbooks, and reports are advised for assessing the competency of students and teachers [23,33]. Teachers are also suggested to focus on the use of inscriptions, structuring and representing data, detecting meaningless information, acting to eliminate unnecessary features, and conserving ideas in which the meta-representation competence emerges [18,35]. However, such research lacks concrete indicators to measure teachers' and students' mathematical modelling competency. Anhalt et al. proposed a concept of mathematical modelling thinking (MMT), a foundation for success in mathematical modelling that includes six practices: (1) recognising assumptions; (2) approximating and estimating to reason quantitatively; (3) prioritising factors that affect the solutions as a means of simplifying the problem; (4) using multiple representations to express the mathematical idea; (5) reflecting on the solution, its meaning, and its reasonableness within the original context; and (6) reconsidering, revising, and refining the solution [48] (pp. 307–330). Therefore, mathematical modelling competency may refer to those components as references in order to determine it.

6. Conclusions

In teaching mathematics at school, mathematical modelling is not new, and modelling ideas are also emphasised in various subjects (e.g., engineering, physics, and medicine). To promote concurrent STEM education in schools, we argue that mathematical modelling can be a bridge to nurture students' STEM competence early, i.e., at the primary level. However, mathematical modelling activities are essential in STEM teaching and learning. Because modelling activities are considered a means to increase the relevance of STEM disciplines, even though STEM education includes multiple disciplines barely connected in content and pedagogy, modelling activities can still bring them together [8,49]. In this systemic review, we screened ten empirical studies from three databases to examine the effects of mathematical modelling on primary mathematics classrooms. We acknowledge that there were limitations that occurred in this review, including issues regarding time and human resources. Only online resources and English versions were adopted to reduce the time cost. Thus, we ignored some articles that do not have an online version and some articles that have an online version but are not written in English are ignored. Because modelling activities can be conducted in different cultural contexts and subjects, a broader and deeper exploration should be considered in the future.

In conclusion, mathematical modelling has advantages in developing primary school students' mathematical thinking, independence in learning mathematics, and critical thinking and communication skills. Teachers can design activities to improve their students' mathematical modelling skills by identifying and analysing variables in realistic problems, solving mathematical problems, generalising models for the solved problems, using suitable representations, and communicating the problems mathematically.

Professional development interventions can enhance teachers' mathematical modelling competency (MMC). These interventions should provide teachers with basic knowledge about mathematical modelling, such as the pedagogy of mathematical modelling and the role of teachers and students in mathematical modelling practice. Intervention experiments with teachers can improve their mathematical modelling skills, eliminate possible negative emotions that they have regarding mathematical modelling, and improve the quality of their classroom teaching. These interventions can also make primary school teachers aware of the importance of mathematical modelling in regard to the goals of primary mathematics education. To develop more effective strategies for teaching mathematical modelling in teacher education programmes, more experimental research on interventions with teachers should be conducted in the future.

Mathematical modelling includes the modelling process and its outcomes. Although primary school students can receive and benefit from teaching related to mathematical modelling [41] (p. 10), how to assess teachers' and students' MMC is still an emerging issue in this field of research. Our findings suggest that formative assessments may be a solid approach. If we want to apply the assessment method to a large sample, future research should focus on specific assessment indicators of mathematical modelling for teachers and students, and the criteria of assessment can be quantitative and standardised.

Because mathematical modelling plays an important role in STEM integration and training, the idea of mathematical modelling cycles should be used in STEM education at the primary level [8,31]. Our findings also suggest that developing mathematical modelling at the primary level is necessary and valuable. Moreover, most mathematical modelling topics for primary students are exhibited in mathematical stories [19]. Researchers and educators should consider whether storytelling in mathematics learning plays an important role in STEM or STEAM (Science, Technology, Engineering, Arts, and Mathematics) education when designing mathematical modelling activities. However, cultural variability in the results exists due to limitations in the length of the experiments and the number of participants, including the lack of generalisability of the modelling topics chosen for the interventions [19,23,34–36]. Researchers should consider the magnitude of the impacts of cultural differences on students' MMCs. A substantial gap in cross-cultural research still exists in mathematical modelling at the primary level, which should be studied further.

Funding: This research was funded by the Education University of Hong Kong Seed Funding Grant, grant number R4248; partially funded by the Hong Kong Research Grants Council, Hong Kong SAR, China, grant number ECS-28609121.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. The Selected Papers for Analysis.

Study (Author, Year)	Participants (Country)	Study Period	Methodology	Data	Results
Baker & Galanti (2017)	8 mathematics coaches from each of the 8 division schools. (United States)	A 4-day summer institute on STEM integration and one coaching meeting a month for one year	Qualitative analysis: In-depth interviews Focus groups	Participant reflections (written documents, recording); Conversations	8 participants expressed a willingness to use MEAs as a tool for STEM integration in their future curriculum design in grades K-6.
English (2006)	A class of 5th graders and their teachers. (Australia)	A three-year, longitudinal teaching experiment	Qualitative analysis: Case study In-depth interviews Focus groups	Videotapes and audiotapes; Responses to the problem activity; Worksheets and final reports; The class presentations; The children's written critiques; Field notes of classroom observations	Elementary school students of all achievement levels contribute and benefit from mathematical modeling. Children will build their own mathematical ideas, hone their communication and teamwork abilities.
English (2010)	Three classes of first-grade children (25 to 26 students each class) and their teachers. (Australia)	The first year of a three-year, longitudinal teaching experiment; Regular half-day teacher meetings	Teaching experiment involving multilevel collaboration; Case study; In-depth interviews; Focus groups	Videotapes and audiotapes; The artefacts of all student groups; Whole-class presentations and discussions	Children's abilities to focus their attention on the qualities of items and create a broad range of models in organizing, structuring, and representing their data. Children's development of meta-representational knowledge.
English (2012)	Three classes of first-grade children (25 to 26 students each class) and their teachers. (Australia)	The first year of a three-year, longitudinal teaching experiment; Regular half-day professional development meetings	Teaching experiment involving multilevel collaboration; Qualitative analysis: Case study In-depth interviews Focus groups	Videotapes, audiotapes, and digital photographs; The artefacts of all student groups; Whole-class discussions and group presentations	Results included the various ways in which children represented and re-represented collected data, and the meta-representational competence they displayed in doing so. The children who dealt with informal inference were also reported.

Table A1. *Cont.*

Study (Author, Year)	Participants (Country)	Study Period	Methodology	Data	Results
English & Watson (2018)	A total of 89 students from 6th grade: 45 girls and 44 boys. (Australia)	At the end of a three-year (last half-year) longitudinal study	Intervention experiments; Design-based research; Content analysis; Statistical analysis; Case study; Focus groups	Documented in the students' workbooks; Students' annotations on the table of data; Transcripts of all whole class discussions and presentations	Modeling with data could develop primary school students' statistical literacy. This modeling framework comprises four components. Currently, research received limited attention in the primary grades about modeling and inferential reasoning processes.
English & Watters (2005)	All four 3rd-grade classes and their teachers. The principal and assistant principal attended some debriefing meetings with the teachers. (Australia)	Activities were organized weekly for a semester. More than 4 to five 40-min lessons for each problem. The teachers had several workshops on introduction, the pre- and post-meetings.	Data analysis; Ethnomethodological; Case study; In-depth interviews; Focus groups	Videotapes and audiotapes; Classroom field notes; Children's artifacts (including their written and oral reports); Children's responses to their peers' feedback in the oral reports.	The modeling problems encouraged young children to develop important mathematical ideas, the metacognitive and critical thinking skills. The study has also highlighted the contributions of these modeling activities to young children's development of mathematical description, explanation, justification, and argumentation.
Hodgson & Wilkie (2021)	18 in-service teachers from four primary schools (Australia)	A 5-month intervention program. Four stages of intervention (2 hours each). Another 60-min co-teaching planning meeting was appointed.	Multiple case studies; Interpretive accounts-of-practice methodological approach; In-depth interviews; Intervention experiment	Questionnaires; Audio recordings; Written documents; Research journal.	This study impelled teachers to reflect on and set goals for improving their practice. Suggestions for pre- and post-lesson protocols for productive discussions are proposed, and designing school-based professional learning processes using modeling.
Leavy & Hourigan (2018)	24 participants from 2nd and 3rd grade, 2 teacher educators, and 25 pre-service teachers (Ireland)	A 4-day data modeling investigation including 4 60-min lessons.	Three-tiered teaching experiment; In-depth interviews	Digital dialog; Video recording; PST's notes, observations, and transcripts of the group conversation.	This study extended previous research as it examines young children's modelling potential. The extension of the study over a longer duration will reap interesting findings.

Table A1. *Cont.*

Study (Author, Year)	Participants (Country)	Study Period	Methodology	Data	Results
Shahbari & Peled (2015)	Two sixth-grade classes, 65 students from 6th grade in a primary school (Israel)	Three lessons	Qualitative analysis: Case study In-depth interviews Focus groups	Students' worksheets, notes, and final models; Videotapes of student groups' discussion.	Results showed that students utilized their knowledge of fractions in constructing mathematical models. Students had some difficulties in their knowledge of fractions.
Yasa & Karatas (2018)	24 preservice elementary math teachers studying their second year in the university (Turkey)	In the spring semester of one academic year. The duration of the lessons was 3 lesson hours each week.	Mixed method: Single group pretest-posttest design; Intervention experiments;	A pre-mathematical modelling test; A post-mathematical modelling test	The result showed that the instruction based on mathematical modelling improved pre-service teachers' mathematical modelling performance.

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