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Promoting Interdisciplinary Research Collaboration among Mathematics and Special Education Researchers

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Abstract: This manuscript provides a theoretical framing of a collaborative research design effort among mathematics education and special education researchers. To gain insight into the current state of research on mathematics learning, we drew on how researchers in mathematics education and special education have defined and operationalized the term 'mathematical concept' related to the learning of fractions. Using this information, we designed a future study that focuses on and connects prior research in mathematics and special education. We conclude by discussing the implications of such collaborative research efforts.

Keywords: special education; mathematics education; student learning

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1. Introduction

As researchers in mathematics education and special education, a critical goal in our fields is to improve the teaching and learning of mathematics by providing insights into student learning. Knowledge of how students learn mathematical concepts is an essential component of learning mathematics [1,2] (Throughout this manuscript, we use the definition of a concept as "an abstract or generic idea generalized through particular instances" rather than the use as "something conceived in the mind" (Merriam-Webster, n.d.)). The authors, as interdisciplinary researchers, desire to work effectively to build knowledge that improves learning, particularly for students with disabilities or who exhibit difficulties in learning mathematics.

Interdisciplinary work is widely considered a source of innovation, a means of addressing critical research questions, and an approach that can yield a broader understanding of topics by integrating diverse perspectives [3]. Yet interdisciplinary research is also known for its challenges, as disciplines' theories, methods, and language often conflict [4]. Thus, an effective interdisciplinary research collaboration requires an intentional work atmosphere—one in which collaborators establish key components that inform and guide the collaborative work for addressing a shared mission, including (a) common ground concerning a research area of interest, (b) cocreation of research aims and approaches through participatory processes, and (c) shared language [5–7]. Each component has its own unique challenges, including the need to value different research designs and theoretical perspectives [8]. Developing a shared language is critical, as clear communication is exacerbated by the fact that people, in general, lack shared meaning for many common terms [9].

In addition, an important goal of the research community is to provide insights into student knowledge of mathematical concepts that allow us to support the application of research findings in K–12 classrooms. Indeed, many researcher insights serve as foundational studies that researchers and practitioners draw on each day. These include research on the importance of focusing on student conceptual explanations [10] and the various types of student mathematical justifications [11], the student schemas that support calculus success [12] and those that children draw on when solving word problems [13], the impact of instructional interventions on student learning with ratios [14] and nonroutine mathematical situations [15], and the challenges that students with learning disabilities face in the learning of mathematics [16] as well as the specific challenges that all students face when learning mathematical topics, such as fractions [17]. These research findings are useful because they provide frameworks for analyzing and building student mathematical understanding. Such clarity and focus in these frameworks are powerful for researchers and classroom teachers as researchers allow classroom teachers to recognize students' initial conceptions, suggest instructional strategies and mathematical tasks that can improve student learning, and increase understanding of common mathematical difficulties. However, as we will explore and discuss in this manuscript, such clarity is currently lacking in how the research community operationalizes mathematical concepts.

For mathematics education and special education researchers, effective collaboration may involve drawing on various theoretical approaches and methodological designs. In this manuscript, we focus on "mathematical concept" as a key construct that mathematics education researchers and special education researchers use to examine student thinking. We focus on this construct as a way to demonstrate the different ways in which this construct is addressed by research from the different disciplines and, importantly, as a way to demonstrate how we can work together for shared meaning. We encourage others to continue to take what we discuss and identify ways for collaborative research efforts in our disciplines.

We attempt to stand at the intersection of mathematics education and special education research to bring together these two research groups. To achieve this, we begin by sharing aspects of effective interdisciplinary research collaborations. We then discuss the strengths and challenges in the current operational definitions. Next, we describe how we plan to operationalize the concepts related to initial fraction meaning in a future study. At the conclusion of this manuscript, we discuss implications for researchers as they design research studies on mathematical learning and discuss recommendations for effective interdisciplinary work for mathematics and special education researchers.

1.1. Shared Mission: The Importance of Mathematical Concepts for Student Learning

Special education and mathematics education researchers agree that learning mathematical concepts is critical for all students, including students with disabilities–and this serves as our common ground for our interdisciplinary work as researchers. Various entities [18] have emphasized that educators establish an environment that provides opportunities for various learners to develop an understanding of concepts [19]. However, in the United States, learning mathematics has focused on mathematical procedures [20], and the conceptual underpinning of the procedures has received less attention in the classroom [21]. Researchers [1] view mathematical concepts and mathematical procedures as intertwined, and, therefore, they cannot be learned in isolation. Providing opportunities to learn the ideas underlying mathematical procedures is the key to better performance and retention of mathematical ideas. But this requires a common understanding of mathematical concepts and how these can be assessed and supported through instruction that enables educators to provide opportunities that align with students' instructional needs.

Recognizing that mathematics education and special education researchers often use differing theoretical approaches that influence the design of the research, we established at the onset of our collaboration that we would co-create our research aims and approaches as a research team [5]. However, the goal of using a shared language to guide our interdisciplinary work focused on mathematical concepts presented a larger challenge, as the meaning of a mathematical concept "does not appear to have a shared understanding in the research literature" [22] (p. 2). We have thus established that fruitful first endeavors for our collaboration were to (a) explore how "concept" is conceived and assessed across and within the special education and mathematics education research literature and (b) use the information to cocreate a shared definition of a "mathematical concept" to anchor our work.

1.2. Theoretical Framing of Mathematical Concepts

Many researchers in mathematics education and special education draw on the *Strands of Mathematical Proficiency* from the National Research Council's framework as a way to emphasize the importance of mathematical concepts [1,23]. This framework supports the development of mathematical concepts along with other goals for mathematical learning (e.g., learning of procedures and strategic competence) and was reinforced by the National Mathematics Advisory Panel [2]. As discussed by the NRC [1], "conceptual understanding refers to an integrated and functional grasp of the mathematical ideas" (p. 141) for a given mathematical topic.

Simon [24] challenged researchers to further clarify their theoretical views of the mathematical concepts they study. The definition that we apply in our work, and the one that Simon includes as his evolving definition of a mathematical concept, consists of "mathematical (mental) objects and the relationships among those objects" (p. 121). For example, under Simon's definition, we could determine that 47 > 23, as 47 comes later in the counting sequence than 23. That is, drawing on the idea that for each natural number, the further a whole number is in the number sequence, the greater the magnitude of that number. From Simon's perspective, developing a deep mathematical understanding involves developing a normative understanding of these mathematical ideas, as learners often apply non-normative ideas as they attempt to develop meaning for a particular topic [24]. As researchers, we find Simon's framing particularly powerful. Stating the mathematical concept in specific terms allows us to be explicit about the overarching idea(s) that support specific mathematical procedures and relationships. We also recognize that researchers have various ways of framing the process of developing mathematical concepts (e.g., reflective abstraction) [25]. We see the process of developing mathematical concepts as beyond the current scope of this manuscript and restrict our focus to identifying specific mathematical concepts.

Simon [24] also stated that mathematics education researchers sometimes refer to the 'concept of fraction,' noting that this is a reference to a mathematical object and not to an essential relationship related to mathematical objects of fractions. In addition, Simon noted that instructional goals in which "students learn to solve a particular type of problem" (p. 128) need further clarification, as learning a particular procedure "offers no description of the intended student knowledge and therefore may be inadequate for focusing instruction, assessment, and research" (p. 129).

Our shared goal in mathematics education and special education is to support the notion that "both conceptual understanding and procedural fluency requires that the primary concepts underlying an area of mathematics be clear to the teacher or become clear during the process of teaching for mathematical proficiency" [23] (p. 233). Thus, the theoretical application of mathematical concepts is critical for developing a shared understanding of student mathematical understanding.

2. Overview of Concept Definitions from the Research Literature

As part of our work to understand the current research, we conducted an initial review of mathematics education and special education research journals to gain insights into the conceptualizations of "concept." Our specific goals were (a) to identify examples of how researchers have operationalized "concept" in mathematics education and special education publications and (b) to consider whether the identified examples suggest or do not suggest a shared understanding of "concept" between and within the fields. As this is a theoretical manuscript, it was beyond our scope to conduct a systematic review of all the existing literature. We did, however, identify top-ranked journals in both fields. For special education, the research journals included the *Journal of Learning Disabilities*, *Exceptional Children, Learning Disabilities Research and Practice, Remedial and Special Education*, *Learning Disability Quarterly*, *Learning and Individual Differences,* and *The Journal of Special Education*. The mathematics education research journals included *Educational Studies in Mathematics, Journal for Research in Mathematics Education*, *Mathematical Thinking and Learning*, and *ZDM—Mathematics Education*. Once the journals were identified, we used the following criteria to find articles: (a) published within the last 10 years (2014–2023), (b) involved students from K–12 grades, (c) conducted an empirical study that was either experimental or descriptive (quantitative, qualitative, or mixed methods), and (d) focused on fractions as a mathematical topic. To ensure that we had identified all possible research studies in the selected journals, two authors examined each journal. Our initial search resulted in approximately 245 articles. We then removed duplicates and articles that did not meet the criteria following a review of the abstract. Dividing the remaining articles among the team, we each read them to ensure they met the criteria. This resulted in a final set of 88 articles.

Using the final set of articles, we read a selection of articles from each journal and identified and recorded how "fractions" as a concept was defined and/or operationalized. As a team, we reviewed the different ways fraction as a concept was operationalized and/or defined and identified common patterns that were then grouped and, following discussion, categorized. We identified three categories for how concepts were defined in the published manuscripts on student fraction knowledge. These categories, which overlap and are not necessarily mutually exclusive, are (a) concepts stated/defined, (b) concepts as a topic, (c) and concepts characterized by what students can/cannot do. In the following sections, we discuss each category and provide examples from both the mathematics education and special education journals. It is important to note that the examples we share do not provide a thorough understanding of how authors operationalize the term "concept" in their full program of research, especially considering that authors' views may evolve over time.

2.1. Concepts Stated/Defined (Relationship among Mathematical Objects)

In a few manuscripts that we reviewed, specific fraction concepts were identified that connect to Simon's [24] definition of a mathematical concept as consisting "of mathematical (mental) objects and the relationships among those objects" (p. 121). For example, Jordan et al. [26] stated that their research involved the meaning of fraction, which

> "refers to understanding that the relation between the fraction's numerator and denominator determines its magnitude rather than either number alone; that fraction magnitudes increase with the numerator size and decrease with the denominator size; that the closer the numerator is to the denominator, the closer the fraction is to 1; that a fraction with a numerator larger than the denominator is always > 1 and vice versa; and that all fractions can be represented on the number line". (p. 627)

Jordan et al. refer to particular fraction concepts (e.g., fractions increase in value when the numerator increases). The reference to particular specific concepts leads to discussion about when these statements are true (e.g., what happens to all fractions when the numerator is one less than the denominator?).

Similarly, Simon [24], in his discussion of the importance of delineating mathematical concepts, provided the following fraction concept related to the meaning of a fraction:

> "Partitioning a unit into n equal parts creates parts one of which will iterate n times to make the whole. Iterating a small quantity n times produces a specific large quantity that is n times as large. So partitioning a unit into n equal parts creates parts of a particular size". (p. 129)

Simon described the meaning of a unit fraction (i.e., that a unit fraction can be created by partitioning the unit into n parts) and a nonunit fraction (i.e., iterating a unit fraction).

Both Simon and Jordan et al. explicitly state the specific mathematical ideas that serve as their foci for student fraction learning. For example, Jordan et al. note that comparing fractions involves coordinating the meaning of the numerator and the denominator. The larger the denominator, the smaller the partitions for a fractional amount (e.g., 3/5 is greater than 3/7, as fifths are larger parts than sevenths), and the numerator provides the number of parts designated by the denominator. Thus, we can compare 14/35 and 13/36 by recognizing that 14/35 has more parts (e.g., 35ths) than 13/36, and those parts are larger than 36ths; thus, 13/35 is greater. Such identification of specific mathematical ideas can support both researchers and teachers by parsing the details of particular ideas that students must apply to have a deep understanding of fractions. Note that the relationship between mathematical objects (in this case, the relationships between the symbolic representation of the denominator to quantities and the relationship between the meaning of the numerator and the denominator) are delineated in this definition of "concept."

2.2. Concepts as a Mathematical Topic (e.g., Reference to the Common Core)

Our second category included manuscripts that referred to the scope and sequence set by professional organizations (i.e., Common Core State Standards, National Council of Teachers of Mathematics Standards). For example, Bottge et al. [27] referred to CCSS-M when discussing how they define fraction concepts: "The Common Core State Standards for Mathematics [28] recommends that students develop understanding of fractions in Grade 3, equivalence in Grade 4, and fluency with adding and subtracting fractions by the end of Grade 5" (p. 424). The researchers used these standards to design measures to collect data on fraction understanding.

Some researchers, such as Hunt [29], extended the information provided in the Common Core State Standards (CCSS). Hunt stated:

> "In terms of content, the CCSS define equivalence as the understanding of two fractions that are the same size or at the same point on a number line. Evidence of a student's growing understandings of rational number equivalence also includes the ability to recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3) and explain why the fractions are equivalent (e.g., by using a visual fraction model)". (pp. 135–136)

In this example, Hunt noted the need to explain why two fractions were equivalent, though not elaborating on what explanation would be sufficient for fractions equivalence.

In both of these examples, the researchers use the term "concept" broadly, relating to a particular fraction topic (such as fraction equivalence). Note that there are likely many important ideas that support student thinking about fraction equivalence. For example, a deep understanding of fraction equivalence would include knowing that the two fractions must reference the same whole/unit and recognizing that equivalent fractions are generated by splitting fractional amounts (e.g., partitioning one-fourth into two parts creates two-eighths) or by joining fraction parts (e.g., joining three-twelfths creates one fourth). In addition, we want students to recognize the relationship between joining and partitioning fractional parts and the procedure for generating equivalent fractions symbolically (e.g., why you can multiply the numerator and denominator of 2/5 by 3 and generate an equivalent fraction, 6/15). Thus, this category of "concepts as a mathematical topic" takes a broad view of the meaning of the term "concept."

2.3. Concepts Characterized by What Students Can/Cannot Do

The final category for how fraction concepts were defined included skills or descriptions of what a student can or cannot perform to demonstrate their conceptual understanding of fractions. We saw this in two ways.

First, in some of the manuscripts, these descriptions were described as a student's "schema" (i.e., a pattern of thinking and behavior). The schema outlines expectations as to what students should be able to carry out to complete a specific mathematical task. An example of a schema was provided by Norton et al. [30], where they refer to a partitive fraction scheme: "Determining the size of a proper fraction relative to a given unpartitioned whole by partitioning the proper fraction to produce a unit fraction and iterating the unit fraction to reproduce the proper fraction and the whole." (p. 213).

Similar to Norton et al. [30], Hackenberg and Lee [31] refer to actions that students would employ when addressing problems involving nonunit fractions, identifying a schema for how students can partition the whole and iterate a unit fraction in various problem situations.

> For example, students who have constructed an iterative fraction scheme view 3/7 as 3 times 1/7 and 7/5 as 7 times 1/5, which means these students have constructed fractional numbers (Steffe & Olive, 2010). One situation of an iterative fraction scheme is a request to make a length that is seven-fifths of a given length. For this request to be sensible, students have to be able to posit a length that stands in relation to the given length yet is freed from relying on being part of a whole for meaning. Students who have constructed an iterative fraction scheme can make the posited length by partitioning the given length into five equal parts, disembedding one part, and iterating it seven times. For these students, the result is a multiple of a unit fraction (7/5 is 7 times 1/5) as well as one whole (5/5) and 2/5. (p. 204)

Shin and Bryant [32] demonstrate a different example in a similar vein, but from the perspective of the skills a student who is at risk for a math disability has yet to master and, therefore, cannot demonstrate a conceptual understanding of fractions. These authors focus on particular actions/responses that these students provide.

> "Specifically, these students demonstrate difficulties with rank-ordering fractions and identifying equivalent fractions (Grobecker, 2000; Hecht & Vagi, 2010; Mazzocco & Devlin, 2008), signifying a lack of conceptual understanding of fractions that can lead to misconceptions with fractions that impinge on the successful performance of fractions. For example, students incorrectly name fractions when equal parts are not shown in the figure (e.g., rectangular), failing to visualize equally sized parts in the whole (Barnett-Clarke, Ramirez, & Coggins, 2010). Also, lack of conceptual understanding of fractions can, in turn, limit students' ability to apply routine computational procedures involving fractions (Siegler et al., 2010)". (p. 375)

Second, other researchers in this category employed researcher-developed or standardized instruments that were used to assess what students could or could not do. In these studies, the instrument was identified as addressing fraction concepts and used to assess student conceptual understanding. For example, Fuchs et al. [33] described their instrument as follows,

> "*Generalized learning about fractions*. Our third measure indexed generalized learning about fractions, with a strong focus on fraction concepts. This measure was comparably different from the focus of instruction in both conditions and addressed fractions magnitude understanding and the part-whole interpretation of fractions with equal emphasis. We administered 19 released items from the 1990–2009 National Assessment of Educational Progress (NAEP): easy, medium, or hard fraction items from the fourth-grade assessment and easy from the eighth-grade assessment". (p. 634)

As a final example, Ennis and Losinski [34] used a fraction pretest.

> "The test included four items related to fractions concepts and vocabulary and six items each on adding, subtracting, and comparing fractions. All problems consisted of proper fractions with one- or two-digit numerators and denominators. This test was administered prior to the beginning of baseline fractions instruction". (p. 404)

In each of these examples, researchers employed an instrument that assessed what students could or could not do, their actions, to assess student understanding of what they characterized as "fraction concepts".

2.4. Reflecting on Mathematics Education and Special Education Researchers' Concept Definitions

At the beginning of this manuscript, we discussed the common ground that mathematics education and special education should establish for understanding and improving student learning of mathematical concepts and the various approaches that mathematics education and special education researchers have employed to operationalize fraction concepts. We are encouraged that researchers have moved beyond mathematical procedures toward focusing on the underlying concepts. Mathematical concepts are powerful, as they cut across and connect problem situations, representations, and contexts. However, considerable variation was found in how the concepts are defined and assessed both within and between the mathematics education and special education literature.

One question that emerged among our interdisciplinary group is whether researchers may be missing critical information about students' mathematical thinking when fraction concepts are operationalized as topics, particular actions, or fraction representations. For example, an emphasis on concepts as a mathematical topic (i.e., Category 2) can be beneficial for certain research goals and research questions but is likely too broad for identifying how students think about aspects of fraction learning. If a study's focus is on the Common Core State Standard of comparing fractions using fraction benchmarks (e.g., comparing $7/8$ and $5/6$ to 1), researchers may want to assess the ideas that children need to apply to correctly employ this comparison strategy. We found that researchers who focused on actions that students applied to fraction situations were useful, though we desired further clarity on the specific mathematical relationships that these students applied to fraction situations. For example, when a child can find $3/4$ by partitioning the unit into 4 equal parts and iterating 3 of those parts, we would want the child to explain a general idea about how partitioning the unit leads to establishing a unit fraction (in this case $1/4$) and how iterating relates to the construction of nonunit fractions from unit fractions.

3. Operationalizing Fraction Concepts for Designing Our Future Study

As part of our work, we next provide an example of what a possible supportive mathematics education and special education collaboration could look like using the context of student fraction learning. We began by examining the landscape of mathematics education and special education research, and we asked: What would help us work collaboratively to further our understanding of student thinking related to fraction concepts? As previously noted, a key aspect of effective interdisciplinary research is communication, including shared meaning. As the authors of this manuscript can attest, we need to consistently share the meaning of terms such as "mathematical concept", "mathematical object", "explicit instruction", and many more. We echo the call of Simon [24] for researchers to clearly delineate the mathematical concepts (defined as a relationship among mathematical objects) in their work and to clearly connect these concepts to how they operationalize these concepts in their work. Below, we provide an example of how we plan to accomplish this in the clinical interviews that we will conduct with third-grade (ages 8–9) students.

Drawing on the work of Simon [24], the mathematical concept we will focus on involves the fundamental meaning of a fraction, including the meaning of the numerator and the denominator. Specifically, we define our concept in terms of the connection between unit nonunit fractions and the whole/unit:

> Partitioning a unit into n equal parts (each $1/n$ of the unit) creates parts, one of which will iterate n times to make the whole (called n/n or 1). Iterating a subunit ($1/n$ of a unit) m times produces a specific quantity that is m times as large (m/n of the unit), and partitioning m/n of the unit into m equal parts will create the subunit ($1/n$ of a unit).

To assess student thinking related to this concept, we plan to explore children's thinking using various mathematical representations, including a set of objects, an area model, and a length (linear) model. The area, length, and set situations that we designed are provided in Figures 1–3. These tasks require students to apply the relationship between the unit and nonunit fractions. In particular, this involves recognizing that iterating a subunit ($1/n$ of a unit) m times produces a specific quantity that is m times as large (m/n of the unit). We were interested in determining the extent to which students apply this mathematical concept across each of these representations.

The cake of (something) below is three-fourths of the amount of the original cake. Draw a diagram of what the original cake would look like. Explain your reasoning.

Figure 1. The Cake Problem.

The length of the rope below is three-fourths of the length of the original rope. Draw a diagram of what the original rope would look like. Explain your reasoning.

Figure 2. The Rope Problem.

The box of cookies below contains three-fourths of what was in the original box. Draw a diagram of what the original box would look like. Explain your reasoning.

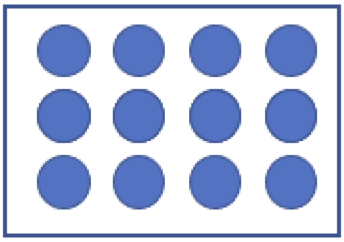

Figure 3. The Box of Cookies Problem.

When we piloted these items with learners in grades 3–5, some learners noted the similarity in the structure, stating that these problems were the same. As they approached each problem, they stated that they needed to determine what one-fourth of the whole was by partitioning the 3/4 into thirds, then iterating one more 1/4 to represent the whole. Note that these learners applied the idea that partitioning a unit into 4 equal parts (each 1/4 of the unit) creates parts, one of which will iterate 4 times to make the whole (called 4/4 or 1). Iterating a subunit (1/4 of a unit) 3 times produces a specific quantity that is 3 times as large (3/4 of the unit), and partitioning 3/4 of the unit into 3 equal parts will create the subunit (1/4 of a unit). Further, the power of this concept was applied across various contexts to these problems. As noted earlier, those learners whom we piloted this with stated these problems were the "same." We also pondered whether learners would see a problem where they are shown 4/5 of the whole and asked to determine the whole as "the same" idea. Our next step is to examine children's thinking across such situations and work to make specific concepts clear to students.

This initial work demonstrates how mathematics education and special education researchers can collaborate to build a shared vision of fraction concepts. Our team feels that the process we engaged in to operationalize specific mathematical concepts could be a beneficial process for other interdisciplinary teams as well. These intentional efforts can help move both mathematics education and special education research forward, allowing us to understand the particular fraction concepts in our study, support other researchers in building on our work, and clarify fraction concepts for classroom teachers.

4. Implications for Effective Interdisciplinary Research in Special Education and Mathematics Education

In this manuscript, we share a vision of what effective collaboration between special education and mathematics education researchers could look like. We recommend that researchers delve into the prior work of mathematics education and special education researchers and find common ground as well as shared meaning for important theoretical constructs. In our review of the research on fraction concepts conducted in mathematics education and special education, we encountered considerable variation in how the concepts were defined and operationalized, leading to different insights into student thinking. This is only one step in such collaborative research efforts, as we need to continue to develop shared philosophy and goals for student learning. Both mathematics education researchers and special education researchers need to continue to develop and grow, as each member of these research communities provides valuable insights into student learning, the mathematics content, and the interactions between the two. As such, we offer the following three implications.

First, we suggest that researchers take time to explicitly frame the concept(s) they are focusing on, as this can further the research of both mathematics education researchers and special education researchers. To accomplish this, researchers could build on our use of Simon's definition [24] or Jordan et al.'s [26] example of a mathematical concept, as they provide an explicit framing of particular mathematical concepts that can move forward both mathematics education and special education research on student thinking. Further, we implore researchers to connect these mathematical concepts to the methodology of their studies, whether these be connected to intervention/instructional tasks or assessments. Such transparency regarding the mathematical concepts and research methodology will reinvigorate discussion and connections among various researchers in mathematical learning. For example, we know that some learners, particularly those who are viewed as "struggling" with mathematics, can experience challenges in recognizing that mathematical concepts apply across representations and contexts [35]. We can address critical gaps in the research by further understanding the specific concepts that such learners apply or do not yet apply in particular situations and tailor interventions to support such connections. In addition, broad mathematical ideas, what Scheiner [36] refers to as "conceptions," can be compressed into a small number of powerful ideas to support learners who may have more limitations with working memory.

Second, we see communication about mathematical concepts as an essential component of effective interdisciplinary work in mathematics and special education. This may mean that reviewers and journal editors hold researchers to high standards regarding their communication about the mathematical concepts that are studied by requiring such definitions and elaborations on the specific mathematical concepts in their studies.

Finally, we also must value the various research paradigms and perspectives that we bring to research design and implementation. Interdisciplinary teams that involve mathematics education and special education researchers should become more common when researching the mathematical learning of students who are experiencing difficulties with mathematics. Healthy dialogue regarding research design and data analysis can lead interdisciplinary teams of researchers that develop innovative ways to improve mathematical learning. We have common ground regarding our goals to improve learning. We must engage in collaborative research design and strive for clarity of communication within and between our disciplines.

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