



Article Analysis of Industrial Engineering Students' Perception after a Multiple Integrals-Based Activity with a Fourth-Year Student

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Abstract: In industrial engineering degrees in Spain, mathematics subjects are usually taught during the first two academic years. Consequently, it is often the case that students sometimes do not feel motivated to learn subjects such as Mathematics II (calculus). Nevertheless, this subject is fundamental for understanding other subjects in the degree study plan, as well as for the graduate's future professional career as an engineer. To address this, a problem-based teaching methodology was carried out with the help of a fourth-year student who explained an activity to first-year students in a manner which was both friendly and approachable. In this experiment, the student went through a series of practical problems taken from different engineering subjects, which required multivariable integrals to be calculated and which he had learned in mathematics as a first-year student. In addition, a method based on pre-test and post-test assessments was applied. From this work, various benefits were observed in terms of learning, as well as an increase in the level of motivation of first-year students. There was a greater appreciation of the usefulness of calculus and computer programs to solve real-life problems, and the students generally responded positively to this type of activity.

Keywords: multivariable integrals; interdisciplinary activities; engineering and science students; problem-based teaching

MSC: 97I10; 97I50

1. Introduction

Engineering degrees involve studies that are extremely oriented toward problem solving. As such, problem-based learning (PBL) strategies were introduced after the Second World War as a means to reform universities, involving new educational models established between 1965 and 1975 [1]. This specific approach uses constructivist principles that encourage the application of prior knowledge, collaborative learning, and active engagement [2], and it feeds into another important methodology directly applicable in engineering degrees, namely, project-based learning. These two methodologies serve as inspiration for each other [3]. Therefore, the activities to be developed in the context of PBL should be associated with recurrent topics related to work the students may encounter in their future careers or a real situation with, for example, missing information or unclear answers [4–6].

In teaching engineering, it is important to integrate activities that reflect real-life situation rather than purely theoretical aspects [7]. In this regard, the use of PBL to teach mathematic subjects has also been called realistic mathematics education [8–12], which has been shown to have a positive effect on student motivation and participation in the classroom [13–16].

Mathematics is usually seen as being one of the most abstract and difficult subjects [17] and can cause students to experience negative feelings such as self-doubt and anxiety [18]. Thus, the manner in which an individual perceives their own competence in relation to the subject can influence final outcomes [19–21].



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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). Intrinsic motivation can be defined as the satisfaction inherent to the performance of an activity, as opposed to that which is caused by the consequences or profits gained by the activity. When intrinsically motivated, a person is driven to act for reasons of satisfaction or challenge, rather than external rewards, pressures, or products [22]. By contrast, extrinsic motivation is that provoked by external factors. Intrinsic motivation is beneficial for the student because it favors student involvement in learning through satisfaction and enjoyment [23,24].

In this work, a learning methodology based on problem solving is proposed, using software to improve mathematical competence and an approach similar to that proposed by other authors [13,14]. The study was carried out on students enrolled in the subject entitled Mathematics II (calculus in one and several variables), which is a compulsory subject in the first academic year for the following degrees: mechanical engineering, electrical engineering, and electronic and automatic engineering. It has been observed that students do not always understand the reason why they must study such complicated mathematics, as they tend to prefer to solve more applied problems related to engineering. In addition, the students often do not realize they are being provided with tools for building a solid foundation for future studies.

The flow work employed for this activity is unique in that a fourth-year student, supervised by the teacher, was asked to teach an activity to the students. In doing so, they shared their experiences and perspective about the usefulness of math throughout a particular degree course. In this way, the learning experience became more relevant and real and could help to change our students' opinion and level of motivation regarding the importance of mathematics in engineering. This is the main focus of this research.

In Section 2, the methodology used to carry out this work is explained, including a description of the context and participants. In Section 3, the main details of the activity undertaken are provided, and the perception that our students had about calculus is examined, in addition to whether the strategy helped them. The questionnaire and its results are analyzed in Section 4, and our conclusions are presented in Section 5.

2. Methodology

2.1. Context and Participants

During the academic year 2020–2021, 63 students were enrolled in the subject Mathematics II (calculus) distributed among the following three degrees: 21 students were studying for a bachelor's degree in electrical engineering, 13 were studying for a bachelor's degree in electronical and automatic engineering, and 29 were studying for a bachelor's degree in mechanical engineering. University of Salamanca students can also study for double degrees such as electrical and mechanical engineering or electrical and electronic engineering. It should be noted that all these degrees fall within what in Spain is called "degrees of the field of industrial engineering" due to previous professional regulations maintained in the new context of the European Higher Education Area. All these degrees in Spain are distributed over 4 years, and Mathematics I (algebra) and Mathematics II (calculus) are always taught in the first year.

Even though the students were enrolled in different degree programs, they were grouped together in the same classroom, since Mathematics II is a compulsory subject for all students. This educational context (with students taking different degrees but all related to industrial engineering) is ideal for educational research. For this reason, other studies have been successfully conducted on a multidisciplinary group of students such as the one used here [25].

To this end, we asked the students to participate in the activity and to take the preand post-test described in Section 4. Additionally, we explained to the students that all of their answers would be treated anonymously according to the ethical code for carrying out questionnaires used by our university. Furthermore, participation in the study was totally optional; thus, only 17 of the 63 students decided to answer the survey. They were 13 men and four women within the age range of 18 to 22 years old. None of the students were excluded from the study.

Consequently, the study sample comprised 17 students out of the 63 students (27%) enrolled in Mathematics II (Table 1). Double degree corresponds to students in electrical and mechanical engineering.

			Students			
Degree Program	ECTS *	Academic Year	Number	Percentage of the Total Study Group		
Bachelor in Mechanical Engineering	240	4	9	52.9%		
Bachelor in Electrical Engineering	240	4	1	5.9%		
Bachelor in Electronic and Automatic Engineering	240	4	4	23.5%		
Double degree	276	4	3	17.6%		

Table 1. Distribution of the sample.

* ECTS is the number of credits according to the European Credit Transfer System. It serves to measure the work that students must complete in order to acquire the knowledge, skills, and abilities necessary to pass their curriculum. One credit according to ECTS corresponds to 25–30 h of student work. Thus, 240 credits according to ECTS corresponds to 4 years, and most of our students in the double degree require at least 5 years to complete their studies.

Prior to conducting the experiment and to gather more information about the sample, a questionnaire was filled out by the students, yielding the following results:

- On a Likert scale [26] of 1–5, students answered the following question: "I think my level of digital competence is high". The average response value was 3, and the standard deviation was 0.75.
- Students were asked if they had previous experience using the package Wolfram Mathematica [27], of which 94.1% answered no.
- Finally, students were asked if they were currently enrolled in any second, third, or fourth subjects, of which 88.2% answered no.

The competencies to be acquired specific to this subject are based on Spanish regulations that are drawn from the regulations of the European Higher Education Area (EHEA). The three university degrees programs belong to the field of industrial engineering; hence, the first two academic years are common to all degrees. By contrast, the subjects taken during the first 2 years are specific to each specialization and, therefore, are different.

In addition, double degrees are also offered at our school. In this case, students enrolled in the double degree program in electrical and mechanical engineering are included in the sample as a different group.

According to Spanish regulations for engineers, some engineering projects and tasks can be performed by any of the engineers in the industrial field. However, other more specialized projects and tasks can only be carried out by specialists in the field; for example, low-voltage electrical projects can be carried out by any industrial field engineer, but high-voltage electric projects can only be carried out by electrical engineers. Please note that a degree in chemical engineering belongs to degrees within the branch of industrial engineering but is not offered by the School of Industrial Engineering of the University of Salamanca and, as such, was not included in this research.

2.2. Analysis of the Methodology

2.2.1. Interdisciplinary Problems to Teach Calculus

The students studying our degree programs will be future engineers. In [28], it is stated that the work of an engineer "is predominantly intellectual and varied and not of a routine mental or physical character. It requires the exercise of original thought and judgement and the ability to supervise the technical and administrative work of others". For this reason, we consider teaching competences through interdisciplinary problems to be beneficial. Additionally, we are of the opinion that the first academic year of an engineering

degree should provide general tools that allow engineers to solve a large array of daily problems: "their education and training will have been such that they will have acquired a broad and general appreciation of the engineering sciences, as well as thorough insight into the special features of their own branch" [28].

Students are constantly asking questions such as the following: "Why is this concept or topic interesting?" "Why do I need to study this part of the subject?" "Am I going to use this in the future and, if so, how?"

Additionally, some of our subjects are taught in the first 2 years of the degree program, which is the case of the subject calculus taught in the industrial engineering degree program in Béjar, Spain. This subject is included in the first year, and most of our students are not yet aware of the real situations that require engineers to know calculus in several variables, for example. The usefulness of some of the operations learnt in calculus does not become obvious until later on during the final stages of the degree.

Since multivariable (calculus) integrals are used by engineers in many situations to solve real problems, we decided it would be useful for third- and fourth-year students, as well as professors teaching other specialty subjects, to share their experiences with the use of calculus (in this case multivariable (calculus) integrals in several variables) to younger students. Obviously, this subject is only a tool for the students and not a main topic of study; therefore, it needs to be remembered that the students' appreciation of integrals in several variables may differ greatly from that of a mathematician's.

For this reason, multivariable (calculus) integrals were initially taught to the students in the normal fashion: theory, properties of integrals, and the most common ways to calculate some types of integrals. Additionally, some exercises in one dimension were shown, followed on by integrals in several variables, as well as solving some common exercises of integrals in several variables. Forming part of the experiment, an interdisciplinary activity was included, together with another teacher and a fourth-year student. The workflow for this activity is shown in Figure 1.

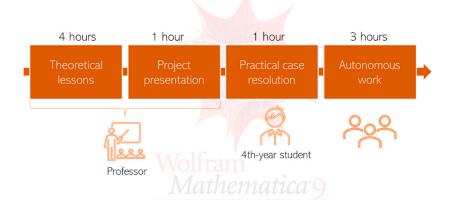


Figure 1. Workflow of the activity.

The aim of this work was to analyze the results of this activity, where it was proposed to evaluate not only how real problem solving can improve (or decrease) but also the motivation of engineering students in relation to the learning of mathematical subjects. In addition, we also wanted to measure the effect of the use of mathematical software in calculus classes. Computers are useful for assisting in two-step problem solving: first, consider the steps needed to solve the problem; then, use the acquired technical skills to control the computer to help solve the problem [29]. In this way, using computer tools, we can solve more complex problems; additionally, we are teaching our students the use of mathematical and computation software that they will have to use in other subjects and possibly also in their future profession as engineers, allowing the development of mathematical thinking. Doing so moves away from more theoretical exercises, permitting

the student to have a change from classes that they consider to be more tedious. In addition, the group of researchers who carried out this activity included a professor from the Department of Applied Mathematics, a professor from the Department of Mechanical Engineering and a fourth-year student who had an interest in educational research. Consequently, a broader view was achieved, from the perspective of mathematical education, the specific technological application of integrals to mechanical engineering, and the student's experience, perception, and interaction with the other students. The instructor teaching mechanical engineering and the student shared their experiences with using integrals in other engineering subjects from both points of view. Moreover, we asked the students to anonymously answer a pre-test and post-test survey as a means to measure whether this new method was helpful in increasing their motivation and becoming more active in the teaching–learning process.

2.2.2. Computer Tools

Many of the problems related to mathematics in the world of engineering are advanced. Thus, engineers need to use several competencies/skills to resolve these kinds of problems, such as the need to formulate the problem mathematically and knowledge about useful tools. However, currently, in many cases, this is not enough, and, depending on the difficulty of the problem at hand, in most cases, specific mathematical software is required to calculate some difficult operations.

In [13], the authors used operations and properties of matrices (which are complex and abstract concepts for our students) with the help of an application in digital imaging and data processing. Obviously, the resulting matrices have very large dimensions, and it would make no sense to do these calculations by hand. Today, computers are very much a part of the life of our students, and more and more computer tools are being used not only in teaching engineering but also by professionals.

At the University of Salamanca, during the activity, we had a license for Wolfram Mathematica [29], as well as MATLAB[®]. Additionally, Wolfram Alpha is available for use in the classroom for carrying out some small and fast calculations. Since our students solve many more simple exercises, we also teach them how to check their results using this software. Hence, the students are able to use these computer mathematical tools together with codes such as those described in Appendix A.

These codes and corresponding manuals are uploaded to the Moodle Learning Platform [30] and are available to students. Thus, students and teachers can access the work and activities at any time, and students have all resources available, allowing them to organize their work.

In this way, codes are provided to enhance the learning of students, but the goal is for them to develop the code that allows them to solve the problem. Codes can be uploaded to the Campus Virtual Studium (our Moodle platform) for the students to see, and the teacher can directly explain real problems and discuss how they should be solved. Therefore, the students come into contact with much more complicated calculations.

Wolfram Mathematica allows the student to work with code which is executed line by line. Modifications can be made in previous lines without the need to repeat introducing all the commands or generate scripts. In addition, the definition of functions is simpler and does not need to be raised separately. Therefore, this program was considered more suitable for carrying out this activity rather than other alternatives like MATLAB or similar free applications such as Octave or SciLab. These programs are very useful and can be used in other activities and subjects, especially those in which the use of matrices is necessary. However, Wolfram Mathematica was chosen in this case owing to its intuitiveness and the ability to introduce functions. In the past, other authors used Mathematica for very similar activities to teach algebra or calculus [13,14].

3. Development of Our Activities and Results

An increasingly technological and competitive environment requires the industrial engineer to pose very diverse problems. In its modeling and resolution, one of the most frequently used mathematical tools is multivariable (calculus) integrals. Therefore, engineering students must understand its importance and practical applications, as well as the computer tools available to facilitate their work.

However, many students, mainly in the first year, do not understand the importance of multivariable (calculus) integrals in engineering and limit themselves to learning mostly repetitive problems. Subsequently, this situation ends up causing the students to become less focused and motivated throughout the course.

Thus, we decided to introduce different real-life engineering problems where multivariable integral calculus is used and to show its different applications. Obviously, solving some of these problems can be quite complex and iterative; hence, the use of mathematical software was required.

The problem-based activity was carried out after having taught all the theoretical lessons on the basic principles of calculus in several variables, Thereby helping the students to link the knowledge acquired through real-life engineering cases.

This session lasted 2 h, with a 10 min break in the middle. In the first half (Section 3.1), the fourth-year student explained to the class several real-life exercises that he faced during the engineering degree, in which multivariable integration was needed to find the solution. For each problem, he first introduced the basic principles required to fully understand the path to the solution and, then, the answer was explained step by step. Finally, the fourth-year student talked about the different degree subjects each problem belonged to and what the students would learn from each one. In the second half of the class (Section 3.2), a complex topography problem was addressed, in which not only interpolation in several variables and integration was needed, but also computational mathematical tools (this exercise is described in Section 3.2 and solved in Appendix A with the help of Mathematica).

3.1. Real-Life Engineering Problems

Industrial engineering studies encompass very diverse areas of knowledge structured in different subjects, some of which were used in this activity.

Fluid mechanics is a second-year subject in which students learn the scientific fundamentals of the design and calculation of fluid systems and installations. To delve into these problems, it is first necessary to be able to characterize the properties of a given fluid. For this reason, the exercise proposed consisted of designing a coaxial cylinder viscosimeter to determine the properties of a lubricant. This apparatus works by rotating an internal cylinder immersed in some given fluid and measuring the power needed to turn it at a certain speed. By knowing the rotation speed of the cylinder and the properties of the lubricant, students are easily able to calculate the power required by the device using the shear stress of the fluid. To do so, double integrals are needed: one for the lateral surface of the cylinder and another one for the bottom surface.

Theory of machines and mechanisms is also a second-year course in which students learn about the modeling, calculation, and design of mechanisms and machines from a static, kinematic, and dynamic point of view. The suggested problem in this area consists of balancing the crankshaft of an engine with the help of CAD/CAE Software such as Autodesk Inventor. An initial model of a simplified and unbalanced engine crankshaft is given, which means that its center of gravity is not aligned with its axis of rotation. Using the design software mentioned and multivariable integrals, students come up with a fixed design by changing the geometry of the initial mechanism to move the centroid. As the center of gravity is above the axis, a semi-cylindrical volume is added in the lower bar, forcing it downward. Therefore, they calculate the diameter of the added volume so that the centroid matches the rotation shaft. For that, triple integrals are needed to obtain the center of mass of the half cylinder. Other real engineering examples mentioned during the first half of the activity were about the following:

- Industrial robots and manipulators. One of the most important features of these automatons is their work range, i.e., the surface or volume represented by all the robot's positions in space. To calculate this, a manipulator position study is carried out obtaining a cloud of points with its limit positions. These data are interpolated, and, with multivariable integration, the range surface or volume can be obtained.
- Self-driving cars and LIDAR sensors. These active sensors scan their surroundings with an infrared laser, mapping thousands of points. The point cloud obtained is treated in a similar way to the previous problem.
- Metrology. Laser interferometers can be used to measure roughness. These perform a
 micrometric scan of the part to be analyzed creating a point cloud that approximates
 its surface. To calculate the volume of material that needs to be removed in a grinding
 operation to improve its finish, it is necessary to apply multivariable integration.
- Calculation of structures. It is increasingly common to see constructions with a quadric surface shape, since they have very interesting properties. For instance, the cooling towers of nuclear power plants are shaped like hyperboloids, to boost natural convection and expel hot gases outside.

All of these cases were described in different subjects using mechanical and electrical engineering examples. Anuar R. Giménez (the first author of this paper) made a collection and briefly described them in the first hour of the practical case activity (third section in the whole activity; see Figure 1). During the second hour of the activity, Anuar described and explained in detail how he solved the problem that appears in Section 3.2. This was a problem the students had already solved in the subject topography which is compulsory in the mechanical engineering degree. However, in this subject, they solved it with computer tools, and the mathematical aspects were not explained in detail.

3.2. Estimating the Volume of a Building Excavation, from a Point Cloud Collected by Topography Measurements

Among the attributes and competencies of an industrial engineer is the ability to design, build, and operate structures and buildings for industrial use. The construction process consists of several phases. One of the most important stages is the taking of measurements directly on the ground and precisely transferring them to paper. This basic operation is known as topographic surveying. That is why topography is part of the curricula in these studies.

The activity that we developed consists of estimating the volume of excavated material needed to raise an industrial building, from a series of measurements made with a theodolite on a plot of a nearby town. The provided data are the Cartesian coordinates (X, Y, Z) of a series of points (identified with signals on the ground) referred to an origin, which will determine the depth of emptying.

Indeed, the position of each point is obtained in polar coordinates (r, φ, θ) . The device used gives, for each point, an azimuthal and a zenithal angle measurement, as well as the distance between the instrument and the point of sight, using the stadia method. Applying basic trigonometry, the projections of each point are obtained on cardinal directions axes. Therefore, cartesian coordinates are calculated.

To ease understanding and simplify the calculations, only 13 terrain signals were set, as can be seen in Figure 2.

The procedure used to solve the suggested problem can be structured in three steps:

(i) Finding an interpolating polynomial that fits the point cloud set up by the previous coordinates. For this, the least squares adjustment method is used. We calculate the coefficients of the polynomial (introduced in Appendix A) so that they minimize the quadratic error between the interpolating function and the data points. The approximation obtained is shown in Figure 2. This is a fourth-order polynomial in *x* and *y*, with 12 terms (12 unknown coefficients determined).

- (ii) Determining the equations of the different lines between the perimeter points, in order to establish the limits of integration with which the volume will be calculated later.
- (iii) Subdividing the prismoid formed by the coordinates of each plot point and the origin plane in different integrable prisms. Its edges are defined by the equations of the line obtained previously. The total volume of the excavation is calculated as the sum of the volumetric integrals of these bodies.

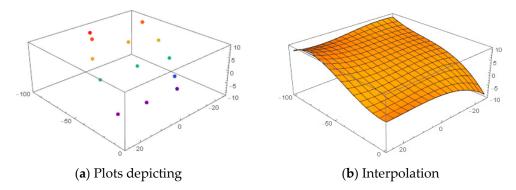


Figure 2. (a) Representation of the plot's relief, with a 13-point cloud in cartesian coordinates. (b) Its numerical approximation with an interpolating polynomial obtained with the least squares method.

When it comes to developing the scripts for doing the calculations, interesting setbacks arose that were successfully solved. The values of the obtained coordinates were measured on the field using a reference origin with a position of (500 m, 500 m, 500 m). This resulted in very high values during the calculation which the program truncated. Therefore, we obtained an inconsistent final result. To solve this, the points were transferred to an equivalent reference system centered at (0, 0, 0), obtaining more congruent values. If the coordinate origin was not changed, some numbers in matrix A were O (10^{10}). These types of difficulties and related mishaps appear in the daily work of an engineer, and the results provided by software may be erroneous if they are not correctly interpreted.

There are commercial programs available to the engineer such as TopoCal 2016 [31], capable of resolving the issue raised. However, students are not aware of the mathematics behind this type of software. After carrying out the activity, it is interesting that students acquire a general idea of the different calculation tools used by these programs, such as multivariable integration or interpolation, as seen with the mentioned example.

It is also important to familiarize the students with mathematical software from the subjects in the first year, as they are very powerful tools that are used to solve some real problems in engineering, also in addition to complementing their training. That is why, in Appendix A, some small scripts examples are included, so that they can complete calculations in mathematics.

3.3. Questionnaire

Since the purpose of this paper was to analyze the methodology explained above, we conducted a pre-test and post-test questionnaire on students in different bachelor's degrees of industrial engineering. This included 15 questions about different items that were to be measured. The questionnaire was carried out in Salamanca before and after the use of the activities proposed in this study.

Students were able to answer each one of these questions using a scale from 1 to 5, where 1 meant the student totally disagreed and 5 meant they totally agreed. Question Q1 measures the student's opinion about mathematics. Q2 and Q3 measure extrinsic motivation, while Q4 measures intrinsic motivation. Q5 measures the usefulness of the practical class using software with respect to the theoretical one. Q6 measures the student's opinion about using a computer. Q7 and Q8 measure the student's perception about how useful the activity is to solve real problems. Q9 addresses the scalability of the activity. Q10 assesses the student's perception about their computer skills. Finally, Q11 and Q12

address student's the opinion on the part of the activity taught by the fourth-year student. This questionnaire was completed by the students before (pre-test) and after the activity (post-test) in order to identify possible differences in the students' responses.

The questions included in the survey are shown in Table 2, and the descriptive statistics for the responses to the pre-test and post-test questionnaire are shown in Table 3.

Table 2. All questions included in the questionnaire with a Likert scale.

Code	Question	Category	
Q1	I think mathematics is a field closely related to engineering.	Motivation	
Q2	I think that integral calculus is related to other subjects in my degree course.	Motivation	
Q3	I think that integral calculus will be important in my professional career.	Motivation	
Q4	The math classes are fun and catch my interest.	Motivation	
Q5	I think a math class with practical applications, like this one, is more useful than a conventional class.	Usability	
Q6	I think computers are needed when teaching math.	Usability	
Q7	The activities of this subject serve to organize my learning and to be able to approach solving problems of different types in a structured way (and not just learn mathematical content).	Critical thinking	
Q8	Knowing the formulas of the surface and the length of the line in different types of coordinates (cartesian, parametric, and polar) can help me solve real-life problems.	Critical thinking	
Q9	Activities that mix engineering and mathematics should also be included when learning other subjects.	Usability	
Q10	I think my computer skills are high.	Usability	
Q11	I think it is a good idea for a senior classmate to teach a math class.	Usability	
Q12	When I am a fourth-year student grade and see the usefulness of mathematics in other subjects, I would like the opportunity to give a class to first-year students.	Usability	

Table 3. Questions included in the questionnaire. All of them were evaluated using a Likert scale.

Que	estion	Mean	Standard Deviation	Standard Error	
01	Pre-test	4.29	0.77	0.19	
Q1	Post-test	4.47	0.62	0.15	
\mathbf{O}	Pre-test	3.77	1.09	0.26	
Q2	Post-test	4.35	0.62 0.15		
02	Pre-test 3.06 1.25 Post-test 4.12 0.78 Pre-test 2.24 1.15 Post-test 3.18 1.13 Pre-test 3.76 1.03 Post-test 4.06 0.75 Pre-test 3.41 0.87 Post-test 3.65 0.93 Pre-test 3.35 0.79 Post-test 3.59 0.87 Pre-test 2.71 0.99 Post-test 3.82 1.01 Pre-test 3.77 0.97 Post-test 4.18 0.81 Pre-test 3.00 1.00	0.30			
Q3	Post-test	4.12	4.47 0.62 0.15 3.77 1.09 0.26 4.35 0.49 0.12 3.06 1.25 0.30 4.12 0.78 0.19 2.24 1.15 0.28 3.18 1.13 0.27 3.76 1.03 0.25 4.06 0.75 0.18 3.41 0.87 0.21 3.65 0.93 0.22 3.35 0.79 0.19 3.59 0.87 0.21 2.71 0.99 0.24 3.82 1.01 0.25 3.77 0.97 0.24 4.18 0.81 0.20 3.00 1.00 0.24 2.88 1.11 0.27 4.00 0.71 0.17 4.24 0.75 0.18 2.65 1.37 0.33	0.19	
04	Pre-test	2.24	1.15	0.28	
Q4	24Post-test3.1825Pre-test3.76Post-test4.06Pre-test3.41Post-test3.65	1.13	0.27		
05	Pre-test	3.76	1.03	0.25	
Q5	Post-test	4.06	0.75	0.18	
06	Pre-test	3.41	0.87	0.21	
Qo	Q6 Post-test	3.65	0.93	0.22	
Q7	Pre-test	3.35	0.79	0.19	
Q/	Post-test	3.59	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.21	
0	Pre-test	2.71	0.99	0.24	
Q8	Post-test	3.82	$\begin{array}{ccccc} 0.49 & 0.12 \\ 1.25 & 0.30 \\ 0.78 & 0.19 \\ 1.15 & 0.28 \\ 1.13 & 0.27 \\ 1.03 & 0.25 \\ 0.75 & 0.18 \\ 0.87 & 0.21 \\ 0.93 & 0.22 \\ 0.79 & 0.19 \\ 0.87 & 0.21 \\ 0.99 & 0.24 \\ 1.01 & 0.25 \\ 0.97 & 0.24 \\ 1.01 & 0.25 \\ 0.97 & 0.24 \\ 0.81 & 0.20 \\ 1.00 & 0.24 \\ 1.11 & 0.27 \\ 0.71 & 0.17 \\ 0.75 & 0.18 \\ 1.37 & 0.33 \\ \end{array}$	0.25	
00	Pre-test	3.41 0 3.65 0 3.35 0 3.59 0 2.71 0 3.82 1 3.77 0 4.18 0 3.00 1 2.88 1	0.97	0.24	
Q9	Post-test	4.18	0.81	0.20	
Q10	Pre-test	3.00	1.00	0.24	
Q10	Post-test	2.88	1.11	0.27	
011	Pre-test	4.00	0.71	0.17	
Q11	Post-test	4.24	0.75	0.18	
012	Pre-test	2.65	1.37	0.33	
Q12	Post-test	2.71	1.31	0.32	

Through these questions, the authors attempted to measure several important issues related to the teaching–learning process such as motivation, usability, and critical thinking.

4. Discussion on the Results

4.1. Questionnaire Given to First-Year Students

In the case of the pre-test, a standardized Cronbach's alpha coefficient of 0.71955 was obtained. In the post-test, a standardized alpha equal to 0.83614 was obtained. These results can be considered good and are in line with other results reported for questionnaires of a similar type [13].

In general terms, the students, after carrying out the activity, considered that mathematics is close to engineering ($\bar{x} = 4.294$, SE = 0.151), and that integrals are related to other subjects of their degree course ($\bar{x} = 4.353$, SE = 0.120).

The students also believe that multivariable (calculus) integrals will be useful for their future professional career ($\bar{x} = 4.118$, SE = 0.190), and that lessons based on practical applications are more useful than theoretical classes ($\bar{x} = 4.06$, SE = 0.181). They also are of the opinion that activities that mix both mathematics and engineering should be included when teaching or learning other subjects ($\bar{x} = 4.060$, SE = 0.181). The students also considered the proposed methodology (shown by a fourth-year student) to be a good idea ($\bar{x} = 4.000$, SE = 0.171). However, they were not interested in participating as a fourth-year student in the future ($\bar{x} = 2.7059$, SE = 0.318). In this last question, the dispersion results were noticeably higher than in other questions; hence, a high polarization in terms of the answer was expected (hypothetically motivated by the fact that some students have teacher vocation and others do not). Lastly, there is no clear position regarding the consideration of mathematics as fun and attractive ($\bar{x} = 3.177$, SE = 0.274), although the score was higher after the activity was carried out.

It can be observed that the mean results of the post-test were more favorable than those of the pre-test for all questions raised, except for question Q10. According to the students' responses, the activity caused students to consider their computer skills to be inferior. This may hypothetically be due to the complexity of the software and the need to use programming code to solve the activity.

Once the descriptive analysis of both pre-test and post-test responses were analyzed (Table 3), a hypothesis contrast based on Levene's test for equality of variances (to check the homoscedasticity) and on the *t*-test for equality of means was applied.

Statistically significant differences were detected between the pre- and post-test results for questions Q3, Q4, and Q8. The results showed that the activity changed the students' perception about the importance of mathematics for their professional career (Q3) (T(28.86) = 2.964, *p*-value = 0.006) and about the perception that math could be fun and appealing (Q4) (T(32) = 2.409, *p*-value = 0.022). Please note that question Q4 was related to the intrinsic motivation of the student, while questions Q2 and Q3 were related to the extrinsic motivation. Lastly, differences were detected regarding specific issues about the activity (Q8) (T(32) = 3.258, *p*-value = 0.003).

Since the parametric statistical treatment of Likert type data can be a controversial issue when an ordinal scale is used, two approaches were applied: parametric and nonparametric tests [32]. An interval scale is handled using parametric statistics [33], but some authors have argued that nonparametric statistics should be used [34]. In this way, different nonparametric tests (median test, Mann–Whitney U test, Kolmogorov–Smirnov test, and Kruskal–Wallis test) were also applied. The results of these tests are shown in Table 4.

Table 4. The *p*-values of all questions included in the survey were rated using a Likert scale. Results in bold are those where the *p*-value was below the significance level (0.05).

Test	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
Kolmogorov– Smirnov	1.00	0.454	0.046	0.240	0.954	0.954	0.954	0.046	0.954	1.00	0.954	1.00
Median test	1.00	0.707	0.396	0.120	1.00	0.493	0.493	0.016	0.463	0.463	1.000	1.000
Mann–Whitney U	0.586	0.131	0.012	0.024	0.540	0.433	0.413	0.003	0.259	0.786	0.375	0.838
Kruskal–Wallis	0.542	0.087	0.008	0.200	0.495	0.403	0.366	0.002	0.223	0.774	0.332	0.830

First, the Kolmogorov–Smirnov test was applied, which showed that questions Q4 and Q8 did not conform to a normal distribution. The Mann–Whitney test (like the *t*-test for the parametric approach) also showed the existence of statistically significant differences between pre-test and post-test for questions Q3 (*p*-value = 0.012), Q4 (*p*-value = 0.024), and Q8 (*p*-value = 0.003), while the median test only detected the existence of significant differences in the median for question Q8 (*p*-value = 0.016). The Kruskal–Wallis test showed statistically significant differences for Q3 (*p*-value = 0.008) and Q8 (*p*-value = 0.002). According to the results, the existence of statistically significant differences in the questions Q3, Q4, and Q8 was demonstrated for a nonparametric approach based on different statistics. These results are compatible with those obtained by the parametric tests.

4.2. Participant Observation

Through the results obtained from the questionnaires, a quantitative analysis of the proposed methodology was carried out. The fourth-year student taking part in the activity and the professor teaching the subject assessed the feedback received from a qualitative point of view. Once the activity was completed, the student wrote a report on the observations made.

The fourth-year student found the activity rewarding, as it allowed him to empathize with his fellow students by sharing his experience and making it easier for them to face difficult mathematical concepts in a more enjoyable and practical way. He stated the following:

"I consider that these teaching methodologies are very enriching for both students and teachers. The mathematics subjects taught in the first years of engineering are usually very theoretical and overwhelm many students. In my opinion, exploring real-world problems and challenges from early grades drives students to obtain a deeper knowledge of the subjects they are studying and encourages them to develop confidence and motivation. Personally, I would have loved to participate in this type of activity when I first entered college.

At the beginning of the activity, I noticed how the students seemed curious and were excited to do something new, outside the usual routine. As the session progressed, although some of them were rather clueless, most students remained attentive and really seemed to be interested in what we were doing. Overall, the atmosphere was quite welcoming. I would highly recommend other senior students to continue carrying out these experiences."

It is worth stressing that the exercise that seemed to motivate the students the most was about balancing the crankshaft of an engine, perhaps because many of them were enrolled in mechanical engineering. When the class finished, the fourth-year student had a good overall feeling. Even a few days later, a couple of students asked him questions about different subjects they would be taking the following year.

The fourth-year student also mentioned how his perspective on calculus in several variables had changed during each subject over the course of this university degree. When he was first introduced to this area, he had problems understanding its theoretical basis and found the subject complex; therefore, he became discouraged. Moreover, he did not find it useful in other engineering subjects. It was not until third year, while studying surveying, that he realized its importance, as he needed multivariable integration to estimate the volume of an industrial building excavation.

In sum, he considered these activities to be helpful for most students, not only in terms of realizing the relationship between mathematical tools taught in first year subjects and other engineering areas, but also in appreciating the relevance of this subject in terms of their future career. As a student in his last year of a mechanical and electrical engineering double degree, he realized that calculus is a key tool for solving many real problems in all branches of industrial engineering. Therefore, associating real engineering applications with mathematical concepts that might seem complex could help first-year students to better understand them. Since some of the activities carried out included complex and iterative operations, mathematical software was required. This could also help the students understand the mathematical operations behind certain functions of some commercial programs that they might use in their studies and professional career, drawing their attention to programing.

This teaching experience allowed the fourth-year student to gain mathematical knowledge as he had to return to studying calculus in several variables, in a more in-depth way and from a different point of view. The experience also permitted him to improve his public communication skills and sparked an interest in teaching. For this reason, he suggested that it might be very enriching for future fourth-year students to take part in similar activities.

At the end of the activity, the professor asked several students their opinion about the experience. The students who responded affirmed their preference for this type of activity rather than solving more theoretical exercises, and they liked hearing from a more experienced peer about the usefulness of calculus in other types of subjects normally associated with their engineering degrees. His comments confirmed the results later observed in the questionnaire described above.

5. Conclusions

Students should be able to solve daily mathematical-related problems throughout their professional careers. Therefore, as instructors, we are obliged to provide them with the tools for solving real problems and teach students how to apply them for the benefit of society. Additionally, new university programs and requirements, as well as societal demands, require the subjects to be as practical as possible. This in turn has encouraged teachers to redesign and rethink how subjects are taught, where subjects are made more attractive to students. This is particularly true for a topic like mathematics, since students generally have problems understanding the concepts and do not like the theoretical part of math-related subjects. For this reason, in this work, an activity was carried out aimed at improving motivation for learning mathematics in a group of students studying in different degrees associated with the field of industrial engineering. The activity involved the participation of a fourth-year student, who is a coauthor of this paper, who explained how to resolve problems to first-year students. During his explanation of the activity, the fourth-year student also tried to explain to the students his perception regarding the importance of mathematics in terms of the subjects he had taken over the course of his degree. At the same time, the teacher responsible for the subject observed the behavior of the students and the progress of the activity.

The activity was based on solving different real-life engineering problems in which multivariable integrals were needed to find the solution, enriching the knowledge acquired in a competency-based mathematics subject using a problem-based teaching method. The exercises carried out covered quite different areas, such as fluid mechanics or construction and topography, some of them involving several calculations; therefore, mathematical software was required.

First, the students completed a pre-test questionnaire, and, once the activity had been completed, the students then completed a post-test questionnaire with the same questions as those presented in the pre-test questionnaire. The results of the questionnaire indicated that the students on a general level do not perceive mathematics as a fun subject. This response is in line with other published reports on the topic [14]. Additionally, the results of the questionnaire were consistent with what both the fourth-year student and the professor observed in the classroom during the activity, showing that, overall, the activity was satisfactory and motivating in general terms. The students indicated that they would like to carry out more activities of this type, which they considered to be helpful for problem solving, although the results also indicated that the students did not wish to participate in the future as a fourth-year student.

The results of pre-test and post-test questionnaire were compared using parametric and nonparametric inferential techniques. Both tests provided similar results, the most remarkable being that statistically significant differences were found for questions measuring intrinsic motivation, extrinsic motivation, and the usefulness of the activity to solve real problems similar to those reported in other studies [13,14,34].

Lastly, protocols should be developed if we want that students in the latest years of our degrees show specific tasks/problems to the first-year students related to the practical application of mathematics in engineering.

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Appendix A

Estimating the volume of a building excavation

The comments in the code used in the classroom are in Spanish for academic purposes. In this work, they have been translated into English to allow readers to understand them.

(* From the Excel file, we import data that the students previously calculated in another third-year subject. *)

data = Import["C:\\data_palomares.xlsx"][[1]]

 $\{\{-0.937085, -5.90457, 0.492498\}, \{13.5512, -7.28953, 0.326174\}, \{25.4282, -9.473, 0.168468\}, \\ \{18.6554, -40.5825, 3.69048\}, \{-6.18117, -16.894, 1.41553\}, \{-13.2269, -33.8948, 3.37329\}, \\ \{-7.95319, -66.4691, 6.04395\}, \{12.8675, -61.5239, 5.9983\}, \{2.85255, -84.6279, 7.25412\}, \\ \{-0.12187, -93.8704, 7.74539\}, \{-26.3122, -86.2181, 7.02484\}, \{-19.8547, -56.455, 5.19855\}, \\ \{1.05622, -39.5746, 3.50618\}\}$

 $\begin{aligned} &\text{data} = \{\{-0.93708456^{\circ}, -5.90457219^{\circ}, 0.49249848^{\circ}\}, \{13.551175^{\circ}, -7.28952502^{\circ}, 0.32617372^{\circ}\}, \\ &\{25.4281851^{\circ}, -9.47299653^{\circ}, 0.16846814^{\circ}\}, \{18.6554001^{\circ}, -40.5824811^{\circ}, 3.69047693^{\circ}\}, \\ &\{-6.18117493^{\circ}, -16.8939747^{\circ}, 1.41553061^{\circ}\}, \{-13.2268731^{\circ}, -33.8947756^{\circ}, 3.37329311^{\circ}\}, \\ &\{-7.95318955^{\circ}, -66.4691303^{\circ}, 6.04395385^{\circ}\}, \{12.8674808^{\circ}, -61.5238784^{\circ}, 5.99830393^{\circ}\}, \\ &\{2.85254857^{\circ}, -84.6279358^{\circ}, 7.25411933^{\circ}\}, \{-0.12186975^{\circ}, -93.8704137^{\circ}, 7.74539119^{\circ}\}, \\ &\{-26.312194^{\circ}, -86.2180903^{\circ}, 7.02483645^{\circ}\}, \{-19.8547003^{\circ}, -56.4550142^{\circ}, 5.1985514^{\circ}\}, \\ &\{1.056217^{\circ}, -39.574576^{\circ}, 3.506184^{\circ}\}\}; \end{aligned}$

(* We must choose how we want the polynomials to be. We want the polynomials to be like the one below (for example). *)

 $pol[x_, y_] = a1 + a2^{*}x + a3^{*}y + a4^{*}x^{2} + a5^{*}x^{*}y + a6^{*}y^{2} + a7^{*}x^{3} + a8^{*}x^{2}y + a9^{*}x^{*}y^{2} + a10^{*}y^{3} + a11^{*}x^{*}y^{3} + a12^{*}x^{2}y^{2};$

(* Thus, the system of vectors, the generators of our vector space, is formed by means of the below phi functions. *)

phi[i_, x_, y_] = Which[i == 1, 1, i == 2, x, i == 3, y, i == 4, x^2, i == 5, x*y, i == 6, y^2, i == 7, x^3, i == 8, x^2*y, i == 9, x*y^2, i == 10, y^3, i == 11, x*y^3, i == 12, x^2*y^2];

(* Now, we can calculate the coefficients of the polynomial that reaches (or is closest to all the points from our Excel file, depending on the dimensions of our problem). We evaluate our basis in all the points and solve the least squares problem. *)

A = Table[phi[j, data[[i, 1]], data[[i, 2]]], {i, 1, Length[data]}, {j, 1, 12}];

b = Table[data[[i, 3]], {i, 1, Length[data]}];

LeastSquares[A, b]

 $\{ 0.0565949, \ 0.0797015, \ -0.070381, \ -0.00583912, \ 0.00981652, \ 0.000672554, \ 0.000107678, \ -0.000217221, \ 0.00022677, \ 5.86501^{*}10^{-}6, \ 1.46394^{*}10^{-}6, \ -1.51735^{*}10^{-}6 \}$

(* This problem can also be solved as an optimization problem, and Plot3D can be used to draw our polynomial and compare it with the real points as shown in Figure A1. *)

NMinimize[dist, {a1, a2, a3, a4, a5, a6, a7, a8, a9, a10, a11, a12}]

{0.00415069, {*a*1 -> 0.0565949, *a*2 -> 0.0797015, *a*3 -> -0.070381, *a*4 -> -0.00583912, *a*5 -> 0.00981652, *a*6 -> 0.000672554, *a*7 -> 0.000107678, *a*8 -> -0.000217221, *a*9 -> 0.00022677, *a*10 -> 5.86501*10^-6, *a*11 -> 1.46394*10^-6, *a*12 -> -1.51735*10^-6}}

```
 \ln[11] = Z[x_, y_] = pol[x, y] /. \{a1 \rightarrow 0.05659488760297747^{,} a2 \rightarrow 0.07970150315538359^{,} a3 \rightarrow -0.07038101175614282^{,} a4 \rightarrow -0.005839117879023703^{,} a5 \rightarrow 0.009816523263871687^{,} a6 \rightarrow 0.0006725538797118747^{,} a7 \rightarrow 0.0001076776122910887^{,} a8 \rightarrow -0.00021722071036437375^{,} a9 \rightarrow 0.00022677038687355584^{,} a10 \rightarrow 5.865014189925957^{,} *^{-6}, a11 \rightarrow 1.463936133060352^{,} *^{-6}, a12 \rightarrow -1.5173512154792845^{,} *^{-6} \}
```

```
\begin{array}{l} \text{Out[11]}= & 0.0565949 + 0.0797015 \ x - 0.00583912 \ x^2 + 0.000107678 \ x^3 - 0.070381 \ y + 0.00981652 \ x \ y - 0.000217221 \ x^2 \ y + 0.000672554 \ y^2 + 0.00022677 \ x \ y^2 - 1.51735 \times 10^{-6} \ x^2 \ y^2 + 5.86501 \times 10^{-6} \ y^3 + 1.46394 \times 10^{-6} \ x \ y^3 + 1.46394 \times 10^{-6} \ x \ y^3 + 1.46394 \times 10^{-6} \ x^3 \ y^3 + 1.46394 \times 10^{-6} \ y^3 \ y^3 + 1.46394 \times 10^{-6} \ y^3 \ y
```

```
 \ln[71] = \text{Plot3D}[\ Z[x, y], \{x, -27, 27\}, \{y, -100, 0\}, \text{PlotRange} \rightarrow \{-10, 10\}, \text{ColorFunction} \rightarrow "Rainbow"] 
 |\text{representación gráfica 3D} |\text{rango de representación} |\text{función de color} |
```

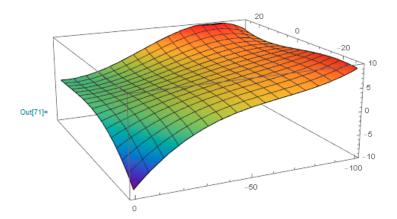


Figure A1. Approximation of the plot's relief with an interpolating polynomial obtained previously with the least squares method.

(* Finally, we are ready to approximate the volume. *)

ord1 = Ordering[data]; (* Order of the coordinates *)

 $\begin{aligned} & data2 = \{\{-26.312194`, -86.2180903`, 7.02483645`\}, \{-19.8547003`, -56.4550142`, 5.1985514`\}, \\ & \{-13.2268731`, -33.8947756`, 3.37329311`\}, \{-0.93708456`, -5.90457219`, 0.49249848`\}, \\ & \{13.551175`, -7.28952502`, 0.32617372`\}, \{25.4281851`, -9.47299653`, 0.16846814`\}, \{18.6554001`, -40.5824811`, 3.69047693`\}, \\ & \{-6.18117493`, -16.8939747`, 1.41553061`\}, \{12.8674808`, -61.5238784`, 5.99830393`\}, \\ & \{2.85254857`, -84.6279358`, 7.25411933`\}, \{-0.12186975`, -93.8704137`, 7.74539119`\}, \{-26.312194`, -86.2180903`, 7.02483645`\}\}; \end{aligned}$

(* We calculate the lines that will be used in the integration. *) For[i = 1,i < Length[data2],i++, m[i] = (data2[[i + 1,2]]-data2[[i,2]])/(data2[[i + 1,1]]-data2[[i,1]]); b[i] = data2[[i,2]]-m[i]*data2[[i,1]]]; Integrate[$Z[x, y], \{x, data2[[1,1]], data2[[2,1]]\}, \{y, (m [11]*x + b [11]), (m [1]*x + b [1])\}] +$ Integrate[$Z[x, y], \{x, data2[[2,1]], data2[[3,1]]\}, \{y, (m [11]*x + b [11]), (m [2]*x + b [2])\}] +$

 $Integrate[Z[x, y], {x, data2[[3,1]], data2[[4,1]]}, {y, (m [11]*x + b [11]), (m [3]*x + b [3])}] + Integrate[Z[x, y], {x, data2[[4,1]], data2[[5,1]]}, {y, (m [11]*x + b [11]), (m [4]*x + b [4])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [5]*x + b [5])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[11,1]]}, {y, (m [11]*x + b [11]), (m [11]*x + b [11])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[5,1]]}, {y, (m [11]*x + b [11]), (m [11]*x + b [11])}] + Integrate[Z[x, y], {x, data2[[5,1]], data2[[5,1]]}, {y, (m [11]*x + b [11])}] + Integrate[Z[x, y], {y, (m [11]*x + b [11]), (m [11]*x + b [11])}] + Integrate[Z[x, y], {y, (m [11]*x + b [11])}] + Integrate[Z[x, y], {y, (m [11]*x + b [11])}] + Integrate[Z[x, y], {y, (m [11]*x + b [11])}] + Integrate[Z[x, y], {y, (m [11]*x + b [11])}] + Integr$

$$\begin{split} & \text{Integrate}[Z[x, y], \{x, \text{data2}[[11,1]], \text{data2}[[10,1]]\}, \{y, (m \ [10]^*x + b \ [10]), (m \ [5]^*x + b \ [5])\}] + \\ & \text{Integrate}[Z[x, y], \{x, \text{data2}[[10,1]], \text{data2}[[9,1]]\}, \{y, (m \ [9]^*x + b \ [9]), (m \ [5]^*x + b \ [5])\}] + \\ & \text{Integrate}[Z[x, y], \{x, \text{data2}[[9,1]], \text{data2}[[6,1]]\}, \{y, (m \ [8]^*x + b \ [8]), (m \ [5]^*x + b \ [5])\}] + \\ & \text{Integrate}[Z[x, y], \{x, \text{data2}[[6,1]], \text{data2}[[8,1]]\}, \{y, (m \ [8]^*x + b \ [8]), (m \ [6]^*x + b \ [6])\}] + \\ & \text{Integrate}[Z[x, y], \{x, \text{data2}[[8,1]], \text{data2}[[7,1]]\}, \{y, (m \ [7]^*x + b \ [7]), (m \ [6]^*x + b \ [6])\}] \end{split}$$

10070.1

(* The result is similar to others obtained using engineering software. *)

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